Numerical Modeling of the Influence of Water Suction on the Formation of Strain Localization in Saturated Sand

X. Liu and A. Scarpas¹

Abstract: Numerical investigations of strain localization have been performed on 3D dense fully saturated sand specimens subjected to triaxial loading and simultaneous inflow or outflow conditions. The role of the water suction field on the formation and evolution of strain localization is addressed computationally. It has been shown that, in a porous medium, the fluid (water) phase plays indeed an important role in strain localization. The formation and evolution of strain localization are influenced both by the material behaviour of the solid component and the interaction between components. In this contribution, after a presentation of the incremental formulation of the coupled problem and the discretization in time, a nonlinear constitutive model is presented for the solid component. In the last part of the contribution, results are presented for the influence of water inflow or outflow on the development of strain localization in a laboratory specimen.

keyword: Strain localization, constitutive modeling, soil hardening and softening, flow surfaces, porous medium, solid fluid phase interaction, water suction, cavitation.

1 Introduction

Localized deformations in the form of narrow shear bands are often observed to develop after large inelastic deformation in materials. Within the shear band, the material behaviour is inelastic. Typical examples of material that are prone to strain localization are frictional materials such as concrete, rocks and soils. These materials show a reduction of load carrying capacity, after reaching the limit load, accompanied by increasing localized deformations. The mechanisms responsible for strain localization can vary widely from one material to another. Normally, strain localization is treated as the result of local inhomogeneities, stress concentrations or the onset of some physical mechanism that degrades abruptly the strength of the material at a point. Alternatively, strain localization can be considered as a bifurcation from a smoothly varying pattern of deformation, which arises as a result of instability in the inelastic behaviour of the material.

The analysis of strain localization is of importance in engineering practice because localization is a precursor to sudden failure. Localized deformations in the form of narrow shear bands are often observed to develop after large inelastic deformation in materials. Within this shear band, the material behaviour is inelastic.

Strain localization has been extensively studied in recent years, in particular in connection with single phase solids. Among others Needleman (1988), Loret and Prevost (1990), Sluys (1992) have investigated the problem of dynamic strain localization in single phase solids.

Since the end of the 70's, several authors have studied strain localization in multiphase materials. Rudnicki (1984) analyzed a fluid saturated rock mass with an embedded weakened layer. Rice (1985) studied the effect of material dilatancy on strain localization in fully saturated frictional material. Vardoulakis (1986) showed the importance of dynamic stability analysis in case of undrained simple-shear deformation of water-saturated granular soil. Loret and Prevost (1991) studied the formation of localization in a fully saturated soil specimen using dynamic strain localization theory. A model based on the general framework of averaging theories and capable of simulating shear band dominated processes in saturated porous media was presented by Schrefler, Majorana and Sanavia (1995), Schrefler, Sanavia and Majorana (1996).

However, due to the complexities associated with solid fluid phase interactions, the mechanisms responsible for strain localization in a porous medium can vary significantly from case to case. Several problems need to be

¹ Section of Structural Mechanics, Faculty of Civil Engineering and Geosciences, Delft University of Technology, The Netherlands

addressed such as: the role of the fluid component, the influence of test conditions on strain localization, the regularizing effect of the phase interactions on the constitutive equations of the multiphase medium.

An important observation by Liu (2003), Liu, Scarpas and Blaauwendraad (2004b) is that the suction field in a specimen under undrained conditions can influence significantly the onset and development of strain localization.

In this investigation, in order to provide further insight into the role of water suction in the phenomenon of strain localization in fluid saturated sands, a fully saturated 3D sand specimen subjected to true triaxial loads was chosen. It is of interest to investigate what happens on onset and development of strain localization in the specimen when the naturally build suction field in the specimen is disturbed. To achieve this, in this study, by introducing artificially the inflow or outflow at the center of the specimen, the influence of the water suction on the formation and evolution of strain localization are identified.

In order to simulate the elastoplastic characterization of a porous medium properly, the establishment of governing equations for the solid and fluid phase and the development of the constitutive model will be presented first. The solution procedure for solving the unsymmetric coupled governing equations with the help of Sherman-Morrison formula will be illustrated. Also, the characteristics of the utilized elsatoplastic constitutive model will be elaborated. At the last section of the paper, by utilization of the numerical examples, the contribution of water suction to the initiation and formation of strain localization in fully saturated 3D sand specimen will be presented in detail.

2 Governing equations

The governing equations for the description of the deformation and the motion of a porous medium are carried out on the basis of modern mixture theory, see Bowen (1976), Ehlers (1989), Lewis and Schrefler (1998) and Liu (2003). The porous medium is postulated to be a mixture, consisting of two basic continua, superpositioned in time and space. The first continuum represents the solid phase and second one represents the fluid phase. In the following, the governing equations are briefly recalled.

2.1 Incremental equations and discretization in time

The equilibrium equation for the solid skeleton can be written as:

$$\int_{V} (\mathbf{N}^{s})^{T} \cdot \boldsymbol{\rho} \cdot {}^{t} \mathbf{a} \cdot dV + \int_{V} \mathbf{B}^{T} \cdot \int_{t-\Delta t} \dot{\boldsymbol{\sigma}}' dt \cdot dV$$
$$+ \int_{V} \mathbf{B}^{T} \cdot \left(\int_{t-\Delta t}^{t} \tilde{\boldsymbol{\alpha}} \cdot \mathbf{I} \cdot \dot{\boldsymbol{p}}_{s} \cdot dt \right) \cdot dV = \int_{\Gamma} (\mathbf{N}^{s})^{T} \cdot {}^{t} \mathbf{t} \cdot d\Gamma$$
$$+ \int_{V} (\mathbf{N}^{s})^{T} \cdot \boldsymbol{\rho} \cdot \mathbf{g} \cdot dV - \int_{V} \mathbf{B}^{T} \cdot {}^{t-\Delta t} \boldsymbol{\sigma} \cdot dV \qquad (1)$$

where \mathbf{N}^s is the interpolation (or shape) function for the displacement and **B** is termed the strain-displacement matrix. **t** is the vector of the applied surface tractions. **a** is the acceleration vector. g is the gravity. $\rho = (1-n)\rho_s + nS_w\rho_w$ in which ρ_s and ρ_w is the material density of the solid and water constituent respectively. n represents the porosity as defined in classical soil mechanics.

 σ' in Eq. (1) is called the "modified effective stress" which represents the stress associated with the total deformation of the porous medium and **I** is the identity tensor. $\tilde{\alpha} = (1 - K_T/K)$ is Biot's constant and *K* and K_T are the bulk and tangential bulk modulus of the solid grains, see Biot (1941) and Zienkiewicz and Shiomi (1985).

