

# Nonlinear Dynamic Response Analysis of Steel Frames under Seismic Action

T.Yoda<sup>1</sup> and N.Kodama<sup>2</sup>

**Abstract:** A nonlinear dynamic response analysis of a box section steel rigid frame under seismic action is proposed on the basis of a beam model. The average stress-strain relation of the beam model can be formulated for transverse stiffener spacing, in which stress-strain relation after local buckling is assumed. As a result of the present study, the maximum lateral displacements and the residual displacements of a box section steel rigid frame were well estimated by the proposed beam element model that considers the deterioration effect due to local buckling.

**keyword:** Nonlinear dynamic analysis, Steel frame, Seismic.

## 1 Introduction

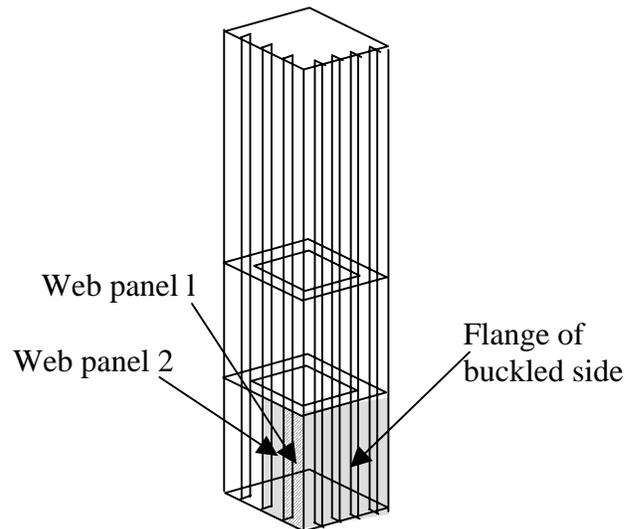
After the disastrous Hyogoken-Nanbu earthquake in 1995, the Japan Society of Civil Engineers introduced a performance-based design method [Japan Road Association (1996)]. Several problems in adopting the performance-based design remain to be unsolved. One of those is an evaluation method for dynamic behavior of steel structures in the post-peak region.

Simple evaluation methods such as the ductility design method, which is based on the equal energy assumption of an elasto-plastic single degree-of-freedom structure [Newmark (1969)], or the time-history response analysis method using the beam moment-curvature model have been used. However, application of these simple methods to steel rigid frames cannot be rationalized, because steel rigid frames have often complicated shapes, and strong ground motion may cause frames severe inelastic deformation with local buckling. In the conventional analyses using beam elements, a general stress-strain model called a fiber model is used so that structures to be analyzed may have arbitrary shapes [Isoe, Ominami, Yoshikawa, Kishida and Ishige (1998)]. However, the stress-strain model that can evaluate residual displacements of locally

buckled steel rigid frames with adequate precision has not been established.

This paper is concerned with a nonlinear dynamic analysis of buckled steel frames under seismic action. The objective of the present analysis is to demonstrate a stress-strain hysteresis model with deterioration effect due to local buckling and to perform an elasto-plastic nonlinear dynamic response analysis.

## 2 Local Buckling Behavior of Box Section Steel Columns



**Figure 1 :** Box section steel column

In order to establish a beam element model that considers the effect of local buckling, time-history response analyses using a shell FE analysis were performed for the local buckling problem of box section steel columns. Approximate formulas for the mean axial stress-mean axial strain relation of locally buckled columns can be derived from the results of shell FE analyses. The plastic moduli  $H$  after local buckling are approximated by using the least squares solutions [(Kodama and Yoda (2004))].

