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On Foundations of the Ultrasonic Non-Destructive Method of Determination of Stresses in Near-the-Surface Layers of Solid Bodies

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Abstract: The ultrasonic non-destructive method of determination of stresses in near-the-surface layers of solid bodies is based on the regularities of elastic surface wave propagation in bodies with initial (residual) stresses. Above mentioned regularities are received in the framework of the 3-D linearized theory of waves propagation in bodies with initial (residual) stresses. Computational methods are used for solution of the dispersion equations as applied to problems under consideration. Description of the non-destructive method and information on instruments and devices for measurements are presented. Some examples of non-destructive determination of stresses in near-the-surface layers of materials are presented also as applied to the residual stresses arising at electric welding and to the operating stresses arising at loading.

keyword: ultrasonic non-destructive method, regularities of surface waves propagation, bodies with initial (residual) stresses, main relationships of method, instruments and devices for measurements.

1 Introduction

Main results of this paper were received in the framework of the 3-D linearized theory of elastic waves propagation in bodies with initial (residual) stresses. This theory is presented in joined form for the theory of finite initial deformations and for two variants of the theory of small initial deformations. More detailed information on above mentioned approach may be received in [Guz (2004), Guz, Makhort (2000), Guz (2002)]. All results of this paper are presented according to following Table of Contents.

2. On three-dimensional theory of elastic waves in bodies with initial (residual) stresses. 2.1. Principles of the theory construction. 2.2. Main relationships. 2.3. General solutions under homogeneous initial (residual) states.

3. Main regularities of Rayleigh waves propagation in bodies with initial (residual) stresses. 3.1. Planar problem. Influence of initial (residual) stresses. 3.2. Axisymmetrical problem. Influence of initial (residual) stresses. 3.3. Rayleigh waves on a circular cylinder. Influence of initial (residual) stresses. 3.4. Rayleigh waves on a sphere. Influence of initial (residual) stresses. 3.5. General regularity.

4. Ultrasonic non-destructive method of determination of stresses in near-the-surface layers of solids. 4.1. Description of the non-destructive method. 4.2. On instruments and devices for measurements. 4.3. Verification of the non-destructive method. 4.4. Examples of non-destructive determination of uniaxial and two-axial stresses in near-the-surface layers of materials.

5. Conclusion. The approach of this paper and obtained results may be considered as the joined approach corresponding to solid mechanics, computational mechanics and experimental mechanics.

2 On three-dimensional theory of elastic waves in bodies with initial (residual) stresses

All results were received by linearization of the three-dimensional non-linear theory of elasticity in cases of finite and small deformations.

2.1 Principles of the theory construction.

Three state of the hyperelastic materials are considered.

First state corresponds to natural state (stresses and strains are absent).

Second state corresponds to initial or residual state (all values of this state are marked by index “0”).

Third state corresponds to disturbanced state. The values of third state are sums of the corresponding values of second state and the disturbances of the corresponding

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values. The values of disturbances are not marked by index. *It is assumed* the disturbances are small values and procedure of linearization is realized.

Above mentioned approach are considered as applied to any value x, value y and relationship $y = f(x)$ of non-linear theory of elasticity. These values and relationship for second state have the following form

$$y_0\,,\ x_0\,,\ y_0 = f(x_0). \tag{1}$$

These values and relationships for third state have the following form

$$y_0 + y,\ x_0 + x,\ y_0 + y = f(x_0 + x). \tag{2}$$

Inequalities for the disturbances have following form

$$|y_0| \gg |y|\,,\ |x_0| \gg |x| \quad . \tag{3}$$

Linearizing the expressions (2) and taking into account the expressions (1) and (3) the following relationship for the disturbances are received approximately

$$y = \left[\left(\frac{df}{dx} \right) \Big|_{x=x_0} \right] x \quad . \tag{4}$$

All relationships of the three-dimensional linearized theory of elastic waves propagation in bodies with initial (residual) stresses were received in accordance with expression (4). All results were received in general united form for theory of *finite initial* deformation and two variants of theory of *small initial* deformations. Needed information on this subject is presented in [Guz (2004)].

In general case of isotropic hyperelastic compressible material the elastic potential Φ is used in form

$$\Phi = \Phi(A_1, A_2, A_3)\ ;\ A_1 = \varepsilon_{nn}$$
$$A_2 = \varepsilon_{nm}\varepsilon_{mn}\,,\ A_3 = \varepsilon_{nm}\varepsilon_{mk}\varepsilon_{kn} \tag{5}$$

Notations: $\Phi-$ elastic potential; A_1, A_2 and A_3- first, second and third algebraic invariants of Green strains tensor. Analytical results were received for isotropic materials with Murnaghan type elastic potential which is presented in form

$$\Phi = \frac{1}{2}\lambda A_1^2 + \mu A_2 + \frac{a}{3}A_1^3 + bA_1A_2 + \frac{c}{3}A_3. \tag{6}$$

Notations: λ and $\mu-$ Lame constants, $\mu \equiv G-$ shear modulus; a, b and $c-$ elastic constants of the third order, the values of constants a, b and c for 39 various materials are presented in [Guz (2004)].

Analytical results were received for quasiisotropic materials with insignificant orthotropy, in this case elastic potential was presented in form of [Guz (2004)]

$$\Phi = \frac{1}{2}E_{ijnm}\varepsilon_{ij}\varepsilon_{nm} + \frac{a}{3}A_1^3 + bA_1A_2 + \frac{c}{3}A_3. \tag{7}$$

Square part of potential (7) corresponds to anisotropic material in the framework of linear theory of elasticity, cubic part of potential (7) corresponds to isotropic material in the framework of non-linear, for example (6), theory of elasticity.

Elastic potential (7) gives the possibility to take into account the insignificant orthotropy of materials which arises as result of some technological processes, for an example, rolling process. Information on this subject was presented in [Guz (2004)].

Regularities of elastic waves propagation in bodies with initial or residual stresses can be described by elastic potentials depending on *third invariant* A_3 *also*, elastic potentials depending *on first* A_1 *and second* A_2 invariants only can not describe above mentioned regularities.

$$\Phi = \Phi(A_1, A_2, A_3) \quad \text{describes,}$$
$$\Phi(A_1, A_2) \quad \text{does not describe.} \tag{8}$$

Statement (8) was proved strictly, information on this subject was presented in [Guz (2004)].

