State-of-the-Art, Trends, and Directions in Computational Electromagnetics

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1 Introduction

Electromagnetic devices are ubiquitous in present day technology. Indeed, electromagnetism has found and continues to find applications in a wide array of areas, encompassing both civilian and military purposes. Among the former, applications of current interest include those related to communications (e.g. transmission through optical fiber lines), to biomedical devices and health (e.g. tomography, power-line safety, etc), to circuit or magnetic storage design (electromagnetic compatibility — EMC—, hard disc operation), to geophysical prospecting, and to non-destructive evaluation (e.g. crack detection), to name but just a few. Equally notable and motivating are applications in defense which include the design of military hardware with decreased signatures ("virtual prototyping"); automatic target recognition — ATR— (e.g. bunkers, mines and buried ordnance, etc); propagation effects on communications and radar systems (e.g. over complex terrains); tactical antenna design; etc. Although the principles of electromagnetics are well understood (see §2), their application to practical configurations of current interest, such as those that arise in connection with the examples above, is significantly complicated and far beyond manual calculation in all but the simplest aspects. These complications typically arise from geometrical and/or compositional complexity in the underlying structures (e.g. circuits, military hardware, biological tissue), from the intricacies of the electromagnetic fields (especially at higher frequencies), or from both. The significant advances in computer modeling of electromagnetic interactions that have taken place over the last two decades, on the other hand, have made it possible to shift the classical "trial and error" design paradigm for electromagnetic devices to one that heavily relies on computer simulation. *Computational Electromagnetics* has thus taken on great technological importance and, largely due to this, it has become a central problem in present-day computational science.

In fact, industrial and engineering requirements have motivated advances in computational electromagnetics at a rapid pace. In particular, a number of significant breakthroughs have been attained in just the last five years. These include, for instance, the design of high-order finite-element methods (greatly improving over their finite-volume counterparts), the introduction of high-order quadrature schemes for the integral equations modeling wave-propagation (finally overcoming the difficulties associated with the singular character of the Green's functions), the proposition of "Eulerian" geometrical-optics solvers (i.e. ones that avoid the need for explicit wave-front tracking), the implementation of general-purpose codes based on Fast Multipole Algorithms (FMA) and other acceleration strategies (e.g. those based on arrays of equivalent-sources), the design of efficient and/or error-controllable hybrid algorithms (especially for high-frequency applications), etc; see §4. In light of these recent advances, and motivated by pressing Army needs, we have recently conducted a *Workshop on Recent Advances, State-of-the-Art, and Future Directions in Computational Electromagnetics* (June 27-28, 2002, Adelphi, MD), sponsored by the Army Research Laboratory (ARL) and the Army High Performance Computing Research Center (AHPCRC), and aimed at reviewing the state-of-the-art in the area. The objective of the Workshop was two-fold: on the one hand, the workshop sought to recognize immediate and long-term needs for computational electromagnetics, especially as they relate to the armed forces; and, at the same time, the meeting was geared towards identifying promising formulations, algorithms and implementations for electromagnetic simulations of interest to DOD.

To achieve the desired goals participants of the Workshop consisted of a mixture of Army laboratory (computational and experimental) researchers and managers,

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and computational scientists from academic and industrial environments. In this introductory paper, we summarize the issues that were raised, and the conclusions that were reached at the Workshop through extensive discussions within this diverse group; representative papers from a significant proportion of the speakers follow as the core of this Special Issue. As we further detail in §4, these papers cover a substantial part of the spectrum of state-of-the-art simulation methods for electromagnetics, and were contributed by some of the most recognized experts in the area. In view of this, we hope and anticipate that the present volume will become a standard reference in computational electromagnetics, and we further expect that it will be a valuable resource in attempts at identifying future trends and needs. To facilitate the reading, we begin our discussion below with a review of the basic mathematical model, and of the difficulties associated with its numerical resolution (§2). Then (§3) we also review some of the most popular and classical numerical techniques that have been used in the simulation of electromagnetic interactions, and we comment on their advantages and shortcomings. Sections 4 and 5 constitute the core of this introductory paper: in §4 we briefly discuss the state-of-the-art, particularly in relation to the papers that follow; and in §5, finally, we summarize our views on the conclusions reached at the Workshop relating to future needs and on the consequent suggestions for basic and applied research directions within the area of computational electromagnetics.

