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Direct and Indirect Approach of a Desingularized Boundary Element Formulation for Acoustical Problems

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Abstract: In standard boundary element formulations, singular integrals need to be solved as soon as the considered sources coincide with the collocation points at the boundary. Using a desingularized boundary element approach, the sources are distributed on a surface outside the acoustic domain which means that they are never located at the boundary. Consequently, all the resulting kernels are nonsingular which reduces the complexity of the numerical treatment of the boundary integral equations considerably. In the current contribution a desingularized formulation is given for both, the direct and the indirect boundary element method used to solve acoustical problems. Three basic examples are shown which demonstrate how the new approach can be used and which accuracy can be expected.

keyword: Desingularized Boundary Element Method, Singular Integral, Numerical Acoustics

1 Introduction

The Boundary Element Method (BEM) is one of the most popular methods used in numerical acoustics. Employing a so-called "fundamental solution" [von Estorff (2000)], it allows the representation of a harmonic acoustical problem, usually described by the *Helmholtz* equation, by means of boundary integrals. The acoustic fundamental solution describes analytically the pressure distribution in a fullspace due to a unique sound source. In general, it can be handled conveniently for most parts of the considered domain. However, as soon as the observation point (where the pressure is calculated) and the source point are identical, the solution becomes singular and a special treatment of the resulting singular integrals is needed, which usually leads to rather expensive numerical calculations. More details of the problem can be found in the references [von Estorff (2000), Chen and Liu (1999), Koopmann and Cunefare (1988), Beskos (1997), Sladek, V. and Sladek, J. (1998)].

In the literature, several approaches have been presented which try to reduce the order of the singularity before hand. An interesting approach, for instance, has been proposed by Qian, Han and Atluri (2004), where the gradients of the fundamental solution are used as vector test functions in order to obtain the weak form of the original Helmholtz equation for potential. In this way, the third order singularity of the boundary integral equation is reduced to a second order singularity. Other approaches handle the singularities by special numerical integrations. Thus Agnantiaris and Polyzos (2003) propose a method which allows to perform a rather accurate numerical integration of nearly singular integrals. Particularly, it has been shown that an accurate solution can be obtained using a special way of selecting Gaussian integration points. The number of Gaussian points depends on the minimum distance between the field points and the boundary element in which the integration is performed in relation to the biggest edge of an element.

A very promising way to avoid singularities can be seen in so-called „desingularized formulations" of the BEM, where the sources are placed in such a way that they cannot coincide with the observation points. For the well known Laplace equation, various investigations are available. For steady flow about arbitrary three dimensional bodies, Webster (1975) has examined a finite element method where triangular patches of sources are "submerged" inside the body surface. He concludes that the "submergence" of the triangular singularity patches greatly improves the accuracy as long as the "submergence" is not too far. Schultz and Hong (1988) have employed the method for the two dimensional flow and mainly discuss numerical convergence. The three dimensional case has been examined by Cao, Schultz and Beck

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(1991). In this work, the flow due to a resting dipole and due to a moving dipole source underneath a free surface has been investigated, respectively. Also the numerical aspects of the resulting sets of equations have been discussed. In particular a desingularization distance of $L_d = l_d \cdot D_m^{\alpha}$ has been proposed where l_d is a parameter determining how far the integral is desingularized. D_m denotes the local mesh size and α is a chosen parameter. For a modified Gram Schmidt procedure, α must be in the range $0<\alpha<1$. It was found that in terms of computational effort the indirect method is significantly more efficient than the direct approach. Mahrenholtz and Markiewicz (1999) show that the desingularized Euler-Lagrange approach is a robust method well suited for nonlinear water wave problems as well for deep as for shallow water. Ochmann (2000) presented a multipole method by using spherical wave functions. It has been employed for acoustical radiation and scattering problems and a weighted residual technique has been used to minimize the error.

With respect to the investigations of the authors mentioned above, the current study applies a desingularized boundary element formulation to acoustical problems governed by the Helmholtz equation. It is based on a positioning of the source points outside of the considered acoustic fluid and its boundary. For the direct BEM the sought boundary values are evaluated by solving a set of linear equations with square dimensions. For the indirect BE approach the respective source properties are evaluated such that the quantities at the boundaries are satisfied. Furthermore, a case study where numerical aspects are discussed has been performed.

