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Elastic wave propagation in fiber reinforced composite materials with non-uniform distribution of fibers

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Abstract: In the present work the iterative effective medium approximation (IEMA) is appropriately used for wave dispersion and attenuation predictions in fiber-reinforced composites that microscopically exhibit a non-uniform fiber distribution. Two types of composites with such irregular topology of fibers are considered. The first contains a regular distribution of clusters of fibers embedded in a composite matrix with uniformly distributed fibers, and the second a uniform distribution of matrix-rich inclusions embedded in a fiber-rich regular composite medium. The resulting from the application of the IEMA scattering problems are solved numerically by means of a two dimensional boundary element method. The obtained dispersion and attenuation curves are compared to those for the corresponding fiber-reinforced composite with a uniform distribution of fibers.

keyword: Dispersion, attenuation, fiber composites, BEM, wave propagation, homogenization.

1 Introduction

Among the existing nondestructive methodologies, ultrasonics can be considered as the most widely used technique for characterizing composite materials. Reconstruction of the effective material constants and determination of the material damage state (matrix micro cracking, interfacial cracks between matrix and fibers e.t.c.) are two essential parts of the nondestructive ultrasonic composite material characterization. The inspection process is usually based on the inversion of mathematical models that relate directly the ultrasonic data, i.e. ultrasound phase or group velocities and/or ultrasound attenuation, to the microstructure or the effective macrostructure of the composite medium. Thus, understanding of how a stress wave propagates through an elastic medium containing a random distribution of particles or fibers is of great importance for this kind of nondestructive evaluation.

Due to the presence of the embedded inhomogeneities, an elastic wave traveling through a composite medium undergoes multiple scattering and becomes both dispersive and attenuated. Dispersion means that the phase velocity of the wave is frequency dependent, while the term attenuation is related to the frequency dependent decay rate of its amplitude. Within the last forty years, many investigators have studied wave dispersion and attenuation phenomena in particulate and fiber-reinforced composites, either analytically or numerically. Since a review on the subject is beyond the scope of this paper, more details on the elastic multiple scattering simulation one can find in the papers of Anson and Chivers (1993) and Kim, Ih, and Lee (1995) for particulate composites and Kim (1996), Verbis, Kattis, Tsinopoulos and Polyzos (2001) for fiber-reinforced materials. Recently, the authors presented a new iterative effective medium approximation (IEMA) for predicting wave dispersion and attenuation in particulate and fiber composites [Tsinopoulos, Verbis, Polyzos (2000) and Verbis, Kattis, Tsinopoulos and Polyzos (2001)]. The proposed there IEMA makes use of a single inclusion self-consistent condition, first considered by Soven (1967) for substitutional alloys, assumes that the effective stiffness of the composite being the same with the corresponding static one and evaluates iteratively an effective and frequency dependent dynamic density for the composite. The complex value of the effective dynamic density and the static effective stiffness of the composite determine, eventually, the wave speed and the attenuation coefficient of a plane wave propagating through the composite medium. The solution of the single inclusion scattering problems, imposed by the self-consistent condition, is accomplished

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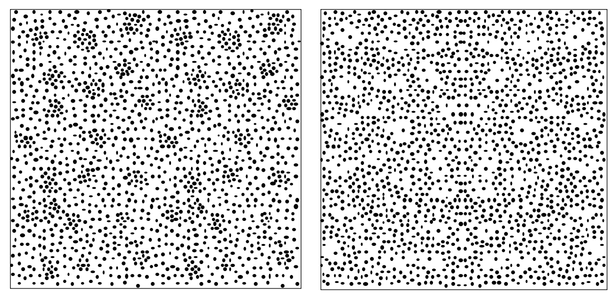
by means of an advanced and highly accurate boundary element method (BEM) described in the works of Polyzos, Tsinopoulos and Beskos (1998), Kattis, Polyzos and Beskos (1999) and Verbis, Kattis, Tsinopoulos and Polyzos (2001) for two and three dimensional compressible and incompressible dynamic elastic applications and in the work of Tsinopoulos, Kattis, Polyzos and Beskos (1999) for problems dealing with axisymmetric scatterers.

Almost all theories, appearing to date in the literature and dealing with the propagation of an elastic wave in a composite medium, are based on the assumption that the embedded inhomogenities have regular topology for their spatial distribution. In macroscopic and microscopic level this assumption ensures constant volume fraction of the particles/fibers over the composite volume and the problem can be reduced to the analysis of a representative unit cell containing, in most cases, only one inhomogeneity. However, due to fabrication process, composite materials often present non-uniform distribution of particles or fibers, thus forming clusters of inclusions or matrix-rich zones both responsible for damage initiation. The problem appears to be more pronounced in particulate composites where a control on the spatial distribution of particles is a very difficult task. In fiber composites the irregular distribution of fibers is observed in the plane being perpendicular to the fibers and in most cases in microscopic level. Thus, although real fiber composites appear macroscopically to have a transversally isotropic behavior, their representative volume element is not a fiber with the surrounding matrix medium but a non-uniform distribution of many fibers embedded in the matrix of the composite. To the authors' best knowledge, theoretical or experimental results concerning wave propagation in composites with irregular distribution of particles/fibers have not been reported so far. Only papers dealing with the static behavior of such kind of composites have appeared to date in the literature. Some representative works are those of Paipetis (1984), Pyrz (1997) and Li, Zhao and Pang (1999) for particulate composites and Axelsen and Pyrz (1997) for fiber composites.

The goal of the present paper is to study, through an appropriate use of IEMA, the dispersion and the attenuation of longitudinal and transverse plane waves propagating in unidirectional fiber composites that in their microscopic level present non-uniform fiber distribution such as clusters of fibers or matrix-rich inclusions. Since the mechanism of forming clusters of fibers is different from that of forming matrix-rich inclusions, two types of composites will be examined. The first contains a uniform distribution of clusters of fibers embedded in a composite matrix with uniformly distributed fibers and the second considers a regular distribution of matrix-rich inclusions embedded in a fiber-rich regular composite medium. In the plane being perpendicular to the fibers the aforementioned two types of composites are depicted graphically in Figs 1(a) and 1(b), respectively.