2   The model: Maxwell's equations

The central (direct) problem in computational electromagnetics relates to the prediction of electromagnetic field values in a given, typically complex, configuration of conductors and/or dielectrics. The basic model is thus provided by the (time-domain) Maxwell's equations, which read

$$
\begin{aligned}
\nabla \times \mathbf{E} &= -\frac{\partial \mathbf{B}}{\partial t} \\
\nabla \cdot \mathbf{D} &= \rho_f \\
\nabla \times \mathbf{H} &= \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t} \\
\nabla \cdot \mathbf{B} &= 0.
\end{aligned}
\tag{1}
$$

Here $\mathbf{E}$ is the electric field, $\mathbf{H}$ is the magnetic field, $\mathbf{D}$ is the electric displacement, $\mathbf{B}$ is the magnetic induction, $\mathbf{J}$

is the free current density, and ρ_f is the free charge density. These field equations are supplemented with *constitutive relations* that describe the behavior of media under the influence of electromagnetic fields. Their most general form can be rather complicated but if the material is isotropic they take on the simple form

$$
\begin{aligned}
\mathbf{D} &= \varepsilon \mathbf{E} \\
\mathbf{B} &= \mu \mathbf{H} \\
\mathbf{J} &= \sigma \mathbf{E}
\end{aligned}
\tag{2}
$$

where ε is the permittivity, μ is the magnetic permeability, and σ is the conductivity of the medium. More generally, for anisotropic materials these parameters are second rank tensors. Finally, across interfaces Γ_{ij} between two media Ω_i and Ω_j, the fields must satisfy the transmission conditions

$$
\begin{aligned}
\hat{\mathbf{n}} \cdot (\mathbf{D}_j - \mathbf{D}_i) &= \sigma_f \\
\hat{\mathbf{n}} \times (\mathbf{E}_j - \mathbf{E}_i) &= 0 \\
\hat{\mathbf{n}} \cdot (\mathbf{B}_j - \mathbf{B}_i) &= 0 \\
\hat{\mathbf{n}} \times (\mathbf{H}_j - \mathbf{H}_i) &= \mathbf{K}_f
\end{aligned}
\tag{3}
$$

where, $\hat{\mathbf{n}}$ is the unit normal to the interface, pointing from region Ω_i to region Ω_j, and σ_f, $\mathbf{K}_f$ are the surface charge density and surface current density at the interface, respectively.

Of particular interest in many applications, especially in defense, is the *scattering problem*, wherein incoming radiation interacts with a structure (the scatterer) and generates a scattered (reflected, transmitted) field. Mathematically, the problem can be posed in terms of the *total field*

$$
\begin{aligned}
\mathbf{E} &= \mathbf{E}^{\text{inc}} + \mathbf{E}^{\text{scatt}} \\
\mathbf{H} &= \mathbf{H}^{\text{inc}} + \mathbf{H}^{\text{scatt}}
\end{aligned}
\tag{4}
$$

consisting of the sum of the known incoming field ($\mathbf{E}^{\text{inc}}, \mathbf{H}^{\text{inc}}$) and the unknown scattered field ($\mathbf{E}^{\text{scatt}}, \mathbf{H}^{\text{scatt}}$), and which must satisfy equations (1). For impenetrable scatterers Ω, the conditions (3) translate into boundary conditions at their surface $\Gamma = \partial \Omega$; for instance for a perfect electric conductor (PEC) we have

$$
\mathbf{E} \times \hat{\mathbf{n}} = 0 \quad \text{on } \Gamma.
\tag{5}
$$

Finally, an additional condition must be imposed away from the scatterer to guarantee that the modeled solution coincides with the physically realizable one. Indeed, in the decomposition (4) it is natural to demand

