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Shape Optimization of Elastic Structural Systems Undergoing Large Rotations: Simultaneous Solution Procedure

Adnan Ibrahimbegovic[1] and Catherine Knopf-Lenoir[2]

Abstract: In this work we present an unconventional procedure for combining the optimal shape design and nonlinear analysis in mechanics. The main goal of the presented procedure is to enhance computational efficiency for nonlinear problems with respect to the conventional, sequential approach by solving the analysis and design phases simultaneously. A detailed development is presented for the chosen model problem, the 3d rod undergoing large rotations.

1 Introduction

Ever increasing demands to achieve a more economical design of a structural system result with the need to exploit and analyze the nonlinear behavior of such a system. Optimization methods can be called upon to guide the design procedure and achieve desired reduction in required mechanical and/or geometric properties, which is formally defined as minimization of the cost or objective function depending upon chosen design variables. Traditionally (e.g. see Kleiber et al. [1997] for a very recent review), the two fields directly concerned by this task, nonlinear mechanics and shape optimization, are studied separately and when brought to bear on the same problem their application is done in a sequential manner, typically using two different computer codes, one for mechanics and another for optimization. In this manner the communication requirements are reduced since each computer code gets only the minimum information from the other one: so-called design sensitivity (e.g. Tortorelli and Michaleris [1994], Rousselet [1992]) for optimization code, or design variables for the finite element code for mechanics. It is clear that such a standard approach to analysis and design will sacrifice the computational efficiency for the case of practical interest where both cost function and mechanical problem are nonlinear and require each an iterative procedure to be solved.

The main idea elaborated upon in this work relates to an alternative method of analysis and design where those two phases are formulated and solved simultaneously. In that respect, the interdependence of analysis and design variables is no longer assumed so that one can iterate simultaneously on both of them. In particular, design sensitivity analysis need no longer be performed separately, but it is naturally integrated as a part of the global solution procedure.

In mathematical terms the combined analysis and design of this kind can be represented as a constrained minimization problem (e.g. see Luenberger [1984] or Strang [1986]), where the constraints are nonlinear, i.e. governing equations of non linear mechanics problem. At each design iteration, the analysis variable should be constraint consistent.

The simultaneous procedure would seek the solution of such a problem by making use of the Lagrange multiplier procedure to remove the presence of constraints and makes it possible to enlarge the admissible space and, more importantly, to iterate simultaneously on both analysis and design variables. Iterative intermediate values of analysis variables are no longer consistent with the constraint, except at the convergence, where basically the same solution is obtained as for the standard sequential solution but with a (significantly) reduced number of iterations. The simultaneous solution procedure gives the same solution providing the latter is unique. One such problem of shape optimization for geometrically nonlinear rods is presented to illustrate these ideas in detail.

The outline of the paper is as follows. In Section 2 we briefly recall the governing equations of the chosen model, geometrically exact 3d curved rod (e.g. see Ibrahimbegovic [1995]). Simultaneous solution proce-

[1] Ecole Normale Supérieure de Cachan
LMT - 61, avenue du Président Wilson
94235 Cachan, France
e-mail: ai@lmt.ens-cachan.fr
[2] Université de Technologie de Compiègne
Laboratoire Roberval - BP 60529
60205 Compiègne, France

dure for analysis and design leading to optimal shape is presented in Section 3 and compared against the standard sequential solution. Some comments on numerical implementation are given in Section 4. In Section 5, we state the closing remarks.

2 Model Problem: 3d Curved Rod

In this section, we briefly review the governing equations of the chosen model problem of a structure undergoing large rotations, a 3d curved rod. For a more thorough discussion of the chosen model we refer to Simo and Vu-Quoc [1986], Iura and Atluri [1989], Simo [1992] or Ibrahimbegovic, Frey and Kozar [1995], among others.

We assume that the initial configuration of the rod is internal force free and that can be described by a 3d vector $\varphi_0(s)$ identifying the position of each point of the neutral fiber (an inextensible fiber in the pure bending) and the corresponding placement of the cross-section of the rod which is carried out by choosing a local orthonormal triad of vectors. The vector triad of this kind can be obtained by simply rotating the global triad by an orthogonal tensor, $\mathbf{\Lambda}_0$. For a usual choice of normal coordinates with the first vector of the triad being orthogonal to the cross-section and the remaining two placed in the plane of the cross section, this orthogonal tensor becomes a known function of the initial configuration, $\mathbf{\Lambda}_0(s)$. For the case of a curved rod studied here, 's' is chosen as the arc length.