 $p_s = S_w \cdot p_w + S_a \cdot p_a$ in which p_w and p_a represent water and air phase pressure². S_w and S_a are the degree of water and air saturation.

In the context of this contribution, p_a remains at atmospheric pressure. Neuman first used this approach in 1975 for a rigid porous medium and later Schrefler and Simoni in 1988 for partially saturated elastoplastic media. The assumption of the air phase being at atmospheric pressure in the partially saturated zone of the porous medium is feasible in soil mechanics and it enables to simplify the governing equations.

Therefore, by setting $p_a = 0$, the fluid pressure p_s in Eq. (1) can be expressed as:

$$p_s = S_w p_w \tag{2}$$

and its time derivative as:

$$\dot{p}_s = S_w \cdot \frac{dp_w}{dt} + p_w \cdot \frac{dS_w}{dt} = \left(S_w + \frac{w_s}{n} \cdot p_w\right) \cdot \dot{p}_w \tag{3}$$

² Following typical solid mechanics notation, tension is considered positive

where $w_s = n \cdot (\partial S_w / \partial p_w)$ is termed the specific moisture content, Lewis and Schrefler (1998).

The constitutive equation of the solid component is of the type

$$\sigma' = \mathbf{D} \cdot \boldsymbol{\varepsilon} \tag{4}$$

where \mathbf{D} is the tangential stiffness matrix. The characteristics of the utilized elastoplastic constitutive model will be introduced in section 3.

Substituting the constitutive relation Eq. (4) and the time derivative of the fluid pressure Eq. (3) into Eq. (1) and considering the interpolation (or shape) function \mathbf{N}^{w} for the fluid pressure, the equilibrium equation (1) can be rewritten as:

$$\int_{V} (\mathbf{N}^{s})^{T} \cdot \boldsymbol{\rho} \cdot \mathbf{N}^{s} \cdot {}^{t} \ddot{\mathbf{d}} \cdot dV + \int_{V} \mathbf{B}^{T} \int_{t-\Delta t}^{t} t^{-\Delta t} \mathbf{D} \cdot \mathbf{B} \cdot \dot{\mathbf{d}} \cdot dt \cdot dV$$
$$+ \int_{V} \mathbf{B}^{T} \int_{t-\Delta t}^{t} \tilde{\boldsymbol{\alpha}} \cdot \mathbf{I} \cdot \mathbf{N}^{w} \cdot \left(S_{w} + \frac{w_{s}}{n} \cdot \mathbf{N}^{w} \cdot \mathbf{p} \right) \cdot \dot{\mathbf{p}} \cdot dt \cdot dV$$
$$= \int_{\Gamma} (\mathbf{N}^{s})^{T} \cdot {}^{t} \mathbf{t} \cdot d\Gamma + \int_{V} (\mathbf{N}^{s})^{T} \cdot \boldsymbol{\rho} \cdot \mathbf{g} \cdot dV$$
$$- \int_{V} \mathbf{B}^{T} \cdot {}^{t-\Delta t} \boldsymbol{\sigma} \cdot dV$$
(5)

in which \mathbf{d} and \mathbf{p} are the nodal displacement and pore fluid pressure respectively.

The continuity equation of the pore fluid is the following:

$$-\int_{V} (\nabla \mathbf{N}_{w})^{T} \cdot \mathbf{k} \frac{k_{rw}}{\mu_{w}} \nabla \mathbf{N}_{w} \cdot {}^{t} \mathbf{p} dV$$

$$-\int_{V} (\nabla \mathbf{N}_{w})^{T} \cdot \mathbf{k} \frac{k_{rw}}{\mu_{w}} \nabla \rho_{w} g dV$$

$$+\int_{\Gamma_{q}} (\mathbf{N}_{w})^{T} \cdot \hat{q} \cdot d\Gamma + \int_{V} (\mathbf{N}_{w})^{T} \cdot Q^{w} \cdot \mathbf{N}_{w} \cdot {}^{t} \dot{\mathbf{p}} \cdot dV$$

$$+\int_{V} (\mathbf{N}_{w})^{T} \cdot \tilde{\alpha} \cdot S_{w} \cdot \mathbf{I}^{T} \cdot \mathbf{B} \cdot {}^{t} \dot{\mathbf{d}} \cdot dV = 0$$
(6)

where $Q^w = \frac{(n-\tilde{\alpha})}{K} \cdot S_w \left(S_w + p_w \frac{w_s}{n}\right) + \frac{nS_w}{K_w} + w_s$, K_w is the bulk modulus of water, **k** is the absolute permeability matrix of the medium which depends only on the current geometry of the porous network through which the fluid flow occurs, k_{rw} is the relative permeabilities of water

phase which depends in general on the relative saturation state of the medium, Neuman (1975). μ_w is the dynamic viscosity of water phase. \hat{q} is the imposed mass flux normal to the boundary.

The shape functions N^s and N^w in Eq. (5) and (6) are expressed in terms of the local coordinates. In the following, the same shape function N is utilized to define both displacements and pore pressures within the element.

Furthermore, the incremental forms of nodal displacements and pore pressures can be expressed as:

$$\Delta^{t} \mathbf{d} = {}^{t} \mathbf{d} - {}^{t-\Delta t} \mathbf{d} = \int_{t-\Delta t}^{t} \dot{\mathbf{d}} \cdot dt$$
(7)

$$\Delta^{t} \mathbf{p} = {}^{t} \mathbf{p} - {}^{t-\Delta t} \mathbf{p} = \int_{t-\Delta t}^{t} \dot{\mathbf{p}} \cdot dt$$
(8)

Substituting Eq. (7) and (8) into (5), the new form of the equilibrium equation is obtained as:

$$\int_{V} \mathbf{N}^{T} \cdot \mathbf{\rho} \cdot \mathbf{N} \cdot {}^{t} \ddot{\mathbf{d}} \cdot dV + \int_{V} \mathbf{B}^{T} \cdot {}^{t-\Delta t} \mathbf{D} \cdot \mathbf{B} \cdot \Delta^{t} \mathbf{d} \cdot dV$$
$$+ \int_{V} \mathbf{B}^{T} \cdot \tilde{\alpha} \cdot \mathbf{I} \cdot \mathbf{N} \cdot \left(S_{w} + \frac{w_{s}}{n} \cdot \mathbf{N} \cdot \mathbf{p} \right) \cdot \Delta^{t} \mathbf{p} \cdot dV$$
$$= \int_{\Gamma} \mathbf{N}^{T} \cdot {}^{t} \mathbf{t} \cdot d\Gamma + \int_{V} \mathbf{N}^{T} \cdot \mathbf{\rho} \cdot \mathbf{g} \cdot dV$$
$$- \int_{V} \mathbf{B}^{T} \cdot {}^{t-\Delta t} \mathbf{\sigma} \cdot dV \qquad (9)$$