that the scattered field be *outgoing*, that is, that it consist of waves that travel *away* from the (bounded) scatterers. Mathematically, the precise condition (the so-called Silver-Müller radiation condition) is

$$\lim_{|\mathbf{x}|\to\infty} \left| \left(\frac{\mu}{\varepsilon}\right)^{1/2} \left(\mathbf{H}^{\text{scatt}} \times \mathbf{x}\right) + |\mathbf{x}|\mathbf{E}^{\text{scatt}} \right| = 0. \tag{6}$$

Although the most general form of Maxwell's equations can entail nonlinear constitutive equations, linear relations such as those in (2) provide accurate models up to moderate powers, and are therefore often applicable and of particular interest. In this case, Maxwell's equations are themselves *linear* and, as such, they are amenable to treatment via Fourier (or Laplace) transforms in time. The (Fourier) transformed equations read

$$\begin{aligned} \nabla \times \widehat{\mathbf{E}} &= i\omega\,\widehat{\mathbf{B}} \\ \nabla \cdot \widehat{\mathbf{D}} &= \widehat{\rho_f} \\ \nabla \times \widehat{\mathbf{H}} &= \widehat{\mathbf{J}} - i\omega\,\widehat{\mathbf{D}} \\ \nabla \cdot \widehat{\mathbf{B}} &= 0 \end{aligned} \tag{7}$$

where ω denotes the frequency and $i^2 = -1$. These equations provide an alternative model for simulation (which may, in fact, be advantageous if the frequency content of the fields is not very broad).

The main difficulties related to the numerical solution of Maxwell's equations become clear in their form (7). Indeed, it follows from this that, for instance, the electric field satisfies

$$\nabla \times \left(\nabla \times \widehat{\mathbf{E}}\right) = i\omega\,\nabla \times \widehat{\mathbf{B}}.$$

Then, using (2) and (7), yields

$$-\nabla^2 \widehat{\mathbf{E}} + \nabla\left(\nabla \cdot \frac{1}{\varepsilon}\widehat{\mathbf{D}}\right) = i\omega\,\nabla \times \mu\widehat{\mathbf{H}}.$$

which, in the absence of currents and charges, delivers

$$-\nabla^2 \widehat{\mathbf{E}} - \nabla \cdot \left(\nabla\log(\varepsilon) \cdot \widehat{\mathbf{E}}\right) = \\ -(i\omega)^2 \mu\varepsilon\,\widehat{\mathbf{E}} + (\nabla\log(\mu)) \times \left(\nabla \times \widehat{\mathbf{E}}\right).$$

For homogeneous media, and letting

$$n = c\sqrt{\varepsilon\mu}$$

denote the refractive index and

$$\kappa = \frac{\omega}{c}$$

the wavenumber in free space, this last equation reduces to the vector Helmholtz equation

$$\nabla^2 \widehat{\mathbf{E}} + \kappa^2 n^2 \widehat{\mathbf{E}} = 0$$

which clearly indicates the oscillatory nature of the fields, on the scale of the wavelength

$$\lambda = \frac{2\pi}{\kappa\,n}.$$

Thus, any numerical method must, at least in principle, deal with the need to resolve these scales which, for applications of interest, may be much smaller than the scale of a scatterer. For example, a military aircraft may have characteristic lengths on the order of tens of meters which, when illuminated in the 8 to 12 GHz range (X band) with wavelengths on the order of 2 to 4 centimeters, translates into several hundred wavelengths in a linear direction, for a total (in three dimensions) on the order of several million wavelengths. With only a few degrees of freedom per wavelength, a simulation on such configurations can easily entail tens of millions of unknowns, with consequently high demands on memory and computational time to solution. Accordingly, most of the current research in CEM focuses on the need to design efficient (fast) algorithms with reduced storage requirements.

