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Application of MBPE Method to Frequency Domain Hybrid Techniques to Compute RCS of Electrically Large Objects

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Abstract: This paper presents an efficient algorithm to evaluate multi-spectral and multi-angular monostatic radar cross section (RCS) of large objects with very fine increments. The technique is based on the combination of Model Based Parameter Estimation (MBPE) method with hybrid frequency domain formulations. A general approach to formulation of MBPE is presented along with a similar approach called the Asymptotic Waveform Evaluation (AWE). Various numerical examples are presented for multi-spectral response calculations using method of moments (MoM) and the hybrid Finite Element-MoM technique in conjunction with MBPE. Example application of MBPE for hybrid MoM-Physical Optics approach for multi-angular calculations is also presented.

keyword: Model Based Parameter Estimation, MBPE, Hybrid Methods, Computational Electromagnetics (CEM), Radar Cross Section (RCS).

1 Introduction

Frequency domain techniques in electromagnetics rely on integral or differential equation approaches and have been very successful in recent years for RCS calculations of targets in low frequency to midband applications [Miller, Mitschang and Newman,(1992)]. More recently, hybrid integro-differential techniques have been developed to take advantage of both approaches and thus model materials and surfaces with greater efficiency [Volakis, Chatterjee and Kempel, (1998)]. Also more efficient methods using hierarchical vector finite elements using p-type multiplicative Schwarz method (*p*MUS) are developed for arbitrary shapes [Lee, Lee and Teixeira, (2004)]. This technique results in one order of magnitude speed up compared to the previous finite element approaches. For electrically large objects with material treatments it is necessary to hybridize first principle CEM techniques with asymptotic high frequency methods, such as Physical optics (PO) [Jakobus and Landstorfer (1995)]. Hybrid methods in frequency domain, though suitable for computing RCS of large objects require repeated calculations over a frequency band of interest. These methods require an iterative process to determine the illuminated and shadow regions for each angle of incident plane wave. Electrically large objects with sharp edges and corners exhibit large variations in RCS with nulls and peaks. Sharp nulls are normally observed within a small angular range. The CPU requirements to compute RCS at fine frequency and angular increments are normally prohibitive. In this paper, Model Based Parameter Estimation (MBPE) technique [Miller and Burke (1991)] is presented to accurately compute multi-spectral and multi-angular responses with a few direct calculations. In MBPE technique, the electric current or field is expanded as a rational function. The coefficients of the rational function are obtained using the either frequency/angular data or the related derivative data. Once the coefficients of the rational function are obtained the RCS can be computed using the rational function at any fine frequency or angle increments. A brief description of another technique called the Asymptotic Waveform Evaluation (AWE) [Pillage and Roherer (1990)] is also presented. AWE is similar to MBPE and is used for microwave circuit analysis [Tang, Nakhala and Griffith (1991)]. An approach similar to AWE is presented in [Jose, Kanapady and Tamma, 2004] using a novel hybrid finite element and Laplace transform formulation for the computation of transient electromagnetic fields.

The rest of the paper is organized as follows. In section 2, MBPE method implementation is described. A brief description of AWE technique is also presented. Numerical results to validate application of MBPE method to hybrid techniques are presented in Section 3. The numerical data are compared with the exact solution over frequency/angle range. Concluding remarks on the ad-

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vantages and disadvantages of MBPE are given in Section 4.

2 Model Based Parameter Estimation (MBPE)

Frequency domain techniques to compute RCS often result in a system matrix equation such as

$$A(k)x(k) = v(k) \tag{1}$$

Where $A(k)$ is the system matrix, $x(k)$ is the solution vector and $v(k)$ is the excitation vector due to the plane wave incident.