For brevity, Eq. (9) can be written in matrix form:

$$\mathbf{M} \cdot {}^{t} \ddot{\mathbf{d}} + {}^{t-\Delta t} \mathbf{K} \cdot \Delta^{t} \mathbf{d} + \mathbf{J} \cdot \Delta^{t} \mathbf{p} = {}^{t} \mathbf{F} - {}^{t-\Delta t} \mathbf{f}_{int}$$
(10)

in which:

$$\mathbf{M} = \int\limits_{V} \mathbf{N}^{T} \cdot \boldsymbol{\rho} \cdot \mathbf{N} \cdot dV \tag{11}$$

$${}^{t-\Delta t}\mathbf{K} = \int\limits_{V} \mathbf{B}^{T} \cdot {}^{t-\Delta t}\mathbf{D} \cdot \mathbf{B} \cdot dV$$
(12)

$$\mathbf{J} = \int_{V} \mathbf{B}^{T} \cdot \tilde{\boldsymbol{\alpha}} \cdot \mathbf{I} \cdot \mathbf{N} \cdot \left(S_{w} + \frac{w_{s}}{n} \cdot \mathbf{N} \cdot \mathbf{p} \right) \cdot dV$$
(13)

$${}^{t}\mathbf{F} = \int_{\Gamma} \mathbf{N}^{T} \cdot {}^{t}\mathbf{t} \cdot d\Gamma + \int_{V} \mathbf{N}^{T} \cdot \mathbf{\rho} \cdot \mathbf{g} \cdot dV$$
(14)

$${}^{t-\Delta t}\mathbf{f}_{int} = \int\limits_{V} \mathbf{B}^{T} \cdot {}^{t-\Delta t} \boldsymbol{\sigma} \cdot dV \tag{1}$$

Similarly, the continuity equation (6) can be also expressed in matrix form as:

$$\mathbf{L}^T \cdot {}^t \dot{\mathbf{d}} + \mathbf{G} \cdot {}^t \dot{\mathbf{p}} + \mathbf{H} \cdot {}^t \mathbf{p} = \mathbf{F}^w$$
(16)

in which:

$$\mathbf{L}^{T} = \int_{V} \mathbf{N}^{T} \cdot \tilde{\boldsymbol{\alpha}} \cdot S_{w} \cdot \mathbf{I}^{T} \cdot \mathbf{B} \cdot dV$$
(17)

$$\mathbf{G} = \int_{V} \mathbf{N}^{T} \cdot \boldsymbol{Q}^{w} \cdot \mathbf{N} \cdot dV \tag{18}$$

$$\mathbf{H} = -\int\limits_{V} (\nabla \mathbf{N})^{T} \cdot \mathbf{k} \frac{k_{rw}}{\mu_{w}} \nabla \mathbf{N} \cdot dV$$
(19)

$$\mathbf{F}^{w} = \int_{V} (\nabla \mathbf{N})^{T} \cdot \mathbf{k} \frac{k_{rw}}{\mu_{w}} \nabla \rho_{w} g dV - \int_{\Gamma_{q}} \mathbf{N}^{T} \cdot \hat{q} \cdot d\Gamma$$
(20)

By now, the generalized semi-discrete governing equations for simulating solid-fluid interaction have been developed. The complete solution may now be obtained by means of an appropriate time integration method.

By applying Newmark's scheme for Eq. (10) and (16), the governing equations can be discretized in the time domain. Then, at time t, the dynamic equilibrium condition of Eq. (10) can be rearranged in matrix form as:

$$\mathbf{\hat{K}} \cdot \Delta^t d + \mathbf{J} \cdot \Delta^t p = {}^t \mathbf{S} - {}^{t - \Delta t} \mathbf{f}_{int}$$
(21)

in which

$$\hat{\mathbf{K}} = c_0 \cdot \mathbf{M} + {}^{t - \Delta t} \mathbf{K}$$
(22)

$${}^{t}\mathbf{S} = {}^{t}\mathbf{F} - \mathbf{M}\left(-c_{1} \cdot {}^{t-\Delta t}\dot{d} - c_{2} \cdot {}^{t-\Delta t}\dot{d}\right)$$
(23)

and $c_0 = \frac{1}{\beta \Delta t^2}$, $c_1 = \frac{1}{\beta \Delta t}$, $c_2 = \frac{1}{2\beta} - 1$ and β is Newmark's parameters.

Similarly, at time t, the dynamic continuity condition of Eq. (16) can be rearranged as:

$$\mathbf{L}^{T} \cdot \Delta^{t} \mathbf{d} + (\mathbf{G} + \mathbf{H} \cdot \boldsymbol{\gamma} \cdot \Delta t) \cdot \frac{\beta}{\gamma^{2}} \cdot \Delta^{t} \mathbf{p} =$$

$$\mathbf{\tilde{F}}^{\mathbf{w}} \cdot \Delta t - \mathbf{H} \cdot {}^{t - \Delta t} \mathbf{p} \cdot c_{5} \cdot \Delta t \qquad (24)$$

in which:

$$\widetilde{\mathbf{F}}^{w} = \left[\mathbf{F}^{w} - \left(c_{3} \cdot^{t - \Delta t} \ddot{\mathbf{d}} + c_{4} \cdot^{t - \Delta t} \dot{\mathbf{d}}\right) \cdot \mathbf{L}^{T} + \mathbf{G} \cdot (1 - \gamma) \cdot^{t - \Delta t} \dot{\mathbf{p}}\right] \cdot c_{5}$$
(25)

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5)
$$c_3 = \left(1 - \frac{\gamma}{2\beta}\right)\Delta t; \ c_4 = 1 - \frac{\gamma}{\beta}; \ c_5 = \frac{\beta}{\gamma}$$
 (26)

and γ is Newmark's parameters.