The paper is structured as follows: In section 2, the IEMA as it is applied to fiber composites with uniform distribution of fibers is described in brief.



(a) (b)

Figure 1 : Representation of the microstructure of two fiber-composites appearing (a) inclusions of clustered fibers and (b) inclusions of matrix-rich inclusions.

Section 3 presents the BEM used for the numerical solution of the imposed by the IEMA single scattering problems. In section 4, it is numerically proved that a cluster of fibers can be replaced by an equivalent homogeneous fiber the effective material properties of which can be predicted by the IEMA. This is accomplished by comparing the BEM solutions of the two corresponding problems. In section 5, the IEMA is appropriately used for the composite media of Fig. 1 and dispersion as well as attenuation curves concerning longitudinal and transverse horizontal plane waves are provided. The obtained results are compared to those taken for a corresponding fiber-reinforced composite with uniform distribution of fibers.

2 IEMA for fiber reinforced composite with uniform distribution of fibers

In this section, the IEMA proposed recently by Verbis, Kattis, Tsinopoulos and Polyzos (2001) for fiber reinforced composite with uniform distribution of fibers, is presented in brief.

The starting point of the IEMA is a self-consistent condition first considered in the coherent potential theory of Soven (1967). According to this theory, any wave propagating in a composite medium can be considered as a sum of a mean wave propagating in a medium having the dynamic effective properties of the composite and a number of fluctuating waves coming from the multiple scattering of the mean wave by the uniformly and randomly distributed material variations from these of the effective medium. On the average, the fluctuating field should be vanished at any direction within the effective medium, i.e.,

$$\left\langle \hat{\mathbf{k}} \cdot \tilde{\mathbf{T}} \cdot \hat{\mathbf{k}} \right\rangle = 0 \tag{1}$$

where $< >$ denotes average over the composition and the shape of the scatterers, $\tilde{\mathbf{T}}$ is a matrix corresponding to the total multiple scattering operator for the fluctuating waves and $\hat{\mathbf{k}}$ is the propagation direction of the mean wave. Eq. 1 is well known as self-consistent condition and can be used to determine the dynamic effective properties of the composite material. However, due to the prohibitive computational cost of the evaluation of the operator $\tilde{\mathbf{T}}$ Soven (1967) proposed, instead of Eq. 1, the use of the following simplified self-consistent condition

$$\left\langle \hat{\mathbf{k}} \cdot \tilde{\mathbf{t}} \cdot \hat{\mathbf{k}} \right\rangle = 0 \tag{2}$$

with $\tilde{\mathbf{t}}$ being a single scattering operator coming from the diffraction of the mean wave by each composition, i.e. matrix and fibers, embedded in an infinitely extended effective medium. Devaney (1980) proved that Eq. 2 could also be written as a function of the far-field scattering amplitudes in forward direction. Thus, for the case of a unidirectional fiber composite, where identical homogeneous elastic fibers are embedded in a homogeneous elastic matrix, Eq. 2 assumes the following form

$$n_1 g^{(1)}(\hat{\mathbf{k}},\hat{\mathbf{k}}) + (1-n_1)\, g^{(2)}(\hat{\mathbf{k}},\hat{\mathbf{k}}) = 0 \tag{3}$$

where n_1 represents the volume fraction of the fibers and $g^{(1)}(\hat{\mathbf{k}},\hat{\mathbf{k}})$, $g^{(2)}(\hat{\mathbf{k}},\hat{\mathbf{k}})$ are the forward scattering amplitudes taken by the solution of the two single wave scattering problems illustrated in Fig. 2.

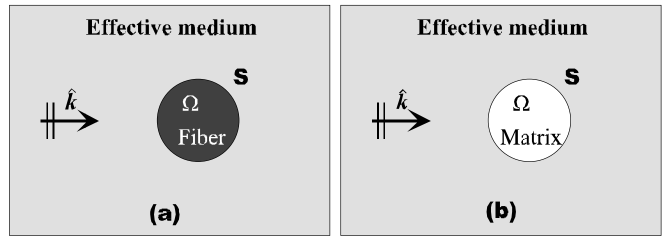


Figure 2 : A plane mean wave propagating in the effective to the composite medium and scattered by (a) a fiber inclusion (problem 1) and (b) a matrix inclusion with identical to the fiber geometry (problem 2).

According to the IEMA the self-consistent condition (3) is satisfied numerically through an iterative procedure, which can be summarized as follows:

Consider a harmonic elastic plane wave with circular frequencyω, either longitudinal (P) or transverse (SH or SV), traveling through a unidirectional composite with its propagation vector being perpendicular to the fibers. Due to the presence of the fibers, multiple scattering occurs and thereby the considered wave becomes both dispersive and attenuated and its complex wavenumber $k_d^{eff}(\omega)$ can be written as

$$k_d^{eff}(\omega) = \frac{\omega}{C_d^{eff}(\omega)} + i\alpha_d^{eff}(\omega) \tag{4}$$

with $C_d^{eff}(\omega)$ and $\alpha_d^{eff}(\omega)$ being the frequency dependent wave phase velocity and attenuation coefficient, respectively. The subscript d denotes either longitudinal ($d \equiv p$) or transverse ($d \equiv s$) wave.

Next, the composite material is replaced by an elastic homogeneous and isotropic medium with effective shear and Young moduli μ^{eff} and E^{eff}, respectively, given by the static model of Halpin and Tsai [Ashton and Halpin (1969)]

$$\mu^{eff} = \mu_2 \frac{\mu_1(1+n_1)+\mu_2(1-n_1)}{\mu_1(1-n_1)+\mu_2(1+n_1)}\ ,$$
$$E^{eff} = E_2 \frac{E_1(1+n_1)+E_2(1-n_1)}{E_1(1-n_1)+E_2(1+n_1)} \tag{5}$$

Subscripts 1 and 2 indicate fiber and matrix material properties, respectively.