Solution of equation (1) at any frequency f_o gives the solution vector $x(k_o)$, where k_o is the free space wavenumber at f_o. Instead of directly solving for $x(k_o)$, it can be written as a rational function,

$$x(k) = \frac{P_L(k)}{Q_M(k)} \tag{2}$$

where

$$P_L(k) = a_o + a_1 k + a_2 k^2 + a_3 k^3 + \ldots.. + a_L k^L \tag{3}$$

$$Q_M(k) = b_o + b_1 k + b_2 k^2 + b_3 k^3 + \ldots.. + b_L k^L \tag{4}$$

b_o is set to 1 as the rational function can be divided by an arbitrary constant. The coefficients of the rational function are obtained by matching the frequency derivatives of $x(k)$. If equation (2) is differentiated t times with respect to k, the resulting equation can be written as [Miller and Burke, (1991)]

$$xQ_M = P_L$$

$$x'Q_M + xQ'_M = P'_M$$

$$x''Q_M + 2x'Q'_M + xQ''_M = P''_L$$

$$x'''Q_M + 3x''Q'_M + 3x'Q''_M + xQ'''_M = P'''_L$$

$$\vdots$$

$$\vdots$$

$$x^{(t)}Q_M + tx^{(t-1)}Q_M^{(1)} + \cdots\cdots + C_{t,t-m}x^{(m)}Q_M^{(t-m)} + \cdots \\ \cdots\cdots + xQ_M^{(t)} = P_L^{(t)}$$

where $C_{r,s} = \frac{r!}{s!(r-s)!}$ is the binomial coefficient. The system of (t+1) equations provides the information from which the rational function coefficients can be found if $t \geq L+M+1$. If the frequency derivatives are available at only one frequency f_o, the variable in the rational function can be replaced with $(k-k_o)$ i.e.,

$$x(k) = \frac{P_L(k-k_o)}{Q_M(k-k_o)} \tag{5}$$

The derivatives are evaluated at $k = k_o$. The coefficients of the rational function can be obtained from the following equations:

$$a_o = x(k_o) \tag{6}$$

$$\begin{bmatrix} 1 & \cdots & -x_o & 0 & \cdots & 0 \\ 0 & \cdots & -x_1 & -x_o & \cdots & 0 \\ 0 & \cdots & -x_2 & -x_1 & \cdots & 0 \\ 0 & \cdots & -x_3 & -x_2 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & \cdots & -x_{L+M-1} & -x_{L+M-2} & \cdots & -x_L \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ \cdots \\ a_L \\ \cdots \\ b_M \end{bmatrix} \\ = \begin{bmatrix} x_1 \\ x_2 \\ \cdots \\ x_L \\ \cdots \\ x_{L+M} \end{bmatrix} \tag{7}$$

Where $x_m = \frac{x^{(m)}}{m!}$.

If the frequency derivatives are known at more than one frequency, then the expansion about $k = k_o$ cannot be used and the system matrix to solve the rational function coefficients takes a general form [Miller and Burke, (1991)]. For the sake of simplicity, let us examine a two-frequency model. Assume that at two frequencies, f_1 (with free space wavenumber k_1) and f_2 (with free space wave number k_2), four derivatives are evaluated at each frequency. Hence 10 samples of data are needed (two frequency samples and a total of eight derivative samples) to form a rational function with L=5 and M=4.

$$x(k) = \frac{a_o + a_1 k + a_2 k^2 + a_3 k^3 + a_4 k^4 + a_5 k^5}{1 + b_1 k + b_2 k^2 + b_3 k^3 + b_4 k^4} \tag{8}$$

Equation (8) can be written as

$$\left(1 + b_1 k + b_2 k^2 + b_3 k^3 + b_4 k^4\right) x(k) \\ = a_o + a_1 k + a_2 k^2 + a_3 k^3 + a_4 k^4 + a_5 k^5 \tag{9}$$

Differentiating equation (9) four times at each frequency, the matrix equation for the solution of the coefficients of the rational function (equation (9)) can be written as

$$\begin{bmatrix} M_{11} & M_{12} & \cdots & \cdots & M_{19} & M_{110} \\ M_{21} & M_{22} & \cdots & \cdots & M_{29} & M_{210} \\ M_{31} & M_{32} & \cdots & \cdots & M_{39} & M_{310} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ M_{101} & M_{102} & \cdots & \cdots & M_{109} & M_{1010} \end{bmatrix} \begin{bmatrix} a_o \\ a_1 \\ a_2 \\ \cdots \\ b_3 \\ b_4 \end{bmatrix} = \begin{bmatrix} x_1^{(0)} \\ x_1^{(1)} \\ x_1^{(2)} \\ \\ x_2^{(3)} \\ x_2^{(4)} \end{bmatrix} \tag{10}$$

where $x_1^{(m)} = \frac{d^m}{dk^m} x(k)|_{k=k_1}, x_2^{(m)} = \frac{d^m}{dk^m} x(k)|_{k=k_2}$

Matrix elements (M_{11}, M_{12} etc) are given in [Reddy (1998a)].