By applying the normal loading $\mathbf{f}_t$, parameterized by pseudo-time 't' ('pseudo' in the sense that the inertia effects are neglected) we obtain the rod deformed configuration defined by the position vector $\varphi_t(s)$ and the orthogonal tensor $\mathbf{\Lambda}_t(s)$. The latter is an accordance with the usual kinematic hypothesis that the cross-section of the rod would not deform, which, along with the hypothesis that the first vector in the triad remains orthogonal to it (with other two within the plane of the cross-section) fully determines $\mathbf{\Lambda}_t(s)$. See Figure 1 where the initial and deformed configuration of the rod are presented.

In short, one can state that the configuration space of the described model of 3d rod consists of

$$\mathcal{C} := \{\phi_t = (\varphi_t, \mathbf{\Lambda}_t) \,|\, \varphi_t \in \mathbf{R}^3, \mathbf{\Lambda}_t \in SO(3)\} \tag{1}$$

where $\mathbf{R}^3$ and $SO(3)$ are spaces of 3d vectors and special orthogonal tensors, respectively.

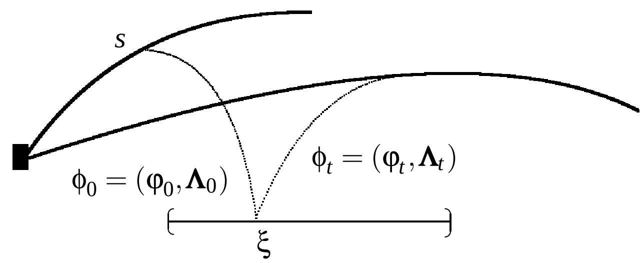

Figure 1: Initial and deformed configuration of the rod.

The main difficulty in numerical solution of the problems featuring the rods of this kind stems from the presence of $SO(3)$ group in its configuration space (e.g. see Argyris [1982], Atluri and Cazzani [1995], Ibrahimbegovic, Frey and Kozar [1995], Ibrahimbegovic [1997] for a more thorough discussion of these issues). In short, in performing a standard task of computing the virtual work principle or the consistent linearization, where a small rotation described by skew-symmetric tensor $\delta\mathbf{\Theta} \in so(3)$, ought to superposed on a large rotation described by an orthogonal tensor $\mathbf{\Lambda} \in SO(3)$ one must first make use of exponential mapping

$$\mathbf{\Lambda}_\varepsilon = \mathbf{\Lambda} \exp[\varepsilon\, \delta\mathbf{\Theta}]$$

$$\exp[\delta\mathbf{\Theta}] = \cos\delta\theta\, \mathbf{I} + \frac{\sin\delta\theta}{\delta\theta}\delta\mathbf{\Theta} + \frac{1-\cos\delta\theta}{\delta\theta^2}\delta\theta \otimes \delta\theta \tag{2}$$

where $\delta\theta$ is the axial vector of the skew-symmetric tensor $\delta\mathbf{\Theta}$ i.e. $\delta\mathbf{\Theta}\, \mathbf{v} = \delta\theta \times \mathbf{v}, \quad \forall \mathbf{v} \in \mathbf{R}^3$.

We compare the last expression, rather involved, with respect to a simple additive update of virtual displacement field $\delta\varphi \in \mathbf{R}^3$ superposed on the deformed configuration $\varphi \in \mathbf{R}^3$

$$\varphi_\varepsilon = \varphi + \varepsilon\, \delta\varphi \tag{3}$$

The results in (2) and (3) can be presented in an equivalent form by stating that the tangent space of the chosen rod model is

$$T\mathcal{C} := \{\delta\phi := (\delta\varphi, \delta\theta) \,|\delta\varphi \in \mathbf{R}^3, \delta\theta \in \mathbf{R}^3\} \tag{4}$$