2 Desingularized Boundary Element Formulation

The distribution of the acoustic pressure p in an acoustic fluid (domain) V, which is bounded by the boundary Γ can be described by the *Helmholtz* equation

$$\nabla^2 p + k^2 p = 0, \tag{1}$$

where $k = \omega/c$ is the wave number. The sound wave velocity is denoted by c and the angular frequency by ω. Considering the general case of a mixed boundary value problem, the boundary conditions for the solution of equation (1) can be written as

$$p = \overline{p} \quad \text{on} \quad \Gamma_1 \quad \text{and} \quad \frac{\partial p}{\partial n} = -i\rho\omega\,\overline{v}_n \quad \text{on} \quad \Gamma_2. \tag{2}$$

The normal vector of the boundary Γ, is denoted by n and "-" marks the known boundary values on Γ_1 and Γ_2, respectively.

The boundary integral equation of the **Direct BEM formulation (DBEM)** can be derived from equation (1) by using the *Green*'s identity [von Estorff (2000)]. One obtains

$$\int\int_{\Gamma} \left[p(Y)\,\frac{\partial G(X,Y)}{\partial n} - G\,(X,Y)\,\frac{\partial p(Y)}{\partial n} \right] d\Gamma = \int\int\int_{V} p(Y)\,\delta\,(X,Y)\,dV, \tag{3}$$

where $G(X,Y)$ is the 3D fundamental solution, given by

$$G(X,Y) = \frac{e^{-ikr}}{4\pi r} \quad \text{with} \quad r = \|X-Y\|. \tag{4}$$

If the source point Y is placed outside the domain V and the computation point X is located on the boundary Γ the volume integral in (3) vanishes due to the definition of the *Dirac* function

$$\delta(X,Y) = 0 \text{ if } X \neq Y \text{ and } \delta(X,Y) = \infty \text{ if } X = Y. \tag{5}$$

Substituting the boundary conditions (2) and the fundamental solution (4) into (3), one obtains

$$\begin{aligned}\int\int_{\Gamma_1} &\left[p(Y)\,\frac{\partial}{\partial n}\left(\frac{e^{-ik\|X-Y\|}}{4\pi\,\|X-Y\|}\right) - \frac{e^{-ik\|X-Y\|}}{4\pi\,\|X-Y\|}\,\frac{\partial p(Y)}{\partial n} \right] d\Gamma_1 \\ +\int\int_{\Gamma_2} &\left[p(Y)\,\frac{\partial}{\partial n}\left(\frac{e^{-ik\|X-Y\|}}{4\pi\,\|X-Y\|}\right) - \frac{e^{-ik\|X-Y\|}}{4\pi\,\|X-Y\|}\,\frac{\partial p(Y)}{\partial n} \right] d\Gamma_2 = 0.\end{aligned} \tag{6}$$

The kernels in equation (6) are nonsingular since X and Y never coincide. In order to find the solution of a given problem, the integral equation (6) must be solved for $\partial p/\partial n$ on Γ_1 and for p on Γ_2.

Once the boundary values are known, the pressure at any location X_D inside the domain D can be determined by using

$$4\pi p\left(X_D\right)=\int\!\!\int_{\Gamma_1}\left[p\left(Y\right)\frac{\partial}{\partial n}\left(\frac{e^{-ik\|X_D-Y\|}}{\|X_D-Y\|}\right)\right.$$
$$\left.-\frac{e^{-ik\|X_D-Y\|}}{\|X_D-Y\|}\frac{\partial p\left(Y\right)}{\partial n}\right]d\Gamma_1$$
$$+\int\!\!\int_{\Gamma_2}\left[p\left(Y\right)\frac{\partial}{\partial n}\left(\frac{e^{-ik\|X_D-Y\|}}{\|X_D-Y\|}\right)\right.$$
$$\left.-\frac{e^{-ik\|X_D-Y\|}}{\|X_D-Y\|}\frac{\partial p\left(Y\right)}{\partial n}\right]d\Gamma_2. \tag{7}$$

By discretizing the surface Γ by means of boundary elements, the following expression is obtained from equation (6)

$$\sum_{m=1}^{m_{\max}}\sum_{\alpha=1}^{\alpha_{\max}}\frac{A_m}{\alpha_{\max}}\cdot p_{m\alpha}\frac{\partial}{\partial n_{m\alpha}}\left(\frac{e^{-ik\left\|X_{m\alpha}-Y_j\right\|}}{4\pi\left\|X_{m\alpha}-Y_j\right\|}\right)$$
$$=\sum_{m=1}^{m_{\max}}\sum_{\alpha=1}^{\alpha_{\max}}\frac{A_m}{\alpha_{\max}}\cdot\frac{\partial p_{m\alpha}}{\partial n_{m\alpha}}\left(\frac{e^{-ik\left\|X_{m\alpha}-Y_j\right\|}}{4\pi\left\|X_{m\alpha}-Y_j\right\|}\right). \tag{8}$$

Where index m is the corresponding element number and index α denotes the four corner nodes of the quadrilateral elements employed here. Therefore the subscript $m\alpha$ refers to each corner node of element m. The summation over each element in equation (8) leads to a simple Newton Cotes quadrature. For each source point Y_j equation (8) is obtained. In order to derive an appropriate set of linear equations, the number of source locations must be equal to the number of nodes at the surface.

