

# A Fully Coupled Finite Element Model of Landfill Gas Migration in a Partially Saturated Soil

W. J. Ferguson<sup>1</sup>, B. Palanathakumar<sup>2</sup>

**Abstract:** Environmental and safety issues associated with landfill gas require the control of off-site migration. Mathematical modelling can assist in the understanding of the processes and mechanisms controlling gas migration from municipal waste disposal sites. This paper presents the development and application of a mathematical model that simulates landfill gas migration within a partially saturated soil. This model accounts for two-phase flow and incorporates multi-component (methane, carbon dioxide, dry air and moisture) transport in the gas and liquid phases together with concomitant heat migration. The governing system of fully coupled non-linear partial differential equations of the model have been derived from a mechanistic approach where the mass and energy conservation laws are defined for a particular phase into which Darcy's law and Fick's law are substituted. Employing the Galerkin finite element method for the spatial discretisation and a finite difference time-stepping scheme for the temporal discretisation, a fully implicit algorithm has been developed for the numerical solution of the governing equations. The model was applied to the Loscoe landfill site to estimate the landfill gas concentration in the vicinity of the gas explosion which occurred in March 1986. An assessment has also been made to determine the relative importance of the gas transport mechanisms (i.e. diffusion, convection and dissolved gas) in this case study.

**keyword:** finite element model, landfill gas migration, partially saturated soil, Loscoe landfill gas explosion, gas transport mechanism

## 1 Introduction

The movement of landfill gas through the earth's surface is now recognised as a major hazard of municipal solid

waste landfills as a consequence of the number of incidents involving landfill gas explosions [Williams and Aitkenhead (1991), Hooker and Bannon (1993), Kjeldsen and Fischer (1995)]. The landfill gas produced during the degradation of organic materials disposed of at landfill sites causes a build up of pressure and an increase in the concentration within the landfill [Farquhar and Rover (1973)]. The high concentration and pressure result in the landfill gas migrating along the path of least resistance into the area surrounding the landfill site. Many investigations have demonstrated that both diffusive and convective transport can be important processes in landfill gas migration [Findikakis and Leckie (1979), Metcalfe and Farquhar (1987)]. Diffusive flow is caused by variations of gas concentrations within the soil, whilst convective flow is a result of a pressure gradient in the fluid. Diffusive flow will always be present because the concentration of landfill gas in atmospheric air is very low. The pressure inside the landfill can be relatively high and can result in a large pressure gradient [Tchobanoglous (1993)] across the site. Variations in barometric pressure also change the pressure gradient particularly in shallow landfills [Young (1992a), Christophersen, Kjeldsen, Holst, and Chanton (2001)]. The temperature gradient is another factor indirectly influencing gas migration across a landfill site [Thomas and Ferguson (1999), El-Fadel, Findikakis and Leckie (1994)] because of its effects on the thermodynamic properties of the fluids.

Depending on the characteristics of the surrounding soil, the gas may migrate laterally many hundreds of metres from the landfill and may have a detrimental effect on public health and the local environment, i.e. explosion hazards, damage to vegetation, global warming and ground water pollution. During the anaerobic methanogenic steady phase of a landfill life cycle, 50-60% of gas released from landfills is methane and 40-50% is carbon dioxide [Farquhar and Rover (1973), Tchobanoglous (1993)]. Methane forms an explosive mixture with air

<sup>1</sup> School of the Built Environment, Heriot-Watt University, Edinburgh, EH14 4AS, United Kingdom

<sup>2</sup> Fault Analysis Group, Department of Geology, University College Dublin, Belfield, Dublin 4, Ireland

when the concentration is between 5% and 15% by volume [Hooker and Bannon (1993)].

A reliable mathematical model represents an efficient tool for the evaluation of landfill gas migration in and around the landfill site. In the past, numerical models of varying degree of complexity have been presented for landfill gas migration. Moore, Rai and Alzaydi (1979) simulated landfill gas migration through a soil which was represented as a collection of parallel capillary tubes of variable radius. Findikakis and Leckie (1979) presented a one-dimensional advection-diffusion model using the finite difference technique. However, coupled liquid flow with gas migration was neglected. Metcalfe and Farquhar (1987) developed a two-dimensional finite element model that solved the advection-dispersion equation. Though this study accounted for the dissolution of the contaminant gas in the soil moisture, the profile of the soil water was assumed to be constant in both time and space. El-Fadel, Findikakis and Leckie (1994) extended the model presented by Findikakis and Leckie (1979) incorporating the basic concepts from microbiology of a landfill ecosystem to simulate the spatial and temporal distribution of gas and heat transport, but its application was limited to only within the confines of the landfill itself.

The review of existing numerical models points out the need for a model to simulate the coupled processes taking place during landfill gas migration. The objective of this study is to develop a two-dimensional, two-phase flow numerical model for the migration of landfill gas in a partially saturated soil. The governing equations of the model are derived from a mechanistic approach where the mass and energy conservation laws are defined for a particular phase into which Darcy's law and Fick's law are substituted. There is some debate about the validity of using the continuum approach to simulate gas migration in very low permeability media such as the engineered clay liner of a landfill site [Impey, Grindrod, Takase and Worgan (1997)]. However, this theoretical approach is very popular and its performance is well examined [Thomas and Ferguson (1999)]. The advantage of the mechanistic approach is that it is possible to easily include a number of physical effects in a properly coupled fashion and to represent geological and geometric features in great detail.

The numerical model presented in this paper employs the finite element method for the spatial discretisation

of the domain of interest. Traditional numerical solution techniques, such as the finite element method, finite difference method and the control volume method, have been effectively applied to a wide range of engineering problems [Zienkiewicz (1997)]. All of these methods require the generation of a mesh. Mesh generation is a potential disadvantage of these methods as it can be far more computationally time-consuming than the assembly and solution of the equations, particularly for a three-dimensional analysis. In recent years, meshless or mesh-free methods have been the subject of much research in an attempt to overcome the potentially burdensome task of meshing the solution domain. Meshless methods such as the diffuse element method [Nayroles, Touzot and Villon (1992)], the element-free Galerkin [Belytschko, Lu and Gu (1994)], smooth particle hydrodynamics [Lucy (1977), Zhang and Batra (2004)], hp clouds [Duarte and Oden (1996)], meshless Galerkin methods using radial basis functions [Mai-Cao and Tran-Cong (2005), Divo and Kassab (2005)] method of characteristics [Iske and Käser (2005)] and the partition of unity finite element method [Melenk and Babuska (1996)] have been developed. These numerical methods are not truly meshless as they require the use of a "background" mesh for the integration of the weak form of the problem. Atluri and Zhu (1998) developed the meshless local Petrov-Galerkin (MLPG) method for solving linear and non-linear boundary problems. The MLPG method is genuinely meshless since no curvilinear mesh is required for the purpose of interpolation. The MLPG method has been employed to solve a variety of problems including heat conduction [Sladek, Sladek and Atluri (2004), Batra, Porfiri and Spinello (2004)], structural analysis [Atluri, Han and Rajendran (2004), Han and Atluri (2004)] and fluid mechanics [Lin and Atluri (2001)]. The analyses carried out in the current work are two-dimensional simulations; consequently, mesh generation is not a time-consuming task. However, it is proposed to extend this research to three-dimensional analysis, incorporating the effect of a deforming soil skeleton, in which case, the MLPG method offers considerable time-savings over the finite element method.

The developed model treats the migration of liquid, heat, air and contaminant gases separately with independent system variables of pore water pressure, temperature, pore air pressure and the molar concentration of the two-contaminant gases, i.e. carbon dioxide and methane. Us-

ing the Galerkin finite element method and applying a finite difference time-stepping scheme, a fully implicit algorithm has been developed for the numerical solution of the governing system of partial differential equations. The model has been used to demonstrate the cause of a landfill gas explosion, which occurred at Loscoe, Derbyshire on 24<sup>th</sup> March 1986. An assessment has also been made to determine the relative importance of the gas transport mechanisms (i.e. diffusion, convection and dissolution) in this incident.

## 2 Theoretical Formulation

Soil, in the unsaturated zone, is a three-phase system comprised of the solid soil particles together with liquid and gas which fill the void space. In the proposed mathematical model, the solid soil skeleton is assumed to be a rigid, non-deforming matrix of soil particles. Hence, the model reduces to a two-phase system of liquid and gas. The liquid phase is considered to be pure water containing both dissolved air and two contaminant gases (methane and carbon dioxide). The gas phase is assumed to be a multi-mixture of water vapour, dry air and the two contaminant gases.