2.2 *Main relationships.*

Main relationships are considered in rectangular Lagrangian coordinates y_n $(n = 1, 2, 3)$ which are introduced in *second* state (initial or residual stress-strain state). In this case equations of motion have the form

$$\left(\frac{\partial}{\partial y_i} \omega'_{ij\alpha\beta} \frac{\partial}{\partial y_\beta} - \rho' \frac{\partial^2}{\partial \tau^2} \right) u_j = 0\ ,\ y_n \in V' \tag{9}$$

and boundary conditions in stresses on surface S_1' have the form

$$Q_j' = P_j'\,,\, y_n \in S_1'\,;\, Q_j' \equiv N_i^0 \omega'_{ij\alpha\beta} \frac{\partial u_\alpha}{\partial u_\beta} \quad . \tag{10}$$

Notations: N_j^0- components of ort of normal to a surface S_1' in second state; $P_j'-$ components of external load vector. In general case tensor ω' is presented in the form

$$\omega'_{ij\alpha\beta} = \omega'_{ij\alpha\beta}\left(\Phi_0, \sigma^0_{nm}\right) \quad . \tag{11}$$

Notations: Φ_0- elastic potential (5)-(7) in *second* state; $\sigma^0_{nm}-$ initial or residual stresses. Concrete structure of expression (11) was presented in [Guz (2004)]. The expressions (9) and (10) do not coinside with corresponding expression of linear theory of elasticity as components of tensor ω' in (9) and (10) do not satisfy to the symmetry conditions of linear theory

$$\omega'_{ij\alpha\beta} \neq \omega'_{ji\alpha\beta}\,;\; \omega'_{ij\alpha\beta} \neq \omega'_{ij\beta\alpha}\,;\; \omega'_{ij\alpha\beta} \neq \omega'_{\alpha\beta ij}\,. \tag{12}$$

Additional information on the theory under consideration (theory of elastic waves propagation in bodies with initial or residual stresses) was presented in [Guz (2004)].

2.3 General solution under homogeneous initial (residual) states.

All concrete results of the theory of elastic waves propagation in bodies with initial (residual) stresses were received in case of homogeneous initial (residual) stresses

$$\sigma^0_{ij} = const \quad \text{at } i = j; \quad \sigma^0_{ij} = 0 \quad \text{at } i \neq j. \tag{13}$$

In case (13) several general solutions of the equations system (9) were received, information on this subject is presented in [Guz (2004)]. As an example, in case

$$\sigma^0_{11} = \sigma^0_{22} = const\,;\; \sigma^0_{33} = const\,;\; \sigma^0_{11} \neq \sigma^0_{33} \tag{14}$$

the general solution is considered for body of arbitrary curvilinear cross-section. The following notations are introduced: N' and $S'-$ normal and tangent (lines) to an arbitrary curvilinear contour in plane $y_3 = const$ (in *second* state); u'_N and u'_S- components of displacement vector along N' and S'. Displacements have the following form

$$u'_N = \frac{\partial}{\partial S'}\Psi' - \frac{\partial^2}{\partial N'\partial y_3}X'\,;$$

$$u'_S = -\frac{\partial}{\partial N'}\Psi' - \frac{\partial^2}{\partial S'\partial y_3}X';$$

$$u_3 = \left(\omega'_{1133} + \omega'_{1313}\right)^{-1} \left(\omega'_{1111}\Delta'_1 + \omega'_{3113}\frac{\partial^2}{\partial y_3^2} - \rho'\frac{\partial^2}{\partial \tau^2}\right)X'; \tag{15}$$

$$\Delta'_1 = \frac{\partial^2}{\partial y_1^2} + \frac{\partial^2}{\partial y_2^2}.$$

Functions Ψ' and X' (15) are determined from equations

$$\left(\Delta'_1 + \xi'^2_1\frac{\partial^2}{\partial y_3^2} - \rho'\frac{1}{\omega'_{1221}}\frac{\partial^2}{\partial \tau^2}\right)\Psi' = 0;$$

$$\left[\left(\Delta'_1 + \xi'^2_2\frac{\partial^2}{\partial y_3^2}\right)\left(\Delta'_1 + \xi'^2_3\frac{\partial^2}{\partial y_3^2}\right) - \rho'\left(\frac{\omega'_{1111} + \omega'_{1331}}{\omega'_{1111}\omega'_{1331}}\Delta'_1 + \frac{\omega'_{3333} + \omega'_{3113}}{\omega'_{1111}\omega'_{1331}}\frac{\partial^2}{\partial y_3^2}\right) + \frac{\rho'^2}{\omega'_{1111}\omega'_{1331}}\frac{\partial^4}{\partial \tau^4}\right]X' = 0. \tag{16}$$

In (16) notations are introduced

$$\xi'_j = \xi'_j\left(\omega'_{mn\alpha\beta}\right)\,; \quad j, m, n, \alpha, \beta = 1, 2, 3. \tag{17}$$

Expression for determination of ξ'_j (17) are given in [Guz (2004)].

In next chapter some materials (metals, alloys and similar materials) will be considered. These compressible materials can be named as relatively rigid materials, in this case the following inequality has place

$$\sigma^0_{ij}/\mu \ll 1, \tag{18}$$

where $\mu \equiv G-$ shear modulus. Taking into account the inequality (18) in next chapters the linear approximation as applied to parameters σ^0_{ij}/μ will be used under analytical and numerical investigations.

Additional information on general solutions was presented in [Guz (2004)]. Similar results for 3-D linearized theory of stability of deformable bodies were presented in [Guz (1999)].

3 Main regularities of Rayleigh waves propagation in bodies with initial(residual) stresses

All results were received in the framework of the previous chapter theory.

3.1 Planar problem. Influence of initial (residual) stresses.

Planar problem is considered in plane $y_1 0 y_2$ (fig.1) for semiplane $y_2 \leq 0$, at fig.1 scheme of loading and wave propagation are presented.