The **Indirect BEM formulation (IBEM)** introduced next, is based on an integration of a source distribution over a surface Ω, which is defined outside the acoustic domain D (see Figure 1)

$$p\left(X\right)=\int\!\!\int_{\Omega}\sigma\left(Y\right)\,G\left(X,Y\right)\,d\Omega, \tag{9}$$

where $\sigma(Y)$ denotes the unknown strengths of simple sources on Ω. In contrast to the direct method, where only physical quantities are deployed, the indirect method uses the source strengths $\sigma(Y)$ as useful auxiliary variable [Brebbia and Butterfield (1978)]. It should be pointed out that the integration of equation (9) has to be performed with respect to surface Ω, where the sources

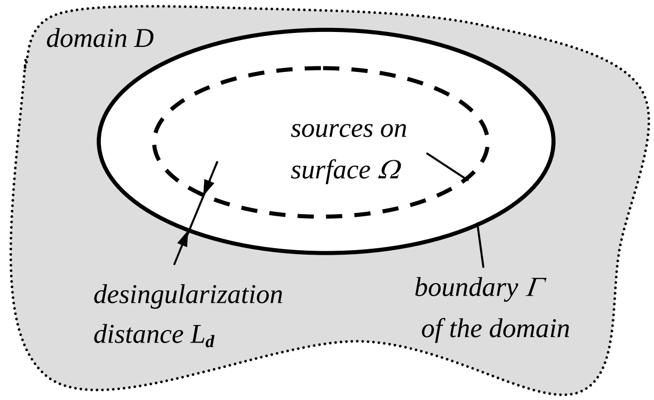

Figure 1 : Definition of an infinite acoustic domain D with sources located on the surface Ω.

$\sigma(Y)$ are distributed, while the integration of equation (6) needs to be conducted with respect to the boundary Γ [Cao, Schultz and Beck (1991)]. Taking into account the general boundary conditions (2) and the fundamental solution (4), the $\sigma(Y)$ values can be calculated using

$$p\left(X\right)=\int\!\!\int_{\Omega}\sigma\left(Y\right)\,\frac{e^{-ik\|X-Y\|}}{4\pi\|X-Y\|}\,d\Omega\quad\text{on}\quad\Gamma_1, \tag{10}$$

$$\frac{\partial p\left(X\right)}{\partial n}=\int\!\!\int_{\Omega}\sigma\left(Y\right)\,\frac{\partial}{\partial n}\left(\frac{e^{-ik\|X-Y\|}}{4\pi\|X-Y\|}\right)d\Omega\quad\text{on}\quad\Gamma_2. \tag{11}$$

By distributing collocation points on the surface Γ and using an array of isolated sources on the surface Ω, equations (10) and (11) are reduced to a simple summation over the influences of each source

$$\sum_{j=1}^{N_1}\sigma_j^{(1)}\frac{e^{-ik\left\|X_{\Gamma i}-Y_j\right\|}}{\left\|X_{\Gamma i}-Y_j\right\|}+\sum_{j=1}^{N_2}\sigma_j^{(2)}\frac{e^{-ik\left\|X_{\Gamma i}-Y_j\right\|}}{\left\|X_{\Gamma i}-Y_j\right\|}$$
$$=4\pi\cdot p(X_{\Gamma i}^{(1)}), \tag{12}$$

$$\sum_{j=1}^{N_1}\sigma_j^{(1)}\frac{\partial}{\partial n_i}\frac{e^{-ik\left\|X_{\Gamma i}-Y_j\right\|}}{\left\|X_{\Gamma i}-Y_j\right\|}+\sum_{j=1}^{N_2}\sigma_j^{(2)}\frac{\partial}{\partial n_i}\frac{e^{-ik\left\|X_{\Gamma i}-Y_j\right\|}}{\left\|X_{\Gamma i}-Y_j\right\|}$$
$$=4\pi\cdot\frac{\partial p(X_{\Gamma i}^{(2)})}{\partial n_i}. \tag{13}$$