The governing system of five fully coupled non-linear partial differential equations, for the two-phase flow through an unsaturated porous medium, are expressed in terms of the five primary system variables; pore water pressure  $P_w$ , temperature  $T$ , pore air pressure  $P_a$ , molar concentration of carbon dioxide  $C_{gi}$  and the molar concentration of methane  $C_{gj}$ .

A mechanistic approach is adopted to derive the governing equations which treats the flow of liquid, heat, air and the contaminant gases independently. The mechanistic approach employs Darcy's law and Fick's law to describe the convective gas flow and diffusive gas flow respectively. By applying the principle of mass conservation, these two laws are combined to produce the governing equations for two-phase flow through a porous medium.

### 2.1 Moisture Transfer

Moisture in unsaturated soil exists in two states; liquid water and water vapour. The volumetric water content,  $\theta$ , is defined as the sum of these two phases. Vapour flow is assumed to flow due to both the influence of a vapour pressure gradient and, also, as part of the bulk flow of air. Therefore, the conservation of mass for moisture transfer

states that;

$$\frac{\partial(\phi\rho_l S_l)}{\partial t} + \frac{\partial(\rho_l \theta_v)}{\partial t} = -\nabla(\rho_l V_l) - \nabla(\rho_l V_v) - \nabla(\rho_v V_g) \quad (1)$$

where  $\phi$ ,  $\rho_l$ ,  $\rho_v$ ,  $S_l$  and  $\theta_v$  denote the porosity, liquid density, water vapour density, liquid saturation and the volumetric vapour content respectively. The velocities of liquid water, water vapour and the pore air are given by  $V_l$ ,  $V_v$  and  $V_g$  respectively.

The principle of thermodynamic equilibrium dictates that at any point, the volumetric liquid water and water vapour are in equilibrium [De Vries (1958)] giving;

$$\theta_v = \frac{(\phi - \phi S_l)\rho_v}{\rho_l} \quad (2)$$

The vapour density is defined by Edelfsen and Andersen (1943) as;

$$\rho_v = \rho_o h = \rho_o \exp \left\{ \frac{P_w - P_g}{\rho_l R_v T} \right\} \quad (3)$$

where  $\rho_o$  is the saturated vapour density,  $h$  is the relative humidity and  $R_v$  is the gas constant for water vapour. The saturated vapour density is a function of the temperature [Mayhew and Rogers (1976)].

According to Darcy's law, the phase velocities for the liquid and gas can be expressed as;

$$V_l = -K_w \left[ \nabla \left( \frac{P_w}{\gamma_l} \right) + \nabla Z \right] \quad (4)$$

$$V_g = -K_g [\nabla P_g + \rho_g \nabla Z] \quad (5)$$

where  $K_w$  and  $K_g$  are the soil water permeability and soil gas permeability respectively and where  $Z$  represents the vertical elevation from datum (positive upward).

The vapour flux is based on the theory first proposed by Philip and de Vries (1957) and then extended by Ewen and Thomas (1989). This flow law states that the velocity of vapour  $V_v$  through an unsaturated soil is defined as;

$$V_v = \frac{-D_{atm} v \alpha \theta_a}{\rho_l} \cdot \nabla \rho_v \quad (6)$$

where  $D_{atm}$  is the molecular diffusivity of vapour through air,  $\alpha$  is the tortuosity factor,  $v$  is a mass flow factor and  $\theta_a$  is the volumetric content of the air.

The final form of the governing partial differential equation for moisture flow can be obtained on substituting the expressions for  $\theta_v$ ,  $\rho_v$ ,  $V_l$ ,  $V_g$  and  $V_v$ , Eqs. 2-6, into the mass balance equation, Eq. 1, and rearranging to give;

$$C_{11} \frac{\partial P_w}{\partial t} + C_{12} \frac{\partial T}{\partial t} + C_{13} \frac{\partial P_g}{\partial t} + C_{14} \frac{\partial C_{gi}}{\partial t} + C_{15} \frac{\partial C_{gj}}{\partial t} \quad (7)$$

$$= \nabla \cdot (K_{11} \nabla P_w) + \nabla \cdot (K_{12} \nabla T) + \nabla \cdot (K_{13} \nabla P_g) + \nabla \cdot (K_{14} \nabla C_{gi}) + \nabla \cdot (K_{15} \nabla C_{gj}) + \nabla \cdot (K_{16} \nabla Z)$$

where,

$$C_{11} = \phi(\rho_l - \rho_v) \frac{\partial S_l}{\partial P_w} + \rho_o \phi S_g \frac{\partial h}{\partial P_w} + \phi S_l \frac{\partial \rho_l}{\partial P_w}$$

$$C_{12} = \phi(\rho_l - \rho_v) \frac{\partial S_l}{\partial T} + \rho_o \phi S_g \frac{\partial h}{\partial T} + \phi S_l \frac{\partial \rho_l}{\partial T}$$

$$C_{13} = \phi(\rho_l - \rho_v) \frac{\partial S_l}{\partial P_g} + \rho_o \phi S_g \frac{\partial h}{\partial P_g} + \phi S_l \frac{\partial \rho_l}{\partial P_g}$$

$$C_{14} = C_{15} = 0$$

$$K_{11} = \frac{K_w}{g} + \rho_l K_{v1}$$

$$K_{12} = \rho_l K_{v2}$$

$$K_{13} = \rho_v K_g + \rho_l K_{v3}$$

$$K_{14} = K_{15} = 0$$

$$K_{16} = \rho_l K_w + \rho_v \rho_g g K_g$$

## 2.2 Heat Transfer

Heat transfer can be categorised into three primary modes of transmission i.e. conduction, convection and radiation [Jakob (1949)]. According to an approach presented by Mitchell (1993), the effect of heat radiation in soil is negligible and, therefore, it is not considered in this study.

Applying the conservation of energy to the heat flow through unsaturated soil gives;

$$\frac{\partial \{H(T - T_r) + \phi(1 - S_l) \rho_v L\}}{\partial t} = -\nabla Q \quad (8)$$

where  $H$ ,  $L$  and  $Q$  represent the heat capacity of the soil at a reference temperature  $T_r$ , the latent heat of vaporisation and the heat flux respectively.

Adopting the approach presented by Ewen and Thomas (1989), the heat capacity of unsaturated soil at the reference temperature can be expressed as;

$$H = (1 - \phi) \rho_s C_{ps} + \phi S_l \rho_l C_{pl} + \phi S_g \rho_v C_{pv} + \phi S_g C_{ga} C_{pga} + \phi S_g C_{gi} C_{pgi} + \phi S_g C_{gj} C_{pgj} \quad (9)$$

where  $C_{ps}$ ,  $C_{pl}$ ,  $C_{pv}$ ,  $C_{pga}$ ,  $C_{pgi}$  and  $C_{pgj}$  are the specific heat capacities of the solid soil particles, liquid, vapour and dry air, carbon dioxide and methane respectively and  $\rho_s$  is the density of the solid soil particles.

The heat flux per unit area  $Q$  is defined as;

$$Q = -\lambda \nabla T + (T - T_r) \{ \rho_l C_{pl} V_l + \rho_v C_{pv} V_v + \rho_v C_{pv} V_g + C_{ga} C_{pga} V_g + C_{gi} C_{pgi} V_g + C_{gj} C_{pgj} V_g \} + (\rho_l V_v + \rho_v V_g) L \quad (10)$$

where  $\lambda$  is the coefficient of thermal conductivity of the partially saturated soil.