3 Numerical Methods: classical techniques, advantages and disadvantages

The continuously increasing industrial and engineering demands for sophisticated electromagnetic modeling, have made computational electromagnetics into an "industry" of its own, involving a large number of researchers in academic, government and industrial laboratories. Not surprisingly then, the number of methods, or variations thereof, is almost as large. Still, the most successful methodologies can be broadly categorized as belonging to one of the following classes:

Variational Methods (MoM, Finite Element Methods, Finite Volume Methods, etc): these are based on the *variational* formulation of Maxwell's equations (1) or (7). They lead to algorithms that present several favorable properties, including great applicability and flexibility, a natural setting for adaptivity and parallelization, sparse matrices, etc. *But*

these approaches typically require large (volumetric) computational domains and the use of approximate radiation conditions (to enforce (6) within a finite computational domain), which may lead to high computational costs and large memory requirements; in addition, low-order implementations (e.g. finite-volume) suffer from significant dispersion and dissipation errors.

Differential Equation Methods (Finite Differences, etc): these methods are based on direct discretization of the differential equations (1). Their most appealing characteristic relates to their ease of implementation. *But* they are also dispersive and costly (typically 10-20 points per wavelength, though somewhat better for MRTD implementations). Moreover they are restricted in applicability to complex geometries, especially in high-order implementations.

Integral Equation Methods (Fast Multipole Methods, Adaptive Integral Methods, FFT-based Methods, etc): these schemes are based on discretization of the integral equation formulation of (1) (or (7)). Similarly to variational approaches, they are versatile and flexible. They can also be made to be very efficient, particularly in applications involving piecewise homogeneous structures where they lead to a lower dimensional problem (posed on the interfaces separating different media). *But* they lead to full matrices and thus can only be made competitive through the use of mechanisms that accelerate the evaluation of fields (e.g. FMA). In addition, the singular character of the integrals imposes substantial challenges which typically result in low-order implementations, with a consequently large computational cost.

Asymptotic methods (ray-tracing, etc): in contrast with the methods above, these *do not* solve the full Maxwell model (1) or (7), but rather an approximation of it. Of particular interest from the point of view of applications (e.g. radar) are those that relate to the high-frequency (geometrical or physical optics) limit of Maxwell's equations. Such methods are extremely efficient since, in contrast with methods that solve the full Maxwell model, they do not involve the resolution of the fields in the scale of the wavelength of radiation. *But,* they are *asymptotic* in

nature and therefore are not error-controllable; as a result, they can give rise to significant inaccuracies for finite (but large) frequencies.

As any review of the literature will reveal, schemes from each of these classes have been very successful at resolving a variety of problems. The same review, will also discover that some methods may be better adapted to specific applications, and that no method can be considered "universally" superior. More importantly perhaps, and as we briefly described above, one will also find that all available methods have very definite limitations in spite of continuous advances in the capabilities of computational algorithms and hardware. In fact, a number of applications continue to challenge *every* approach, and some lie well-beyond today's capabilities (e.g. the rigorous prediction of scattering returns at very-high frequencies). In the next section, and within the context of summarizing the contributions that follow in this Special Issue, we briefly describe the latest algorithmic advances in the area. As we hope will be clear from this description these advances were specifically designed to extend the limits of applicability and/or the reliability of electromagnetic simulations, thus putting us closer to a resolution of the most challenging problems.

4 Numerical Methods: state-of-the-art and this special issue

A large proportion of the speakers contributed papers to this volume, covering a significant part of the spectrum of methodologies in CEM (see §3). Of the fourteen papers that follow, seven deal with variational techniques, two with finite difference schemes, three with integral equations and two with asymptotic methods. Although more pronounced than at the actual workshop where a total of twenty presentations were delivered, the disparity in these numbers does reflect the relative popularity that variational methods (especially all variants of MoM and FEM) enjoy in practice. Below is a brief description of the papers, classified according to the categories described in §3:

- Variational Methods

  The paper [Castillo, Koning, Rieben, and White (2004)] reviews a software class library ("FEMSTER") of high-order finite element basis functions that is based on the language

of differential forms. As explained there, such formulations are well suited to electromagnetics as they naturally lead to discretizations that preserve important symmetry, conservation and spectral properties. Specifically, it is shown that simulations based on these principles result, for instance, in frequency domain solutions free of spurious modes, as well as in time domain solutions that are stable, and charge and energy conserving.