To obtain the unknown source strengths $\sigma_j^{(1,2)}$, the corresponding matrix equation is assembled. The superscript denotes the boundary condition at the associated surface points $X_{\Gamma i}$. After employing equations (12) and (13), the system of linear equations takes the form

$$\begin{pmatrix} A^{(1,1)} & A^{(1,2)} \\ A^{(2,1)} & A^{(2,2)} \end{pmatrix} \cdot \begin{pmatrix} \sigma^{(1)} \\ \sigma^{(2)} \end{pmatrix} = \begin{pmatrix} b^{(1)} \\ b^{(2)} \end{pmatrix}, \tag{14}$$

where the right hand side consists of the known boundary values and the unknown values are the source strengths. Depending on the chosen number of sources, the corresponding matrix **A** is not necessarily quadratic. Once the strengths $\sigma(Y)$ of the sources are determined, the acoustic pressure $p(X_D)$ can be obtained from (9) for any location of $X_D \in D$. Whereby the summation form of equation (9) is given by equation (12).

Using QR-decomposition, the sets of equations obtained from the direct and the indirect method represented by equations (8) and (14) respectively, are solved. The accompanied orthogonal transformations of the system matrix lead to a numerically rather stable solution process and also permit to solve over determined systems of linear equations which may be obtained from the indirect method if the number of sources is less than the number of nodes on the surface. The computational cost for QR-decomposition amounts to $(u^2 \cdot v)$ operations, where the parameter u denotes the number of unknowns and the parameter v refers to the number of equations available. Thus, the computational effort for the solution process of the source strengths for the indirect method depends quadratically, on the number of sources used.

From the literature dedicated to the desingularized integral methods for the Laplace equation [Mahrenholtz and Markiewicz (1999), Cao, Schultz and Beck (1991)], it is known, that the distance between the surface Ω of the sources and the domain boundary Γ, the desingularization distance L_d (see Figure (1)), has a significant influence on the accuracy of the derived desingularized integral equations. Therefore, the proper location of Ω is investigated next solving three numerical examples.

3 Numerical Applications

The performance of the desingularized BEM for acoustical problems will be discussed by means of three general examples including sound radiation into an unbounded domain. In the first example a "breathing sphere" with a radius of 1 m is investigated by applying the indirect and the direct BE formulation. In the second and third examples, only the indirect method has been considered. They deal with a cube, whose edges are 2 m long. For the second example an eccentric monopole source is placed halfway between the center and the top face of the cube. The results from this calculation are used for an according Neumann boundary condition at the surface of the cube. For the third example the Neumann boundary condition is set to zero for all nodes except for one node located in the center of the top face. Thus, the third example deals with a boundary condition of interest for the calculation of Acoustic Transfer Vectors (ATV) as described by Tournour, Cremers, Guisset, Augusztinovicz, Márki (2000) and Zaleski, Cremers, von Estorff (2001).

The desingularized BEM is applied in a frequency range from 10 to 550 Hz. If not deviantly stated, the number of nodal points at the surface is 2402 for all numerical examples. The desingularization distance L_d is varied in a range from 0.05 to 0.95 m and it is assumed that the surfaces of the sources build a sphere and a cube, respectively.

To generate reference solutions, analytical approaches are used for the first two examples. For the third example, the commercial software SYSNOISE has been employed. The reference solutions are used to determine the relative error, which helps to judge about the accuracy of the desingularized BE approach. It is defined by

$$Error_{rel} = \frac{1}{\#\, of\, nodes} \sum_{i=1}^{\#of\,nodes} \frac{|p_{de\,singularized.}(node_i) - p_{exact}(node_i)|}{|p_{exact}(node_i)|}. \tag{15}$$

Figures (2), (3), (4) and (6) depict the relative error over frequency and desingulization distance L_d. In the first case, Figure (2), the direct method was employed. In contrast to the direct method, where the number of source locations needs to be identical to the number of nodes on the surface, Figure (3) and (4) show the relative error for the indirect method. While Figure (3) deals with the same number of equations as in the case of the direct method, Figure (4) shows the relative error for a significantly reduced number of sources leading to an over-determined set of equations. Thereby, the computational effort has been reduced by a factor of four.

From Figure (2) and Figure (3) it is observed, that the relative errors for the direct and the indirect formulation show a similar behavior. However, the error level is ap-

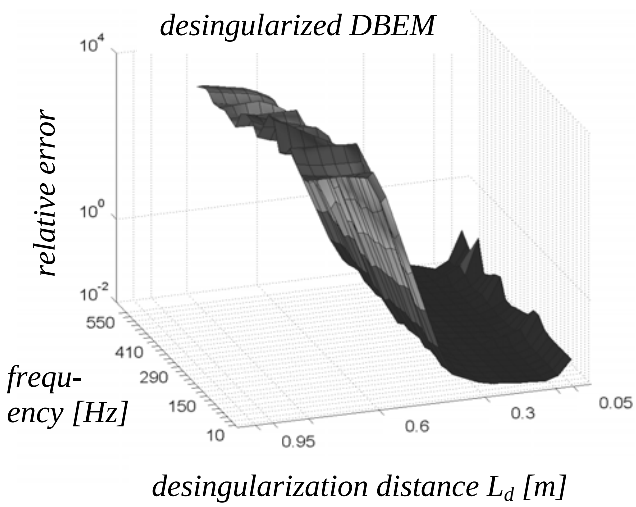

Figure 2 : Relative error for the “breathing sphere” (radius =1m).

