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Construction of Integral Objective Function/Fitness Function of Multi-Objective/Multi-Disciplinary Optimization

Z. Q. Zhu[1], **Z. Liu**[1], **X. L. Wang**[1], **R. X. Yu**[1]

Abstract: To extend an available mono-objective optimization method to multi-objective/multi-disciplinary optimization, the construction of a suitable integral objective function (in gradient based deterministic method-DM) or fitness function (in genetic algorithm-GA) is important. An auto-adjusting weighted object optimization (AWO) method in DM is suggested to improve the available weighted sum method (linear combined weighted object optimizationLWO method). Two formulae of fitness function in GA are suggested for two kinds of design problems. Flow field solution is obtained by solving Euler equations. Electromagnetic field solution is obtained by solving Maxwell equations. Bi-disciplinary optimization computation is carried out by coupling these two solutions with a nonlinear optimization method. Numerical results show that the needed Pareto solutions can be effectively obtained by using these suggested methods to meet the original design requirements.

keyword: multiobjective/multidisciplinary optimization, Euler equations, Maxwell equations, genetic algorithms

1 Introduction

Multi-objective (MO)/Multi-disciplinary (MD) optimization can be applied effectively to general engineering designs, which by its very nature often require tradeoffs between disparate and conflicting objectives, such as modern aircraft design. Stealthy performance has become one of the basic requirements for a modern flight vehicle. Nevertheless, in a practical design, the shape requirements for stealthy performance are generally in conflict with those for aerodynamic performance. The pervasiveness of these tradeoffs in engineering design has given rise to a rich and vast array of approaches for MO/MD optimization. Examples include the weighted sum and compromise programming approaches (Osyczka, 1985; Standler, 1984; Steuer, 1986), genetic algorithm-based approaches (Osyczka et al, 1995; Schaumann et al, 1998,Mathur et al, 2003), Pareto front approximations (Kasprazak et al, 1999; Zhang et al, 1999),response surface method(Levin et al, 2002), and heuristic topology method (Tapp et al, 2004).

Many researchers have pointed out the drawbacks of weighted sum method as: i) it fails to capture the Pareto points where the Pareto frontier is non-convex, and ii) an evenly distributed set of weights fails to produce an even distribution of points in the Pareto solution front. Messac et al (2000, 2001) discussed in detail the necessary conditions for capturing any Pareto point, and the required form of the aggregate objective function to capture the points in a non-convex Pareto frontier.

In recent years, many methods have been shown to overcome the drawbacks of weighted sum method; these include the compromise programming and exponential weighted criteria (Athan and Papalambros, 1996), normal boundary intersection method (Das and Dennis, 1998) and using physical programming method (Messac, 1996; Messac et al, 2001, 2002), surrogate approximation (Wilson et al, 2001) and evolutionary computation (Bramanti et al, 2001) to explore Pareto frontiers efficiently. Those new methods have shown the ability of being effective and efficient multiobjective optimization methods.

It is desired to emphasize that the objective of this paper is not to advocate the generation of the Pareto frontiers as a normal means to reach an optimal design. In some engineering design applications, such as the shape optimization of airfoil and wing, it is desired to obtain the needed Pareto solution at a minimal computing expense, rather than to obtain the solution from the Pareto front, for computing time is huge to generate the Pareto front when the Euler and N-S equations are used to cal-

[1] Institute of Fluid Mechanics, Beijing University of Aeronautics and Astronautics, Beijing 100083

culate the flow field. For this purposes the authors tried to improve the classical optimization method to obtain the needed Pareto solution in different individual design tasks at a minimal computing expense using the Euler equations for flow field calculation or the Maxwell equations for electromagnetic field calculation.

Mono-objective optimization is a scalar optimization, while MO/MD optimization is a vector optimization. In order to apply those available mono-objective optimization algorithms to MO/MD optimization, the vector optimization problem has to be formulated as a problem of scalar optimization. The central issue of doing it is to construct a suitable integral objective function (in DM)/fitness function (in GA) to meet the requirements of the design problem. This issue is discussed in the present paper.