For the case of multi-coated fibers the material properties μ_1 and E_1 appearing in Eqs 5 are obtained by means of the simple mixture rule.

In the first step of the IEMA, the effective density of the composite is assumed to be

$$\left(\rho^{eff}\right)_{step1} = n_1\rho_1 + (1-n_1)\,\rho_2 \quad (6)$$

Then, the effective wave number $\left(k_d^{eff}\right)_{step1}$ is evaluated in as straightforward through the relations

$$\left(k_p^{eff}\right)_{step1} = \omega\left[\frac{\mu^{eff}\left(E^{eff}-4\mu^{eff}\right)}{\left(E^{eff}-3\mu^{eff}\right)\left(\rho^{eff}\right)_{step1}}\right]^{-\frac{1}{2}} \quad (7)$$

for a P-wave and

$$\left(k_s^{eff}\right)_{step1} = \omega\left[\frac{\mu^{eff}}{\left(\rho^{eff}\right)_{step1}}\right]^{-\frac{1}{2}} \quad (8)$$

for a shear wave, respectively.

In the sequel, utilizing the material properties obtained from the first step, the two single wave scattering problems illustrated in Fig. 2 are solved. The solution of these problems is accomplished numerically by means of a 2-D boundary element code described in the next section. Combining the evaluated forward scattering amplitudes $g_d^{(1,2)}(\hat{\mathbf{k}},\hat{\mathbf{k}})$, according to the self-consistent condition (3), i.e.,

$$g_d(\hat{\mathbf{k}},\hat{\mathbf{k}}) = n_1 g_d^{(1)}(\hat{\mathbf{k}},\hat{\mathbf{k}}) + (1-n_1)\,g_d^{(2)}(\hat{\mathbf{k}},\hat{\mathbf{k}}) \quad (9)$$

and making use of the dispersion relation proposed by Foldy (1945), one obtains the new effective wave number of the mean wave

$$\left[\left(k_d^{eff}\right)_{step2}\right]^2 = \left[\left(k_d^{eff}\right)_{step1}\right]^2 + \frac{4n_1 g_d(\hat{\mathbf{k}},\hat{\mathbf{k}})}{\pi a^2} \quad (10)$$

where a is the radius of the fibers. The new complex density $\left(\rho^{eff}\right)_{step2}$ is evaluated from the $\left(k_d^{eff}\right)_{step2}$ and Eqs 7 and 8. The procedure is repeated with the material properties (5) and the new density $\left(\rho^{eff}\right)_{step2}$ until the self-consistent condition (3) is satisfied. This means that $\left(k_d^{eff}\right)_{step(n-1)} = \left(k_d^{eff}\right)_{step(n)}$. Finally, the evaluated k_d^{eff} in conjunction with Eq. 4 determines the frequency dependent, effective phase velocity $C_d{}^{eff}(\omega)$ and the attenuation coefficient $\alpha_d^{eff}(\omega)$ of the propagating wave.

In order to demonstrate the effectiveness of the IEMA, the propagation of a longitudinal wave in a composite material consisting of Titanium (Ti) alloy matrix reinforced with 24% Silicon Carbide (SiC) fibers is studied. Each fiber has a diameter of 142 μm and contains a Carbon core, a SiC shell and a Carbon rich interphasial layer. All the material properties as well as the dimensions of the three phases of the fiber are given in the Tab. 1.

Table 1 : Material properties of each phase in SiC/Ti fiber/matrix composite

Phase (Material)	**E (GPa)**	ν	ρ **(kgr / m^3)**	**Radius (μm)**
Core (Carbon)	41.0	0.250	1700	18
Shell (SiC)	415.0	0.170	3205	68
Interphase (Carbon-rich)	13.0	0.413	2100	71
Matrix (Titanium alloy)	121.6	0.348	5400	-

Huang and Rohklin (1995) utilizing a generalized self-consistent methodology (GSCM) provided theoretical dispersion and attenuation curves, accompanied by experimental results, for a longitudinal wave propagating in the above-mentioned composite material. Their results are displayed in Figs 3 and 4 and compared to the corresponding ones obtained in the present study. As it is observed, the experimental measurements and the present numerical results are in a very good agreement. On the other hand, although the method of Huang and Rohklin (1995) predicts very well the wave phase velocity, it fails to predict with the same accuracy the wave attenuation.

3 BEM evaluation of the scattering amplitudes

In what follows, the outline of a 2-D BEM utilized for the evaluation of the wave scattering amplitudes required in the above-mentioned IEMA is presented.

Consider a long homogeneous elastic fiber of arbitrary cross-section embedded into an infinitely extended, homogeneous isotropic elastic matrix and a Cartesian coordinate system $Ox_1x_2x_3$ with the x_1x_2 plane being perpendicular to the fiber.

A time harmonic plane wave $\mathbf{u}^I$ (with $e^{-i\omega t}$ suppressed) impinges upon the fiber with a circular frequaeny ω. The incident wave propagates across the direction $\hat{\mathbf{k}}$ and is polarized in the $\hat{\mathbf{d}}$ direction. The solution of this scattering problem can be obtained by the solution of a combined system of integral equations, written as

$$\tilde{\mathbf{c}}(\mathbf{r})\,\mathbf{u}^{\text{ext}}(\mathbf{r}) + \int_S \tilde{\mathbf{T}}^{\text{ext}}(\mathbf{r},\mathbf{r}')\,\mathbf{u}^{\text{ext}}(\mathbf{r}')\;dS = \int_S \tilde{\mathbf{U}}^{\text{ext}}(\mathbf{r},\mathbf{r}')\;\mathbf{t}^{\text{ext}}(\mathbf{r}')\;dS + \mathbf{u}^{\mathbf{I}}(\mathbf{r}) \tag{11}$$