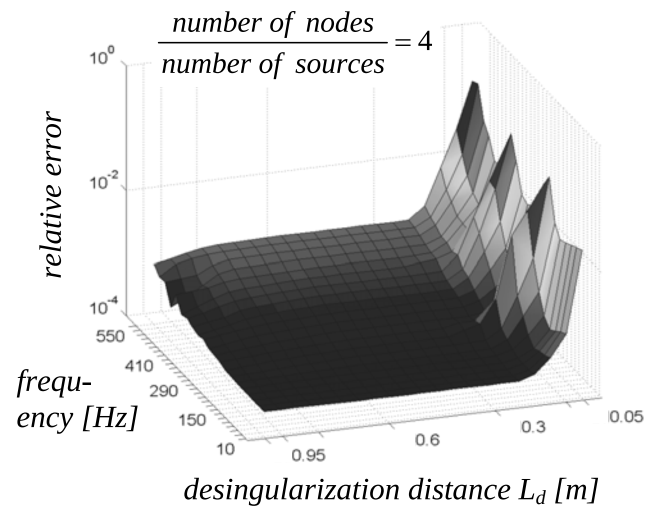

Figure 4 : Relative error for the “breathing sphere” (radius =1m).

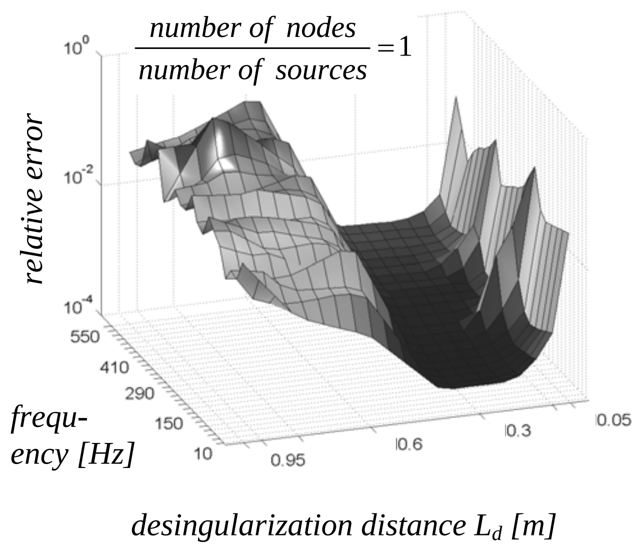

Figure 3 : Relative error for the “breathing sphere” (radius =1m).

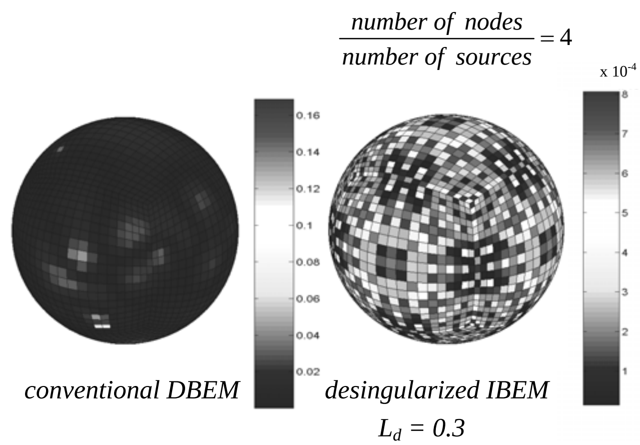

Figure 5 : Local relative errors on the surface of a “breathing sphere” at 250 Hz.

proximately two orders of magnitude larger for the direct formulation. Independently from the formulation it can be observed, that for the whole frequency range the best desingularization distance is about $L_d = 0.3$ m. If the sources are too far away from the surface the system of linear equations (14) for the indirect method, as well as the set of equations resulting from the direct method (8), are poorly conditioned. This leads to uniqueness problems and a drastic increase of the error with increasing L_d. Figure (7) shows how the condition of matrix $\mathbf{A}$ of equation (14) depends on the desingularization distance L_d and on the ratio of the numbers of nodes to the number of sources. The phenomena depicted in Figure (7) hold for the indirect method in all three examples discussed.