The strain measures employed in this rod theory (e.g. Ibrahimbegovic [1995]) can be written in direct form as

$$\varepsilon_t = \mathbf{\Lambda}_t \varphi_t' \tag{5}$$

for the axial and shear strains, and

$$\mathbf{\Omega}_t = \mathbf{\Lambda}_t^T \mathbf{\Lambda}_t'$$

$$\mathbf{\Omega}_t \mathbf{v} = \omega_t \times \mathbf{v}; \forall \mathbf{v} \in \mathbf{R}^3 \tag{6}$$

for bending and torsional strains. In (5), (6) and subsequent equations we denote with superposed prime the derivative with respect to arc-length coordinate in the initial configuration,

$$\frac{\partial}{\partial s}(\cdot) = (\cdot)' \tag{7}$$

We consider the simplest case of linear elastic material model for the rod which allows us to express the constitutive equations in terms of stress resultants as

$$\mathbf{n}_t = \mathbf{C}(\varepsilon_t - \varepsilon_0)\ ; \quad \mathbf{C} = diag(EA, GA, GA) \tag{8}$$

$$\mathbf{m}_t = \mathbf{D}(\omega_t - \omega_0)\ ; \quad \mathbf{D} = diag(GJ, EI, EI) \tag{9}$$

We also consider the simplest case of a circular cross section, with section diameter 'd', and

$$A = \frac{d^2\pi}{4}\ ; \quad I = \frac{d^4\pi}{64}\ ; \quad J = \frac{d^4\pi}{32} \tag{10}$$

as the section area, moment of inertia and polar moment. In order to complete the description of the chosen rod model we state the equilibrium equations in the weak form as

$$G(\phi_t;\delta\phi) := \int (\delta\varepsilon \cdot \mathbf{n}_t + \delta\omega \cdot \mathbf{m}_t)\, ds - G_{ext}(\delta\phi) = 0 \tag{11}$$

where $G_{ext}(\delta\phi)$ is the external virtual work and $\delta\varepsilon$ and $\delta\omega$ are the virtual strains. The latter can be obtained as the Gâteaux derivative of the real strains in (5) and (6), by taking the results in (2) and (3) into consideration. In particular, this leads to

$$\begin{aligned} \delta\widehat{\varepsilon}(\phi_t;\delta\phi) &= D_\phi[\widehat{\varepsilon}(\phi_t)] \\ &= \frac{d}{d\varepsilon}[\Lambda_{t,\varepsilon}^T \varphi_{t,\varepsilon}']\,|_{\varepsilon=0} \\ &= \mathbf{\Lambda}^T \delta\varphi' + \varepsilon_t \times \delta\theta \end{aligned} \tag{12}$$

and

$$\begin{aligned} \delta\widehat{\Omega}(\phi_t;\delta\phi) &= D_\phi[\widehat{\Omega}(\phi_{t,\varepsilon})] \\ &= \frac{d}{d\varepsilon}[\Lambda_{t,\varepsilon}^T \Lambda_{t,\varepsilon}']\,|_{\varepsilon=0} \\ &= \delta\mathbf{\Theta}' + \delta\mathbf{\Theta}^T \Omega + \Omega\delta\mathbf{\Theta} \end{aligned} \tag{13}$$

which can also be written in an equivalent form in terms of axial vectors

$$\delta\omega = \delta\theta' + \omega \times \delta\theta \tag{14}$$

For the chosen model of geometrically non linear elastic rod the weak form of the equilibrium equation in (11) can be obtained as the minimum of the total potential energy

$$\begin{aligned} \Pi(\phi_t) := & \int_l \frac{1}{2}\{(\varepsilon_t - \varepsilon_0)\cdot \mathbf{n}_t + (\omega_t - \omega_0)\cdot \mathbf{m}_t\}\, ds \\ & - \Pi_{ext}(\phi) \rightarrow \min \end{aligned} \tag{15}$$

with

$$\Pi(\phi_t) = \min_{\forall \phi_t^*} \Pi(\phi_t^*) \Longrightarrow \begin{cases} D_\phi[\Pi(\phi_t)] \equiv G(\phi_t;\delta\phi) = 0 \\ D_\phi[D_\phi \Pi(\phi_t)] > 0 \end{cases} \tag{16}$$

3 Simultaneous Solution Procedure for Shape Optimization

The shape optimization procedure is interpreted herein as minimization of so-called cost or objective functional J($\cdot$), a functional which depends not only on mechanical variables ϕ_t but also on design variables d_t which can be written as

$$J(\phi_t, d) = \min_{G(\phi^*, d^*; \delta\phi)=0} J(\phi^*, d^*) \tag{17}$$

Contrary to the minimization of the total potential energy functional in (15), not all mechanical and design variables are admissible candidates, but only those for which (the weak form of) the equilibrium equations are satisfied. In other words, we need to deal with a constrained minimization problem.