For sake of convenience, Eq. (21) and (24) can be combined in matrix form as:

$$\begin{bmatrix} \hat{\mathbf{K}} & \mathbf{J} \\ \mathbf{L}^{T} & (\mathbf{G} + \mathbf{H} \cdot \boldsymbol{\gamma} \cdot \Delta t) \cdot \frac{\beta}{\boldsymbol{\gamma}^{2}} \end{bmatrix} \cdot \begin{cases} \Delta^{t} \mathbf{d} \\ \Delta^{t} \mathbf{p} \end{cases}$$
$$= \begin{cases} {}^{t} \mathbf{S} - {}^{t - \Delta t} \mathbf{f}_{int} \\ \tilde{\mathbf{F}}^{\mathbf{w}} \cdot \Delta t - \mathbf{H} \cdot {}^{t - \Delta t} \mathbf{p} \cdot c_{5} \cdot \Delta t \end{cases}$$
(27)

These are the incremental coupled governing equations that constitute the basis of finite element solutions for porous media under fully or partially saturated conditions. They can be utilized not only for solving dynamic problems but also for static or quasi-static ones without loss of computational efficiency. The governing equations have been implemented into the finite element code INSAP-PM by Liu and Scarpas (2001).

It can be seen that Eq. (27) is not symmetric and hence requires a nonsymmetric solver. Symmetry can be restored, if the condition $w_s p \ll S_w n$ is satisfied or if the material is fully saturated. The general solution techniques for solving the nonsymmetric coupled governing equations will be discussed in the next section.

2.2 Solution procedure

The simplest way to solve the coupled governing equations indicated in Eq. (27) is the so called *monolithic* (or *direct*) approach first proposed by Lewis and Karahanoglu (1981), by which the system of equations is solved simultaneously. However, because of the nonsymmetric characteristics, the matrices at the left side of Eq. (27) need to be evaluated at every time step. Therefore, for partially saturated conditions, the monolithic approach is a time consuming procedure while for fully saturated condition, due to symmetry, it is probably the best choice.

In order to overcome the limitation of the monolithic approach, a partitioned solution procedure can be utilized to restore the symmetry of the coupled governing equations, see Park and Felippa (1983) and Schrefler (1985). Using this procedure, the coupled governing equations are first partitioned into a proper form. Then a suitable predictor scheme is set up to evaluate the partitioned equations and to obtain part of the unknowns. Subsequently the

remaining equations are solved to obtain the rest of the unknowns. This procedure is repeated until the required accuracy is achieved. The efficiency of this solution procedure is sometimes questionable due to the necessity of iterations within each time step. Also the choice of a suitable predictor can influence the numerical stability.

In this study, in order to solve the coupled equations efficiently, the Sherman-Morrison formula (Golub, 1989) is applied. With the Sherman-Morrison formula, the unsymmetric matrix in Eq. (27) can be reformed and decomposed into one symmetric matrix and two vector multiplications. The inversion of the matrix required for the solution is computed only once at the beginning of the calculation. The influences of in time pressure variation on the system solutions can be evaluated at every time step by matrix multiplications.

Based on this solution strategy the coupled governing equation (27) can be reformulated as:

$$\begin{bmatrix} \hat{\mathbf{K}} & \mathbf{L} + \Delta \mathbf{L} \\ \mathbf{L}^{T} & (\mathbf{G} + \mathbf{H} \cdot \boldsymbol{\gamma} \cdot \Delta t) \cdot \frac{\beta}{\boldsymbol{\gamma}^{2}} \end{bmatrix} \cdot \begin{cases} \Delta^{t} \mathbf{d} \\ \Delta^{t} \mathbf{p} \end{cases}$$
$$= \begin{cases} {}^{t} \mathbf{S} - {}^{t - \Delta t} \mathbf{f}_{int} \\ \tilde{\mathbf{F}}^{\mathbf{w}} \cdot \Delta t - \mathbf{H} \cdot {}^{t - \Delta t} \mathbf{p} \cdot c_{5} \cdot \Delta t \end{cases}$$
(28)

in which:

$$\Delta \mathbf{L} = \int\limits_{V} \mathbf{B}^{T} \cdot \tilde{\boldsymbol{\alpha}} \cdot \mathbf{I} \cdot \mathbf{N} \cdot \frac{w_{s}}{n} \cdot \mathbf{N} \cdot \mathbf{p} \cdot dV$$
(29)

$$\mathbf{L} = \int_{V} \mathbf{B}^{T} \cdot \tilde{\boldsymbol{\alpha}} \cdot \mathbf{I} \cdot \mathbf{N} \cdot S_{w} \cdot dV$$
(30)

Furthermore, Eq. (28) may be rearranged in the form:

$$\begin{bmatrix} \hat{\mathbf{K}} & \mathbf{L} \\ \mathbf{L}^{T} & \mathbf{I} \end{bmatrix} \cdot \begin{cases} \Delta^{t} \mathbf{d} \\ \Delta^{t} \mathbf{p} \end{cases} + \begin{bmatrix} 0 & \Delta \mathbf{L} \\ 0 & (\mathbf{G} + \mathbf{H} \cdot \boldsymbol{\gamma} \cdot \Delta t) \cdot \frac{\beta}{\boldsymbol{\gamma}^{2}} - \mathbf{I} \end{bmatrix} \cdot \begin{cases} \Delta^{t} \mathbf{d} \\ \Delta^{t} \mathbf{p} \end{cases}$$
$$= \begin{cases} {}^{t} \mathbf{S} - {}^{t - \Delta t} \mathbf{f}_{int} \\ \tilde{\mathbf{F}}^{\mathbf{w}} \cdot \Delta t - \mathbf{H} \cdot {}^{t - \Delta t} \mathbf{p} \cdot c_{5} \cdot \Delta t \end{cases}$$
(31)

It can be observed that symmetry is restored in the first matrix of Eq. (31). Formally, to solve this large system of linear equation, inversion of matrices is required. According to the Sherman-Morrison formula, if **A** is a non-singular $n \times n$ invertible matrix and $1 + \mathbf{v}^T \cdot \mathbf{A}^{-1} \cdot \mathbf{u} \neq 0$

in which **u** and **v** are two vectors with n entries, then the following identity holds:

$$\left(\mathbf{A} + \mathbf{u} \cdot \mathbf{v}^{T}\right)^{-1} = \mathbf{A}^{-1} - \frac{\mathbf{A}^{-1} \mathbf{u} \mathbf{v}^{T} \mathbf{A}^{-1}}{1 + \mathbf{v}^{T} \mathbf{A}^{-1} \mathbf{u}}$$
(32)

Applying Eq. (32) into Eq. (31), matrix **A** in Eq. (32) becomes:

$$\mathbf{A} = \begin{bmatrix} \mathbf{\hat{K}} & \mathbf{L} \\ \mathbf{L}^T & \mathbf{I} \end{bmatrix}$$
(33)

and the vectors **u** and **v** can be defined as:

$$\mathbf{u} = \begin{bmatrix} \Delta \mathbf{L}, & (\mathbf{G} + \mathbf{H} \cdot \boldsymbol{\gamma} \cdot \Delta t) \cdot \frac{\beta}{\gamma^2} - \mathbf{I} \end{bmatrix}^T$$
(34)

$$\mathbf{v}^T = \begin{bmatrix} 0, & 1 \end{bmatrix} \tag{35}$$

Hence, the solution of the governing equation (31) can be obtained numerically by means of the Newton-Raphson iterative procedure with one time matrix inversion. At every time step, the contribution of the pressure update on the system solution is evaluated by the matrix multiplications of Eq. (32).