$$\left(\tilde{\mathbf{I}} - \tilde{\mathbf{c}}(\mathbf{r})\right)\mathbf{u}^{\text{int}}(\mathbf{r}) + \int_S \tilde{\mathbf{T}}^{\text{int}}(\mathbf{r},\mathbf{r}')\,\mathrm{u}^{\text{int}}(\mathbf{r}')\;\mathrm{dS} = \int_S \tilde{\mathbf{U}}^{int}(\mathbf{r},\mathbf{r}')\;\mathbf{t}^{int}(\mathbf{r}')\;dS \tag{12}$$

where S is the boundary of the space Ω occupied by one fiber, $\mathbf{r}$ is a field point, $\mathbf{r}'$ is a source point lying on the boundary S, $\mathbf{u}$, $\mathbf{t}$ are the displacement and traction vectors, respectively, $\tilde{\mathbf{U}}$ and $\tilde{\mathbf{T}}$ represent the frequency domain fundamental solutions, the indices "*ext*" and "*int*" indicate exterior and interior to the fiber quantities, respectively, and $\tilde{\mathbf{c}}(\mathbf{r})$ is the usual jump tensor. Analytical expressions for $\tilde{\mathbf{U}}$, $\tilde{\mathbf{T}}$ and $\tilde{\mathbf{c}}(\mathbf{r})$ can be found in the books of Manolis and Beskos (1988) and Dominguez (1993).

Far away from the scatterer, the scattered field satisfies the radiation conditions and admits a representation of the form [Dassios and Kiriaki (1984)]

$$\frac{\mathbf{u}^{\mathbf{s}}(\mathbf{r})}{|\mathbf{u}^{\mathbf{I}}|} = g_r\left(\hat{\mathbf{d}};\hat{\mathbf{r}},\hat{\mathbf{k}}\right)\hat{\mathbf{r}}\frac{e^{-ik_p|\mathbf{r}|}}{\sqrt{|\mathbf{r}|}} + g_\varphi\left(\hat{\mathbf{d}};\hat{\mathbf{r}},\hat{\mathbf{k}}\right)\hat{\phi}\frac{e^{-ik_S|\mathbf{r}|}}{\sqrt{|\mathbf{r}|}} + \mathbf{O}\left(\frac{1}{|\mathbf{r}|^{3/2}}\right),\ |\mathbf{r}| \to \infty \tag{13}$$

when the incident waves are of the type P or SV type

$$\frac{\mathbf{u}^{\mathbf{s}}(\mathbf{r})}{|\mathbf{u}^{\mathbf{I}}|} = g_{x_3}\left(\hat{\mathbf{d}};\hat{\mathbf{r}},\hat{\mathbf{k}}\right)\hat{\mathbf{x}}_{\mathbf{3}}\frac{e^{-ik_s|\mathbf{r}|}}{\sqrt{|\mathbf{r}|}} + \mathbf{O}\left(\frac{1}{|\mathbf{r}|^{3/2}}\right) \quad |\mathbf{r}| \to \infty \tag{14}$$

when the incident wave is of the SH type. In the above relations, $\hat{\mathbf{r}}$, $\hat{\phi}$ are the unit vectors of a polar coordinate system, having its origin interior to the scatterer, k_p and k_s are the longitudinal and shear wave numbers, respectively, and the complex functions g_r, g_φ and g_{x3} represent

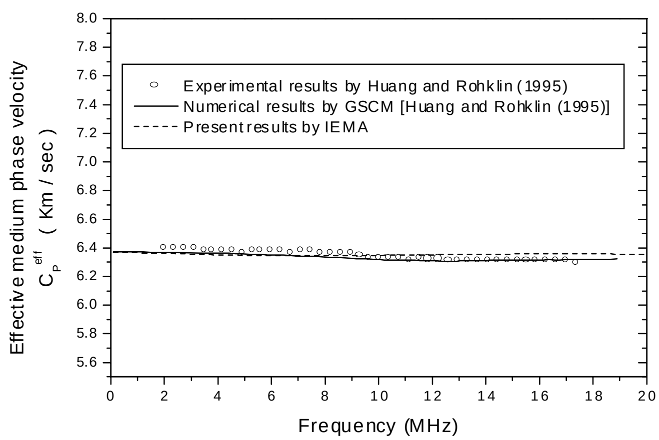


Figure 3 : Phase velocity of longitudinal wave propagating in a SiC/Ti fiber/matrix composite material with a fiber volume fraction n_1=0.24 versus frequency.

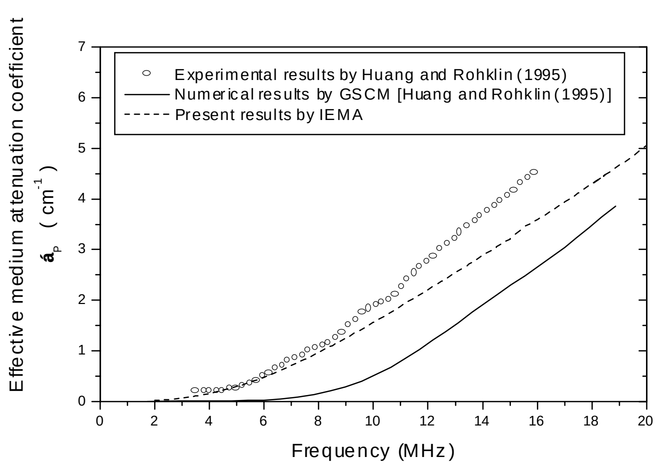


Figure 4 : Attenuation coefficient of longitudinal wave propagating in a SiC/Ti fiber/matrix composite material with a fiber volume fraction n_1=0.24 versus frequency.

the scattering amplitudes, the analytical form of which can be found in the work of Verbis, Kattis, Tsinopoulos and Polyzos (2001).

In the present work, the boundary value problem and the scattering amplitudes are calculated by means of a 2-D frequency domain BEM code, developed by the authors [Verbis, Kattis, Tsinopoulos and Polyzos (2001)]. According to this code, the boundary of the scatterer S is discretized into three-noded quadratic line elements. Collocating the integral equations (11) and (12) at all nodes and satisfying the continuity conditions at the interface of the fiber and the matrix, one obtains a linear system of equations with unknowns all the nodal components of the displacement and traction. As soon as the boundary problem is solved, the evaluation of the scattering amplitudes through their integral representations is straightforward.