Using Dalton's law, the molecular concentration of dry air  $C_{ga}$  can be stated as;

$$C_{ga} = \frac{P_g}{RT} - \frac{\rho_v R_v}{R} - C_{gi} - C_{gj} \quad (11)$$

Substituting Eqs. 9 and 10 into the energy conservation equation, Eq. 8, and introducing the expression for the molecular concentration of dry air, Eq. 11 yields the governing partial differential equation for the flow of heat in a partially saturated soil;

$$C_{21} \frac{\partial P_w}{\partial t} + C_{22} \frac{\partial T}{\partial t} + C_{23} \frac{\partial P_g}{\partial t} + C_{24} \frac{\partial C_{gi}}{\partial t} + C_{25} \frac{\partial C_{gj}}{\partial t} \quad (12)$$

$$= \nabla \cdot (K_{21} \nabla P_w) + \nabla \cdot (K_{22} \nabla T) + \nabla \cdot (K_{23} \nabla P_g) + \nabla \cdot (K_{24} \nabla C_{gi}) + \nabla \cdot (K_{25} \nabla C_{gj})$$

where,

$$C_{21} = (T - T_r) \left\{ A_{T1} \frac{\partial S_l}{\partial P_w} + \phi S_g C_{pga} \frac{\partial C_{ga}}{\partial P_w} + \phi S_g C_{pv} \rho_o \frac{\partial h}{\partial P_w} + \phi S_l C_{pl} \frac{\partial \rho_l}{\partial P_w} \right\} - \phi \rho_v L \frac{\partial S_l}{\partial P_w} + \phi S_g L \rho_o \frac{\partial h}{\partial P_w}$$

$$C_{22} = H + (T - T_r) \left\{ A_{T1} \frac{\partial S_l}{\partial T} + \phi S_g C_{pga} \frac{\partial C_{ga}}{\partial T} + \phi S_l C_{pl} \frac{\partial \rho_l}{\partial T} + \phi S_g C_{pv} (h\beta + \rho_o \frac{\partial h}{\partial P_w}) \right\} - \phi \rho_v L \frac{\partial S_l}{\partial T} + \phi S_g L (h\beta + \rho_o \frac{\partial h}{\partial T})$$

$$C_{23} = (T - T_r) \left\{ A_{T1} \frac{\partial S_l}{\partial P_g} + \phi S_g C_{pga} \frac{\partial C_{ga}}{\partial P_g} + \phi S_g C_{pv} \rho_o \frac{\partial h}{\partial P_g} + \phi S_l C_{pl} \frac{\partial \rho_l}{\partial P_g} \right\} - \phi \rho_v L \frac{\partial S_l}{\partial P_g} + \phi S_g L \rho_o \frac{\partial h}{\partial P_g}$$

$$C_{24} = (T - T_r) \left\{ A_{T1} \frac{\partial S_l}{\partial P_g} + \phi S_g C_{pga} \frac{\partial C_{ga}}{\partial P_g} + \phi S_g C_{pv} \rho_o \frac{\partial h}{\partial P_g} + \phi S_l C_{pl} \frac{\partial P_l}{\partial P_g} \right\} - \phi \rho_v L \frac{\partial S_l}{\partial P_g} + \phi S_g L \rho_o \frac{\partial h}{\partial P_g}$$

$$C_{25} = (T - T_r) \left\{ \phi S_g C_{pgi} + \phi S_g C_{pga} \frac{\partial C_{ga}}{\partial C_{gi}} \right\}$$

$$C_{26} = (T - T_r) \left\{ \phi S_g C_{pgj} + \phi S_g C_{pga} \frac{\partial C_{ga}}{\partial C_{gj}} \right\}$$

And,

$$A_{T1} = \phi \rho_L C_{pl} - \phi C_{pv} \rho_v - \phi C_{pga} C_{ga} - \phi C_{pgi} C_{gi} - \phi C_{pgj} C_{gj}$$

$$K_{21} = \rho_l L K_{v1} + C_{pv} \rho_l K_{v1} (T - T_r) + \frac{C_{pl} \rho_l K_w}{\gamma_l} (T - T_r)$$

$$K_{22} = \lambda + \rho_l L K_{v2} + C_{pv} \rho_l K_{v2} (T - T_r)$$

$$K_{23} = \rho_l L K_{v3} + \rho_v L K_g + C_{pv} \rho_l K_{v3} (T - T_r) + (T - T_r) K_g (C_{pv} \rho_v + C_{pga} C_{ga} + C_{pgi} C_{gi} + C_{pgj} C_{gj})$$

$$K_{24} = K_{25} = 0$$

$$K_{26} = L \rho_v \rho_g K_g + C_{pl} \rho_l K_w (T - T_r) + (T - T_r) \rho_g K_g (C_{pv} \rho_v + C_{pga} C_{ga} + C_{pgi} C_{gi} + C_{pgj} C_{gj})$$

### 2.3 Dry Air Transfer

Dry air in unsaturated soils can be considered to exist in two forms, bulk air and dissolved air [Fredlund and Rahardjo (1993)]. The transfer of bulk air is driven by a gradient in pore air pressure whilst the dissolved air transfer is coupled to the flow of pore liquid. Henry's law is used to define the proportion of dry air contained within the pore liquid.

Applying the law of mass conservation to the flow of dry air within the pores of the soil yields;

$$\frac{\partial \{ \phi C_{ga} (S_g + H_a S_l) \}}{\partial t} = \nabla \{ (D_{ga} + H_a D_{la}) \nabla C_{ga} \} - \nabla \{ (V_g + H_a V_l) C_{ga} \} \quad (13)$$

where  $H_a$  is Henry's volumetric coefficient of solubility,  $D_{ga}$  is the diffusivity coefficient of dry air and  $D_{la}$  is the hydrodynamic dispersion coefficient of dry air

Introducing the expressions for the liquid velocity Eq. 4, gas velocity Eq. 5 and the molecular concentration of dry

air Eq. 11, into the conservation equation Eq. 13 gives the governing equation for dry air transfer;

$$C_{31} \frac{\partial P_w}{\partial t} + C_{32} \frac{\partial T}{\partial t} + C_{33} \frac{\partial P_g}{\partial t} + C_{34} \frac{\partial C_{gi}}{\partial t} + C_{35} \frac{\partial C_{gj}}{\partial t} = \nabla \cdot (K_{31} \nabla P_w) + \nabla \cdot (K_{32} \nabla T) + \nabla \cdot (K_{33} \nabla P_g) + \nabla \cdot (K_{34} \nabla C_{gi}) + \nabla \cdot (K_{35} \nabla C_{gj}) + \nabla \cdot (K_{36} \nabla Z) \quad (14)$$

Where,

$$C_{31} = \phi (S_g + H_a S_l) \frac{\partial C_{ga}}{\partial P_w} + \phi C_{ga} (H_a - 1) \frac{\partial S_l}{\partial P_w}$$

$$C_{32} = \phi (S_g + H_a S_l) \frac{\partial C_{ga}}{\partial T} + \phi C_{ga} (H_a - 1) \frac{\partial S_l}{\partial T}$$

$$C_{33} = \phi (S_g + H_a S_l) \frac{\partial C_{ga}}{\partial P_g} + \phi C_{ga} (H_a - 1) \frac{\partial S_l}{\partial P_g}$$

$$C_{34} = \phi (S_g + H_a S_l) \frac{\partial C_{ga}}{\partial C_{gi}}$$

$$C_{35} = \phi (S_g + H_a S_l) \frac{\partial C_{ga}}{\partial C_{gj}}$$

$$K_{32} = (D_{ga} + H_a D_{la}) \frac{\partial C_{ga}}{\partial T}$$

$$K_{33} = (D_{ga} + H_a D_{la}) \frac{\partial C_{ga}}{\partial P_w} + C_{ga} K_g$$

$$K_{34} = (D_{ga} + H_a D_{la}) \frac{\partial C_{ga}}{\partial C_{gi}}$$

$$K_{35} = (D_{ga} + H_a D_{la}) \frac{\partial C_{ga}}{\partial C_{gj}}$$

$$K_{36} = K_w H_a C_{ga} + K_g C_{ga} \rho_g$$

### 2.4 Contaminant Gas Transfer

The application of the molar concentration of gases is more convenient for gas movement than the mass density of gases. The mass density of the multi-component mixture of gases is highly dependent upon the relative concentrations of the component gases, whereas, the molar concentration is independent of the relative molar concentrations of the component gases [Metcalf and Farquhar (1987)].

The gas phase is a multi-component mixture of dry air, water vapour and the contaminant gases (carbon dioxide and methane). The gas mixture is assumed to be inert, i.e. gas-gas and gas-soil reactions are ignored.

### 2.4.1 Carbon Dioxide (CO<sub>2</sub>)

Invoking the principle of mass conservation for the carbon dioxide phase and assuming that gas and liquid velocities of the contaminant gas equal those of the gas mixture gives,

$$\frac{\partial\{\phi C_{gi}(S_g + H_i S_l)\}}{\partial t} = \nabla\{(D_{gi} + H_i D_{li})\nabla C_{gi}\} - \nabla\{(V_g + H_i V_l)C_{gi}\} \quad (15)$$

where  $D_{gi}$ ,  $D_{li}$ , and  $H_i$  are the effective diffusion coefficient, the hydrodynamic dispersion coefficient and Henry's law coefficient for carbon dioxide respectively.