2 Computational methods of flow field and electromagnetic field solutions

In the flow field calculation, Euler equations are used as governing equations. The finite volume method is used to discretize Euler equations and Van Leer's scheme (Van leer, 1982) is used to discretize the inviscid flux vector. A LU factorization is used for time integration. The local time step and multigrid technique are used in the process of solution to accelerate convergence (Siikonen, et al, 1989). The boundary conditions are treated in a usual way, i.e., zero normal velocity on the wall and nonreflecting boundary conditions in infinite are used. In the following calculation examples, drag C_D is equal to inviscid drag C_{D_W}, because Euler equations are taken as governing equations. Up to now, radar is one of the main measures to detect a flight vehicle, thus reducing radar cross section (RCS) is the most important part of low observable technique of a flight vehicle. The evaluation of RCS can be based on the numerical solution of time domain Maxwell equations, which are the governing equations describing an electromagnetic field. In the present paper Maxwell equations of TM mode are solved by using the flux splitting scheme (Zhu et al, 1998). For the perfectly conducting scatters, the reflecting boundary condition of the electric field on body surface is satisfied. The nonreflecting boundary condition (Mur, 1981) is used in infinite. The solution of the above boundary value problem is called as near-field scattered solution H^s and E^s. By using equivalence principle (Umashankar et al, 1982), the obtained near-field solution is converted into a far-field solution, which is transformed to frequency domain using Fast Fourier Transformation. The RCS usually defined in frequency domain can then be evaluated (Zhu et al, 1998).

In optimization process the airfoil shape varies continuously and a lot of iteration of field solution is required. Thus a fast, robust and high qualitative grid generation method is essential. A transfinite interpolation grid generation method applying B spline curve lines and surfaces is used. The basic considerations are:

i) Origin B spline method can produce a high qualitative curve surface grid, but the grid boundaries are only similar to and not coincident with the initial boundaries. Applying transfinite interpolation method, they can be completely coincident.

ii) Transfinite interpolation method can generate the smoothest grid with continuous second derivatives when B spline is used as blending function, especially when the inverse algorithm of B spline curve surface generation is used, in which the boundary conditions of tangential vector at end points of boundary and torque vector at 4 corners are used. These vectors can be controlled and adjusted to meet the requirement to the grid.

In 3D calculation 2D grid is generated in streamwise direction by using this method and the spanwise grid topogy is of H type.

3 Optimization methods

The available gradient based deterministic optimization methods take successive search. Its optimization speed is high when the design parameters are few, but slow down rapidly as the number of parameters increases. In some cases it leads to local optimization easily. The other type genetic algorithm is not limited by restrictive assumptions about the search space, such as continuity, unimodality and regulation. It is roubst, can arrive to global optimization, and has the nature of parallel computation, though its computational work is huge.

3.1 Auto-adjusting weighted object optimization (AWO) method

A commonly used method (LWO) (Standler, 1984) to formulate a vector optimization as a scalar one is taking the weighted sum of individual objective functions, which is $\sum_{i=1}^{n} c_i f_i(x)$ as the resultant objective function. Here is a weighted coefficient, $0 \leq c_i \leq 1$ and $\sum_{i=1}^{n} c_i = 1$. The disadvantage of LWO is that its solution is very sensitive to the combination of weighted coefficients, which is determined mainly with user's experience. To improve LWO an auto-adjusting weighted object optimization (AWO) method is suggested. In AWO, the weighted coefficients are adjusted automatically according to the information about the improving rates of individual objective functions during the optimization process to protect the further optimization of other objective functions from being hampered by too fast optimization of some individual functions. Thus, in AWO all individual objective functions can be optimized at almost the same speed, and the Pareto optimal solution is achieved.

As an example, let the bi-objective optimization problem be a minimizing one. The procedure of adjustment can be stated as:

$$\Delta obj_i = \frac{obj_i - Reobj_i}{|Reobj_i|}, \quad (i = 1,2) \tag{1}$$

$$Globj = \sum_{i=1}^{2} C_i \cdot obj_i, \quad \sum_{i=1}^{2} c_i = 1 \tag{2}$$

Where $Reobj_i$ is the reference quantity of that objective function. If $\forall (i = 1,2)$, $\Delta obj_i \leq \delta_0$ then the solution is accepted and $Reobj_i = obj_i$. Otherwise the solution is abandoned. Here δ_0 is a control value, usually $\delta_0 \leq 0.1$. If the solution is accepted, the weighted coefficients will be adjusted as:

$$\begin{aligned} C_{better} &= C_{better} - \delta_c \\ C_{worse} &= C_{worse} + \delta_c \end{aligned} \tag{3}$$

Where δ_c is the adjusted step size, C_{better} is the weighted coefficient of better optimized object and C_{worse} is the weighted coefficient of worse optimized object. In the present paper $\delta_c = 0.1 \times 0.9^L$, L is the number of having optimized steps.