Obviously, the required computer time with this solution procedure depends mainly on the speed of matrix multiplications. Since this solution process involves only matrix inversion and multiplications, without choosing any predictor, numerical stability problems, which occur in the partitioned solution procedure, can be avoided.

In algorithmic format, the steps necessary for the above solution procedure of the governing equations are presented in Liu (2003).

3 Constitutive model

In order to be capable of simulating strain localization phenomena, an appropriate constitutive model needs to be specified. In the finite element analysis of geotechnical problems, the choice of an appropriate constitutive model may have a significant influence on the numerical results. The constitutive model should be able to capture the main features of the mechanical behaviour of geotechnical materials under complex states of stress.

In this study, on the basis of the hierarchical approach proposed by Desai (1980), a modified form of the Desai yield function is proposed to simulate the elastoplastic characterization of the geomaterial, see Liu, Cheng, Scarpas. and Blaauwendraad (2004a). The models are general and sufficiently simplified in terms of number of material parameters and every parameter has a clear physical meaning.

In general, the proposed model is applicable for any frictional material. However, in this study, only geotechnical materials will be considered. In this section, the emphasis is placed on the presentation of the model characteristics in both the hardening and the softening ranges of response, and the establishment of model nonassociativity.

3.1 Basic associative model

Every constitutive model has its advantages and limitations. One of the major limitations of commonly used cap or critical state models, is that the yielding is controlled by two separate yield functions that intersect each other with a slope discontinuity. In associated plasticity theory, the incremental plastic strain is assumed to be normal to the flow surface at the loading point. Thus, in case of two intersecting flow surfaces, the direction of the incremental plastic strain is not defined uniquely at the point of surface intersection. Thus the volumetric and shear response of the material cannot be properly predicted.

The single surface plasticity model proposed by Desai (1980) includes most of the currently common used plasticity models as special cases. The surface is continuous (smooth) and hence avoids the above mentioned discontinuity problems of multisurface models.

The particular form of employed yield function is given by:

$$F = \frac{J_2}{p_a^2} - \left[-\alpha \cdot \left(\frac{I_1 + R}{p_a} \right)^n + \gamma \cdot \left(\frac{I_1 + R}{p_a} \right)^2 \right] = 0 \quad (36)$$

where I_1 and J_2 are first and second stress invariants respectively, p_a is the atmospheric pressure with units of stress, parameter R represents the triaxial strength in tension.

The yield function in Eq. (36) can be written also in terms of effective mean normal stress p' and deviator stress q as:

$$F = \frac{q^2}{3p_a^2} - \left[-\alpha \cdot \left(\frac{3p'}{p_a}\right)^n + \gamma \cdot \left(\frac{3p'}{p_a}\right)^2 \right] = 0$$
(37)

The material isotropic hardening/softening is described by means of parameter α in the yield function. The val-



Figure 1 : α influences on F in p' - q space



Figure 2 : n influences on F in p' - q space

ues of α control the size of the flow surface. It is typically defined as a function of deformation history. As α decreases, the size of the flow surface increases, Figure 1. When $\alpha = 0$, the ultimate stress response surface of the material is attained.

The value of n determines the apex of the flow surface on the $I_1 - \sqrt{J_2}$ or p' - q space. Parameter n is related to the state of stress at which the material response changes from compaction to dilation. Its influence on the geometric characteristics of the surface is portrayed in Figure 2. It is worth noticing that not only the shape but also the size of the surface is influenced as well.

Parameter γ is related to the ultimate strength of the material. It denotes the slope of the ultimate stress response surface.

According to experimental observations in Cheng, den Haan and Barends (2001), the ultimate stress response surface of some geomaterials is not always a straight line



Figure 3 : g influences on F in p' - q space

on the $I_1 - \sqrt{J_2}$ or p' - q space, but a curved one. In order to enhance the applicability of the classical Desai yield function in Eq. (36), a modified form of the yield function has been utilized by Liu, Cheng, Scarpas. and Blaauwendraad (2004a) and shown in Eq. (38). In this, the exponent 2 in the multiplier of γ in Eq. (36) is replaced by the parameter g:

$$F = \frac{J_2}{p_a^2} - \left[-\alpha \cdot \left(\frac{I_1 + R}{p_a}\right)^n + \gamma \cdot \left(\frac{I_1 + R}{p_a}\right)^g \right] = 0 \quad (38)$$

Similarity to Eq. (37), the modified yield function of Eq. (38) can be written in terms of effective mean normal stress p' and deviator stress q as:

$$F = \frac{q^2}{3p_a^2} - \left[-\alpha \cdot \left(\frac{3p'}{p_a}\right)^n + \gamma \cdot \left(\frac{3p'}{p_a}\right)^m \right] = 0 \qquad (39)$$

Parameter g controls the shape of the ultimate response surface of the material. The influence of g on the ultimate stress response surface in p'-q space is presented in Figure 3. For g < 2, the ultimate response surface becomes concave. If g = 2 the classical Desai ultimate stress response surface is attained.

3.2 Modeling of hardening and softening behavior

3.2.1 Material hardening

According to experimental evidence, it is generally known that during the process of incremental plastic deformation, the yield surface changes size, shape and location. A law governing this aspect of response is called the hardening rule. The manner in which hardening occurs for geotechnical materials can be quite complicated. For this reason, some simplified assumptions must be made in the view of the numerical implementation.

Mathematically, hardening is characterized by parameters that vary with the plastic loading history. The hardening parameter is often a function of the effective plastic strain or plastic work. There are several hardening rules that have been proposed to describe the growth of subsequent yield surfaces for material hardening. These are: isotropic hardening, kinematic hardening and mixed hardening.

For quasi-static monotonic loading, the isotropic hardening model is appropriate for the representation of material behaviour. In the case of reversals of loading, kinematic mixed hardening may be more appropriate. In this study, only isotropic hardening is considered.

According to the theory of plasticity, for an isotropically hardening material, the plastic deformations are associated with expansion of the flow surface. Therefore, parameter α employed here for the constitutive model can be defined as a function of the plastic deformation history. The actual functional form of α should be determined on the basis of laboratory tests.