Large scattering problems with an emphasis on parallel mesh generation and implementation are the subject of the work in [Hassan, Morgan, Jones, Larwood, and Weatherill (2004)]. The underlying numerical method is based on low- (first)-order finite elements which, as we mentioned, demands significant mesh refinement with increasing frequency. The main contribution here is the presentation of a computational infrastructure that not only parallelizes the solver, but also the mesh generation and visualization stages.

A new high-order accurate finite-element method for time-domain electromagnetic calculations is described in [Hesthaven and Warburton (2004)]. The method is based on a "discontinuous" formulation which, as argued there, yields a highly parallel local scheme. The high-order of convergence, on the other hand, demonstrably translates into significant reductions in memory and execution time requirements.

The work in [Jose, Kanapady, and Tamma (2004)] exemplifies the use of transform methods to reduce time-domain simulations to a series of frequency domain calculations. It is shown that the use of integral (Laplace) transforms in conjunction with a frequency-domain solver (finite-elements, in this case) can provide a viable alternative to direct time-domain calculations, especially in view of the obvious parallelizability of simulations at different frequencies.

The paper [Lee, Lee, and Teixeira (2004)] is concerned with the use of vector finite elements in modeling three-dimensional waveguide components. The emphasis is on the construction of a hierarchical class of high order basis functions, and on their use within a *p*-type Schwarz method for solving the resulting matrix equations. It is shown that

the performance of such a scheme can compare favorably with existing commercial software for electromagnetic simulations (e.g. HFSS).

A new implementation of the Method of Moments for scattering off faceted surfaces is presented in [Mittra and Prakash (2004)]. A basic property of the scheme relates to the choice of basis functions which are designed so that the size of the facets is not limited by the wavelength of radiation. It is shown that, for planar facets, these "Characteristic Basis Functions" (CBF) can be easily constructed (from successive solutions of scattering problems off infinite flat interfaces) and that, for scatterers and frequencies of interest, the resulting matrices can be several orders of magnitude smaller than those associated with classical MoM implementations.

The paper [Reddy (2004)] advocates the use of single- (AWE) and multi-point (MBPE) Padé approximation in frequency and/or angle of incidence to collect multi-spectral/multi-angular simulated data for scattering off electrically large structures. The benefits of the use of these ideas in lowering computational costs is demonstrated within implementations of the MoM, hybrid FEM/MoM and hybrid MoM/PO schemes.

- Differential Equation Methods

In [Fan, Wang, and Steinhoff (2004)] a new finite-difference treatment for problems of wave propagation is presented. A basic property of the method is that it allows for the long-time simulation of the evolution of short pulses with minimal dispersion and dissipation and without the need to fully resolve their shape. The scheme can be made to faithfully capture certain integral quantities of the pulse (e.g. amplitude, centroid, etc) and thus it could prove useful if incorporated into alternative computational procedures to propagate short pulses away from scattering surfaces.

The paper [Namburu, Mark, and Clarke (2004)] discusses a scalable environment for large-scale computational electromagnetics (or acoustics) based on a parallel implementation of a finite difference time-domain scheme. The environment is based on a data format (XDMF) that allows for the development of re-usable pre- and post-processing tools. Indeed, it is shown that, using this (eXtensi-

ble) Data Model and Format, a voxel-based scalable structured grid generator and a parallel visualization strategy that uses Network Distributed Global Memory (NDGM) can be effectively attained.

- Integral Equation Methods

The paper [Bruno (2004)] deals with the general problem of designing efficient and high-order schemes for the solution of the integral equation formulation of scattering problems in the frequency domain. It introduces novel procedures that attain high-order convergence and reduced operation counts for both surface and volumetric scattering problems.