In the above equations, $x^{(t)}$, the tth derivative is obtained using the recursive relationship,

$$x^{(t)} = A^{-1}(k)\left[v^{(t)} - \sum_{q=0}^{t}(1-\delta_{qo})C_{t,q}A^{(q)}(k)x^{(t-q)}(k)\right] \tag{11}$$

where $A^{(q)}(k)$ is the qth derivative with respect to k of $A(k)$ and $v^{(t)}(k)$ is the tth derivative with respect to k of v(k). The Kronecker delta δ_{qo} is defined as

$$\delta_{qo} = \begin{cases} 1 & q = 0 \\ 0 & q \neq 0 \end{cases}$$

The above procedure can be generalized for multiple frequencies with frequency-derivatives evaluated at each frequency to increase the accuracy of the rational function. Alternatively, the two-frequency-four-derivative model can be used with multiple frequency windows.

2.1 Asymptotic Waveform Evaluation (AWE)

AWE is similar to MBPE and was used for the timing analysis of very large scale integration (VLSI) circuits [Pillage and Roherer (1990)]. The AWE technique is also applied to the electromagnetic analysis of microwave circuits [Gong and Volakis (1996), Erdemli, Reddy and Volakis (1999)]. Like MBPE, AWE also results in a rational function approximation. In the AWE technique the electric field or current is expanded in the Taylor series around a frequency. The coefficients of the Taylor series (called "moments") are evaluated using the frequency derivatives of equation (1). Taylor series approximation gives fairly good results. However, the radius of convergence limits the accuracy of the Taylor series and will not converge beyond the radius of convergence. The rational function approach is used to improve the accuracy of the numerical solution. The coefficients of the Taylor series are matched via Padè approximation to a rational function.

To implement AWE, solution vector in equation (1) is expanded in Taylor series as

$$x(k) = \sum_{n=0}^{\infty} m_n(k-k_o)^n \tag{12}$$

With the moments given by

$$m_n = A^{-1}(k_o)\left[\frac{v^{(n)}(k_o)}{n!} - \sum_{q=0}^{n}\frac{(1-\delta_{qo})A^{(q)}(k_o)m_{n-q}}{q!}\right] \tag{13}$$

To obtain Padè approximation, the Taylor series expansion in equation (12) is matched with a rational function

$$\sum_{n=0}^{\infty} m_n(k-k_o)^n = \frac{P_L(k-k_o)}{Q_M(k-k_o)} \tag{14}$$

Since there are $(L+M+1)$ unknowns, $(L+M)$ moments of the Taylor series should be matched. Equating the coefficients for powers $(k-k_o)^{L+1}\ldots\ldots(k-k_o)^{L+M}$, the coefficients of $Q_M(k-k_o)$ can be obtained by solving the matrix equation

$$\begin{bmatrix} m_{L-M+1} & m_{L-M+2} & \cdots & m_L \\ m_{L-M+2} & m_{L-M+3} & \cdots & m_{L+1} \\ \cdots & \cdots & \cdots & \cdots \\ m_L & m_{L+1} & \cdots & m_{L+M-1} \end{bmatrix} \begin{bmatrix} b_M \\ b_{M-1} \\ \cdots \\ b_1 \end{bmatrix} = \begin{bmatrix} m_{L+1} \\ m_{L+2} \\ \cdots \\ m_{L+M} \end{bmatrix} \tag{15}$$

The numerator coefficients can be found by equating the powers $(k-k_o)^0 \ldots\ldots (k-k_o)^L$

$$a_o = m_o$$

$$a_1 = m_1 + b_1 m_o$$

$$a_2 = m_2 + b_1 m_1 + b_2 m_o$$

$$\ldots\ldots\ldots\ldots$$

$$a_L = m_L + \sum_{i=1}^{\min(L.M)} b_i m_{L-i}$$

For a single frequency calculation, the computational effort to construct an AWE model or MBPE model is identical. AWE may be limited to single frequency calculations due to the Taylor series, where as MBPE could be used over many frequencies to construct a rational function spanning a wide frequency band.