The effective diffusion coefficient of carbon dioxide in a multi-component system can be determined from the diffusion coefficient in a binary gas mixture [Reid and Sherwood (1966)] and is defined as;

$$D_{gi} = \frac{1 - X_{gi}}{\left(\frac{X_{ga}}{D_{ai}} + \frac{X_{gv}}{D_{iv}} + \frac{X_{gj}}{D_{ij}}\right)} \quad (16)$$

where  $X_{gi}$ ,  $X_{gj}$ ,  $X_{gv}$  and  $X_{ga}$  are the molar fractions of carbon dioxide, methane, water vapour and dry-air respectively, and  $D_{ij}$ ,  $D_{iv}$  and  $D_{ai}$  are binary diffusion coefficients of methane, water vapour and dry-air in carbon dioxide respectively.

A tortuosity factor is applied to the binary diffusion coefficient [Fredlund and Rahardjo (1993)] to take into account the restrictions within the soil pores. For example,  $D_{ij}$  is determined from the equation,

$$D_{ij} = \tau D_{ij}^o \quad (17)$$

in which the tortuosity factor  $\tau$  is defined as [Millington and Quirk (1961)];

$$\tau = \frac{(\phi S_g)^{10\beta}}{\phi^2} \quad (18)$$

and  $D_{ij}^o$  is defined as [Bird, Stewart and Lightfoot (1960)];

$$D_{ij}^o = D_{ijr}^o \left(\frac{P_{gr}}{P_g}\right) \left(\frac{T}{T_r}\right)^{1.823} \quad (19)$$

where  $P_{gr}$  and  $D_{ijr}^o$  are the reference pressure and reference diffusivity respectively.

The hydrodynamic dispersion coefficient is defined as;

$$D_{li} = D_{lmi} + D_{ldi} \quad (20)$$

where  $D_{lmi}$  and  $D_{ldi}$  are the effective molecular diffusion coefficient of the dissolved carbon dioxide and the coefficient of mechanical dispersion respectively.

Expanding the mass conservation equation, Eq. 15, and introducing the liquid and gas velocities, Eqs. 4 and 5, yields the governing partial differential equation for the movement of carbon dioxide within a porous medium;

$$\begin{aligned} C_{41} \frac{\partial P_w}{\partial t} + C_{42} \frac{\partial T}{\partial t} + C_{43} \frac{\partial P_g}{\partial t} + C_{44} \frac{\partial C_{gi}}{\partial t} + C_{45} \frac{\partial C_{gj}}{\partial t} \\ = \nabla \cdot (K_{41} \nabla P_w) + \nabla \cdot (K_{42} \nabla T) + \nabla \cdot (K_{43} \nabla P_g) \\ + \nabla \cdot (K_{44} \nabla C_{gi}) + \nabla \cdot (K_{45} \nabla C_{gj}) + \nabla \cdot (K_{46} \nabla Z) \end{aligned} \quad (21)$$

Where,

$$C_{41} = \phi C_{gi} (H_i - 1) \frac{\partial S_l}{\partial P_w}$$

$$C_{42} = \phi C_{gi} (H_i - 1) \frac{\partial S_l}{\partial T}$$

$$C_{43} = \phi C_{gi} (H_i - 1) \frac{\partial S_l}{\partial P_g}$$

$$C_{44} = \phi S_g + H_i \phi S_l$$

$$C_{45} = 0$$

$$K_{41} = \frac{K_w C_{gi} H_i}{\gamma_l}$$

$$K_{42} = 0$$

$$K_{43} = K_g C_{gi}$$

$$K_{44} = D_{gi} + H_i D_{li}$$

$$K_{45} = 0$$

$$K_{46} = K_w H_i C_{gi} + C_{gi} \rho_{gg} K_g$$

### 2.4.2 Methane (CH<sub>4</sub>)

The derivation for the governing equation for methane transfer follows the same procedure and is omitted to avoid repetition. Henry's law coefficient for methane is denoted by  $H_j$ . The governing equation is given by;

$$\begin{aligned} C_{51} \frac{\partial P_w}{\partial t} + C_{52} \frac{\partial T}{\partial t} + C_{53} \frac{\partial P_g}{\partial t} + C_{54} \frac{\partial C_{gi}}{\partial t} + C_{55} \frac{\partial C_{gj}}{\partial t} \\ = \nabla \cdot (K_{51} \nabla P_w) + \nabla \cdot (K_{52} \nabla T) + \nabla \cdot (K_{53} \nabla P_g) \\ + \nabla \cdot (K_{54} \nabla C_{gi}) + \nabla \cdot (K_{55} \nabla C_{gj}) + \nabla \cdot (K_{56} \nabla Z) \end{aligned} \quad (22)$$

Where,

$$C_{51} = \phi C_{gj}(H_j - 1) \frac{\partial S_l}{\partial P_w}$$

$$C_{52} = \phi C_{gj}(H_j - 1) \frac{\partial S_l}{\partial T}$$

$$C_{53} = \phi C_{gj}(H_j - 1) \frac{\partial S_l}{\partial P_g}$$

$$C_{54} = \phi S_g + H_j \phi S_l$$

$$C_{55} = 0$$

$$K_{51} = \frac{K_w C_{gj} H_j}{\gamma_l}$$

$$K_{52} = 0$$

$$K_{53} = K_g C_{gj}$$

$$K_{55} = 0$$

$$K_{54} = D_{gj} + H_j D_{lj}$$

$$K_{56} = K_w H_j C_{gj} + C_{gj} \rho_g g K_g$$

### 3 Finite Element Formulation

The variation of the system variables, pore water pressure  $P_w$ , temperature  $T$ , pore gas pressure  $P_g$ , molar concentration of methane  $C_{gi}$  and the molar concentration of methane  $C_{gj}$ , throughout the domain of interest  $\Omega$  can be approximated in terms of the nodal values;

$$\phi \approx \sum_{s=1}^{s=n} N_s(x, y) \phi_s \quad (23)$$

If the approximations given by Eq. 23 are substituted into Eq. 7, a residual is obtained, which is then minimized using the Galerkin method. This requires that the integral of the weighted errors over the domain  $\Omega$  must be zero, with the standard finite element shape functions  $N_s$  being employed as the weighting function. That is,

$$\begin{aligned} & \int_{\Omega} N_r \left\{ \nabla \cdot (K_{11} \nabla P_w) + \nabla \cdot (K_{12} \nabla T) + \nabla \cdot (K_{13} \nabla P_g) \right. \\ & + \nabla \cdot (K_{14} \nabla C_{gi}) + \nabla \cdot (K_{15} \nabla C_{gj}) + \nabla \cdot (K_{16} \nabla Z) \\ & - C_{11} \frac{\partial(P_w)}{\partial t} - C_{12} \frac{\partial(T)}{\partial t} - C_{13} \frac{\partial(P_g)}{\partial t} - C_{14} \frac{\partial(C_{gi})}{\partial t} \\ & \left. - C_{15} \frac{\partial(C_{gj})}{\partial t} \right\} d\Omega = 0 \end{aligned} \quad (24)$$

The application of Green's theorem to reduce the order of the equations, and the introduction of the nodal approximations, Eq. 23, into Eq. 24 a relation which can be expressed in matrix form, when all governing equations are considered, as;

$$[K] \{\phi\} + [C] \{\dot{\phi}\} + [J] = 0 \quad (25)$$

In which typical elements of the matrices  $[K]$ ,  $[C]$ ,  $[J]$  and  $\{\phi\}$  are;

$$K_{ij} = \sum_{s=1}^n \int_{\Omega} K_{ij} \left( \frac{\partial N_r}{\partial x} \frac{\partial N_s}{\partial x} + \frac{\partial N_r}{\partial y} \frac{\partial N_s}{\partial y} \right) d\Omega$$

$$C_{ij} = \sum_{s=1}^n \int_{\Omega} C_{ij} N_r N_s d\Omega$$

$$J_i = \sum_{s=1}^n \int_{\Gamma} \sum_{j=1}^5 K_{ij} N_r N_s \nabla \phi_j' \cdot \bar{n} \cdot d\Gamma$$

$$\{\phi\} = [P_w, T, P_g, C_{gi}, C_{gj}]^T$$

The matrix equation, Eq. 25, describes a system of first order differential equations. The temporal discretisation is achieved by replacing the time derivative with a finite difference approximation using a backward two-level time stepping scheme, to yield;

$$\left\{ [K]^{n+1} + \frac{[C]^{n+1}}{\Delta t} \right\} \cdot [\phi]^{n+1} = \left\{ \frac{[C]^{n+1} [\phi]^n}{\Delta t} - [J]^{n+1} \right\} \quad (26)$$

The superscript  $n$  refers to the time level and  $\Delta t$  represents the time step. It can be seen from Eq. 26 that the solution for  $\phi$  at time level  $n + 1$  can be obtained directly from the stiffness  $K$  and capacity  $C$  matrices, and  $\phi$  at time level  $n$ , which is known.