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Flange of buckled side (Fig. 1):

$$\begin{aligned} H_{pf}^d &= \frac{d\sigma_f}{d\varepsilon_p} = \sigma_{cr}^T \frac{d}{d\varepsilon_p} \left( \frac{\sigma}{\sigma_{cr}} \right)_f \\ &= \frac{-5000\sigma_{cr}^T (\varepsilon_p - \varepsilon_{pcr})}{\left\{ 1 + 10000 (\varepsilon_p - \varepsilon_{pcr})^2 \right\}^{1.25}} \end{aligned} \quad (1)$$

Web panel adjacent to the buckled flange (web panel 1):

$$\begin{aligned} H_{pw1}^d &= \frac{d\sigma_{w1}}{d\varepsilon_p} = \sigma_{crw} \frac{d}{d\varepsilon_p} \left( \frac{\sigma}{\sigma_{cr}} \right)_{w1} \\ &= \frac{-13000\sigma_{crw} (\varepsilon_p - \varepsilon_{pcr})}{\left\{ 1 + 46600 (\varepsilon_p - \varepsilon_{pcr})^2 \right\}^{1.14}} \end{aligned} \quad (2)$$

Web panel next to the buckled web (web panel 2):

$$\begin{aligned} H_{pw2}^d &= \frac{d\sigma_{w2}}{d\varepsilon_p} = \sigma_{crw} \frac{d}{d\varepsilon_p} \left( \frac{\sigma}{\sigma_{cr}} \right)_{w2} \\ &= \sigma_{crw} \left[ \frac{d}{d\varepsilon_p} \left( \frac{\sigma}{\sigma_{cr}} \right)_f \cdot \left( \frac{\sigma}{\sigma_{cr}} \right)_{w1} \right. \\ &\quad \left. + \left( \frac{\sigma}{\sigma_{cr}} \right) \cdot \frac{d}{d\varepsilon_p} \left( \frac{\sigma}{\sigma_{cr}} \right)_{w1} \right] \end{aligned} \quad (3)$$

in which  $\sigma_f$ = axial stress of the flange,  $\sigma_{cr}^T$ = buckling stress of the flange,  $\sigma_{wn}$ = axial stress of web ( $i = 1, 2$ ),  $\sigma_{crw}$ = buckling stress of the web,  $\varepsilon_p$ = axial plastic strain,  $\varepsilon_{pcr}$ = axial plastic strain at buckling,  $\varepsilon_y$ = yield strain.

### 3 Formulation of Beam Elements with Local Buckling

The results of the previous section indicate that the key features of the modeling of the mechanical characteristics of the plastic deterioration hinge are as follows:

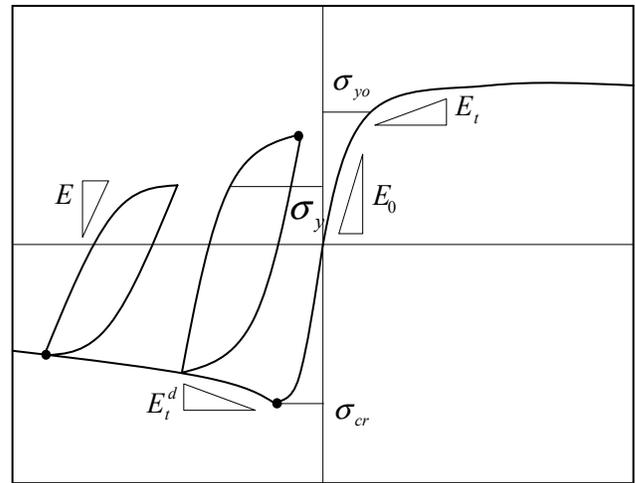
1. Before local buckling, the stress-strain relation follows kinematic hardening rule with strain hardening, and the von Mises yield criterion  $\sigma_y^2 = \sigma^2 + 3\tau^2$  holds true.

2. The local buckling stress of a flange is evaluated as the compressive elasto-plastic buckling stress of a T-section column whose assembly forms the flange.

3. The local buckling stress of each panel in a web is evaluated as the elasto-plastic buckling stress of a simply supported panel subjected to uniform axial compressive stress and in-plane shear stress.

4. The compressive buckling stress of a panel in a web will be smaller than that of the adjacent panel previously buckled.