4 Homogenization of a cluster of fibers through the IEMA

This section aims to show that the forward far-field scattering parameters of a circular cluster of fibers embedded in an infinitely extended matrix medium are almost equivalent to those taken by a dimensionally equal homogeneous fiber for which the effective material properties are provided by the IEMA. To this end, two circular clusters consisting of N=25 and N=100 identical, cylindrical glass fibers of radius α=50μm embedded in an Epoxy matrix are considered. Each cluster occupies a cylindrical area of radius R_{25}=390μm and R_{100}=780μm, respectively, corresponding to a volume fraction n=40.78%. Both glass and Epoxy materials are homogeneous and isotropic and their properties are given in Tab. 2. In the circular area of each cluster, the N fibers have been computationally placed by means of a random number generator function [IMSL (1994)].

Table 2 : Material properties of a Glass/Epoxy fiber/matrix composite

Material	**E (GPa)**	N	**ρ (kgr/m^3)**
Glass	62.620	0.20	2490.
Epoxy	4.742	0.37	1202.

The scattering problems concerning the interaction of each cluster with elastic, longitudinal (P) or transverse horizontal (SH) plane waves are treated numerically through the BEM described in the previous section. For both cluster sizes, the influence of the fiber-randomness on the obtained forward scattering amplitudes is investigated by considering three different cluster configurations depicted in Fig. 5.

The obtained results are compared to those taken from the BEM solution of the corresponding single scattering problems where each cluster is considered as a homogeneous fiber for which the effective material properties are provided by the IEMA for n=40.78%. The evaluated forward scattering amplitudes are presented in Figs 6 and 7 as a function of the dimensionless frequency $k_d^m a$, where k_d^m is the wave number of the incident wave traveling in the matrix medium and α the radius of each fiber.

From Figs 6 and 7 it is apparent that, in the framework of wave scattering, a cluster of fibers can be effectively homogenized by the IEMA for both longitudinal and transverse wave incidences.

5 IEMA for fiber composites with irregular distribution of fibers

In this section, the use of the IEMA for wave dispersion and attenuation predictions in fiber composites with a microstructure characterized by a non-uniform distribution of fibers is described. As it is explained in the introduction, two specific types of composites with non-uniform distribution of fibers will be examined here. Both are represented graphically in Fig. 1. As in section 4, the constituents of the examined composites are glass fibers embedded in an Epoxy matrix with a fiber volume fraction being equal to 24%.

The composite material of Fig. 1(a) concerns a uniform distribution of not equally sized clusters of fibers embedded in a composite matrix with uniformly distributed fibers. The total fiber volume fraction is 24% while 75% of the fibers are enclosed in the clusters. The clusters as well as the medium in which they are embedded are composites with fiber volume fractions 40.78% and 11%, respectively. The radii of the clusters follow a Gaussian distribution of the form

$$P(R) = \frac{1}{\sqrt{2\pi}}\frac{1}{\sigma}e^{\frac{-(R-R_0)^2}{2\sigma^2}}$$
$$\int_{-\infty}^{+\infty} P(R)dR = 1 \qquad (15)$$