The algorithm is iterative due to the non-linear nature of the kinetic and capacity coefficients. A converged solution is deemed to have achieved when:

$$|\phi_{s+1}^{n+1} - \phi_s^{n+1}| < \varepsilon \quad (27)$$

at all nodes where  $\varepsilon$  is a prescribed tolerance and subscript  $s + 1$  is the current iteration number. In order to stabilise and to increase the convergence rate of the iterative procedure, it is often desirable to slow down the changes that occur in the system variables from one iteration to

the next. This process is known as under-relaxation and is given by:

$$\phi_{s+1} = \alpha\phi_s + (1 - \alpha)\phi_{s-1} \quad \text{where} \quad 0 < \alpha \leq 1 \quad (28)$$

The system of simultaneous equations, generated within each iteration, is diagonally dominant and block symmetric and was solved using a skyline solver. The verification of the complete new formulation has been achieved by satisfactory comparison of analytical solutions and experimental results presented in the literature. The validity of the model results has been confirmed against experimental data and it has showed a good agreement with the experimental results [Palanathakumar and Ferguson (2001 a,b)].

#### 4 Results and Discussion

The landfill gas explosion that occurred in 1986 in Loscoe, Derbyshire is a well-known example of the danger caused by the uncontrolled migration of landfill gas. The location and history of the landfill site is presented in Williams and Aitkenhead (1991). The landfill site had previously been used as a brick quarry for over a century and then, consent was given for the disposal of inert waste in 1973 after the closure of the brickworks. During this period, a housing development had come to surround it on all sides. In 1977, a waste disposal license was granted under the provision of the Control of Pollution Act, 1974, allowing 50 tonnes per day of domestic waste to be deposited. Over the years, the waste deposition rate was increased up to 100 tonnes per day and continued at that level until 1982 when all tipping was ceased [Ryan, King and Munday (1988)].

At 06.30 on 24<sup>th</sup> March 1986, a bungalow at 51 Clarke avenue in Loscoe was completely destroyed by a methane gas explosion. Gas samples taken from the wreckage soon after the explosion were found to be generally similar to landfill gas which is typically comprised of 60% methane and 40% carbon dioxide. Attention was drawn to the Loscoe landfill whose boundary lay approximately 60m from the bungalow (see Fig. 1). In 1984, shortly before the explosion, the landfill had been capped with a layer of compacted clay which might have prevented the release of the landfill gas from the surface of landfill and encouraged the lateral migration of the landfill gas [Williams and Aitkenhead (1991)]. However, despite the public inquiry called by Derbyshire Country

Council in November 1986, the exact reason for the explosion has never been determined. In this section, the proposed model for landfill gas migration has been applied to find the landfill gas profile in the subsurface of the surrounding soil in the time leading up to the explosion.

The geological cross section of the landfill site shows there are potential migration pathways for landfill gas through the geo-medium beneath Clarke Avenue. The rectangular section through the centre of the landfill, shown in Fig. 1, with 35m depth and 150m length was investigated using the numerical model with the best available data. This section of the geo-medium has five different types of solid medium i.e. sandstone, coal, seatearth, mudstone and siltstone. Since no measurements of hydraulic properties and thermal properties were available appropriate values were directly taken from the literature and are presented in Tab. 1.

The characteristics of the water retention curve of all the solid media are considered to be comparable to a general rock. The best prediction of the water retention curve and relative hydraulic conductivity was obtained using the van Genuchten-Mualem model [van Genuchten (1980)];

$$Se = [1 + (\alpha h)^n]^{-m} \quad (29)$$

$$k_{rl} = Se^l \left[ 1 - (1 - Se^{\frac{1}{m}})^m \right]^2 \quad (30)$$

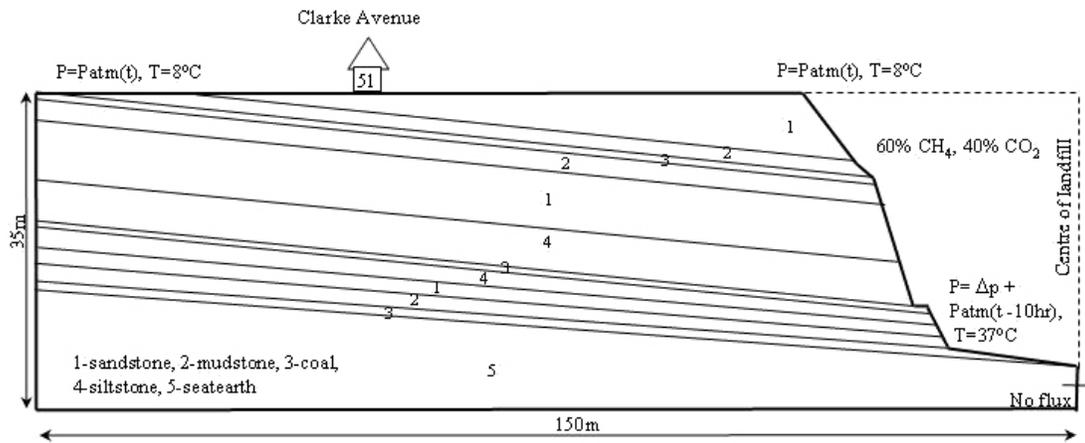
where  $h$ ,  $Se$  and  $k_{rl}$  are the capillary potential (cm), effective water saturation and relative hydraulic conductivity respectively. The constants  $\alpha$ ,  $l$ ,  $m$  and  $n$  are  $0.0058 \text{ cm}^{-1}$ ,  $0.5$ ,  $0.6302$ , and  $2.7044$  respectively. These constants were determined from the water retention characteristic of sandstone, which is shown in Fig. 2, using the inverse modelling technique in RETC developed by van Genuchten, Simunek, Leij and Sejna (1992). The same constants were used to compute the relative gas permeability  $k_{rg}$  following the method adopted by Parker, Lenhard and Kappusamy (1987);

$$k_{rg} = (1 - Se)^l \left[ (1 - Se^{\frac{1}{m}})^m \right]^2 \quad (31)$$

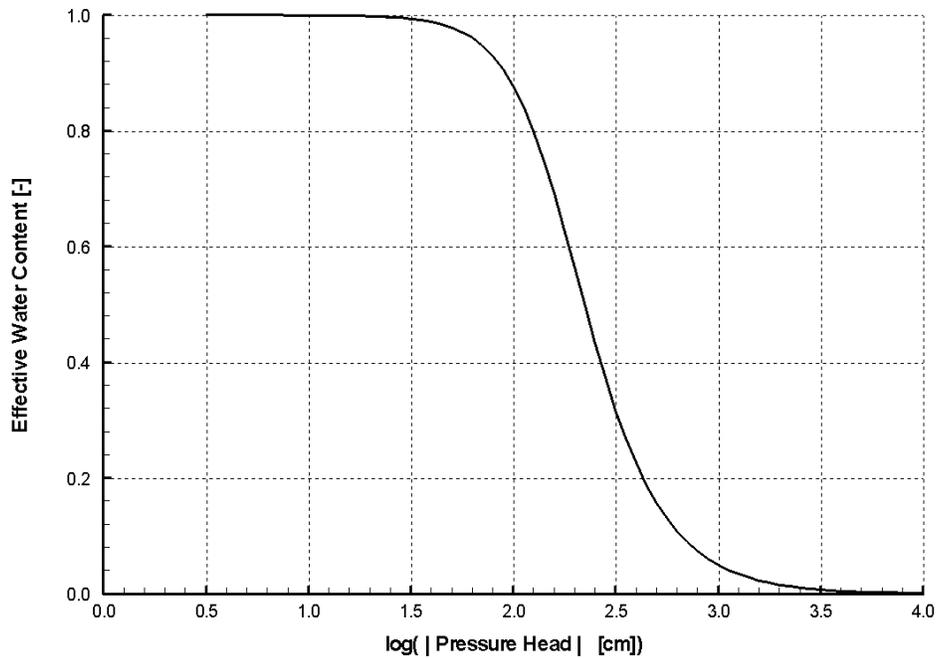
The geometry of the domain of interest and the finite element mesh are shown in Fig. 3. The mesh is comprised of 500 linear quadrilateral elements. Initial conditions throughout the domain under investigation were assumed