3 Numerical Results

MBPE technique as described in the above section is applied to integral equation method such as MoM, hybrid method such as FEM/MoM for RCS calculations over a frequency range. MBPE is also applied to hybrid MoM/PO technique to compute monostatic RCS over a range of incident angles.

3.1 Application of MBPE to MoM

RCS of perfectly conducting three-dimensional objects is calculated using MoM. This method leads to dense, complex matrix system of equations. To compute RCS over a frequency range the system matrix is solved at each frequency, leading to large CPU times if fine frequency increments are needed. To over come this limitation, MBPE is applied to MoM formulation [Reddy (1998b)] to compute RCS over a frequency range with fine increments.

First example is a square plate (1cmX1cm) with the incident electric field, E_ϕ at θ_i=90° and ϕ_i=0°. The frequency response is calculated with one-frequency MBPE ($L = 5$ and $M = 4$) at 30GHz and using nine frequency derivatives. The frequency response is also calculated with two-frequency MBPE ($L = 5, M = 4$), at f_1=24GHz and f_2=36GHz and using four frequency derivatives at each frequency. Figure 1 shows the frequency response along with the discrete calculations with MoM. It can be seen that both one-frequency and two-frequency MBPE agree well with the discrete calculations with MoM. Discrete calculations took 22,258 secs to compute 31 points, whereas one-frequency MBPE tool 1688 secs and two frequency MBPE took 3060 secs. Both one-frequency and two-frequency MBPE calculations are done with 0.1 GHz increment, a total of 300 frequency points.

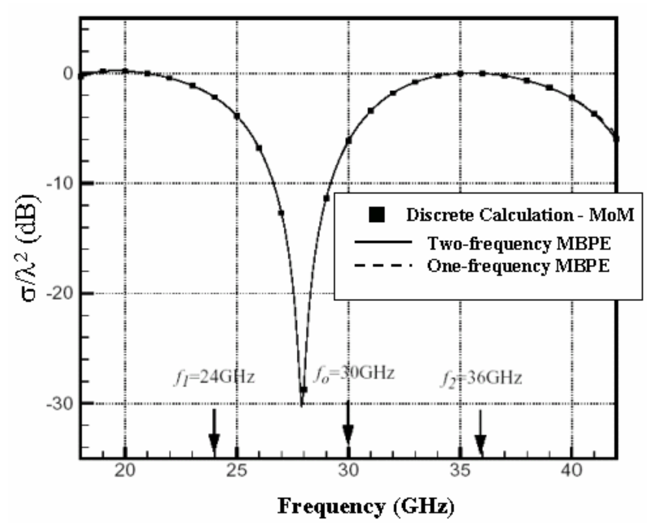


Figure 1 : RCS calculation of a square plate over a frequency range using MBPE.

The second example is a perfectly conducting cube (1cmX1cmX1cm). Normal incidence is assumed. RCS calculations over a frequency range of 2GHz to 22GHz are calculated and shown in Figure 2. One-frequency MBPE calculation is done at 15GHz, whereas the two-frequency MBPE calculations are done at 11GHz and 19GHz. The one-frequency MBPE took 1,143secs, whereas the two-frequency MBPE solution took 2,066secs. Both MBPE calculations are done with 0.1GHz increments (200 frequency points). The discrete calculations are done at 21 frequencies and took 10,500secs of CPU time.

A more challenging problem, an ogive structure (10" in length and 1" in diameter) with plane wave incident at the tip is addressed [Reddy, Cockrell, Beck, Bindignavale, and Sancer (1999)]. The VV-polarized backscatter is computed using MBPE in conjunction with MoM and is plotted in Figure 3. As expected the backscatter shows deep nulls over the frequency band. To exactly locate the nulls, MoM calculations need to be done at very fine