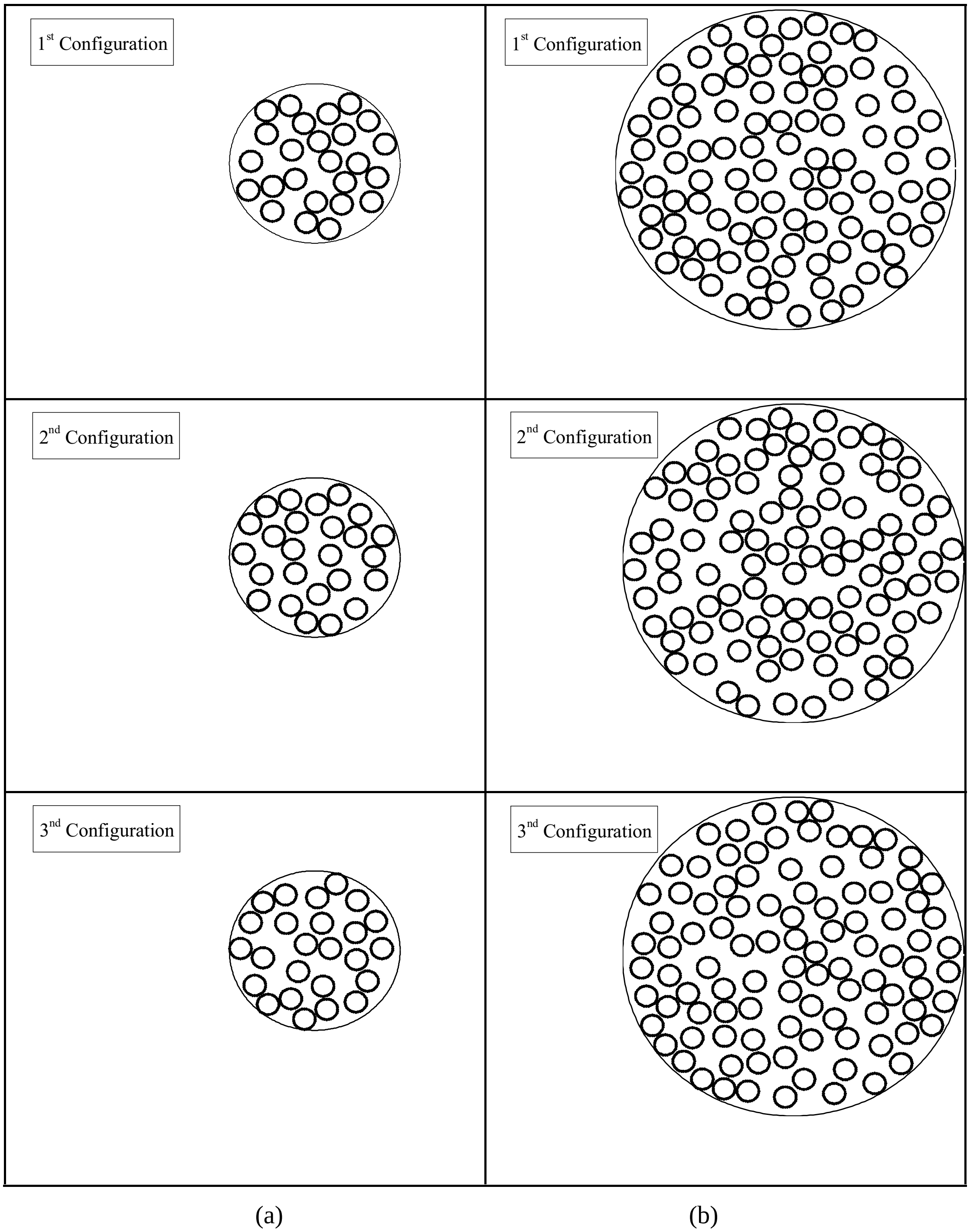


Figure 5 : Three different random fiber distributions for a circular cluster of radius (a) R_{25}=390μm (N=25 fibers) and (b) R_{100}=780μm (N=100 fibers).

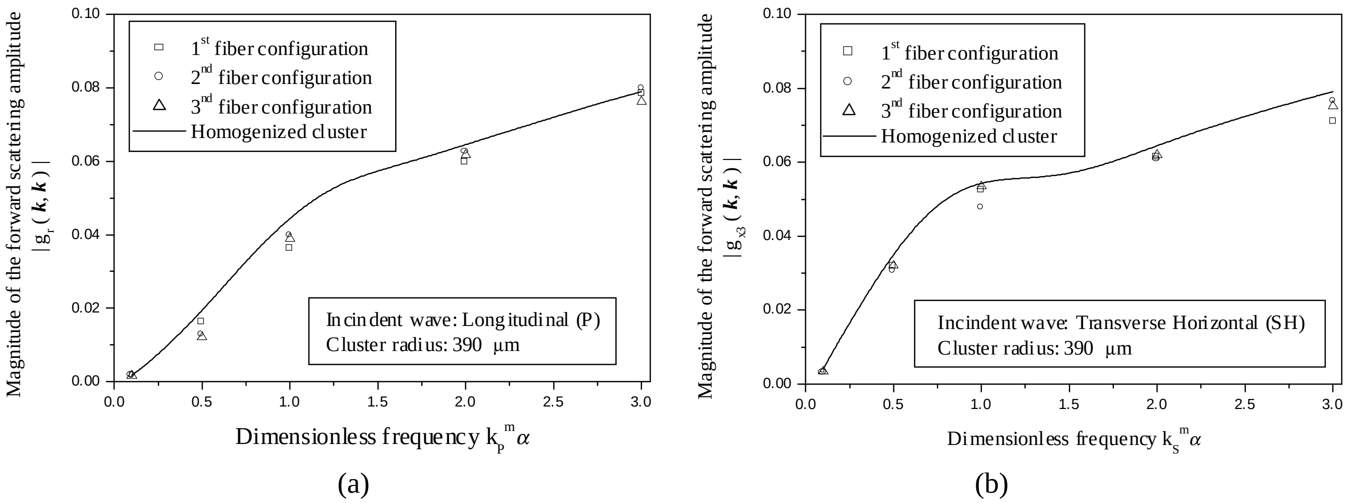


Figure 6 : Forward scattering amplitudes for the three cluster configurations of Fig 5(a) (symbols) and for (a) longitudinal and (b) transverse horizontal incidence. The solid line corresponds to the same cluster (R_{25}=390μm) homogenized by the IEMA.