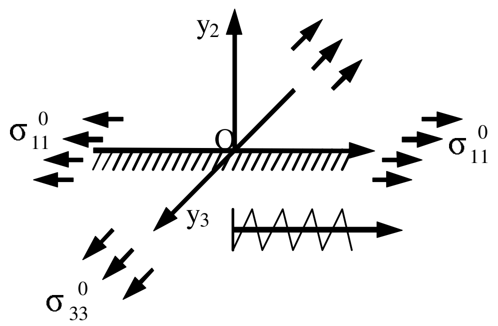


Figure 1 :

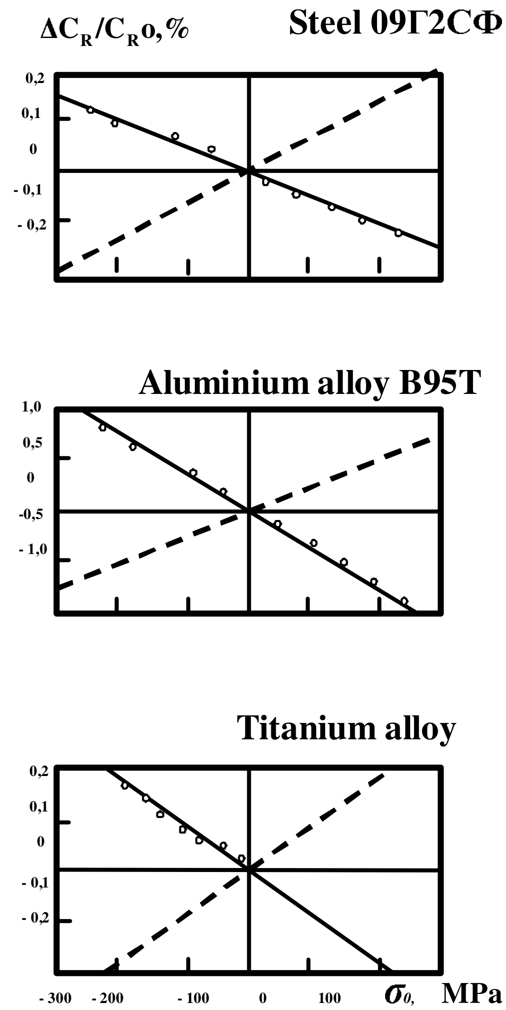


Figure 2 :

Additional condition is used

$$\sigma^0_{22} = 0, \tag{19}$$

thus, the conditions are used

$$\sigma^0_{22} = 0\,;\ \sigma^0_{11} = const\,;\ \sigma^0_{33} = const. \tag{20}$$

Rayleigh waves propagate along the axis $0y_1$ on fig.1.
Solution includes the expressions of following type

$$[\exp(k\alpha_1 y_2)]\,[\exp i\,(ky_1 - \omega\tau)]\,; \quad C_R = \omega/k, \tag{21}$$

where C_R - Rayleigh waves velocity in body with initial stresses.
The dependences of the value $\Delta C_R/C_{R0}$ on initial stresses σ_0 for steel, aluminium alloy and titanium alloy are presented at fig.2 in case of uniaxial loading

$$\sigma^0_{11} = \sigma_0;\ \sigma^0_{22} = 0;\ \sigma^0_{33} = 0. \tag{22}$$

Value $\Delta C_R/\ C_{R0}$ is defined by expression

$$\Delta C_R/C_{R0} = (C_R - C_{R0})\,/C_{R0}. \tag{23}$$

Notations: C_R- Rayleigh waves velocity in material with initial (residual) stresses; $C_{R0}-$ Rayleigh waves velocity in material without initial (residual) stresses; “o” - experimental results; solid lines - theoretical result on the base of Murnaghan type potential (6); dotted lines - theoretical results on the base of square potential

$$\Phi = \frac{1}{2}\lambda A_1^2 + \mu A_2. \tag{24}$$

Results on fig.2 prove that the potential (6) depending on first and second invariants only $(\Phi = \Phi(A_1, A_2))$ does not describe the regularities under consideration. Results on fig.2 correspond to the case of Rayleigh waves propagation in direction of loading as the expressions (21) and (22) take place. The dependences of the value (23) on initial stresses σ_0 for some steels and some aluminium alloys are presented at fig.3 in two cases of uniaxial loading. *First case* corresponds to uniaxial loading along direction of waves propagation, expression (21) and (22) take place. Notations: “o” - experimental results; solid lines with number 1 - theoretical results. *Second case* corresponds to uniaxial loading along perpendicular to direction of waves propagation, expression (21) and following expression

$$\sigma^0_{11} = 0, \sigma^0_{22} = 0\,; \sigma^0_{33} = \sigma_0 \tag{25}$$

take place.

Notations: "•" - experimental results; solid lines with number 2 - theoretical results. All theoretical results on fig.3 were received on the base of Murnaghan type potential (6). *Conclusion.* Elastic potential of Murnaghan type (6) gives the possibility to describe the regularities under consideration.

3.2 Axisymmetrical problem. Influence of initial (residual) stresses.

Axisymmetrical problem is considered for semispace $y_3 \leq 0$. Additional condition is used

$$\sigma_{33}^0 = 0 \quad , \tag{26}$$

thus, the conditions are used

$$\sigma_{11}^0 = \sigma_{22}^0 = const; \quad \sigma_{33}^0 = 0. \tag{27}$$

Phase surface is circular cylinder with $0y_3$ axis.

Solution includes the expression of following type

$$b(y_3)\left[H_0^{(1)}(kr)\right][\exp(-i\omega\tau)]\,;\, C_R = \omega/k \quad . \tag{28}$$

Notations: $r-$ radial coordinate; $H_0^{(1)}(x)-$ Hankel function of first type and zero order, as the waves propagate from $r = 0$ to $r = \infty$

Theoretical results were received, these results are similar to corresponding results for plane problem.

3.3 Rayleigh waves on a circular cylinder. Influence of initial (residual) stresses

Scheme of loading and direction of the surface waves propagation are presented at fig.4. Results were received for two cases.