5. After local buckling, the elastic range contracts in an isotropic manner.



**Figure 2** : Typical mean stress-mean strain curve of a beam element

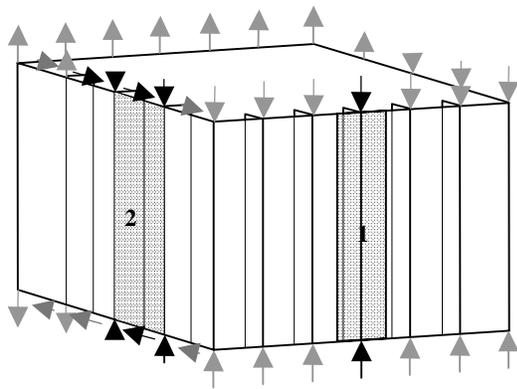
#### 3.1 Evaluation of flange local buckling stress

The orthogonally stiffened plate model is used to evaluate the buckling stress of the stiffened flange plate [Nakai, Kitada, Taido and Fukuoka (1985)]. The buckling stress of the flange plate can be evaluated as the elasto-plastic buckling stress of the T-section column, whose assembly forms the flange. The T-section column can be assumed to be fixed at the fixed end and pin-ended at the nearest transverse stiffener. In addition, the other T-section column is assumed to be pin-ended at the position of the transverse stiffeners. It follows from this that the buckling stress of a T-section flange column becomes

$$\sigma_{cr}^T = \begin{cases} \frac{2.04\pi^2 E_t I_T}{A_T a^2} \\ \frac{\pi^2 E_t I_T}{A_T a^2} \end{cases} \quad (4)$$

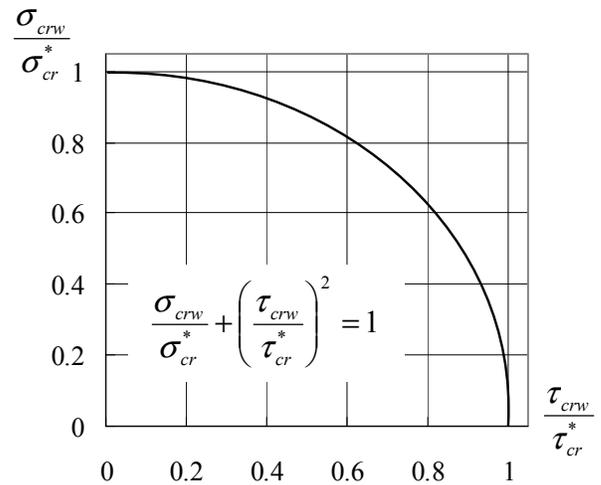
**Table 1** : Material constants in Eq. (5)

Steel type	$C_1$ (MPa)	$C_2$ (MPa)	$\sigma_Y$ (Mpa)	$E_0$ (GPa)
SM400A	264	162	250	206
SM490A	281	98.7	320	
SM490YA	263	-60.2	360	
SM490YB				
SM520B	305	36.4	360	
SM570	246	-94.8	455	



1 Flange: T-section column  
2 Web panel: simply supported plate

**Figure 3** : Modeling for buckling stress evaluation



**Figure 4** : Interaction formula

where  $I_T$ = moment of inertia of the T-section column,  $A_T$ = area of the T-section column,  $a$ = transverse stiffener spacing and  $E_t$ = tangential stiffness at the buckling.

In the present model, the stress-plastic strain curves of the steel materials are approximated by the following equation:

$$\sigma = C_1 \epsilon_p^{1/4} + C_2 \epsilon_p + \sigma_Y \quad (5)$$

Consequently, the plastic modulus  $H_p$  and the tangential stiffness  $E_t$  are given by the following equations:

$$\begin{cases} H_p = \frac{d\sigma}{d\epsilon_p} = \frac{C_1}{4} \epsilon_p^{-3/4} + C_2 \\ E_t = \frac{E_0 H_p}{E_0 + H_p} \end{cases} \quad (6)$$

where  $\sigma_Y$ = uniaxial yield stress of the material,  $C_1$  and  $C_2$ = material constants and  $E_0$ = initial elastic modulus. These parameters are shown in Tab. 1.