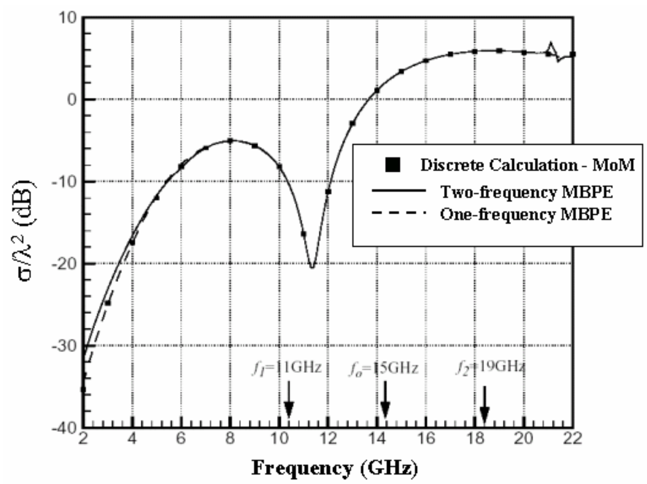


Figure 2 : RCS calculation of a cube over a frequency range using MBPE.

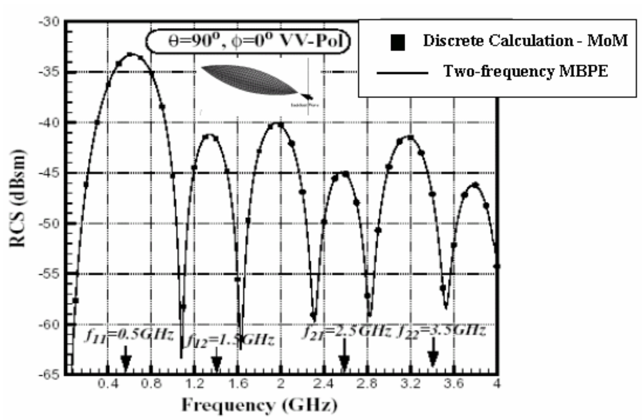


Figure 3 : RCS calculations of an ogive over a frequency range.

increments (0.01GHz at the minimum). Using MBPE, frequency sweep is accomplished with fine increments. MBPE calculations are performed in two windows, using two-frequency MBPE model. For the first window, MBPE coefficients are computed at 0.5GHz and 1.5GHz and for the second window, the coefficients are calculated at 2.5GHz and 3.5GHz. Excellent agreement with discrete calculations can be observed in Figure 3. Discrete calculations at 40 frequency points took approximately 20hrs of CPU time, whereas MBPE calculation for both windows took only 3hrs for a total of 400 frequency points.

3.2 Application of MBPE to Hybrid FEM/MoM technique

Electromagnetic characterization of cavity-backed apertures is of importance in understanding the scattering properties and in electromagnetic penetration/coupling studies. Hybrid FEM/MoM technique is widely used for computing the electromagnetic scattering characteristics of cavity-backed apertures [Reddy, Deshpande, Cockrell, and Beck (1995)]. FEM is used in the cavity volume to compute the electric field, whereas MoM is used to compute the magnetic current at the aperture. This method results in a partly sparse and partly dense complex matrix system of equations. Instead of computing RCS at each frequency, MBPE is used to accelerate RCS calculations with fine frequency increments.

As an example, a square cavity in an infinite ground plane is considered (Cavity depth is 2cm and aperture size is 1cmX1cm). Backscattering calculations are done for normal incidence of the plane wave. Figure 4 shows RCS of the cavity over the frequency range calculated using the Taylor series (equation (12)) for E_θ polarized incident wave. Taylor series moments are calculated at 20GHz. Figure 5 shows RCS calculations using one-frequency MBPE (or AWE with Padè approximation). It can be seen from Figure 4 that Taylor series expansion gives good results over 18GHz to 22GHz. Beyond this frequency range, there is no improvement in accuracy, even by adding more terms to the Taylor series. However, Figure 5 indicates that MBPE (or AWE) with rational function approximation gave good results over the frequency range 15GHz to 25GHz with L=5 and M=5, and good convergence is observed as the orders of numerator and denominator polynomials increase.