First case corresponds to omnidirectional loading of cylinder by "tracking" load or "follower", the following conditions have place

$$\sigma_{11}^0 = \sigma_{22}^0 = \sigma_{33}^0 = \sigma_0 \quad . \tag{29}$$

Second case corresponds to uniaxial loading of cylinder by "dead" load, the following conditions have place

$$\sigma_{11}^0 = \sigma_{22}^0 = 0; \sigma_{33}^0 = \sigma_0. \tag{30}$$

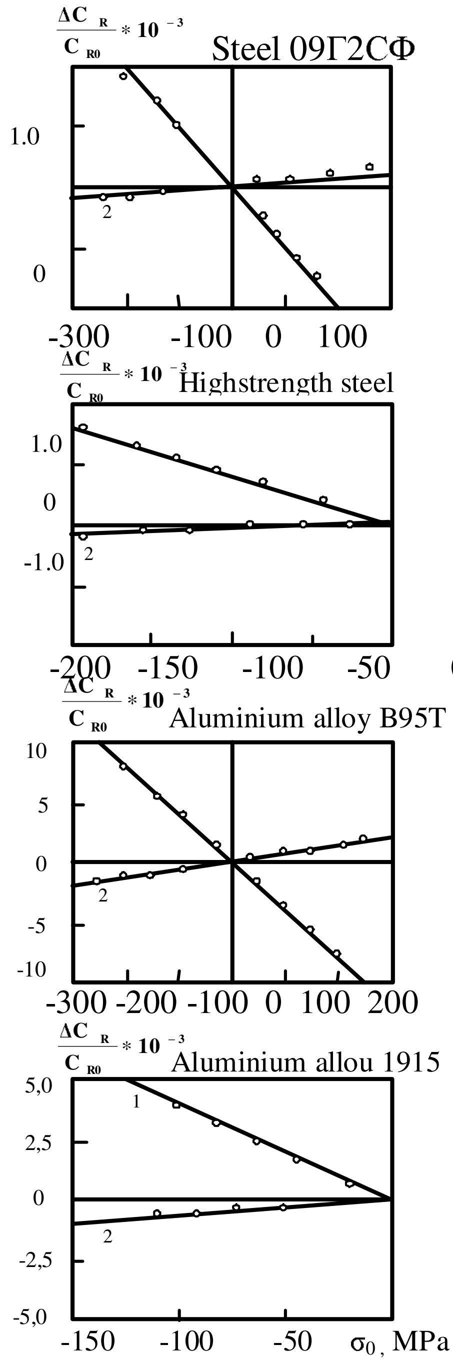


Figure 3 :

Solution includes the expressions of following type

$$[J_p(k_l r)][\exp i(p\theta - \omega\tau)]; \\ [J_p(k_t r)][\exp i(p\theta - \omega\tau)]; \\ C_{R\ddot{o}} = \omega/k = \omega R/p. \tag{31}$$

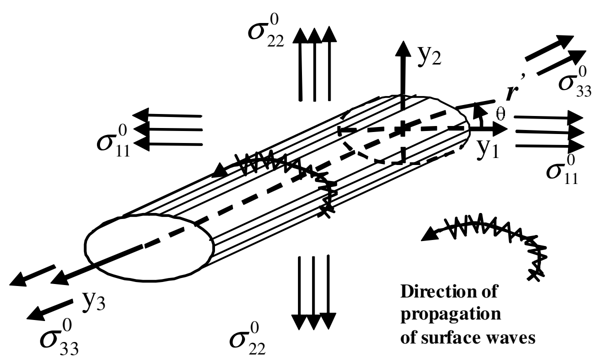


Figure 4 :

Notations: $J_p(x)-$ Bessel function of p-th order; r and $\theta-$ the radial and angular coordinates of the polar system in the plane $y_3 = const$;$p-$ angular wave number, $-\infty \leq p \leq +\infty$; $C_{R\ddot{o}}-$ Rayleigh wave velocity along circular surface in cylinder with initial (residual) stresses. Rayleigh waves along the cylindrical surface in circular cylinder propagate with dispersion. Dispersion equation has very complex structure. Computational methods were used for solution of these dispersion equations.

For a steel cylinder (**Steel 09Г2СФ**) the dependences of the value η on the value $\left(-\sigma_0/\mu \cdot 10^5\right)$ are presented at fig.5 in *first* case (omnidirectional loading of cylinder by "tracking" load or "follower"). In this case the following expressions have place

$$\eta = \left(C_{R\ddot{o}}^0 - C_{R\ddot{o}}\right) \cdot \left(C_{R\ddot{o}}^0\right)^{-1} \cdot 10^4;$$
$$\sigma_{11}^0 = \sigma_{22}^0 = \sigma_{33}^0 = \sigma_0. \tag{32}$$

Notations: $C_{R\ddot{o}}^0-$ Rayleigh wave velocity along circular cylindrical surface in cylinder without initial (residual) stresses; solid lines with numbers 1,2,3 and 4 on fig.5 correspond to dimensionless frequencies

$$k_t^0 R = 8.75; 13.75; 25.00; 35.00. \tag{33}$$

For a steel cylinder (**Steel 09Г2СФ**) the dependences of the value η on the value $\left(\sigma_{33}^0/\mu \cdot 10^5\right)$ are presented at fig.6 in *second* case (uniaxial loading of cylinder along axis $0y_3$ on fig.4 by "dead" load). In this case the following expressions have place

$$\eta = \left(C_{R\ddot{o}}^0 - C_{R\ddot{o}}\right) \cdot \left(C_{R\ddot{o}}^0\right)^{-1} \cdot 10^5;$$
$$\sigma_{11}^0 = \sigma_{22}^0 = 0;\ \sigma_{33}^0 = \sigma_0. \tag{34}$$

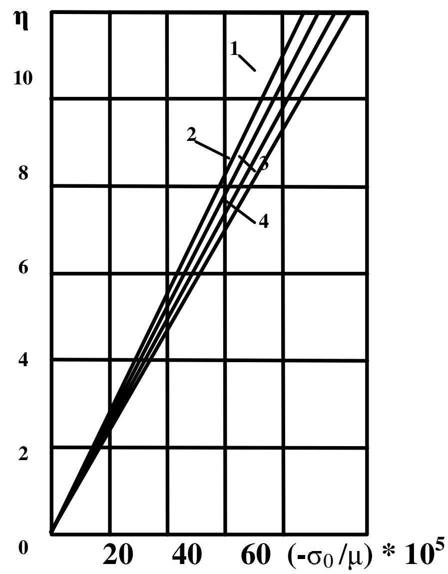


Figure 5 :

Notations: $C_{R\ddot{o}}^0-$ Rayleigh wave velocity along circular cylindrical surface in cylinder without initial (residual) stresses; $\mu-$ shear modulus.