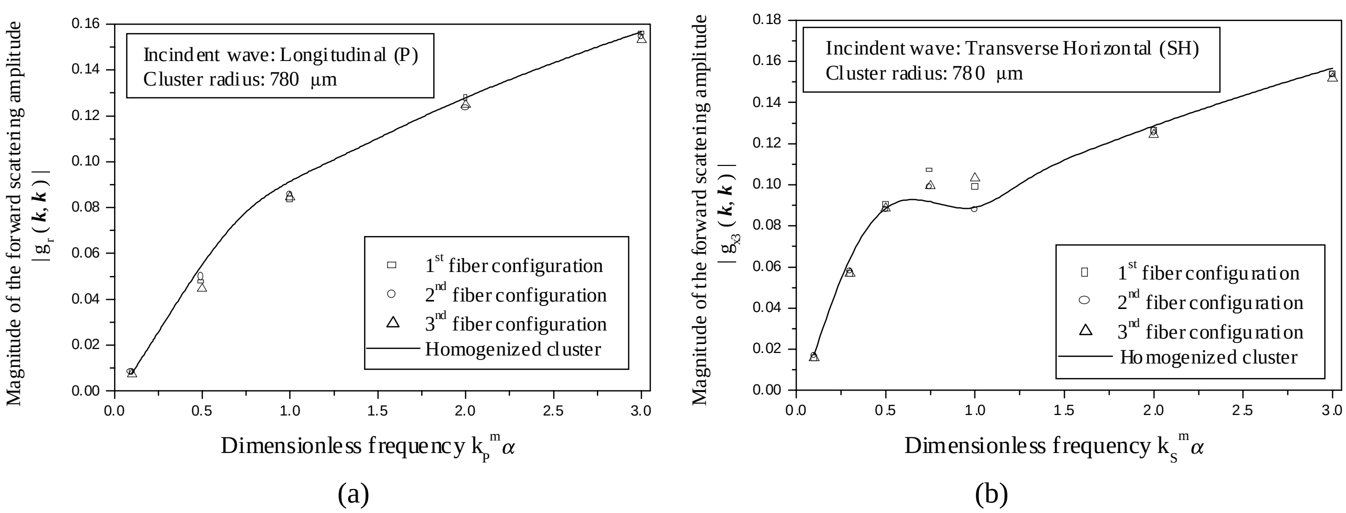


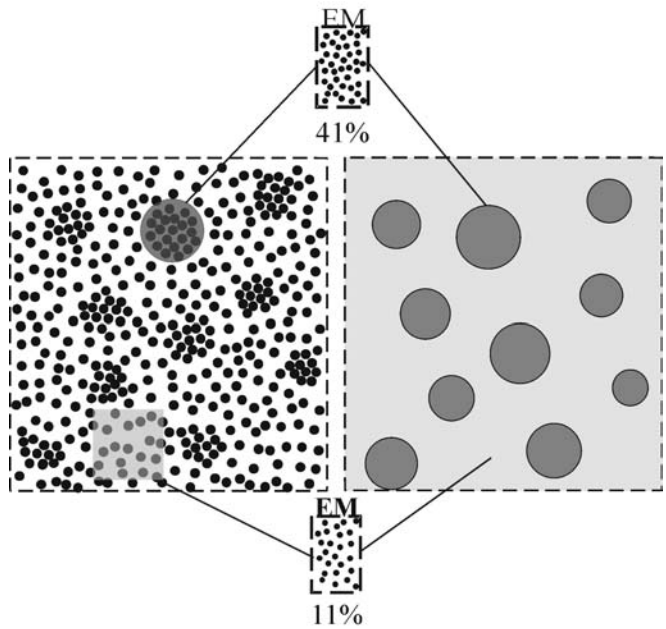
Figure 7 : The same as Fig. 6, for the three cluster configurations shown in Fig. 5(b) (R_{100}=780μm).

with $\sigma = \sqrt{\pi/2}$ and R_0 being the mean radius of the distributed clusters taken in the present study equal to 680 μm.

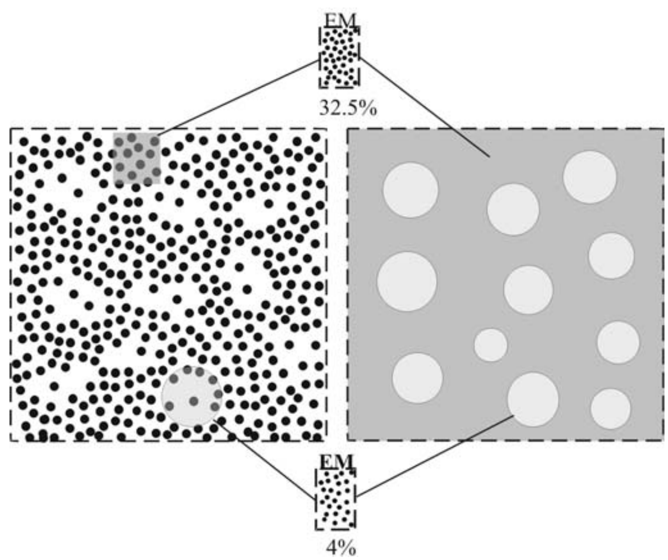
The second composite medium considered here is a random distribution of matrix-rich inclusions embedded in a fiber-rich regular composite medium (Fig. 1(b)). As in the first case the total fiber volume fraction is 24%. Both fiber-rich and matrix-rich areas are assumed to be regular composite media with fiber volume fractions 32.5% and 5%, respectively. The matrix-rich inclusions are circular with a size distribution described again by Eq. 15.

Applying the IEMA for each phase of the above mentioned composites one can derive two artificial composite media with randomly and uniformly distributed circular inclusions as it is illustrated in Fig. 8. The elastic constants of the homogenized constituents of the two new composites are evaluated by means of Eqs 5, while their complex densities are provided by the IEMA for the corresponding fiber volume fractions. The volume fractions of the inclusions in the two artificial composites result in $\overline{n} = 45\%$ for the case of the clustered fibers and $\overline{n} = 30\%$ for the case of matrix-rich inclusions.

The key idea of the present work is that the dispersion and attenuation properties of a plane wave propagating in the two original composites of Fig. 1 can be effectively predicted by applying the IEMA on the two artificial composites of Fig. 8. Since the IEMA is based on a self-consistent condition that combines the forward far-field scattering parameters of the two scattering problems illustrated in Fig. 2, the above idea seems to be realistic if the forward scattering amplitudes taken from the scattering of the incident wave by a cluster of fibers or a matrix-rich inclusion are similar to those taken from the corresponding homogenized inclusions. However, it has been already proved in section 4 that a cluster of fibers can be effectively homogenized by the IEMA for both longitudinal and transverse wave incidences. Thus, applying the IEMA for the artificial composites of Fig. 8, one can effectively make wave dispersion and attenuation predictions for the original composite media of Fig. 1. In the present case, where not equally sized circular inclusions are considered, the scattering amplitudes used in the iteration procedure of the IEMA and given by Eqs 9 are appropriately modified as