3.3 Application of MBPE to Hybrid MoM/PO technique

Despite of the innovative fast algorithms for integral equation [Chew, Jin, Michielssen, and Song (2001)] and differential equation techniques, characterizing electrically large objects requires hybridization with high frequency techniques such as Physical Optics (PO) [Jakobus and Landstorfer (1995)]. Efficient implementation of hybrid MoM/PO technique requires an iterative process to compute the RCS of electrically large objects. Despite the hybridization, multi angular calculations for monostatic RCS are prohibitive due to excessive CPU requirements. MBPE is applied to the hybrid MoM/PO tech-

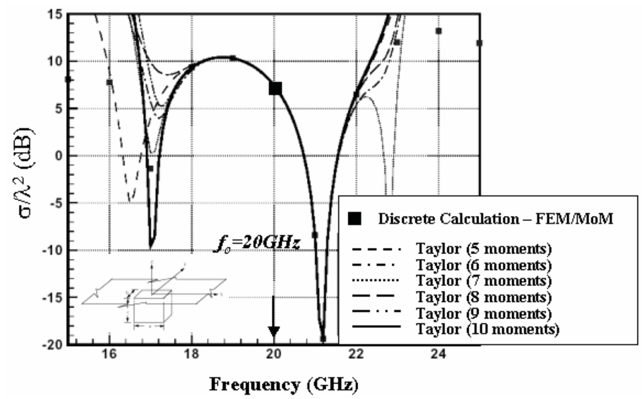


Figure 4 : RCS calculations of an air-filled square cavity-backed aperture using Taylor series approximation applied to hybrid FEM/MoM technique.

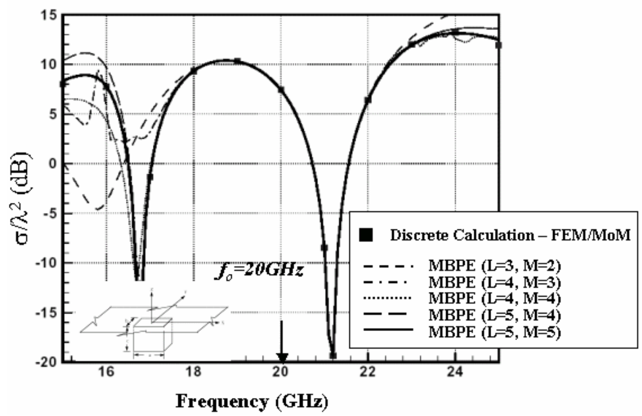


Figure 5 : RCS calculations of an air-filled square cavity-backed aperture using one-frequency MBPE (AWE) approximation applied to hybrid FEM/MoM technique.

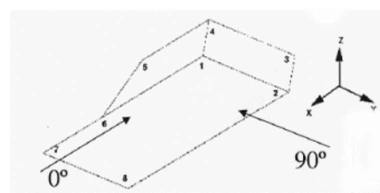
nique for fast multi-angular monostatic RCS calculations.

A rational function of polynomials is constructed as function of angle.

$$x(\phi) = \frac{P_L(\phi)}{Q_M(\phi)} \tag{16}$$

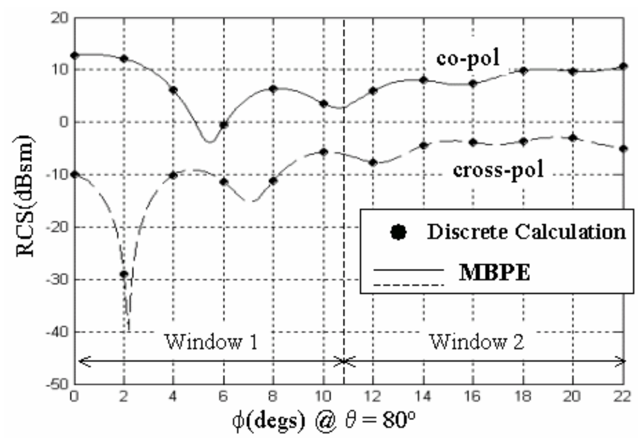
where $P_L(\phi)$ and $Q_M(\phi)$ are polynomial functions of order L and M, respectively. MBPE calculations are carried out for a trihedral geometry (Figure 6, EMCC benchmark [Greenwood (2001)]) at 3GHz with E_θ incidence at θ=80° (10° elevation above grazing).