Solid lines with numbers 1, 2, 3, 4 and 5 on fig.6 correspond to dimensionless frequencies:

$$k_t^0 R = 7.50; 15.00; 22.50; 30.00 \text{ and } \infty. \tag{35}$$

Solid line with number 5 on fig.6 (dimensionless frequence $= \infty$) corresponds to plane surface (Rayleigh waves propagate along plane boundary surface of semispace).

3.4 *Rayleigh waves on a sphere. Influence of initial (residual) stresses.*

Scheme of loading and direction of the surface waves propagation are presented at fig.7 for a solid sphere. Results were received for the case

$$\sigma_{11}^0 = \sigma_{22}^0 = \sigma_{33}^0 = \sigma_0 \tag{36}$$

as applied to the "tracking" or "follower" and "dead" loading. Rayleigh waves propagate along spherical surface of a solid sphere from upper pole to lower pole and in opposite direction. Phase surface is a conical surface of circular cross-section with $0y_3$ axis.

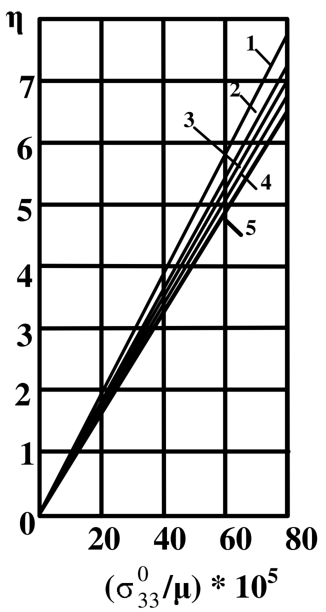


Figure 6 :

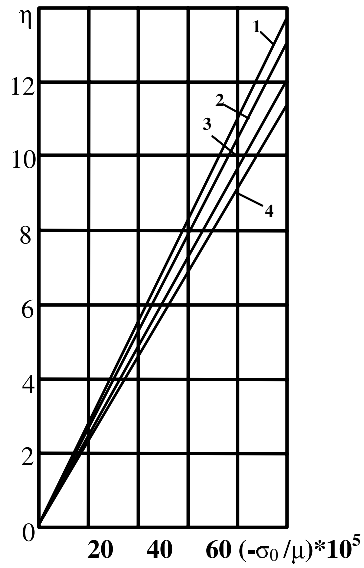


Figure 8 :

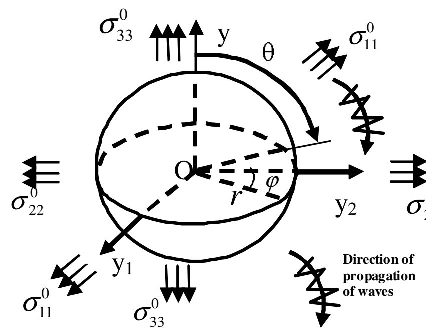


Figure 7 :

Solution includes the expression of following type

$$\left[J_{l+\frac{1}{2}}(k_t r)\ ; J_{l-\frac{1}{2}}(k_t r)\right]\cdot[Y_{lm}(\theta,\varphi)]\,[\exp(-i\omega\tau)] \quad (37)$$
$$C_{RC}=\omega/k=\omega R/l.$$

Notations: $J_{l\pm\frac{1}{2}}(x)-$ Bessel function of $\left(l\pm\frac{1}{2}\right)$-th order; $r, \theta, \varphi-$ coordinates of spherical coordinate system; $\theta = const-$ phase surface; $l-$ angular wave number for sphere; $C_{RC}-$ Rayleigh wave velocity along spherical surface in solid sphere with initial (residual) stresses; $Y_{lm}(\theta,\varphi)-$ spherical harmonic of following type

$$Y_{lm}(\theta,\varphi) = [P_l^m(\cos\theta)]\,[\exp im\varphi] \quad , \quad (38)$$

where $P_l^m(\cos\theta)-$ joined Legendre function of l-th power and m-th order.

For a steel solid sphere (**Steel 09Г2СФ**) the dependences of the value η on the value $\left(-\sigma_0/\mu\cdot 10^5\right)$ are presented in case of loading by the "tracking" load or "follower" at fig.8

$$\eta=\left(C^0_{RC}-C_{RC}\right)\left(C^0_{RC}\right)^{-1}\cdot 10^4; \quad (39)$$
$$\sigma^0_{11}=\sigma^0_{22}=\sigma^0_{33}=\sigma_0$$

Notations: $C^0_{RC}-$ Rayleigh wave velocity along spherical surface in solid sphere without initial (residual) stresses; $\mu-$ shear modulus; solid lines with numbers 1,2,3 and 4 correspond to dimensionless frequencies

$$k^0_t R = 8.75; 13.75; 25.00; 32.00. \quad (40)$$

Case of $R\to\infty$ or $l\to\infty$ corresponds to axisymmetrical Rayleigh waves along plane boundary in semispace.

Rayleigh waves along the spherical surface in solid sphere propagate with dispersion. Dispersion equation has very complex structure. Computational methods were used for solution of these dispersion equations.

3.5 General regularity.

General or main regularity of the Rayleigh waves propagation along planar and curvilinear boundary surface in bodies with initial (residual) stresses are received from analysis of the results presented at fig.2, 3, 5,6, and 8. The results at fig.2 and 3 correspond to the Rayleigh waves along plane boundary surface of semispace. The results at fig.5 and 6 correspond to the Rayleigh waves along circular cylindrical surface of solid circular cylinder. The results at fig.8 correspond to the Rayleigh waves along spherical surface of solid sphere. Above mentioned results correspond to the compressible relatively rigid materials (metals, alloys, ...). Above discussed general or main regularity for these materials may be formulated by following manner.

GENERAL or MAIN REGULARITY

Rayleigh waves propagation velocities depend linearly on initial or residual stresses

Taking into account the linear property of above formulated **GENERAL or MAIN REGULARITY** the **REVERSE FORM of GENERAL or MAIN REGULARITY** can be formulated by following manner.