3.2 Optimization search algorithm

The search algorithm greatly influences the optimization result in DO method. With the fast development of computer technology, direct searching algorithms are becoming favorable to engineers. They are simpler than other ones, such as gradient and second derivative methods. Powell method (Powell, 1964), which does not need to calculate derivatives of design variables and is easily linked with flow solver, is chosen as an optimum searching algorithm in the present paper.

3.3 Genetic Algorithm (GA)

GA is stochastic searching algorithm based on natural selection and evolution behavior (Holland, 1975; Goldberg, 1989). Design variables are coded with some coding techniques to represent as individual members of a population and to form "chromosomes", in which "genes" maintain the features of individual members. Individuals are evaluated for a fitness value, which is a measurement of individual quality and based on which highly fit individuals are more likely to survive and become parents conducted via a selection strategy. The selected parents then mate and produce offspring. In GA, the process of selection, mutation, crossover, evaluation, and reproduction are repeated until the convergence of a suitable solution to the problem is achieved. The procedure of the method can be stated as follows: (1) Encoding the design variables; (2) Initializing the population; (3) Evaluating fitness values; (4) Appling selection strategies; (5) Operating genetic operators; (6) Utilizing the stop rule. In present paper, the decimal coding and game selection strategy are used. The parameters are: population size 40, generation number 40, crossover probality 0.6, and mutation probality 0.4. Considering the large computational work of applying GAs, which is particularly evident if Euler or Navier-Stokes flow analysis is employed, parallel computation is carried out on cluster PCs connected via Ethernet under PVM/MPI environments to allow a reasonable total computing time. A network of 2 or 4 PCs is used in 2D calculation and a net work of 40 PCs is used in 3D calculation.

3.4 Construction of the fitness functions

In nature, the quality of a species is usually evaluated with the degree of fitness to the environment, in GAs also necessary to construct an environment (design problem) for the population and then evaluate the quality of individuals with certain criteria. Obviously, the higher the fitness value, the better the individual's quality. So the quality of the fitness function greatly influences the results of using GAs. How to evaluate the satisfactory degree of the MO/MD optimization results is not easy. In practical tasks most objects have their own physical meanings, such as lift, drag, etc. They are usually conflicting with or restricting to each other, i.e., when one object value becomes better, some others' values may become worse. There is no such a solution, in which all the objects' values approach to their own optimal values simultaneously. While in optimization process only one abstract quantity, which reflects our satisfactory degree to the result, can be used. It is necessary to combine all the objects into a suitable quantity to be used as a fitness function, which has to be constructed according the design problem nature and requirements. Two formulae of fitness function corresponding their own design requirements are suggested at the present paper.

Fitness function I

It is desired to obtain certain expected objective values in many designs, such as expected lift and drag values at given flight conditions in aerodynamic design. Such kind design problem is stated as:

To minimize

$$\min\left|f_j(x)-O_j\right|, \mathrm{j}=1,\ldots,\mathrm{M} \tag{4}$$

Subject to

$$\Phi_q(x)\geq C_q, \mathrm{q}=1,\ldots,\mathrm{m} \tag{5}$$

Then the fitness function can be suggested as:

$$F(x)=\prod_{q=1}^{m}\exp[A_q(\Phi_q(x)-C_q)]/\sum_{j=1}^{M}\delta_j(f_j(x)-O_j)^2 \tag{6}$$

Where $f_j(x)$ is an objective function, M the number of objective functions, $\Phi_q(x)$ a constraint function, m the number of constraints, δ_j the "nondimensional" corresponding coefficient of the objective function (here "nondimensional" is to make the contribution of multi-objective functions to the fitness value at the same order), A_q the penalty coefficient of the constraint function, and O_j, C_q are given constants representing the goal's expected value and the boundary value of the constraint, respectively.

Fitness function II

In some designs the following two requirements should be met:

a) Make the gains of all objects obtained as equal as possible, for example, increase lift 50% and decrease drag 50%.

b) Make total gain of all objects as large as possible.

(i) Biobjective (BO)/Bidisciplinary (BD) case

In BO/BD case, by using the nonlinear objective function combination method (OFCM) a fitness function formula is suggested as (Zhu et al, 2003):

$$FF=\exp R^{\beta}[(1-\alpha)(1-\phi^2)+\alpha],\quad \alpha,\quad \beta=\mathrm{const} \tag{7}$$

$$\phi=\begin{cases}\frac{4}{\pi}\left|\arctan(\frac{GO_2}{GO_1})-\frac{\pi}{4}\right| & , GO_1>0\\ 4-\frac{4}{\pi}\left|\arctan(\frac{GO_2}{GO_1})-\frac{\pi}{4}\right| & , GO_1<0\end{cases} \tag{8}$$

$$R=(GO_1)^2+(GO_2)^2 \tag{9}$$

Where GO_1, GO_2 are the increments of two objects' values, respectively. α=0.1, β=0.4 are taken in the present paper.