The parameter α can be typically expressed in terms of internal variables such as the effective plastic strain, the plastic work, the dissipated energy etc. It was found that use of the effective plastic strain provides a more consistent formulation than that of plastic work (Desai, 2001). Also it is relatively easier to compute the effective plastic strain from available test data.

Hence α is expressed as:

$$\alpha = \alpha \left(\xi, \xi_{\nu}, \xi_{d} \right) \tag{40}$$

in which the effective plastic strain ξ is defined on the basis of plastic strain increments $d\epsilon_{ij}^p$ as:

$$\xi = \int \left(d\varepsilon_{ij}^p d\varepsilon_{ij}^p \right)^{\frac{1}{2}} \tag{41}$$

Obviously, the magnitude of the effective plastic strain ξ never decreases. ξ_V and ξ_d are the volumetric and deviatoric components of ξ respectively. They can be expressed as:

$$\xi_d = \int \left(de^p_{ij} \cdot de^p_{ij} \right)^{1/2} \tag{42}$$

$$\xi_{\nu} = \int \frac{1}{\sqrt{3}} \left(d\varepsilon_{kk}^{p} \cdot d\varepsilon_{kk}^{p} \right)^{1/2} \tag{43}$$

where de_{ij} is incremental deviatoric plastic strain tensor defined as:

$$de_{ij}^{p} = d\varepsilon_{ij}^{p} - \frac{1}{3}d\varepsilon_{kk}^{p} \cdot \delta_{ij}$$

$$\tag{44}$$

and $d\varepsilon_{kk}^{p}$ is the incremental volumetric plastic strain. δ_{ij} is the Kronecker deta.

Several forms of α have been developed for description of the hardening response of various engineering materials, see Desai and Faruque (1984); Desai, Somasundaram and Frantziskois (1986). Based on laboratory observations for various stress paths, material hardening response is influenced both by the coupled and uncoupled actions from volumetric and deviatoric plastic deformations. For example, in a hydrostatic compression test, since the stress path corresponding to this test remains along the hydrostatic axis, only volumetric plastic deformations are created. On the other hand, for purely shear loading, there will be no volume change and the material will experience only large shear deformations.

In order to take these observations into account, in the framework of this study, parameter α of the modified Desai surface in Eq. (38) is expressed as a function of both volumetric and deviatoric hardening components, α_V and α_D :

$$\alpha = \eta_h \cdot \alpha_V + (1 - \eta_h) \cdot \alpha_D \tag{45}$$

where:

$$\alpha_V = a_1 \cdot e^{b_1 \cdot \xi_\nu} \tag{46}$$

$$\alpha_D = c_1 \cdot \left[1 - \frac{(M')^2}{27\gamma} \cdot \left(\frac{\xi_d}{d_1 + \xi_d}\right)^2 \cdot \left(\frac{3p_c}{p_a}\right)^{(2-g)} \right] \quad (47)$$

$$\eta_h = \frac{\zeta_v}{\xi_v + \xi_d} \tag{48}$$

 α_V and α_D are the volumetric and deviatoric hardening components respectively. a_1, b_1, c_1 , and d_1 are hardening parameters. The ratio η_h in Eq. (48) denotes the

contribution of volumetric hardening to the overall material hardening response. M' is the peak stress ratio. p_c is the soil preconsolidation pressure.

When volumetric and deviatoric behaviour are coupled, α can be determined directly by using Eq. (45).

Details of the development of mathematical expressions for α_V and α_D including the determination of the corresponding hardening parameters are presented in Liu, Cheng, Scarpas. and Blaauwendraad (2004a).

3.2.2 Material softening

Typical stress-strain behaviour for a soil under compressive loading is shown in Figure 4. This figure indicates that, for deformations beyond those corresponding to the ultimate strength (denoted by d), the material undergoes softening (degradation) in its strength and stiffness. Nevertheless, it continues to carry load until it approaches its residual strength at the critical state (denoted by c).

In this study, an isotropic measure of response flow surface degradation has been introduced into the model to simulate the softening process. This adaptation of the model is achieved by means of specifying the variation of parameter α , after response degradation initiation, as an increasing function of the monotonically varying equivalent post fracture plastic strain ξ_{pf} :

$$\alpha = \alpha_R + \eta_s \cdot (\alpha_u - \alpha_R) \tag{49}$$

in which:

$$\eta_s = e^{-\kappa_1 \cdot \xi_{pf}} \tag{50}$$



Figure 4 : Stress-strain behaviour

and α_u and α_R are the values of α corresponding to material ultimate stress response and residual stress state respectively, see Figure 5. The parameter κ_1 is a material parameter that determines the material degradation rate.

The definition of ξ_{pf} is similar to the effective plastic strain ξ defined in Eq. (41). The difference is that only incremental plastic strains after response degradation initiation are now taken into account.

The variation of α as a function of ξ_{pf} is shown in Figure 5. At material degradation initiation, due to rapid material softening, α increases quickly. As the state of residual response is approached, the variation of α becomes insignificant and softening gradually ceases.



By relating α to a physically measurable quantity like the plastic strains, its functional form can be determined on the basis of laboratory tests. Also, with this simplified approach, only one parameter κ_1 needs to be determined to characterize the material softening response. The determination procedure of the softening parameter κ_1 is presented in Liu (2003).

3.3 Nonassociative model

The physical soil characteristics such as density, void ratio, water content and mineralogy can greatly influence soil behaviour. Successful prediction of soil response depends on whether the material model used can capture the significant characteristics of response under engineering conditions. The most significant characteristics of soil response are the ultimate strength, the dilation or contraction, the hardening or softening response and the stress path dependence.

Both associated and nonassociated flow rules are commonly used with plasticity models for geotechnical materials. For some materials such as metals and undrained cohesive soils, the use of the associated flow rule is most common. On the other hand, for some frictional and cohesionless soils, material models incorporating the associated flow rule usually exhibit plastic dilation that is larger than the one that is observed in laboratory testing. In this case, it is necessary to employ a nonassociative flow rule for plasticity modelling.

In the hierarchical approach, see Desai, Somasundaram and Frantziskonis (1986), a nonassociative model is obtained by defining the potential function as a correction/modification to the yield function. This correction approach can be used to develop models of various grades for characteristics such as associative and isotropic hardening, nonassociative, isotropic and anisotropic hardening and strain softening response.

By utilizing the notion of correction of the yield function, the potential function Q is expressed as:

$$Q = F + h(I_1, J_i, \xi) \tag{51}$$

in which F is the yield function defined in Eq. (38), $h(I_1, J_i, \xi)$ is a correction function consisting of stress invariants $I_1, J_i (i = 2, 3)$ and ξ is the effective plastic strain defined in Eq. (41).