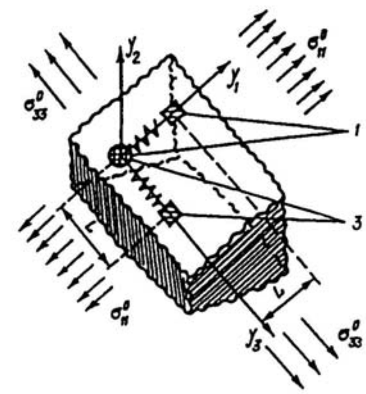
GENERAL or MAIN REGULARITY. REVERSE FORM

Initial or residual stresses depend linearly on Rayleigh waves propagation velocities

In this chapter information on the theory of the Rayleigh waves propagation in bodies with initial (residual) stresses was presented in very short form. Additional information on this subject may be received in [Guz (2004)].

4 Ultrasonic non-destructive method of determination of stresses in near-the-surface layers of solids

This method is intended for determination of uniaxial and two-axial stresses in near-the-surface layers of solids as applied to actual, assembly, operating, initial, residual, preload and other stresses. Theoretical foundation of the ultrasonic non-destructive method under consideration is the GENERAL or MAIN REGULARITY (REVERSE FORM) which was formulated as applied to initial or residual stresses in the end of the previous chapter. In view of its in the method under consideration the actual, assembly, operating, preload, prestress and other stresses must be considered as *initial or residual stresses* in the framework of the theory of first chapter. In this case *the disturbances* (displacements and stresses of the three-dimensional theory of elastic waves in bodies with initial or residual stresses - the theory of first and second chapters) *arise due to ultrasonic vibrations.*



Notations: ○– emitter; - receiver; variant 1 - emitter and receiver along $0y_1$ axis; variant 2 - emitter and receiver along $0y_3$ axis; σ_{11}^0 and σ_{33}^0– stresses, which must be determined.

Figure 9 :

4.1 Description of the non-destructive method.

Scheme of this method is given at fig.9, where L is the distance between the emitter and the receiver of ultrasonic vibrations.

The ultrasonic non-destructive method under consideration is intended for measurements of uniaxial and two-axial stresses σ_{11}^0 and σ_{33}^0 in following cases:

1) relatively rigid elastic materials (metals, alloys and similar materials);

2) near non-loading boundary surface ($y_2 = 0$ at fig.9), the following condition has place

$$\sigma_{22}^0 = 0; \tag{41}$$

3) bodies with planar or slight curved boundary surface;

4) in very thin near-the-surface layers of materials, where the following conditions can be accepted as applied to the

dependences on y_2

$$\sigma_{11}^0 \approx const\,;\ \sigma_{33}^0 \approx const \text{at } |y_2| \ll L; \tag{42}$$

5) for insignificant changing stresses σ_{11}^0 and σ_{33}^0 at distance L in the plane $y_1 0 y_2$, the following conditions can be accepted as applied to the dependences on y_1 and y_3

$$\sigma_{11}^0 \approx const\,;\ \sigma_{33}^0 \approx const \quad \text{at } \min\{\Delta y_1, \Delta y_3\} \le L; \tag{43}$$

6) for "elastic" stresses σ_{11}^0 and σ_{33}^0, but the sources of these stresses can have different nature (electric welding, plastic local loading, operating loading, local irradiation and other sources).

Thus in the method under consideration the following conditions are accepted

$$\sigma_{22}^0 = 0\,;\ \sigma_{11}^0 = const\,; sigma_{33}^0 = const. \tag{44}$$

Conditions 1-6 define the limits of application of the method under consideration. It must be remarked that the conditions (43) (condition 5) are generally accepted in any experimental method as the average value of the measured quantity is determined in the limits of the transducer (sensor) dimension. In the method under consideration the emitter and the receiver at fixed distance L must be considered as the transducer (sensor) (fig.9). Taking into account the **GENERAL or MAIN REGULARITY** (the end of previous chapter) the **MAIN RELATIONSHIP of METHOD** under consideration can be presented in following form as applied to fig.9

$$\begin{aligned} \sigma_{11}^0 - \sigma_{33}^0 &= \left(\frac{c_{R1} - c_R^0}{c_R^0} - \frac{c_{R3} - c_R^0}{c_R^0} \right) A_R\,; \\ \sigma_{11}^0 + \sigma_{33}^0 &= \left(\frac{c_{R1} - c_R^0}{c_R^0} + \frac{c_{R3} - c_R^0}{c_R^0} \right) B_R\,. \end{aligned} \tag{45}$$

Notations: σ_{11}^0 and σ_{33}^0- stresses, which must be determined; c_R^0- Rayleigh waves velocity in material without stresses σ_{11}^0 and σ_{33}^0; $c_{R1}-$ Rayleigh waves velocity along the $0y_1$-axis (fig.9) in material with stresses σ_{11}^0 and σ_{33}^0 ; $c_{R3}-$ Rayleigh waves velocity along the $0y_3$-axis (fig.9) in material with stresses σ_{11}^0 and σ_{33}^0 ; A_R and B_R- constant values for each material.

The **MAIN RELATIONSHIP of METHOD** in form (45) corresponds to two-axial stresses. In case of uniaxial stresses at $\sigma_{33}^0 = 0$ taking into account the first expression (45) the **MAIN RELATIONSHIP of METHOD** can be presented in following form as applied to fig.9

$$\sigma_{11}^0 = \left(\frac{c_{R1} - c_{R3}}{c_R^0} \right) A_R \tag{46}$$

The **MAIN RELATIONSHIP of METHOD** in form (45) for two-axial stresses includes the expression for the difference of two main stresses (first expression (45)) and the expression for the sum of two main stresses (second expression (45)). In view of its two main stresses σ_{11}^0 and σ_{33}^0 can be determined separately from the expression (45). First expression (45) resembles the main relationship of the photoelasticity method for the difference of two main stresses, but the photoelasticity method *does not have the relationship for the sum of two main stresses*. In view of its two main stresses σ_{11}^0 and σ_{33}^0 can not be determined separately by the expression of the photoelasticity method directly. In the photoelasticity method the additional procedures are used in order to separate the main stresses σ_{11}^0 and σ_{33}^0.