The classical shape optimization procedure of solving this constrained minimization problem is carried out in a sequential manner, where for each iterative value of design variables, a new iterative procedure need to be completed leading to $\phi_t(d)$ verifying the equilibrium equations. A considerable computational cost of such a procedure, most of it waited on iterating to convergence on equilibrium equations even for non-converged values of design variables, can be drastically reduced by formulated the minimization problem in (17) using the method of Lagrange multipliers with

$$\max_{\forall\lambda} \min_{\forall(\phi_t^*, d^*)} L(\phi_t^*, d^*;\, \lambda)$$

$$L(\phi_t^*, d^*; \lambda) = J(\phi_t^*, d^*) + G(\phi_t^*, d^*; \lambda) \tag{18}$$

In (18) above $\lambda = (\nu, \mu)$ are the Lagrange multipliers featuring in the weak form of equilibrium equations instead of virtual displacements and rotations which in accordance to the results presented in the previous section can be written as

$$G(\phi_t, d; \lambda) = \int_l \begin{pmatrix} \nu' \\ \mu \\ \mu' \end{pmatrix} \cdot \begin{bmatrix} \mathbf{\Lambda}_t^T & \mathbf{E}_t & \mathbf{0} \\ \mathbf{0} & \mathbf{\Omega}_t & \mathbf{I} \end{bmatrix}^T \begin{pmatrix} \mathbf{n}_t \\ \mathbf{m}_t \end{pmatrix} ds$$

$$- G_{ext}(\lambda) \tag{19}$$

The main difference of (18) with respect to constrained minimization problem in (17) pertains to the fact that state variables ϕ_t and design variables d are now considered independent and they can be iterated upon simultaneously.

The Kuhn-Tucker optimality condition (e.g. Luenberger [1984]) associated with the minimization problem in (18) can be written as

$$\mathbf{0} = D_\phi\left[L(\phi_t, d; \lambda)\right] = D_\phi\left[J(\phi_t, d)\right] + D_\phi\left[G(\phi_t, d; \lambda)\right] \tag{20}$$

where

$$\begin{aligned} &D_\phi\left[G(\phi_t, d; \lambda)\right] \cdot \delta\phi \\ &= \int_l \begin{pmatrix} \nu' \\ \mu \\ \mu' \end{pmatrix} \cdot \begin{bmatrix} \mathbf{\Lambda}_t^T & \mathbf{E}_t & \mathbf{0} \\ \mathbf{0} & \mathbf{\Omega}_t & \mathbf{I} \end{bmatrix}^T \begin{bmatrix} \mathbf{C} & \mathbf{0} \\ \mathbf{0} & \mathbf{D} \end{bmatrix} \\ &\qquad \begin{bmatrix} \mathbf{\Lambda}_t^T & \mathbf{E}_t & \mathbf{0} \\ \mathbf{0} & \mathbf{\Omega}_t & \mathbf{I} \end{bmatrix} \begin{pmatrix} \delta\varphi' \\ \delta\theta \\ \delta\theta' \end{pmatrix} ds \\ &+ \int_l \begin{pmatrix} \nu' \\ \mu \\ \mu' \end{pmatrix} \cdot \begin{bmatrix} \mathbf{0} & \mathbf{\Lambda}_t \mathbf{N}_t^T & \mathbf{0} \\ \mathbf{N}_t \mathbf{\Lambda}_t^T & \mathbf{\Xi}(\varepsilon_t \times \mathbf{n}_t) + \mathbf{\Xi}(\omega_t \times \mathbf{m}_t) & \mathbf{M}_t \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix} \\ &\qquad \begin{pmatrix} \delta\varphi' \\ \delta\theta \\ \delta\theta' \end{pmatrix} ds \end{aligned} \tag{21}$$