**Table 1** : Material properties of the solid medium

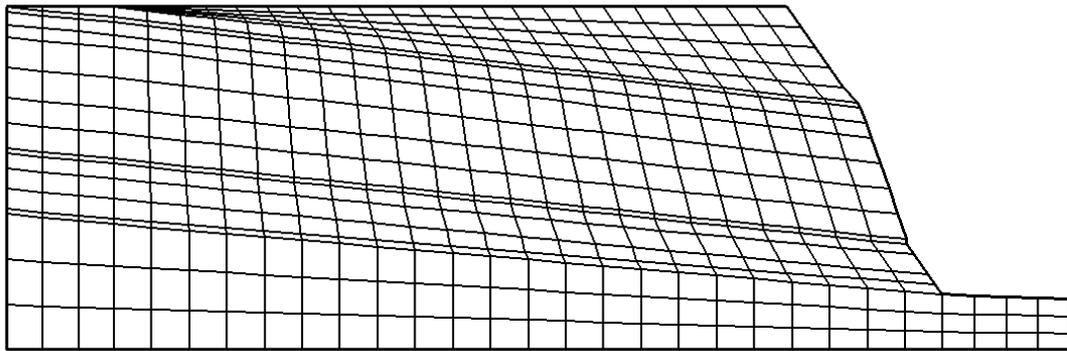
	Porosity $\phi$	Particle density $\rho_s(\text{kg/m}^3)$	Intrinsic permeability $K (\text{m}^2)$	Thermal Conductivity $\lambda_t (\text{W/m.K})$	Heat Capacity $C_{ps}(\text{J/kg.K})$
Sandstone	0.3	2500	$8.0 \times 10^{-13}$	4.326	858
Seatearth	0.1	2150	$10^{-15}$	$1.76+0.3S_l$	838
Coal	0.1	853	$10^{-14}$	$1.76+0.3S_l$	838
Mudstone	0.15	2400	$2.14 \times 10^{-15}$	1.6	700
Siltstone	0.25	2500	$2 \times 10^{-14}$	$0.5+1.5S_l$	838



**Figure 1** : Schematic cross section of the interested domain



**Figure 2** : Variation of effective water content against capillary pressure in the rock



**Figure 3** : Finite element mesh for simulation of gas migration

to be uniform and are pore-water pressure 40000Pa, temperature 8°C, total gas pressure 101325Pa, and the molar concentrations of both methane and carbon dioxide 0%.

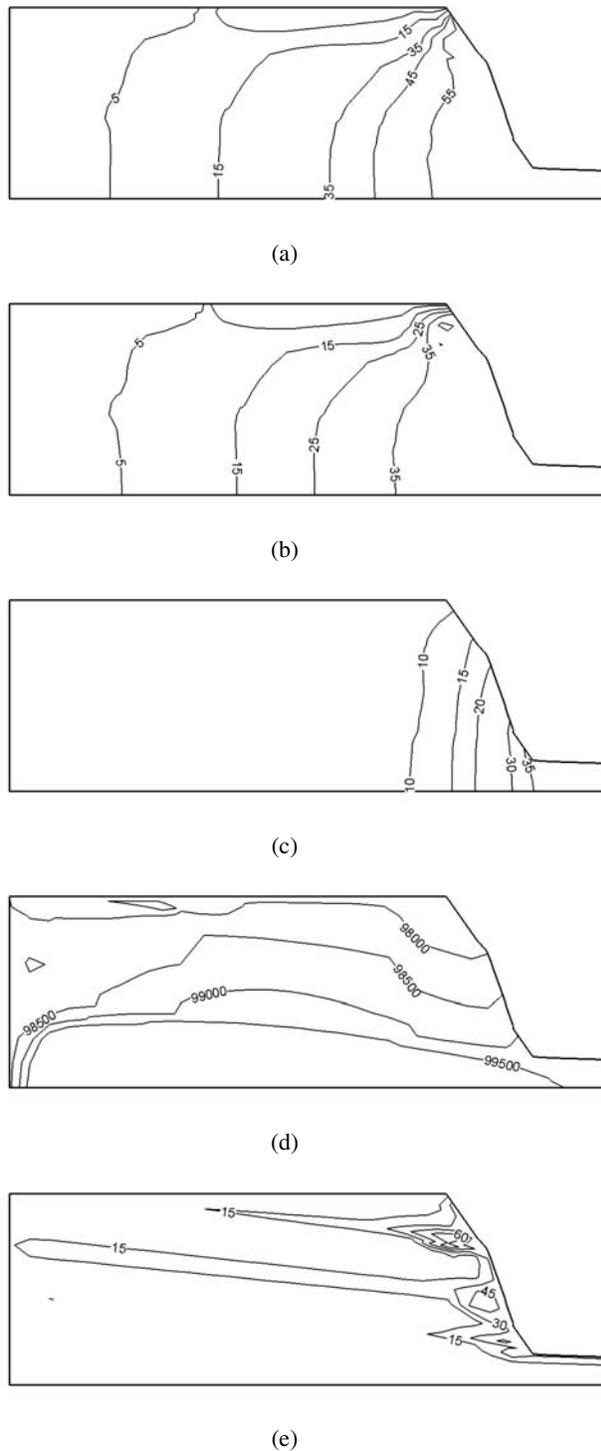
Meteorological office reports showed that the atmospheric pressure,  $P_{atm}$  varied during the period prior to the incident and is shown in Fig. 5. The average temperature was 8°C in the area surrounding the landfill [Parker, Legg and Folland (1992)]. Based on past investigations of landfill sites, the gas concentrations at the edge of the landfill were assumed to be 60% CH<sub>4</sub> and 40% CO<sub>2</sub> [Kjeldsen and Fischer (1995), Farquhar and Rover (1973)]. The gas pressure at the lowest point within the interior of the landfill was assumed to be  $\Delta P + P_{atm}$  (t-10 hrs) where  $\Delta P$  is the average pressure above atmospheric pressure [Poulsen, Christophersen, Moldrup and Kjeldsen (2001)]. In this case, 20 millibars above atmospheric has been reported during the drilling of relief wells at the landfill site to avoid a further explosion [Williams and Aitkenhead (1991)]. The pressure along the right boundary was allowed to vary freely between this value and the atmospheric pressure at the surface. Similarly, the temperature was also assumed to vary between 37°C at the lowest point of the landfill and 8°C at the surface. The gas flux through the right boundary between the lowest point of the landfill and right bottom corner of the domain was assumed to be zero due to symmetry. The left is boundary placed far enough away to have no significant impact on the results and a no flow boundary condition was imposed for all system variables.

The simulation of the Loscoe incident was run for 4 years, for the period 1982-1986, with initial, minimum and maximum time step of 1s, 0.001s and 2 days respectively. The profiles of the molar concentration of

methane and carbon dioxide, pore-air pressure, temperature, water saturation throughout the domain at the time of explosion are shown in Fig. 4. As methane is lighter than air, its tendency of moving upward (Fig. 4a) was faster than carbon dioxide (Fig. 4b). Fig. 4c shows that the temperature of the landfill has an influence on the soil temperature up to 50m away from the centre of landfill. Capillary action caused by low porosity of the underlying soil has developed a high water content along the layer of coal seams and seat earth, Fig. 4d.

The molar fraction of both methane and carbon dioxide at the location of the explosion, which is around 100 m away from the centre of the landfill, is between 5% and 15%, Fig. 4a and 4b. A private company hired by Derbyshire council to carry out gas measurements has reported 32.9 % CH<sub>4</sub> and 30.1% CO<sub>2</sub> on 10<sup>th</sup> March 1986 (a fortnight before the explosion) at 51 Clarke Avenue. On 29<sup>th</sup> March 1986, 5 days after the explosion, the gas concentrations were measured as 13.5% CH<sub>4</sub> and 12.6% CO<sub>2</sub> [Aitkenhead and Williams (1986)]. The first measurement is relatively high whilst the latter is very close to the model prediction shown in Fig. 4a and 4b. The accumulation of landfill gas under the building foundation could be the reason of the high concentration prior to the explosion.