### 3.2 Evaluation of web local buckling stress

The buckling stress of the web panel can be approximated by the elasto-plastic buckling stress of a simply supported plate subjected to compressive stress and shear stress. It is assumed that each web panel is under uniform compression and shear, and the buckling condition is given by the following equation [Timoshenko and Gere (1961)]:

$$\frac{\sigma_{crw}}{\sigma_{cr}^*} + \left( \frac{\tau_{crw}}{\tau_{cr}^*} \right)^2 = 1 \quad (7)$$

where  $\sigma_{crw}$ = compressive stress at buckling,  $\tau_{crw}$ = shear stress at buckling,  $\sigma_{cr}^*$ = elasto-plastic compressive buckling stress of the simply supported plate under uniform compression ( $\sigma_{cr}^* = k_c \cdot \sigma_e$ ) and  $\tau_{cr}^*$ = elasto-plastic shear buckling stress of the simply supported plate under uni-

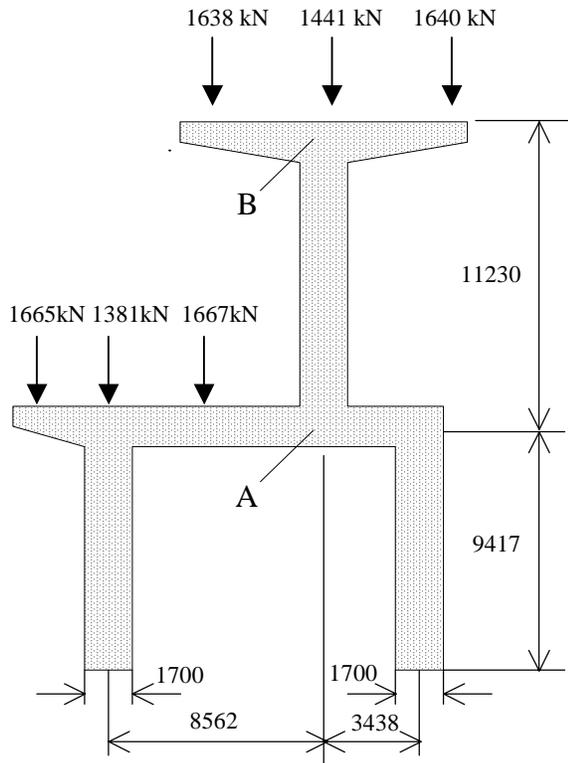


Figure 5 : Box section steel rigid frame (Unit: mm)

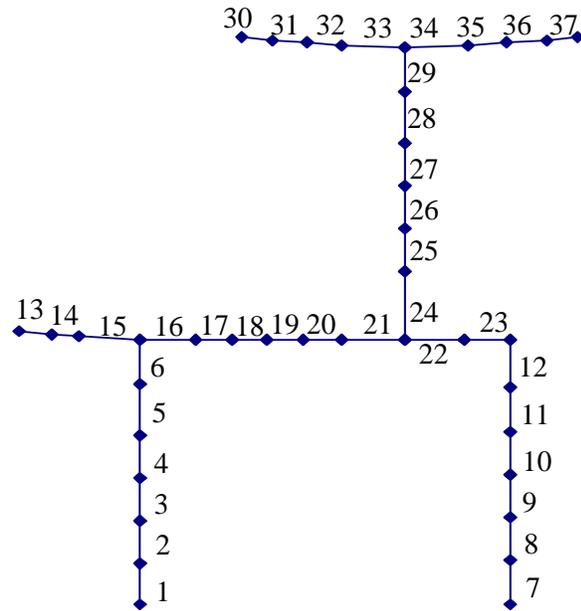


Figure 6 : Beam element discretization

form shear ( $\tau_{cr}^* = k_s \cdot \sigma_e$ ), in which

$$\sigma_e = \frac{\pi^2 E_t}{12(1-\mu^2)} \left(\frac{t_w}{b}\right)^2 \quad (8)$$

$$k_c = \left(\frac{\beta}{m} + \frac{m}{\beta}\right)^2 \quad (9)$$

$$k_s = 5.34 + \frac{4.00}{\beta^2} \quad (10)$$

where  $m$  is the number of waves:  $m=1$  ( $\beta \leq \sqrt{2}$ );  $m=2$  ( $\sqrt{2} \leq \beta \leq \sqrt{6}$ );  $m=3$  ( $\sqrt{6} \leq \beta \leq \sqrt{12}$ );  $m=4$  ( $\sqrt{12} \leq \beta$ ),  $\mu$ =Poisson ratio,  $t_w$ =thickness of web panel,  $b$ =width of web panel,  $\beta=a/b$ ,  $a$ =spacing of transverse stiffeners.