The MBPE representation was used from ϕ=0° to 22°

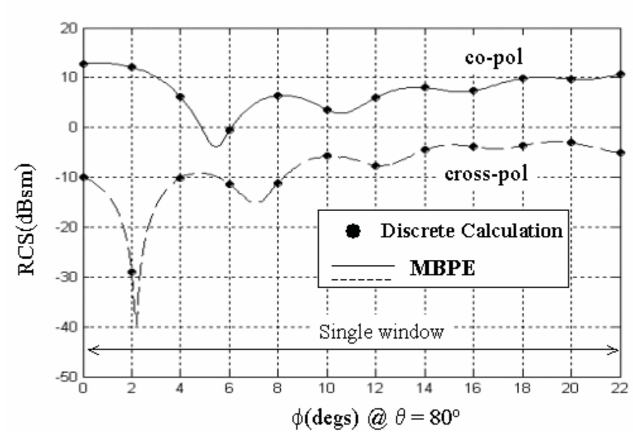


Rectangular Coordinates (x, y, z) in meters	Vertex Number
(0, 0, 0)	1
(0, 0.6096, 0)	2
(-0.0544574, 0.6096, 0.2032)	3
(-0.0544574, 0, 0.2032)	4
(0.515315, 0, 0.2032)	5
(0.867258, 0, 0)	6
(1.36794, 0, 0)	7
(1.36794, 0.6096, 0)	8

Figure 6 : Trihedron geometry



(a)



(b)

Figure 7 : Monostatic RCS calculations of the trihedron geometry using MBPE applied to hybrid MoM/PO technique.

(Figure 7) using discrete calculations at 1° increments. In Figure 7(a) a model with L=5 and M=4 was applied in the 0° to 11° window (window 1) and another model with L=5 and M=4 was applied in the 11° to 22° window (window 2) using 10 discrete solution vectors for each window. Also a model with L=10 and M=9 between 0° to 22° was constructed (Figure 6b). It can be seen that both models produce identical results. With the MBPE model, the monostatic RCS is computed at 0.1° increments and by using L=5 and M=4 only 200secs was required to compute the RCS values at all angles. Similarly, using L=10 and M=9 it took only 221secs. However, direct calculations (using hybrid MoM/PO technique) at 0.1° intervals require 2,382hours on a single processor, that is the MBPE allows in this case for a CPU reduction by a factor of 4000 (3 orders of magnitude). It can also be noted that discrete calculation gives the first null for cross polarization (Figure 7b) at 2°. However with MBPE, the first null can be observed at 2.201°, i.e. more accurately. Further, once the MBPE model is constructed, the RCS values at even finer increments (0.01° for example) could be computed with very little CPU time consumption.

Figure 8 shows monostatic RCS calculations for the trihedron geometry for 0° to 90° azimuthal variation at θ=80° (10° depression angle). The discrete calculations were performed at 2° interval, whereas MBPE calculations were done at 0.1° increments.

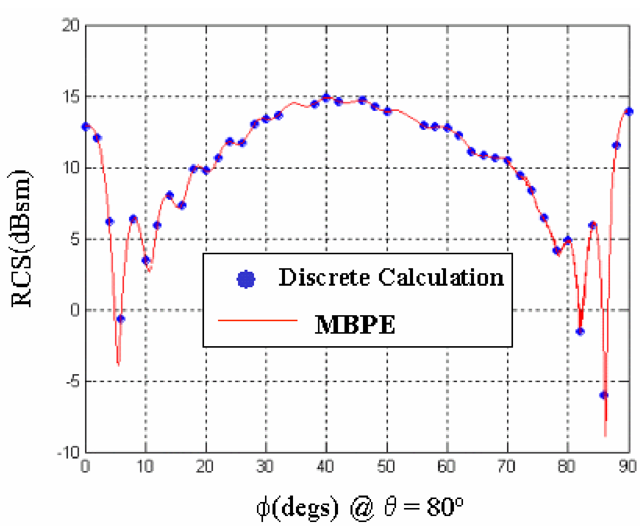


Figure 8 : Monostatic RCS calculations of the trihedron geometry using MBPE at 3GHz.

4 Conclusions

The MBPE technique is applied to frequency domain techniques, with an emphasis on hybrid methods for RCS calculations. Multi-spectral and multi-angular responses are computed using MBPE and compared with discrete calculations. From the numerical examples presented in this paper, the MBPE technique is found essential for efficient multi-spectral and multi-angular calculations. Basically, once the MBPE model is constructed, calculations at finer increments can be calculated with a very minimal cost. To be accurate a reliable error criterion should be developed, which can be used to sample the discrete points to apply MBPE model. Development of such a sampling criterion will make MBPE a very powerful tool for computational electromagnetics.

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