(ii) MO/MD case

In present paper, the OFCM method is extend to MO/MD case, the quantities R and ϕ can be expressed as:

$$R=\sum_{i=1}^{n}(GO_i)^2 \tag{10}$$

$$\phi=\frac{4}{\pi}\arccos(\frac{k_1}{\sqrt{k_1^2+k_2^2}}) \tag{11}$$

where

$$k_1=\frac{1}{\sqrt{n}}\sum_{i=1}^{n}(GO_i) \tag{12}$$

$$k_2=\sqrt{\sum_{j=1}^{n}[GO_j-\frac{1}{\sqrt{n}}(\sum_{i=1}^{n}GO_i)^2]} \tag{13}$$

when n equals 2, Eq. (10)-(13) are regressed to Eq. (7)-(8).

(iii) Effect of the constraints

Usually there are some constraints in an optimization problem. These constraints can be satisfied by adding a penalty function to the resultant objective function F. If the constraints are $\psi_i\geq D_i$, i=1, 2,..., m, the resultant objective function with constraints can be written as:

$$F=F\cdot\prod_{i=1}^{m}P_i,\ P_i=\begin{cases}e^{A_i(D_i-\psi_i)} & , \psi_i<D_i\\ 1 & , \psi_i\geq D_i\end{cases} \tag{14}$$

Table 1 : Calculated results of AWO and LWO methods with initial weighted coefficients (0.5, 0.5)

NACA0012 $M_{¥} = 0.7$, $a = 2.57^\circ$		C_L	C_{D_W}	$RCS_{q=180^\circ}$	C_L / C_{D_W}
	initial	0.48567	0.021880	-2.8480	22.197
LWO	optimal	0.58173	0.024989	-100.89	23.280
	$D = \frac{V_{opt} - V_{ini}}{V_{ini}} (\%)$	19.8	14.1	-3422.4	4.88
AWO	optimal	0.62550	0.018786	-91.688	33.298
	$D = \frac{V_{opt} - V_{ini}}{V_{ini}} (\%)$	28.8	-14.1	-3119.4	50

3.5 Representation of airfoil and wing

The coordinates of airfoil contour are presented as:

$$\overline{y} = \overline{y}_b(\overline{x}) + \sum a_i S_i(\overline{x}) \tag{15}$$

Where $\overline{y}_b(\overline{x})$ is the baseline airfoil, $S_i(\overline{x})$ are analytic shape functions (Hager et al, 1992), which are used to systematically perturb baseline airfoil, and undetermined coefficients α_i are taken as the design variables. 4 design variables are used in the 2D calculation. Five sections are used as control sections in 3D calculation. In each of them four coefficients α_i are used as design variables. The total number of design variables is 20.

4 Numerical results and discussion

4.1 2D calculations

Case 1: Airfoil NACA0012 is chosen as a baseline airfoil. It is desired to modify its shape to minimize its drag C_{D_W} and at the same time to minimize the RCS at the leading edge of airfoil under the condition of Mach number M_∞=0.75, and angle of attack α=2.57^o and the constraint of that the maximum thickness is larger than or equal to the original thickness, i.e., $(t/c)_{\max} \geq (t/c)^0_{\max}$. The calculation is carried out by using both AWO method and LWO method with weighted coefficients (0.5, 0.5). Table. 1 presents the calculated results. In Table 1 C_L is the lift and C_L/C_{D_W} is the ratio of lift to drag.

It is seen that RCS is decreased significantly with both methods, but C_{D_W} is increased too in LWO solution. This is obviously not the design meeting the requirement. However, C_{D_W} is decreased by 14% in AWO solution, making it a required Pareto solution. Though another Pareto solution can be obtained using LWO method with coefficients (0.8, 0.2), but choosing the suitable weighted coefficients needs the user's experience or more trail and calculations are used. The calculated results are given in table 2. As seen is table 2, its integral performance is worse than that of the AWO solution. It means that with AWO method not only the requirement of having experience choosing initial weighted coefficients is not needed, but also a better compromised solution can be achieved.