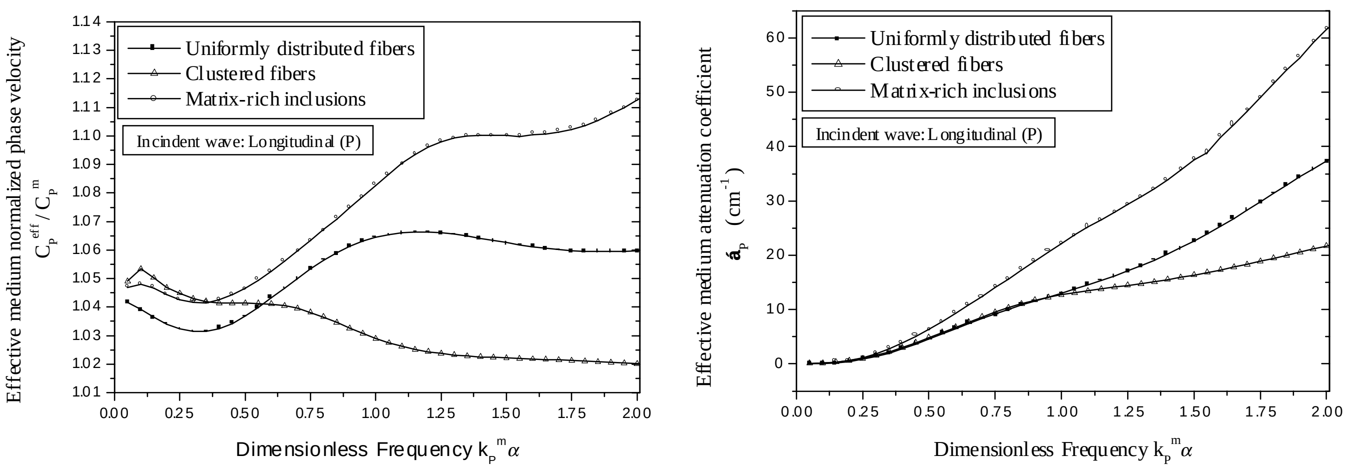


(a)



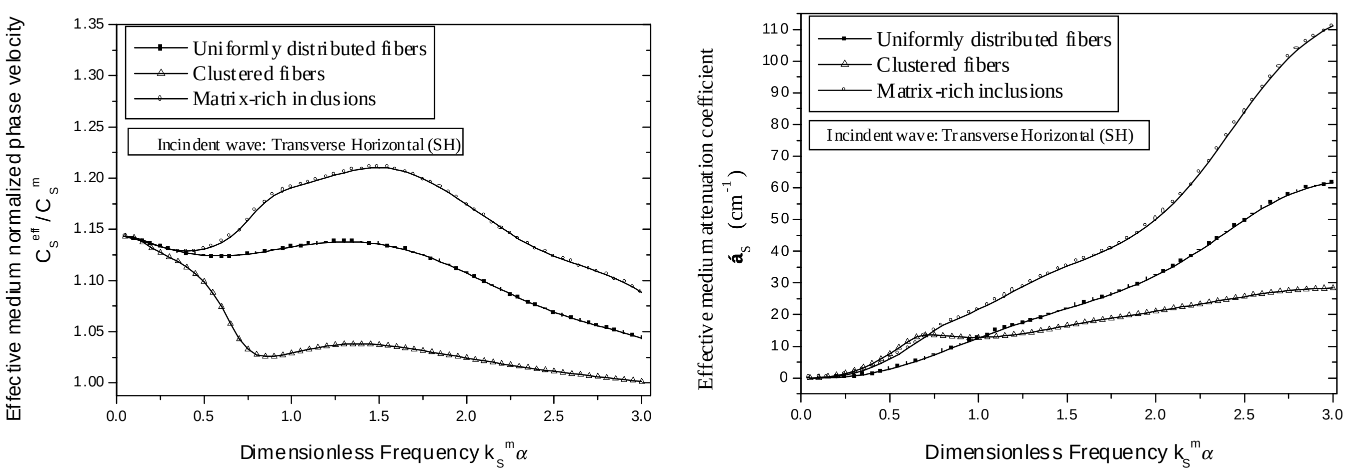
(b)

Figure 8 : Homogenization of the two different phases of the composite by means of the IEM approximation; (a) clustered fibers (b) matrix-rich areas.



(a) (b)

Figure 9 : Phase velocity (a) and attenuation coefficient (b) of a longitudinal (P) wave propagating in a glass/Epoxy composite with uniformly distributed fibers (squares), clustered fibers (triangles) and matrix-rich inclusions (circles).



(a) (b)

Figure 10 : Same as Fig.9 for transverse horizontal (SH) incidence.

$$g(\hat{\mathbf{k}},\hat{\mathbf{k}}) = \overline{n}\left\langle g^{(1)}(R;\hat{\mathbf{k}},\hat{\mathbf{k}})\right\rangle + (1-\overline{n})\left\langle g^{(2)}(R;\hat{\mathbf{k}},\hat{\mathbf{k}})\right\rangle$$
$$\left\langle g(R;\hat{\mathbf{k}},\hat{\mathbf{k}})\right\rangle = \int \left(g(R;\hat{\mathbf{k}},\hat{\mathbf{k}})\right) P(R)dR \quad (16)$$

where the scattering amplitudes $g^{(1)}(R;\hat{\mathbf{k}},\hat{\mathbf{k}}), g^{(2)}(R;\hat{\mathbf{k}},\hat{\mathbf{k}})$ are taken from the solution of the two single scattering problems of Fig. 1, with the radius of the corresponding circular scatterers being equal to R. The integral in the right hand side of the third relation of Eq. 16 is treated numerically by means of Gauss quadrature utilizing 10 integration points.

In Figs. 9 and 10 the evaluated phase velocity as well as the attenuation coefficient of a longitudinal and a transverse horizontally plane wave, respectively, traveling in the two types of composites considered in this paper are presented. The calculated phase velocities are normalized with the corresponding ones of a wave traveling in the Epoxy matrix medium. The obtained results are compared with dispersion and attenuation curves corresponding to wave propagation in glass/Epoxy fiber/matrix composite with randomly and uniformly distributed fibers. The results of this comparison are presented in the conclusions section, which follows.

6 Conclusions

On the basis of the results of Figs 9 and 10, one can draw the following conclusions concerning wave dispersion and attenuation in composites with clustered fibers and matrix-rich inclusions as compared to composites with a uniform distribution of fibers:

1. For $k_p^m\alpha > 0.6$ both longitudinal and transverse waves become faster when they propagate in the composite with the matrix-rich inclusions. The opposite is valid in the composite with the clustered fibers.

2. For all cases and for $k_p^m\alpha < 0.6$ there are small differences among the evaluated phase velocities.

3. For all the considered frequencies both longitudinal and transverse waves propagating in the composite with the matrix-rich inclusions are more attenuated than in the case of uniformly distributed fibers.

4. For $k_p^m\alpha > 1$, both longitudinal and transverse waves propagating in the composite with the clustered fibers are less attenuated than in the case of uniformly distributed fibers.

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