For a constant number of nodes, Figure (7) suggests to decrease the number of sources in order to improve the matrix condition. Even for symmetrical geometries as used here, the matrix looses symmetry as the desingularization L_d distance increases. Comparing Figure (3) and Figure (4) confirms for the indirect method that the qual-

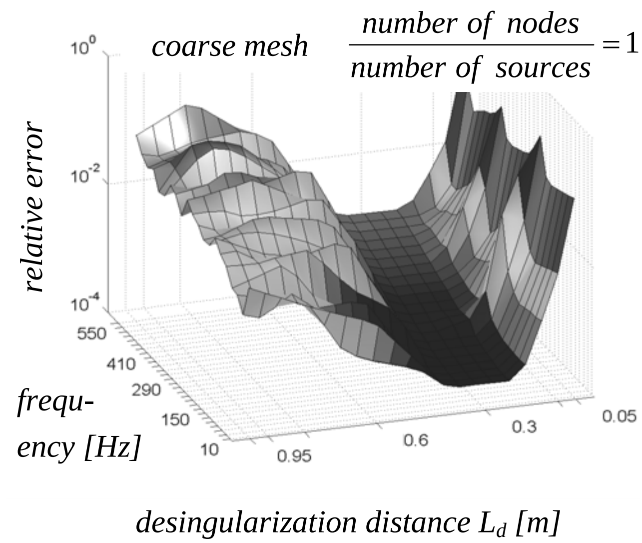

Figure 6 : Relative error for the "breathing sphere" (radius =1m).

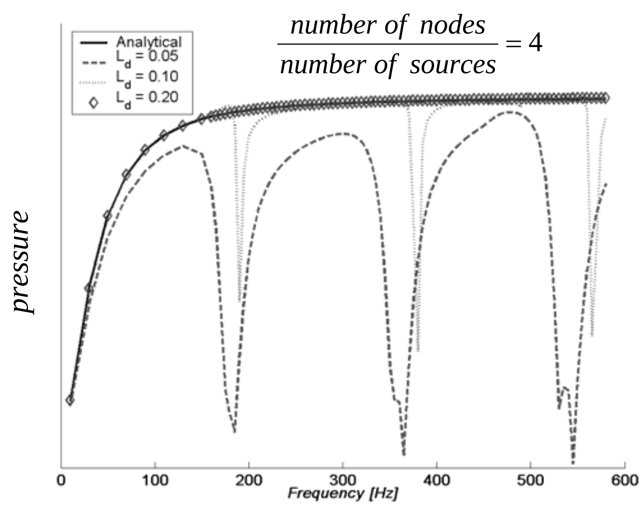

Figure 8 : Irregular frequencies for the "breathing sphere".

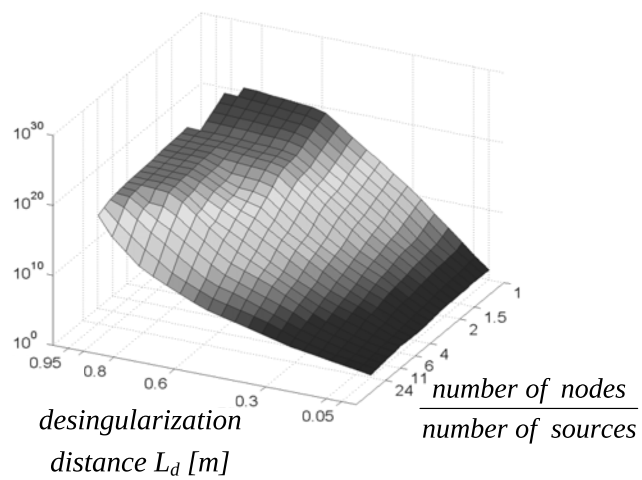

Figure 7 : Matrix condition for varying source number and desingularization distance.

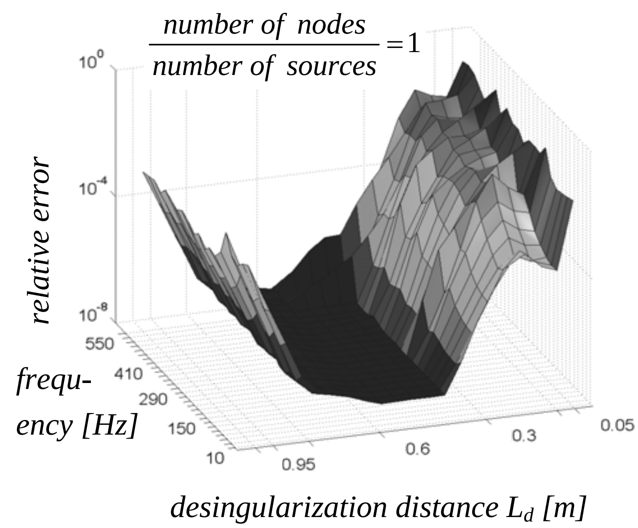

Figure 9 : Relative error for the "cube" (edge length = 2m) with eccentric monopole b.c..

ity of the results achieved even for L_d values up to 0.95 can be improved significantly, if the number of sources is much smaller than the number of nodes.