In the proposed model, the size of Q is controlled by a hardening/softening parameter α_0 defined as:

$$\alpha_Q = \alpha + \alpha_c \tag{52}$$

in which α_c is a correction function expressed as:

$$\alpha_c = \kappa_c \left(\alpha_0 - \alpha \right) \left(1 - \chi_\nu \right) \tag{53}$$

It is determined on the basis of experimental evidence. The parameter α_0 in Eq. (53) is the value of α at the initiation of nonassociativeness. The parameter χ_{ν} controls the contribution of volumetric plastic deformation to the expansion of the potential surface and is defined by:

$$\chi_{\nu} = \frac{\xi_{\nu}}{\xi} \tag{54}$$

where ξ_{ν} is the volumetric component of effective plastic strain ξ . The parameter κ_c in Eq. (53) is the only extra material parameter that needs to be determined to capture material nonassociative behaviour.

uata									
E	79	b ₁	-3044	g	1.795				
(Mpa)									
ν	0.29	c ₁	1.17e-03	k _w	1.0e-06				
				(m/s)					
n	5.04	d1	5.53e-03	L (mm)	100				
γ	0.155	κ_c	0.47	B (mm)	50				
a ₁	1.17e-	κ ₁	3.5	W	10				
	03			(mm)					

 Table 1 : Material parameters and specimen geometry

Thus, the plastic potential Q in Eq. (51) is written in $I_1 - J_2$ space as:

$$Q = \frac{J_2}{p_a^2} - \left[-\alpha_Q \cdot \left(\frac{I_1 + R}{p_a}\right)^n + \gamma \cdot \left(\frac{I_1 + R}{p_a}\right)^g \right]$$
(55)

It can be observed that for isotropically hardening material subject to hydrostatic compression loads, $\chi_v = 1$ in Eq. (54)and hence $\alpha_Q = \alpha$ in Eq. (52). This means that nonassociativeness does not occur under a hydrostatic compression condition. For the case of $\kappa_c = 0$, the potential function Eq. (55) yields Q = F indicating that the associative model is a special case of the nonassociative one.

In the following sections, application of the nonlinear model for simulation of strain localization in saturated sands will be presented.

4 Numerical examples

4.1 3D specimen subjected to triaxial load

4.1.1 Specimen geometry and material characteristics

A fully saturated cubic 3D specimen with impermeable boundaries was selected for the numerical simulations. The finite element mesh consisted of 20-nodded brick elements, Figure 6. The geometry parameters of the specimen are given in Table 1. Because of symmetry, only half of the specimen thickness was simulated.

A confining pressure of 150 kPa was applied to all boundaries of the specimen and kept constant throughout the analysis. Incremental displacements are applied on a rigid platen at the top of the specimen. The left, right and front planes of the specimen could move freely in the normal direction of each plane. The bottom plane of the specimen was constrained in the y-direction. In order to



Figure 6: 3D specimen geometry and loading conditions

simulate the real test conditions, interface elements were introduced at the top and the bottom of the sand specimen between the rigid platens and the specimen. By adjusting the bond stiffness of the interfaces, the influence of the roughness of the platen on strain localization within the specimen could be simulated.

In this study, dense sand was chosen for the numerical investigation. The nonassociative constitutive model described in section 3 was chosen to simulate the material nonlinear response. The material parameters have been derived on the basis of triaxial tests on 'Eastern Scheldt' dense sand, see Cheng, den Haan and Barends (2001), and are shown in Table 1.

4.1.2 Correlation between suction and strain localization

According to laboratory observations, in a drained test, depending on the stress level, granular materials exhibit a dilatant or contractive response. In an undrained test, due to the incompressibility of the fluid, the tendency to dilate or to contract will induce pore water pressure variations. The dilatancy of the material can build up positive excess pore water suction in the specimen and hence increase the shear strength. In this section, emphasis lies on the relation between positive excess pore water (suction) and the occurrence of strain localization.







Figure 8 : Plots at vertical displacement = 1.5025mm



Figure 9 : Plots at vertical displacement = 1.6297mm

The specimen in Figure 6 consisting of 800 elements, with height L=100mm, width B=50 mm and thickness W=10 mm was investigated.

Figure 7 through Figure 9 present the plots of excess water pressure, water flow direction, as well as the corresponding effective plastic strains in the specimen. It can be observed that initially, the distribution of the water suction field in the specimen was nearly homogenous and only minor concentrations of plastic strain occured at the corners of the top and bottom surfaces of the specimen, Figure 7. In such conditions, the undrained specimen is constrained to isochoric deformation (i.e. deformation without volume change).

At a vertical displacement of 1.5025 mm, four regions of high effective plastic strain developed symmetrically around the central part of the specimen, Figure 8(c). These caused uneven gradients of excess pore water pressure, Figure 8(a), and hence flow towards the regions of higher plastic strains, Figure 8(b).

At a vertical displacement of 1.6297 mm, two pairs of dominant shear bands formed and propagated to the boundaries of the specimen, Figure 9(c). During shear band development, large material dilatancy occured inside the shear bands. As a consequence, high excess pore pressure gradients develop within the dilated regions, Figure 9(a), hence water flowed mostly towards them, Figure 9(b).

The variations of excess pore water pressure inside and outside the shear bands are compared in Figure 10. It is observed that, due to the contraction of sand at the beginning of loading, negative excess pore water pressure (compression) built up initially. As soon as the vertical displacement of the loading platen increased to a certain value, the sand started to dilate and hence positive excess water pressure (suction) appeared. At the onset of localization, an abrupt jump of the positive excess water pressure inside the shear band was observed. This jump of positive water pressure may relate to the fluid cavitation phenomena as indicated by Schrefler, Sanavia, and Majorana (1996), Gawin, Sanavia and Schrefler (1998) and Mokni and Desrues (1998).

At cavitation, the fluid phase changes to vapour, a phenomenon that occurs when the water pressure decreases below the vapour saturation pressure. In such condi-



Figure 10 : Comparison of excess pore water pressure inside and outside shear band



Figure 11 : Comparison of load-deformation and excess water pressure inside shear band for three undrained specimens



Figure 12 : The variation of excess water pressure inside shear band

tions, the undrained specimen cannot be constrained to isochoric deformation.

Figure 11 presents the variations of excess water pressure at the intersection point of the shear bands and the corresponding load-deformation curves for three dense sand specimens (indicated by cases 1 to 3). It can be seen that, in all three cases, the abrupt jump of the positive excess pressure (suction) tends toward the same final value, which means that all tests start to cavitate at the same pore pressure and also that, the abrupt degradation of the material strength takes place the moment cavitation occurs.