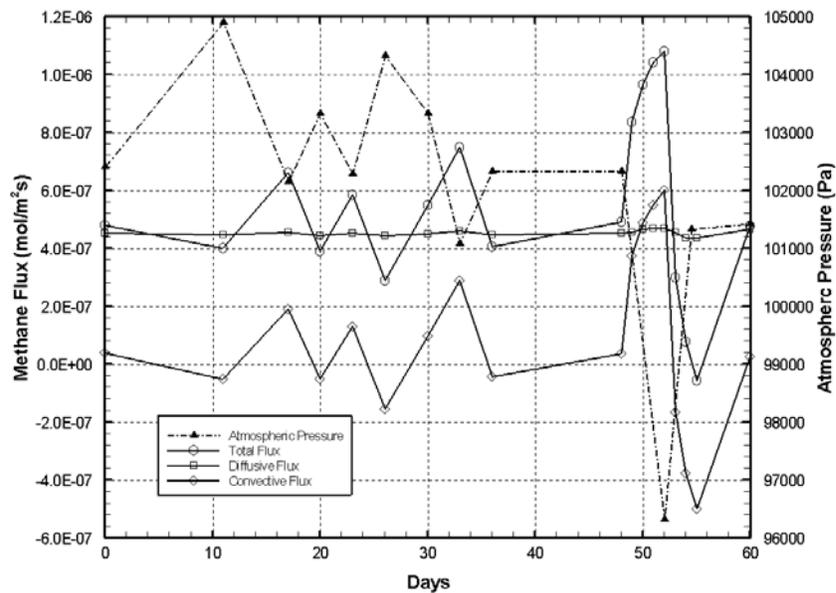
Fig. 5 shows the predicted total methane flux in an upwards vertical direction and its contributions from the different gas transport mechanisms, located at 1.5m below the explosion site, simultaneously with the variation of atmospheric pressure [Young (1992b)] during the last 60 days before the explosion. Convection, diffusion and dissolution are the basic transport mechanisms of gas considered in this model. The contribution of dis-



**Figure 4 :** (a) Molar fraction of CH<sub>4</sub> (%), (b) Molar fraction of CO<sub>2</sub> (%), (c) Temperature (°C), (d) Pressure (Pa) and (e) Water saturation (%) at the time of explosion

solved gas in the total flux is negligible at the surface ( $\approx 10^{-11}$  mol/m<sup>2</sup>/s, Fig. 6c) and is not shown in Fig. 5. The convective gas flux proved very sensitive to the variation in the atmospheric pressure, hence, the total gas flux varied with changes in the barometric pressure. A change in atmospheric pressure creates a temporary pressure gradient between the inner soil and the surface soil layers until the pore air pressure adjusts to the new atmospheric pressure [Christophersen and Kjeldsen (2001)]. Fig. 5 clearly shows that diffusion is the dominant transport mechanism of the surface gas flux during periods of relatively constant atmospheric pressure. This is due to a large difference in methane concentration between the surface layer and the ambient air. At the time of the explosion (52<sup>nd</sup> day in Fig. 5), the sudden drop of barometric pressure resulted in the convective gas flux becoming the dominant gas transport mechanism and, consequently, the total methane flux has been escalated to more than two times of the average gas flux. This provides very strong support for the belief that the pressure drop was the primary trigger factor for the explosion at Loscoe.

Diffusive, convective and dissolved flux of methane throughout the domain of interest at a time of constant atmospheric pressure are shown in Fig. 6 where arrows illustrate the directions of flow whilst contours illustrate the magnitudes of flow in mol/m<sup>2</sup>/s. Fig. 6a shows that the diffusive flux is dominant at the surface layers which are distant from the landfill. In contrast, Fig. 6b shows that the convective flux is dominant close to the landfill and negligible within low permeability layers. Close to the ground surface, the directions of both diffusive and convective flux are vertically upwards. Fig. 6c shows that the dissolved gas moves downwards and shows little influence on the lateral migration of methane. Hence, it is evident that convection is the primary mechanism in releasing gas from the landfill due to the high-pressure gradient between inner and outer edges of the landfill. Diffusion and dissolution in soil moisture carry the released gas away from the landfill boundary. The construction of a basal, perimeter and capping liner with low permeability materials around landfills to avoid convection, the dominant mechanism of gas release from landfills, can minimise the uncontrolled migration of methane gas to the area surrounding the landfills.



**Figure 5 :** Vertical flux of methane ( $\text{mol}/\text{m}^2/\text{Sec}$ ) at beneath of the exploded bungalow and atmospheric pressure during 60 days prior to gas explosion.

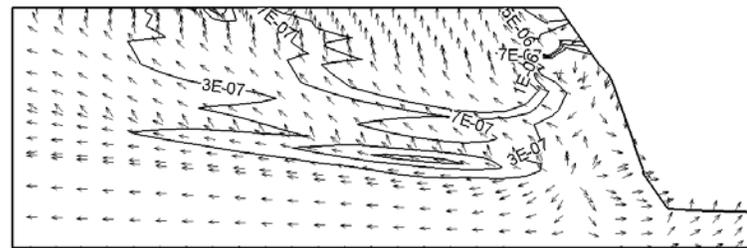
## 5 Conclusions

The objective of this paper was to present a fully coupled heat and mass transfer model for the movement of land-fill gas through partially saturated soil. The model incorporates four fluid components (methane, carbon dioxide, dry air and moisture) and an energy component (heat). A mechanistic approach was adopted to develop the governing system of five coupled non-linear partial differential equations. The resulting system of equations, in terms of pore water pressure, temperature, total gas pressure and the molar concentrations of methane and carbon dioxide, was solved numerically using the Galerkin finite element method to describe spatial variations and a finite difference recurrence relationship for the temporal discretisation.

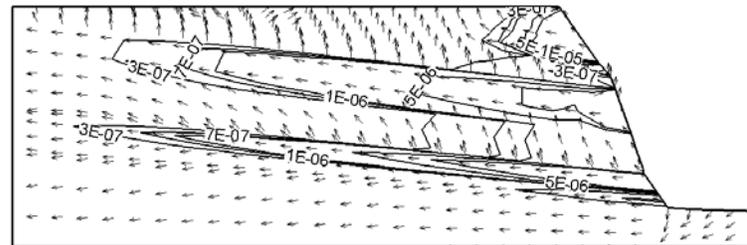
The proposed model was used to simulate the subsurface of the Loscoe landfill for the period leading up to the explosion. The subsurface gas profiles at the site of Loscoe landfill gas explosion were predicted. The result clearly showed that the methane concentration at the explosion location was within the explosive range, i.e. 5 - 15%. In the area immediately surrounding the landfill site, convection was the dominant gas transport mechanism, whereas, further afield, diffusion was the dominant mechanism.

## 6 References:

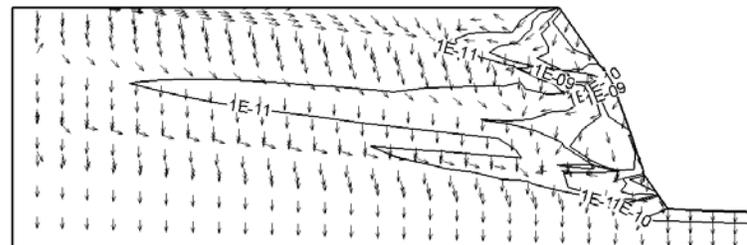
- Aitkenhead, N.; Williams, G.M.** (1986): *Geological Evidence to the Public Inquiry into the Gas Explosion at Loscoe*. British Survey, Report no. FP/87/8/83AS.
- Atluri, S.N.; Han, Z.D.; Rajendran, A.M.** (2004): A new implementation of the meshless finite volume method, through the MLPG "mixed" approach, *CMES: Computer Modeling in Engineering and Sciences*, vol. 6, no. 6, pp. 491-513.
- Atluri, S.N.; Zhu, T.** (1998): A new meshless Local Petrov-Galerkin (MLPG) approach in computational mechanics, *Computational Mechanics*, vol. 22, pp. 117-127.
- Batra, R.C.; Porfiri, M.; Spinello, D.** (2004): Treatment of material discontinuity in two meshless Petrov-Galerkin (MLPG) formulations of axisymmetric transient heat conduction, *International Journal for Numerical Methods in Engineering*, vol. 61, pp. 2461-2479.
- Belytschko, T.; Lu, Y.Y.; Gu, L.** (1994): Element-free Galerkin methods, *International Journal for Numerical Methods in Engineering*, vol. 37, pp. 229-256.
- Bird, R.B.; Stewart, W.E.; Lightfoot, E.N.** (1960): *Transport Phenomena*, John Wiley, New York.
- Christophersen, M.; Kjeldsen, P.** (2001): Lateral gas transport in soil adjacent to an old landfill: Factors gov-



(a)



(b)



(c)

**Figure 6 :** (a) Diffusive, (b) Convective (c) Dissolved flux of methane throughout the domain of interest (mol/ m<sup>2</sup>/s)

erning gas migration, *Waste Management and Research*, vol. 19, no. 6, pp. 579-594.