Figures (2), (3), (4) and (6) show, that for sources too close to the surface of the sphere, problems within the numerical integration can be observed, leading to less accurate results with decreasing L_d. So-called irregular frequencies of the interior acoustic problem occur only if a multiple of the wave length is equal to the opposing source distance. For all four cases discussed above, the according errors are observed, when a rather small desingularization distance is chosen. As depicted in Figure (9), this phenomenon is not observed if the desingularization distance is chosen larger than 0.2 while employing a reasonable mesh density.

The above clearly shows that the error is depending on the desingularization distance as well as on the number of sources taken into account. An interesting result is shown in the Figure (5), where in this particular case the desingularized BEM is even more accurate than the results from a commercial software package where the conventional BEM formulation is implemented [von Estorff (2000) and LMS International (2000)].

For a rather coarse grid consisting of half the number of nodes as for all other cases investigated, Figure (6) shows the relative error. The error level ranges around the same level as for the finer grid shown in Figure (3) and shows the same characteristics. Therefore, it could not be found that the coarseness of the mesh surface has a significant influence on the proper desingularization distance to be chosen.

Next, the second model which deploys a cube is investigated. It should be noted that a cube is, with respect to its acoustic radiation, more complex than the sphere, since its surface is not smooth anymore. Moreover, the given boundary conditions (eccentric monopole) are more complicated than in the previous example. In Figures (9) and (10) again the relative error over frequency and distance L_d is plotted. One can observe that the relative error evaluated for L_d = 0.3 m reaches the same satisfying low level as obtained for the "breathing sphere" (see Figures (3) and (4)). By comparing Figures (9) and (10) it is also interesting to observe, that the results can be improved by decreasing the number of sources taken into account. Figure (11) shows that the method yields as expected, a symmetric sound pressure distribution which is almost identical with the analytical solution.

Finally, the relative error for the third example is shown in Figure (12). Due to geometry and the boundary condition, this is the most challenging of the examples depicted here. In accordance with the previous examples, the lowest error level is reached for L_d = 0.3 m. Anyhow, the magnitude of the error is significantly higher than for the previously discussed examples. By further tuning the desingularization distance and the number of sources, further optimization is possible. If the ratio of the number of nodes to the number of sources is set to 3/2, (see Figure (13)), the relative error can be reduced by more than 15 % at L_d = 0.3 m. Figure (14) yields an example for the distribution of the sound pressure level and the local relative error across the cube.

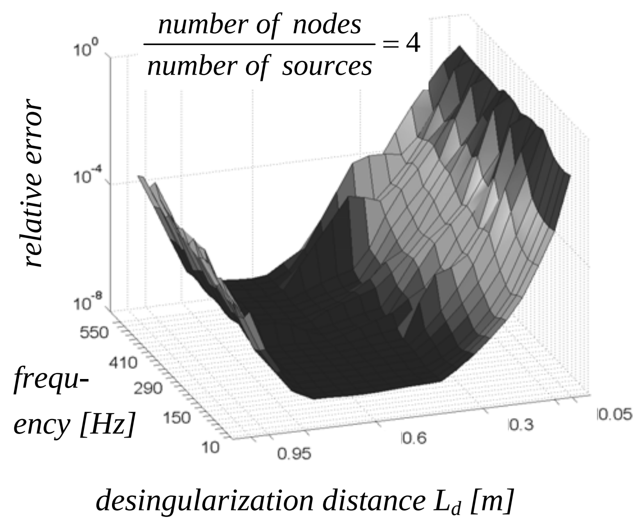

Figure 10 : Relative error for the "cube" (edge length = 2m) with monopole b.c..