These observations cannot be attributed to accidental phenomena, because in such a case there would be no reason for the pore pressure value at localization to be the same in the three cases. The results obtained allow us to conclude that for dense sands under undrained conditions it is cavitation of the pore water that leads to abrupt degradation of the strength of the material, and not vice versa. These numerical observations coincide with the experimental observations carried out on biaxial undrained tests for Hostun RF dense sand by Mokni, and Desrues (1998). They stated that in these tests localization starts only after cavitation took place.

The magnitude of confining pressure can influence water suction in the specimen. For instance, the variations of excess water pressure inside the shear band for a specimen with two confining pressures, 150kPa and 400kPa, are shown in Figure 12. It can be observed that in specimens with higher confining pressure, higher suction developed in the shear band.

4.2 3D specimen subjected to triaxial load and in or out flow conditions

4.2.1 Specimen geometry and material characteristics

It was already observed in the previous section, that the onset and the development of strain localization are both strongly influenced by the level of water suction in the specimen.

It is of interest to investigate what happens at the onset and development of strain localization in a specimen when the pore water suction field is disturbed. To achieve this, the same specimen that was utilized in the previous section was chosen,

however this time water outflow or inflow was specified in the center four elements of the specimen during the

E (MPa)	79	a ₁	1.17e-03	κ ₁	3.5
ν	0.29	b1	-3044	$k_w(m/s)$	1.0e-09
n	4.36	c ₁	1.17e-03		
γ	0.155	d ₁	5.53e-03	inflow rate $(mm^3 / (s^*mm^2))$	-0.1
g	1.795	κ_c	0.47	outflow rate $(mm^3 / (s*mm^2))$	0.1

 Table 2 : Material parameters

triaxial load test, see Figure 13. The material parameters are shown in Table 2.

4.2.2 Comparison of the evolution of strain localization

Figure 14 through Figure 15 present diagrams of the nodal pore pressure and the effective plastic strain in the specimen when inflow is applied in the central part. Three stages of the analysis are shown: before, at and after the onset of localization.

Figure 14 (a) shows a contour plot of the nodal pore pressures in the specimen before initiation of strain localization. As a result of the application of inflow in the specimen, a field of negative pore pressures can be observed in the centre of the specimen. Due to the material dilatancy, in the remaining part of the specimen, positive excess pore water pressure (suction) was generated. The applied inflow led to a decrease of the shear strength in the center and hence larger effective plastic strains developed at this localization, Figure 14 (b).

As the vertical displacement is increased, the negative pore pressure was restricted only to a small zone from the



Figure 13 : Schematic of 3D Specimen subjected to in/out flow

centre of the specimen, Figure 15 (a). Most part of the specimen was subjected to positive pore pressure. The appearance of the shear bands in the centre of the specimen can be observed in Figure 15 (b).

After the formation of the shear bands was completed, the suction field gradually diminished and negative pore pressure appeared in the entire specimen, Figure 16 (a). The effective plastic strains evolved along the same lines as in the previous stage, Figure 16 (b). The magnitudes of the effective plastic strains in the central part of the specimen are now almost twice as large as in Figure 15 (b). The induced inflow direction can be clearly seen in Figure 17

Figure 18 through Figure 20 present the diagrams of nodal pore pressure and effective plastic strain in the specimen when outflow was applied in the central part of the specimen. Similar to the previous cases, the three stages of the analysis: before, at and after onset of localization are shown.

Figure 18 (a) shows the nodal pore pressure diagram before the shear bands occured in the specimen. It can be observed that, due to the induced outflow, higher values of suction concentrated in the centre of the specimen and hence less effective plastic strains were generated in this part of the specimen, Figure 18 (b).

As the vertical displacement was increased, higher suction pressures are generated in the centre of the specimen, Figure 19.

As a result, the shear strength of the specimen in this region increased and, in contrast to the previous case, the shear bands shifted more towards the loading platens. The magnitude of the effective plastic strain in the shear bands was more than twice value of the previous stage, Figure 19 (b).

Continuous vertical displacement of the specimen led to a gradually decreasing suction field and negative pore pressure around the central part of the specimen, Figure 20 (a). As a consequence, higher effective plastic strains in the shear bands near the loading platens were gener-



(a). Nodal pore pressure (b). Effective plastic strains

Figure 14 : Plots before initiation of strain localization (inflow at the center)



(a). Nodal pore pressure

(b). Effective plastic strains

Figure 16 : Plots after initiation of strain localization (inflow at the center)

ated, Figure 20 (b).

Figure 21 presents the direction of outflow in the specimen. It can be clearly seen that the direction of outflow was influenced by the development of the shear bands in the specimen.



Figure 15 : Plots at initiation of strain localization (inflow at the center)



Figure 17 : Plots direction of flow inside the specimen (inflow at center)

Figure 22 shows the load-deformation curves for the three cases. It can be observed that for the specimen subject to inflow, localization in the specimen occurs at a lower vertical displacement. The variation of shear strength of the material depends on the magnitude of the applied inflow/outflow rate.



Figure 18 : Plots before initiation of strain localization (outflow at the center)



(a). Nodal pore pressure (b). H

(b). Effective plastic strains

Figure 20 : Plots after initiation of strain localization (outflow at the center)

5 Conclusions

Strain localization phenomena have been observed in many geotechnical engineering problems. Gaining a better understanding of the mechanics and physics of strain localization is important for design purposes.



(a). Nodal pore pressure

(b). Effective plastic strains

Figure 19 : Plots at initiation of strain localization (outflow at the center)



Figure 21 : Plots direction of flow inside the specimen (outflow at center)

Based on the numerical studies, the following conclusions can be drawn:

The governing equations and the constitutive model are capable of simulating the elastoplastic characteristics of a porous medium.

In undrained conditions, due to the incompressibility of



Figure 22 : Load-deformation curves

the fluid, the tendency to dilate will build up water suction in the specimen. The induced water suction leads to soil shear strength increases. However, water suction can only delay but not preclude the development of strain localization in the specimen.

The water suction field in sand under undrained conditions can influence significantly the onset and development of strain localization.

The strain localization in specimens under undrained conditions always coincides with fluid cavitation. There is a physical connection between these two phenomena. It is cavitation of the pore water that leads to abrupt degradation of the strength of the material, and not vice versa

Additional outflow and inflow applied to a specimen can greatly modify the pattern of the pore water pressure and effective plastic strains development. In the case of outflow induced test, the shear bands move towards the platens and in the case of inflow induced test, the shear bands develop in the centre of the specimen.

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