Case 2: Airfoil NACA0012 is taken as baseline airfoil. Two expected values of objects are: $RCS_{\theta=180^\circ} \rightarrow -35.0$ and $C_L/C_{D_W} \rightarrow 35.0$. The constraint is $(t/c)_{\max} \geq (t/c)^0_{\max}$. The calculated results using GA with fitness function I are given in Table. 3.

Table 3 shows that result realizes the expected design values of 2 objects and a required Pareto solution is obtained.

Case 3: Airfoil RAE2822 is chosen as a baseline airfoil. It is desired to modify its shape to minimize its drag C_{D_W} and at the same time to maximize its lift C_L under the condition of $M_\infty = 0.75$, $\alpha = 0^\circ$. The goal is $C_L -> 0.45$, $C_{D_W} -> 0.004$, and the constraints are $(t/c)_{\max} \geq (t/c)^0_{\max}$ and $(C_L/C_{D_W}) > (C_L/C_{D_W})^0$. Table 4 gives the calculated results.

The results show that the required solution is obtained. Optimization increases lift by 15%, decreases drag by

Table 2 : Calculated results of AWO and LWO methods with initial weighted coefficients (0.8, 0.2)

NACA0012 $M_{\infty} = 0.7$, $\alpha = 2.57^{\circ}$		C_L	C_{D_W}	$RCS_{\theta=180^{\circ}}$	C_L / C_{D_W}
	initial	0.48567	0.021880	-2.8480	22.197
LWO	optimal	0.65712	0.018293	-68.618	32.923
	$D = \frac{V_{opt} - V_{ini}}{V_{ini}}(\%)$	35.3	-16.4	-2309.3	48.3
AWO	optimal	0.69044	0.016796	-16.669	41.099
	$D = \frac{V_{opt} - V_{ini}}{V_{ini}}(\%)$	42.2	-23.2	-485.3	85.2

Table 3 : Calculated results of GA method with fitness function I

NACA0012 $M_{\infty} = 0.7$, $\alpha = 2.57^{\circ}$		C_L	C_{D_W}	$RCS_{\theta=180^{\circ}}$	C_L / C_{D_W}
	initial	0.48567	0.021880	-2.8480	22.197
GA fitness function I	optimal	0.68583	0.018645	-36.727	36.784
	$D = \frac{V_{opt} - V_{ini}}{V_{ini}}(\%)$	41.2	-14.8	-1189.6	65.7

Table 4 : Calculated results of GA with fitness function I

RAE2822 $M_{\infty} = 0.75$, $\alpha = 0^{\circ}$	C_L	C_{D_W}	C_L / C_{D_W}
initial	0.39839	0.0042563	93.599
optimized	0.46170	0.0041817	110.41
$D = \frac{V_{opt} - V_{ini}}{V_{ini}}(\%)$	15.9	-1.75	18.0

Table 5 : Calculated results of GA with fitness function II

NACA0012 $M_{\infty} = 0.7$, $\alpha = 2.57^{\circ}$		C_L	C_{D_W}
	initial	0.30767	0.0043288
OFCM	optimal	0.35486	0.0038774
	$D = \frac{V_{opt} - V_{ini}}{V_{ini}} (\%)$	15.3	-10.4
LWO	optimal	1.0067	0.014081
	$D = \frac{V_{opt} - V_{ini}}{V_{ini}} (\%)$	227.2	225.3

1.75% at the same time, and increases the ratio of lift to drag to 110.409 (the increment is 18%) even though the baseline airfoil has already had a high ratio.

This case is a multi-objective optimization of a single disciplinary and case 2 is a multi-disciplinary optimization. The results of both cases show that the fitness function I can solve the desired design task.

Case 4: The initial airfoil is NACA0012. It is desired to increase lift and decreases drag at the same time and as equally as possible under the condition of $M_\infty = 0.7$, $\alpha = 1.0^\circ$, and the constraint of $(t/c)_{\max} \geq (t/c)^0_{\max}$. Table 5 gives the calculated results of GA with fitness function II. The results of GA with fitness function constructed by using LWO (0.5, 0.5) method are also given in Table. 5.

It is seen from Table. 5 that C_L(LWO) is increased significantly and drag C_{D_W} is increased too. This does obviously not meet the design requirement. However, OFCM method gives the desired solution.