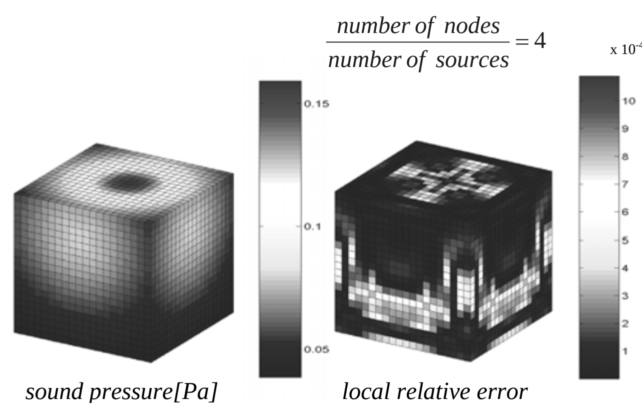

Figure 11 : Results of a desingularized IBEM calculation with eccentric monopole b.c. at 250 Hz.

4 Conclusion

A desingularized boundary element formulation, which was known for the solution of problems governed by the Laplace equation, has been extended to acoustical investigations. Locating the source points on an additional surface outside the acoustic domain, the evaluation of singular integrals could be avoided completely.

By means of three examples, using a sphere and a cube as basic geometries, it could be shown that the desingularized BEM leads to very accurate results as long as the distances between the source points and the boundary are chosen properly. An error calculation has been employed

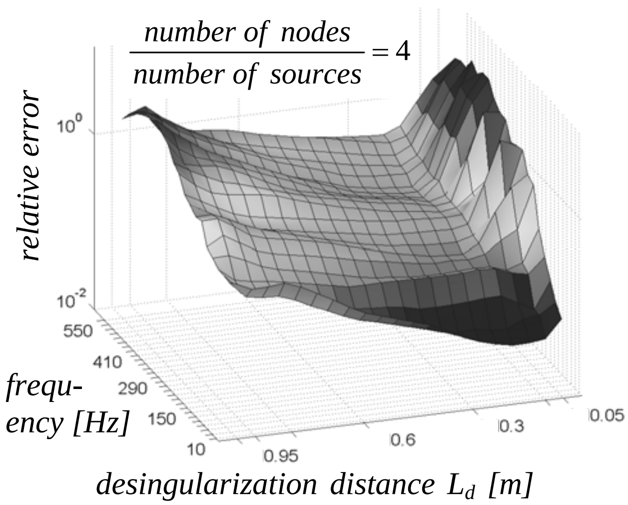

Figure 12 : Relative error for the “cube” (edge length = 2m) with a single “vibrating” node.

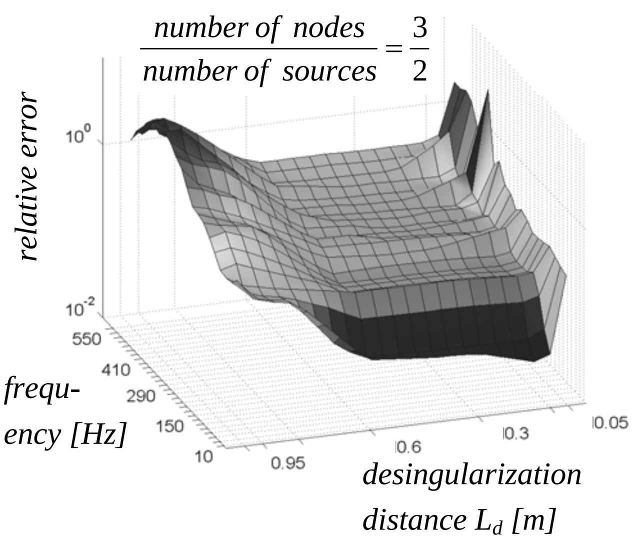

Figure 13 : Relative error for the “cube” with a single “vibrating” node.

in order to find the optimal configuration.

By applying the indirect desingularized method, the number of source locations could be reduced significantly without a loss in the accuracy of the results. Thereby, the computational effort could be reduced.

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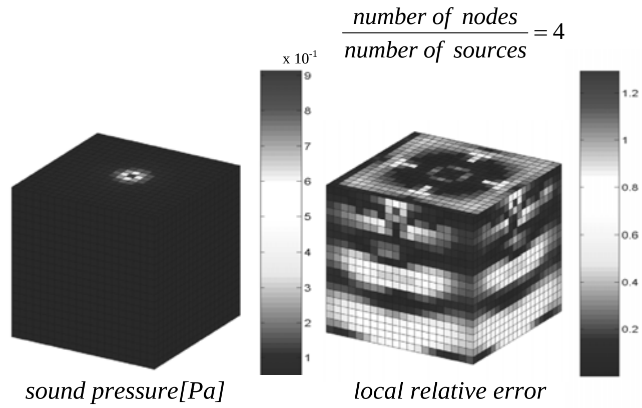

Figure 14 : Results of a desingularized IBEM calculation with a single “vibrating” node at 250 Hz.

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