The constant values A_R and B_R (45) can be determined for each materials by two ways. First way is theoretical determination, in this case the constant values A_R and B_R (45) were defined by expression

$$A_R = A_R\left(c_R^0, \lambda, \mu, a, b, c\right); B_R = B_R\left(c_R^0, \lambda, \mu, a, b, c\right) \tag{47}$$

Second way is experimental determination which involves the following. For the material under consideration, from experimental studies the value c_R^0 is determined. Then for specified values σ_{11}^0 and σ_{33}^0 (arbitrary values which are realized convenient under experimental studies), also from experimental studies the values c_{R1} and $c_{R3}$are determined. Then for specified values of σ_{11}^0 and σ_{33}^0 and experimentally determined values c_R^0, c_{R1} and c_{R3} from expressions (45) the values of A_R and B_R are determined. Perhaps the experimental method of determination of values of A_R and B_R for each materials is preferable as these results do not depend on the theory under consideration and take into account microinhomogeneous for each material.

4.2 On instruments and devices for measurements.

General view of the device for ultrasonic measurements is presented at fig.9. Experimental studies were done at the E.O.Paton Electric Welding Institute of National Academy of Sciences of Ukraine under the supervision

Figure 10 :

of DSc O.I.Gushcha. The device (fig.10) used for the measurements is based on the pulse recirculation method and provides a relative measurement error no higher than 10^{-5}.

Excitation and reception of Rayleigh waves is done using a “wedge” type transducer, it operates at frequency 5MHz.

Transducer includes CTS-19 piezocerzmic plates (sizes 10×4 mm, resonant frequency 3 MHz) rigidly fastened to each other (at the distance L on fig.9). The general view of portable device (acoustic transducer with electromagnets) is present at fig.11. Before the ultrasonic measurement the surface of the specimen or the structure element should be polished. *The merit of the method* under consideration is the possibility to make measurements not only on models but also *on structure elements*.

The drawback of the method under consideration is the necessity of carrying out measurements of velocity with high degree of precision.

Block-scheme of the precise device is presented at fig.12, general view of this device is presented at fig.10.

Additional information on the instruments and the devices for ultrasonic measurements as applied to the method under consideration may be received in [Guz (2004)] and in publications cited in [Guz (2004)].

4.3 Verification of the non-destructive method.

Verification of the non-destructive ultrasonic method of determination of two-axial stresses in near-the-surface layers of material was carried out for circular disk

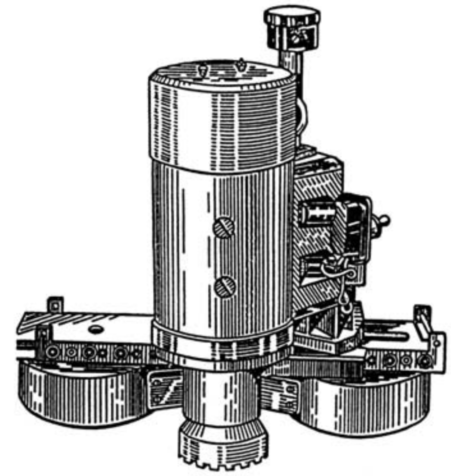
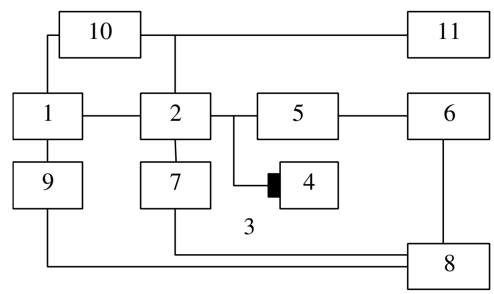


Figure 11 :



Notations: 1 - generator; 2 - high-power generator; 3 - acoustic transducer (transformer); 4 - specimen; 5 - key device; 6 - amplifier; 7 - regulated delay line; 8 - coincidence scheme; 9 - delay line; 10 - discriminator; 11 - counting type electronic frequency meter.

Figure 12 :

(fig.13). For originate of the two-axial stress state at fig.13 the circular disks were compressed by concentrated load along the vertical diameter. The measurements were carried out along the horizontal diameters of steel and aluminium alloy discs. The experimental results for above mentioned discs were received by two ways according to the end of first part of this chapter. First way corresponds to case when constant values A_R and B_R in the expressions (45) were determined by theoretical way. Second way corresponds to case when constant values A_R and B_R in the expressions (45) were de-

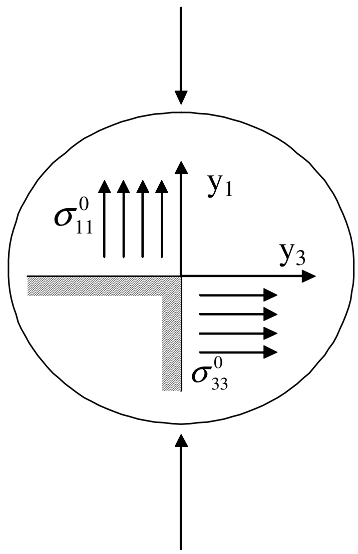


Figure 13 :

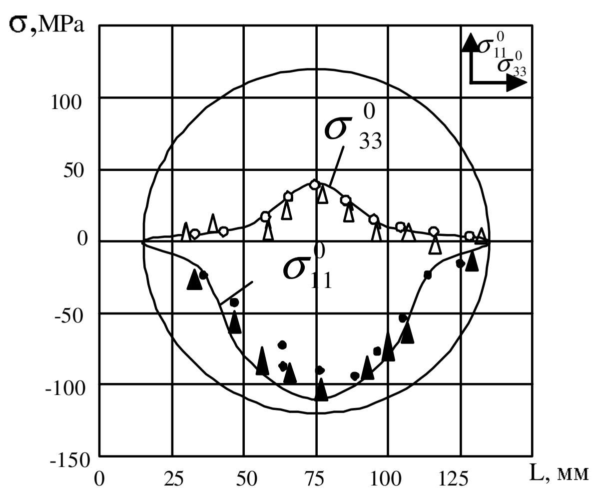
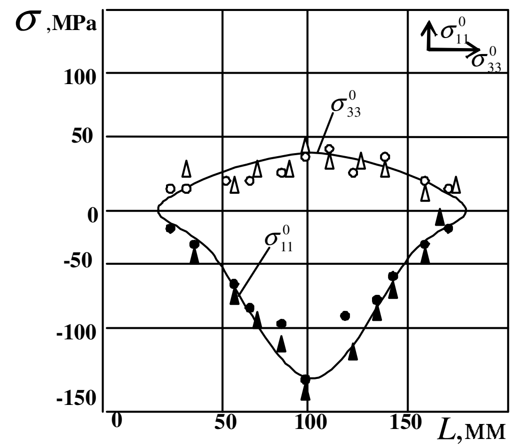


Figure 14 :

termined by experimental way. The theoretical solutions in the framework of classical linear theory of elasticity for the situation at fig.13 were considered also.