where we denoted $\mathbf{\Xi}(\mathbf{a} \times \mathbf{b}) = (\mathbf{a} \otimes \mathbf{b}) - (\mathbf{a} \cdot \mathbf{b})\mathbf{I}$, as well as $\mathbf{M}_t \mathbf{v} = \mathbf{m}_t \times \mathbf{v}$ and $\mathbf{N}_t \mathbf{v} = \mathbf{n}_t \times \mathbf{v}$, $\forall \mathbf{v} \in \mathbf{R}^3$; Moreover, we have

$$0 = D_d\left[L(\cdot)\right] = D_d J(\cdot) + D_d G(\cdot) \tag{22}$$

where

$$\begin{aligned} &D_d\left[G(\cdot)\right] \cdot \delta d \\ &= \int_l \begin{pmatrix} \lambda' \\ \mu \\ \mu' \end{pmatrix} \cdot \begin{bmatrix} \mathbf{\Lambda}_t^T & \mathbf{E}_t & \mathbf{0} \\ \mathbf{0} & \mathbf{\Omega}_t & \mathbf{I} \end{bmatrix}^T \begin{pmatrix} \frac{\partial \mathbf{n}_t}{\partial d} \\ \frac{\partial \mathbf{m}_t}{\partial d} \end{pmatrix} \cdot \delta d\, ds \end{aligned} \tag{23}$$

and finally

$$\begin{aligned} &0 = D_\lambda[L(\cdot)] \cdot \delta\lambda \\ &= \int_l \begin{pmatrix} \delta\lambda' \\ \delta\mu \\ \delta\mu' \end{pmatrix} \cdot \begin{bmatrix} \mathbf{\Lambda}_t^T & \mathbf{E}_t & \mathbf{0} \\ \mathbf{0} & \mathbf{\Omega}_t & \mathbf{I} \end{bmatrix}^T \begin{pmatrix} \mathbf{n}_t \\ \mathbf{m}_t \end{pmatrix} ds \end{aligned} \tag{24}$$

We note for illustration that if the rod diameter is chosen as the design variable we can express the result in (23) explicitly as

$$\frac{\partial \mathbf{n}_t}{\partial d} = \frac{\partial \mathbf{C}}{\partial d}(\varepsilon_t - \varepsilon_0); \quad \frac{\partial \mathbf{C}}{\partial d} = diag(E\frac{\partial A}{\partial d}, G\frac{\partial A}{\partial d}, G\frac{\partial A}{\partial d}) \tag{25}$$

$$\frac{\partial \mathbf{m}_t}{\partial d} = \frac{\partial \mathbf{D}}{\partial d}(\omega_t - \omega_0); \quad \frac{\partial \mathbf{D}}{\partial d} = diag(G\frac{\partial J}{\partial d}, E\frac{\partial I}{\partial d}, E\frac{\partial I}{\partial d}) \tag{26}$$

In order to provide a similar explicit result for directional derivative of the cost function, we consider a simple choice given as the rod volume (or its weight for a constant density),

$$J(\phi_t, d) = \int_l A\, ds \tag{27}$$

In such a case the contribution of the cost function to the Kuhn-Tucker optimality conditions can be written as

$$\begin{aligned} D_\phi J(\phi_t, d) \cdot \delta\phi &= 0 \\ D_d J(\phi_t, d) \cdot \delta d &= \int_l \frac{\partial A}{\partial d} \delta d\, ds \\ D_\lambda J(\phi_t, d) \cdot \delta\lambda &= 0 \end{aligned} \tag{28}$$

The shape optimization can also be carried out with respect to the rod axis form in the initial configuration. A reference configuration is selected in such a case (see Figure 1) and the design variable is given in terms of the position vector describing the rod initial configuration with respect to this reference configuration $\mathbf{d} \equiv \varphi_0(\xi)$. The cost function in (25) can now be described as

$$\begin{aligned} J(\phi, \mathbf{d}) &:= \int_l A\, ds \\ &= \int_{\xi_1}^{\xi_2} A\, j(\xi)\, d\xi\,; \qquad j(\xi) = \left\| \frac{\partial d(\xi)}{\partial \xi} \right\| \end{aligned} \tag{29}$$