Case 5: Airfoil RAE2822 is taken as a baseline airfoil. It is desired to increase C_L/C_{D_W} and decrease C_{D_W} under the condition of $M_\infty = 0.73$, $\alpha = 0^\circ$ with the constraint of $(t/c)_{\max} \geq (t/c)^0_{\max}$. In this case, C_L/C_{D_W} and $1/C_{D_W}$ are taken as two optimization objects. Fitness function II is used in the calculation with both GA and Powell method. Table 6 gives the calculated results. Although RAE2822 is a supercritical airfoil and has high ratio of lift to drag, from table 6 it can be seen that design goal is still reached, i.e. C_L/C_{D_W} is increased by 3% and drag is decreased by 1% using GA. This shows that present method (OFCM) can be used in a fine design case. The obtained solution using GA is better than the solution using Powell method. Both solutions are required solutions.

Case: 6 As a MO example, NACA65006 airfoil is taken as a baseline airfoil. Object 1 and 2 are C_L and $1/C_{D_W}$ at $M_\infty = 0.7$, $\alpha = 2^\circ$ respectively, and object 3 is $1/C_{D_W}$ at $M_\infty = 1.5$, $\alpha = 0^\circ$. It is obvious that the camber of the airfoil has to be increased to improve subsonic aerodynamic performance. But this will increase supersonic drag, too. Optimization goal in this case is to get a suitable compromised camber to obtain a subsonic performance as high as possible and a supersonic drag as low as possible at the same time. Table 7 gives the calculated results.

Table 7 shows that nearly the same satisfactory solutions are obtained by using GA and Powell method. This illustrates that OFCM method can be used in multiobjective optimization to obtain a better compromised solution.

4.2 Bi-objective aerodynamic optimization of a 3D wing

A 3D wing plane is taken as the baseline, which has a sweep angle of leading edge $x = 35^\circ$, an aspect ratio $\lambda = 3.5$, and a taper ratio $\eta = 0.17$. The NACA65006 airfoil is taken as the wing-section profile. It is required to increase and to reduce the drag C_{D_W} at the same time under the condition at Ma=0.6, $\alpha = 2^\circ$. The calculated results are given in Table 8, in which the results using LWO (0.5, 0.5) method are also given.

Table 6 : Calculated results of RAE2822 airfoil

RAE2822 $M_{\infty} = 0.73$ $a = 0^{\circ}$		C_L / C_{D_W}	$1/C_{D_W}$
	initial	117.3	330.5
Powell	optimal	117.84	332.22
	$D = \frac{V_{opt} - V_{ini}}{V_{ini}} (\%)$	0.46	0.52
GA	optimal	120.9	334.134
	$D = \frac{V_{opt} - V_{ini}}{V_{ini}} (\%)$	3.06	1.1

Table 7 : Calculated results of MO optimization

NACA65006		Subsonic C_L	Subsonic $1/C_{D_W}$	Supersonic $1/C_{D_W}$
	initial	0.30820	268.19	37.34
Powell	optimal	0.37250	328.7	35.20
	$D = \frac{V_{opt} - V_{ini}}{V_{ini}} (\%)$	20.9	22.6	-5.7
GA	optimal	0.37345	308.6	35.26
	$D = \frac{V_{opt} - V_{ini}}{V_{ini}} (\%)$	21.2	15.1	-5.56

It is seen from Table 8 that C_L/C_{D_W} (LWO) is increased significantly (54.3%) at the sacrifice of drag increase (35.69%). The solution of OFCM method gives the desired design. The optimization effect are not obvious in this case, since the C_{D_W} of initial wing is already very small.

5 Concluding Remarks

Traditional mono-objective optimization is a scalar optimization, while MO/MD optimization is a vector one. In order to apply numerous available mono-objective optimization methods to MO/MD optimization, the most important thing is to construct a suitable integral objective function (in DOM) or fitness function (in GA). AWO method and fitness function I/II which are suggested in the present paper can be effectively used in DOM or GA. They can be used not only in aircraft design but also in general engineering design problems. Numerical results of BO/BD presented in this paper show that their solutions can meet the desired design requirements, can obtain the needed Pareto solutions, and are better than the compromised solutions of the available LWO method in both 2D and 3D cases.

Table 8 : Calculated results of a 3D wing

3D wing Ma=0.6 $a = 2^\circ$		C_L	C_{D_W}	C_L / C_{D_W}
	initial	0.045801	0.0051553	8.884
OFCM	optimal	0.047546	0.0051509	9.231
	$D = \frac{V_{opt} - V_{ini}}{V_{ini}} (\%)$	3.8	-0.1	3.9
LWO	optimal	0.095876	0.0069952	13.706
	$D = \frac{V_{opt} - V_{ini}}{V_{ini}} (\%)$	109	35.69	54.3

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