The basic ideas behind the Multilevel Fast Multipole Algorithm (MLFMA) for the solution of the integral formulation of Maxwell's equations are presented in [Chew, Song, Cui, Velamparambil, Hastriter, and Hu (2004)]. Some large-scale examples produced by an implementation (FISC) of MLFMA are presented. Additionally, some recent advances are discussed, including an acceleration strategy based on the use of ray optics in the translation step between well-separated clusters, a parallelization scheme that reduces communication costs, and the extension of MLFMA to complex configurations that include layered media (e.g. scatterers on the ground).

The discussion in [Volakis, Sertel, Jorgensen, and Kindt (2004)] is concerned with volumetric scattering simulations. Specifically, both volume integral equations (VIE) and finite element/boundary integral (FE-BI) formulations are considered and compared. Additional topics discussed here include: parallelization of MLFMA (where it is argued that the high rate of inter-processor communication needed at the translation step remains a bottleneck for optimal performance); and higher order basis functions (where it is shown that their choice can significantly affect the conditioning properties of the resulting matrices).

- Asymptotic methods

The paper [Bleszynski, Bleszynski, and Jaroszewicz (2004)] presents an extension of the "wavefront (WF) evolution method", a scheme that provides an improvement over classical SBR

approaches to ray-tracing by concurrently evolving a family of surfaces (the "wavefronts") defined by triangular faces (whose vertices are points on the rays). Here, the WF approach is expanded to account for edge diffraction and a new procedure for evaluating currents and fields on scattering surfaces is provided. In addition, it is shown that these evaluations (including diffraction effects) could prove useful in the development of a high frequency integral equation (HFIE) approach previously introduced by O. Bruno et al.

Finally, a new Eulerian approach to geometrical optics (GO) calculations is introduced in [Cheng, Kang, Osher, Shim, and Tsai (2004)]. The method effectively deals with some of the central issues related to GO simulations, including those that concern resolution (i.e. divergent rays) and multivaluedness (i.e. caustics). The approach is based on a reformulation of the (eikonal) equation in "phase space" (with coordinates given by spatial location and local wavefront direction) and the use of the Level Set Method to resolve the resulting (Liouville) equations. The paper reviews the overall strategy and emphasizes a new, efficient implementation of reflecting boundary conditions.

5 Future directions

As we said, extensive discussions took place at the Workshop geared towards the identification of the needs in and consequent desirable directions for computational electromagnetics research, especially in connection with defense applications. A number of areas were identified where further work is needed to enable a most useful computational infrastructure with predictive capabilities. These needs encompass all aspects of CEM, including modeling and validation, algorithm development and implementations. Specifically, they include

- Modeling and Validation:

    - Complex terrains (urban, foliage, etc).

    - Atmospheric and environmental effects.

    - Target roughness (especially relevant at high-frequencies).

    - Design of appropriate "performance metrics" for simulators.

– More extensive comparison/validation with experimental data.

- Algorithms:

  – Concurrent development of time- and frequency- domain solvers.

  – Emphasis on high-order methods (to lower computational and memory requirements).

  – Development of a capability for the accurate, error-controllable and efficient simulation of scattering processes in the X through W bands.

- Implementations:

  – Efficient parallelization, adaptivity and grid generation.

  – CAD interfacing.

  – Efficient tools for:

    ∗ Pre-Processing (CAD repair, variable terrain descriptors, etc).

    ∗ Post-Processing (visualization, error metrics, etc)

This list was compiled from consideration of some of the most pressing needs of DOD in the area of CEM, as perceived by the group assembled at the Workshop. While perhaps not comprehensive, the list does include very significant challenges for computational scientists. Many of these challenges have already been taken on by the computational community, as the articles that follow will attest; others, in contrast, remain largely unexplored (e.g. the effects of target roughness at high-frequencies). In any case, significant advances in any of these will entail a substantial and concerted effort, which we hope the state-of-the-art review in the present issue will further motivate and facilitate.

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