**Christophersen, M.; Kjeldsen, P.; Holst, H.; Chanton, J.** (2001): Lateral gas transport in soil adjacent to an old landfill: Factors governing emissions and methane oxidation, *Waste Management and Research*, vol. 19, no. 6, pp.595-612.

**De Vries, D.A.** (1958): Simultaneous transfer of heat and moisture in porous media. *Transactions of the American Geophysical Union*, vol. 39, no. 5, pp. 909-916.

**Divo, E.; Kassab, A.J.** (2005): A meshless method for conjugate heat transfer problems, *Engineering Analysis with Boundary Elements*, vol. 29, pp. 136-149.

**Duarte, C.A.; Oden, J.T.** (1996): H-p clouds - an hp meshless method, *Numerical methods for Partial Differ-*

*ential Equations*, vol. 12, pp. 673-705.

**Edlefsen, N.E.; Andersen, A.B.C.** (1943): Thermodynamics of soil moisture. *Hilgardia*, vol. 15, no. 2, pp. 231- 298.

**El-Fadel, M.; Findikakis, A.N.; Leckie, J.O.** (1994): Numerical modelling of generation and transport of gas and heat in sanitary landfills: 1 Model formulation. *Waste Management & Research*, vol. 14, pp. 483-504.

**Ewen, J.; Thomas, H.R.** (1989): Heating unsaturated medium sand. *Geotechnique*, vol. 39, no. 3, pp. 455-470.

**Farquhar, G.J.; Rover, F.A.** (1973): Gas production during refuse decomposition. *Water, Air and Soil Pollution*, vol. 2, pp. 483-495.

**Findikakis, A.N.; Leckie, J.O.** (1979): Numerical sim-

- ulation of gas flow in sanitary landfills. *Journal of Environmental Engineering Division*, vol. 105, pp. 927-944.
- Fredlund, D.G.; Rahardjo, H.** (1993): *Soil mechanics for unsaturated soils*. John Wiley, New York.
- Han, Z.D.; Atluri, S.N.** (2004): Meshless local Petrov-Galerkin (MLPG) approaches for solving 3D problems in elasto-statics, *CMES: Computer Modeling in Engineering and Sciences*, vol. 6, no. 2, pp. 169-188.
- Hooker, P.J.; Bannon, M.P.** (1993): Methane: Its occurrence and hazards in the construction industry. *Construction Industries and Information Association (CIRIA)*, Report no. 130.
- Impey, M.; Grindrod, P.; Takase, H.; Worgan, K.J.** (1997): A capillary network model for gas migration in low-permeability media. *SIAM Journal of Applied Mathematics*, vol. 57, no. 3, pp. 597-608.
- Iske, A.; Käser, M.** (2005): Two-phase flow simulation by AMMoC, an adaptive meshfree method of characteristics, *CMES: Computer Modeling in Engineering and Sciences*, vol. 7, no. 2, pp. 133-148.
- Jakob, M.** (1949): *Heat transfer: Vol I*. John Wiley.
- Kjeldsen, P.; Fischer, E.** (1995): Landfill gas migration-field investigation at Skellingsted landfill, Denmark. *Waste Management and Research*, vol. 13, pp. 467-484.
- Lin, H.; Atluri, S.N.** (2001): The meshless local Petrov-Galerkin (MLPG) method for solving incompressible Navier-Stokes equations, *CMES: Computer Modeling in Engineering and Sciences*, vol. 2, no. 2, pp. 117-142.
- Lucy, L.B.** (1977): A numerical approach to the testing of fission hypothesis, *The Astronomical Journal*, vol. 82, no. 12, pp. 1013-1024.
- Mai-Cao, L.; Tran-Cong, T.** (2005): A meshless IRBFN-based method for transient problems, *CMES: Computer Modeling in Engineering and Sciences*, vol. 7, no. 2, pp. 149-171.
- Mayhew, Y.R.; Rogers, G.F.C.** (1976): *Thermodynamic and transport properties of fluids*. Blackwell, Oxford.
- Melenk, J.M.; Babuska, I.** (1996): The partition of unity finite element method: Basic theory and applications, *CMES: Computer Methods in Applied Mechanics and Engineering*, vol. 139, pp. 289-314.
- Metcalf, D.E.; Farquhar, G.J.** (1987): Modeling gas migration through unsaturated soils from waste disposal sites. *Water, Air and Soil Pollution*, vol. 32, pp. 247-259.
- Millington, R.J.; Quirk, J.P.** (1961): Permeability of porous media, *Transactions of the Faraday Society*, vol. 57, pp. 1200-1207.
- Mitchell, J.K.** (1993): *Fundamentals of soil behaviour*. John Wiley, New York.
- Moore, C.; Rai, I.S.; Alzaydi, A.A.** (1979): Methane migration around sanitary landfills. *Journal of Geotechnical Engineering Division*, vol. 105, no. 2, pp. 131-144.
- Nayroles, B.; Touzot, G.; Villon, P.** (1992): Generalising the finite element method: diffuse approximation and diffuse elements, *Computational Mechanics*, vol. 10, pp. 307-318.
- Palanankumar, B.; Ferguson, W.J.** (2001a): Migration of landfill gas: a numerical model. *Proceedings of 7<sup>th</sup> International Conference on Environmental Science and Technology*, Greece, pp. 682-689.
- Palanankumar, B.; Ferguson, W.J.** (2001b): A finite element analysis of landfill gas migration in unsaturated soil. *Proceedings of 3<sup>rd</sup> BGA Geoenvironmental Conference*, Edinburgh, pp. 355-360.
- Parker, D.E.; Legg, T.P.; Folland, C.K.** (1992): A new daily central England temperature series: 1772-1991. *International Journal of Climatology*, vol. 12, pp. 317-342.
- Parker, J.C.; Lenhard, R.J.; Kuppasamy, T.** (1987): A parametric model for constitutive properties governing multiphase flow in porous media. *Water Resources Research*, vol. 23, no. 4, pp. 618-624.
- Philip, J.R.; De Vries, D.A.** (1957): Moisture movement in porous materials under temperature gradients. *Transactions of the American Geophysical Union*, vol. 38, no. 2, pp. 2222-2232.
- Poulsen, T.G.; Christophersen, M.; Moldrup, P.; Kjeldsen, P.** (2001): Modeling lateral gas transport in soil adjacent to old landfill. *Journal of Environmental Engineering*, vol. 0127, no. 2, pp. 0145-0153.
- Ryan, G.; King, P.J.; Munday, G.** (1988): *Report of the non-statutory public inquiry into the gas explosion at Loscoe*. County Information Office, Derbyshire County Council.
- Reid, R.C.; Sherwood, T.K.** (1966): *The properties of gases and liquids*. McGraw-Hill, New York.
- Sladek, J.; Sladek, V.; Atluri, S.N.** (2004): Meshless local Petrov-Galerkin method for heat conduction problem in an isotropic medium, *CMES: Computer Modeling in Engineering and Sciences*, vol. 6, no. 3, pp. 309-318.

**Tchobanoglous, G.** (1993): *Integrated solid waste management*. McGraw-Hill, New York.

**Thomas, H.R.; Ferguson, W.J.** (1999): A fully coupled heat and mass transfer model incorporating contaminant gas transfer in an unsaturated porous medium. *Computers and Geotechnics*, vol. 24, pp. 65-87.

**van Genuchten, M.Th.** (1980): A closed-form equation for predicting the hydraulic conductivity of unsaturated soils. *Journal of the Soil Science Society of America*, vol. 44, pp. 892-898.

**van Genuchten, M.Th.; Simunek, J.; Leij, F.J.; Sejna, M.** (1992): *Code for quantifying the hydraulic functions of unsaturated soils, RETC*. US Salinity Laboratory, US-ADA, ARS, Riverside, CA 92507, US.

**Williams, G.M.; Aitkenhead, N.** (1991): Lessons from Loscoe-the uncontrolled migration of landfill gas. *Quarterly Journal of Engineering Geology*, vol. 24, pp. 191-207.

**Young, A.** (1992a): The effects of fluctuations in atmospheric pressure on landfill gas migration and composition. *Water Air and Soil Pollution*, vol. 64(3-4), pp. 601-616.

**Young, A.** (1992b): *CMES: Computer Modelling of Landfill*. DoE report no. CWM 039A/92; 163.

**Zhang, G.M.; Batra, R.C.** (2004): Modified smoothed particle hydrodynamics method and its application to transient problems, *Computational Mechanics*, vol. 34, pp. 137-146.

**Zienkiewicz, O.C.** (1977): *The Finite Element Method*, McGraw-Hill, London.