In this case all the integrals in (20) to (25) must be recomputed with the same change of variables like the one presented in (30), and the derivatives with respect to arc-length coordinate ought to be computed by making use of the chain rule

$$\frac{d}{ds}(\cdot) = \frac{1}{j(\xi)} \frac{\partial}{\partial \xi}(\cdot) \tag{30}$$

The contribution of the cost function to the Kuhn-Tucker optimality conditions for such a choice of design variables can be written as

$$D_d\left[J(\phi_t, \mathbf{d})\right] \cdot \delta\mathbf{d} = \int_{\xi_1}^{\xi_2} A \frac{1}{j(\xi)} \mathbf{d}(\xi) \cdot \frac{\partial \mathbf{d}(\xi)}{\partial \xi} d\xi \tag{31}$$

4 Numerical Implementation

In this section we discuss several important aspects of numerical implementation of the presented theory for analysis and design and related issues which arise in numerical simulations.

The analysis part of the problem,i.e. the state variables are represented by using the standard isoparametric finite element approximations (e.g. see Zienkiewicz and Taylor [2000]). In particular, this implies that the element initial configuration is represented with respect to its parent element placed in the natural coordinate space, corresponding to a fixed interval, $-1 = \xi_1 \leq \xi \leq \xi_2 = +1$, by using

$$\varphi_0(s) \equiv \mathbf{x}(\xi) = \sum_{a=1}^{n_{en}} N_a(\xi)\, \mathbf{x}_a \tag{32}$$

where $\mathbf{x}(\xi)$ is the position vector field with respect to the reference configuration, $\mathbf{x}_a$ are nodal values of an element with n_{en} nodes and $N_a(\xi)$ are the shape functions. The latter can easily be constructed for rods by using the Lagrange polynomials, which for an element with n_{en} nodes can be written as

$$N_a(\xi) = \prod_{b=1, b\neq a}^{n_{en}-1} \frac{\xi - \xi_b}{\xi_a - \xi_b} \tag{33}$$

where ξ_a , $a \in [1, n_{en}]$ are the nodal values of natural coordinates.

With isoparametric interpolations one chooses the same shape function in order to approximate the element displacement field, which allows us to construct the finite element representation of the element deformed configuration as

$$\varphi(\xi) = \sum_{a=1}^{n_{en}} N_a(\xi)\,\varphi_a \tag{34}$$

where φ_a are the nodal values of the position vector in the deformed configuration. Finally, the incremental and virtual displacement field are also presented by isoparametric finite element interpolations

$$\begin{aligned}\delta\varphi(\xi) &= \sum_{a=1}^{n_{en}} N_a(\xi)\,\delta\varphi_a\\ \Delta\varphi(\xi) &= \sum_{a=1}^{n_{en}} N_a(\xi)\,\Delta\varphi_a\end{aligned} \tag{35}$$

which enables that a new (iterative) guess for the deformed configuration be easily obtained with the corresponding additive updates of the nodal values

$$\varphi_a \longleftarrow \varphi_a + \Delta\varphi_a$$

The finite element approximation of the incremental displacement field in (36) above, where at each point $\xi \in [\xi_1, \xi_2]$ the corresponding value is a linear combination of the nodal values are referred to as the continuum consistent (e.g. see Ibrahimbegovic [1994, 1995] for they allow to commute the finite element interpolation and the consistent linearization of nonlinear problem (e.g. Marsden and Hughes [1983]).