Corresponding results for steel disc are presented at fig.14 and for aluminium alloy disc are presented at fig.15.

Conclusion. Acceptable coincidence of the experimental results obtained by first and second ways and the theoret-



Notations at fig.14 and 15: σ^0_{11} (- by the *first* way, ►- by the *second* way, solid line corresponds to the theoretical solution); σ^0_{33} (Δ– by the *first* way, ◯– by the *second* way, solid line corresponds to the theoretical solution).

Figure 15 :

ical results as applied to fig.14 and 15 may be declared.

4.4 Examples of non-destructive determination of uniaxial and two-axial stresses in near-the-surface layers of materials.

In this part the determination of the residual stresses arising at electric welding and the determination of the operating stresses arising at loading are considered.

The determination of the residual stresses arising at electric welding. The residual stresses at electric welding were determined in the case of two rectangular steel of 171 plates butt welded, these results are presented at fig.16 and fig.17. Distribution of σ^0_{11} and σ^0_{33} in perpendicular direction to the weld (along line L) is presented at fig.16. Distribution of σ^0_{11} and σ^0_{33} along weld at the distance from weld (along line L) is presented at fig.17.

The sizes of the butt welded plates and the directions of axis are presented in the upper parts of fig.16 and fig.17. It must be remarked the directions of axis at fig.9 and fig.13-17 coincide.

The determination of the operating stresses arising at loading. The operating stresses arising at loading were determined as applied to a vessel of internal pressure. In this situation the measurements of stresses with applica-

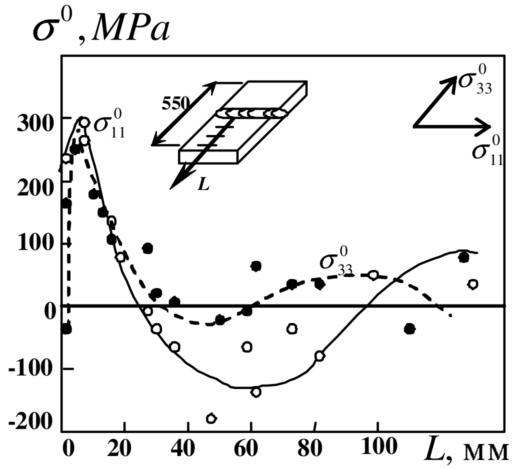


Figure 16 :

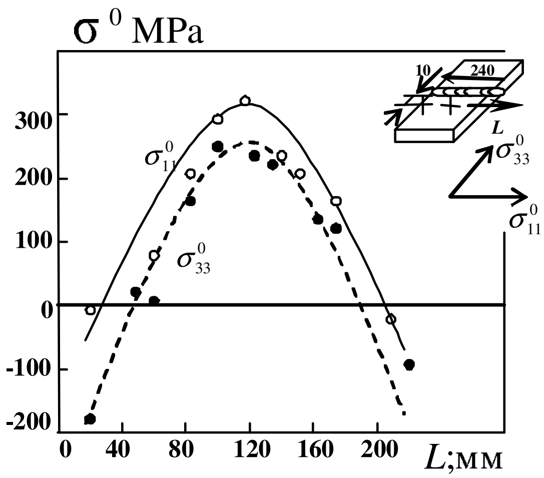
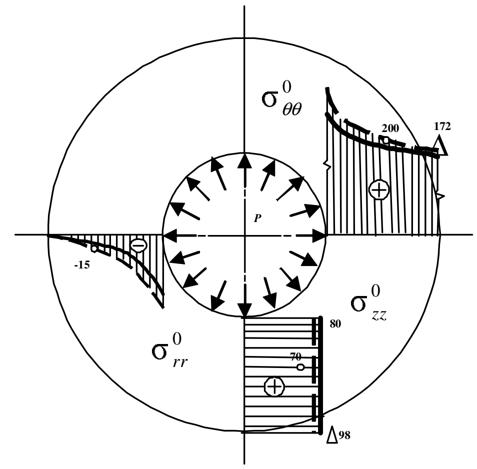


Figure 17 :

tion of Rayleigh surface waves (method of this paper) combined with measurements on the basis of waves in an infinite solid [Guz (2004)]. Such approach allows to determine more complex stresses fields.

Cross-section of a cylindrical closed thick-walled vessel of internal pressure and distribution of three-axial stresses are presented at fig.18.

Conclusion. More acceptable coincidence of the experimental results and the theoretical results was received in the case when the method of Rayleigh surface waves (method of this paper) was used also.



Notations: dotted lines correspond to the results of experimental method of this paper; solid lines correspond to the results of theoretical solution in the framework of classical linear theory of elasticity; index "Δ" corresponds to the experimental results of the joined method (Rayleigh surface waves and waves in an infinite solid [Guz (2004)]); index "○" corresponds to the experimental results of the method of waves in an infinite solid [Guz (2004)] only.

Figure 18 :

5 Conclusion

Taking into account above analyzed results on the foundations of the ultrasonic non-destructive method of determination of stresses in near-the-surface layers of solids the following conclusion may be formulated.

The approach of this paper and obtained results can be considered as the joined approach corresponding to solid mechanics, computational mechanics and experimental mechanics.

Additional information can be received in [Guz (2004)], [Guz, Makhort (2000)] and [Guz (2002)].

Similar problems for linearized solid mechanics were considered in [Guz (2003,a)], [Guz (2003,b)] and [Guz A.N., Guz I.A. (2004)].

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