For the same reason we also choose the isoparametric interpolations for virtual and incremental rotation field with

$$\begin{aligned}\delta\theta(\xi) &= \sum_{a=1}^{n_{en}} N_a(\xi)\,\delta\theta_a\\ \Delta\theta(\xi) &= \sum_{a=1}^{n_{en}} N_a(\xi)\,\Delta\theta_a\end{aligned} \tag{36}$$

The commutativity of the finite element discretization and consistent linearization thus also applies to the rotational state variables. The only difference from the displacement field concerns the multiplicative updates of the rotation parameters, which, when done at nodal points (e.g. see Ibrahimbegovic and Al Mikdad [1998]) can be written as

$$\mathbf{\Lambda}_a \leftarrow \mathbf{\Lambda}_a \exp[\Delta\theta_a] \tag{37}$$

In the combined analysis and design procedure proposed herein one also needs to interpolate the Lagrange multipliers, which is also done by using the isoparametric interpolations with

$$\lambda(\xi) = \sum_{a=1}^{n_{en}} N_a(\xi)\,\lambda_a \Longleftrightarrow \begin{cases} \nu(\xi) = \sum_{a=1}^{n_{en}} N_a(\xi)\,\nu_a \\ \mu(\xi) = \sum_{a=1}^{n_{en}} N_a(\xi)\,\mu_a \end{cases} \tag{38}$$

The corresponding integrals appearing in governing Lagrangian functional in (18) or Kuhn-Tucker optimality conditions in (20), (22) and (24) are computed by numerical integration (e.g. Gauss quadrature, see Zienkiewicz and Taylor [2000]). To illustrate these ideas we further restate a single element contribution to the analysis part of the governing Lagrangian functional in (19) given in the discrete approximation by setting

$$\begin{aligned} G(\phi_a, \mathbf{d}, \lambda_a) :=& \\ & \begin{pmatrix} \lambda_a \\ \mu_a \end{pmatrix} \sum_{l=1}^{n_{ip}} \begin{bmatrix} \frac{dN_a(\xi_l)}{ds}\mathbf{I} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & N_a(\xi_l)\mathbf{I} & \frac{dN_a(\xi_l)}{ds}\mathbf{I} \end{bmatrix}^T \\ & \begin{bmatrix} \mathbf{\Lambda}_t(\xi_l) & \mathbf{E}_t(\xi_l) & \mathbf{0} \\ \mathbf{0} & \mathbf{\Omega}_t(\xi_l) & \mathbf{I} \end{bmatrix}^T \begin{pmatrix} \mathbf{n}_t(\xi_l) \\ \mathbf{m}_t(\xi_l) \end{pmatrix} \\ & j(\xi) w_l - G_{ext}(\lambda(\xi_l)) \end{aligned} \tag{39}$$

where ξ_l and w_l are the abscissas and weights of the chosen numerical integration rule (e.g. see Zienkiewicz nd Taylor [2000] and n_{ip} is the total number of points for a single element.

In order to complete the discretization procedure one must also specify the interpolations of the design variables. If the latter is the thickness or the diameter for the present case of a circular cross-section, or the element nodal coordinates, it is possible to use again the isoparametric finite element approximations. However, the best results are obtained by reducing the number of design variables as opposed to those chosen at the element level, by using the concept of design element and employing, for example, Bézier and B-spline curves for representation of rod shape (e.g. see Kegl [2000] for a detailed discussion of these ideas). What is important to note from the standpoint of the simultaneous solution procedure presented herein is that the design variable at any point is given as a linear combination of the design element interpolation parameters

$$\mathbf{d}(\xi_l) = \sum_{a=1}^{n_{dn}} B_a(\xi_l)\,\mathbf{d}_a \tag{40}$$

Consequently, the finite (or rather design) element discretization and consistent linearization will commute again. This result was already noticed earlier for linear analysis problem by Chenais and Knopf-Lenoir[1989].

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5 Conclusions

In this work we proposed an unconventional solution procedure for shape optimization problems, which solves simultaneously the analysis and design phase. Although the total system to be solved includes both the minimization of the cost function and the equilibrium equation, i.e. solving simultaneously for mechanical and design variables. In this manner we hope to achieve a more efficient resolution with respect to the classical sequential approach in the case of nonlinear problems where the analysis phase requires several iterations to converge. Moreover, the standard optimization tasks, such as sensitivity computation, is automatically integrated in the proposed procedure. The final admissible values of design and mechanics variables are obtained only at convergence. Therefore, we are implicitly targeting the applications in geometrically nonlinear elasticity, where the final result would not be dependent on deformation trajectory. In that sense, it is also possible if needed to modify the presented procedure to account for eventual instability phenomena of structures undergoing large rotations (e.g. Ibrahimbegovic and Al Mikdad [2000]).

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