### 3.3 Modeling of stress-strain relations after local buckling

The axial stress-plastic strain curves after local buckling are determined by using Eqs. (1), (2) and (3). In addition, it is assumed that the yield surface keeps its position and contracts in an isotropic manner during compressive loading after local buckling.

### 3.4 Dividing and choosing elements

A two-node beam element is used between transverse stiffeners in view of the normalized stress-strain relation of the plastic deterioration hinge. Moreover, Timoshenko beam element, which allows shear deformation, is used due to the fact that bending of thin steel members causes substantial shear stress in their webs.

## 4 Application to a Box Section Steel Rigid Frame

The proposed model is applied to a nonlinear dynamic response analysis of a box section steel rigid frame as shown in Fig.5. The frame model is discretized to 37 beam elements (Fig.6). The length of a beam element at a corner is defined as the sum of the transverse stiffener spacing and the distance between the column edge (or the beam edge) and the center of the corner. The buckling stresses of the flange or the web panels are calculated with a view to the transverse stiffener spacing. The acceleration wave (Fig.7) is imposed on the bases perpendicular to the bridge axis. Elasto-plastic finite displacement analyses are carried out using Newmark- $\beta$  method for direct integration.

Figure 8 shows the deformed shape of the rigid frame calculated by the beam model. The comparisons of the

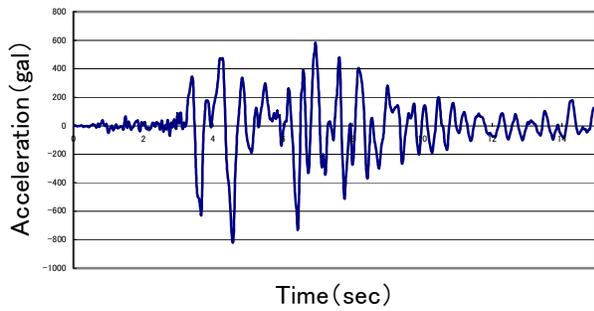


Figure 7 : Input acceleration wave

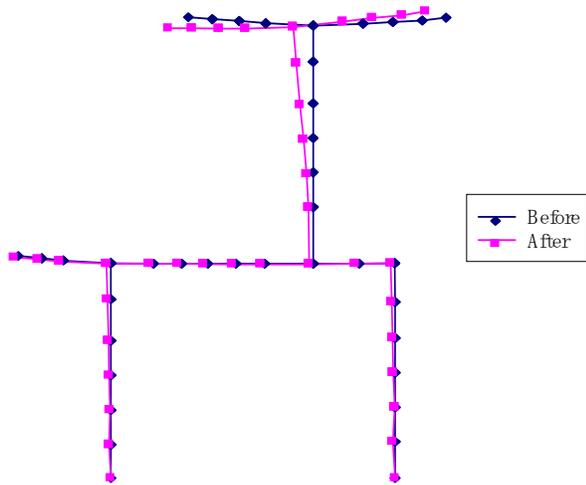


Figure 8 : Deformed shape of the steel rigid frame

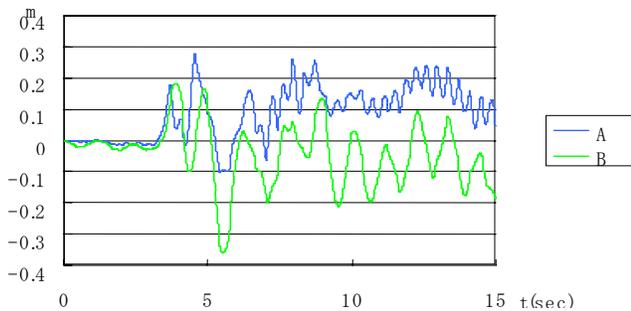


Figure 9 : Time history of lateral relative displacement at corners A, B

lateral relative displacement time histories of the corners (A, B) are shown in Fig.9.

The maximum displacements and the residual displacements of the corners (A, B) of the steel rigid frame were consistent with those of calculated by FE analyses using shell elements [Kodama and Yoda (2004)]. Computation

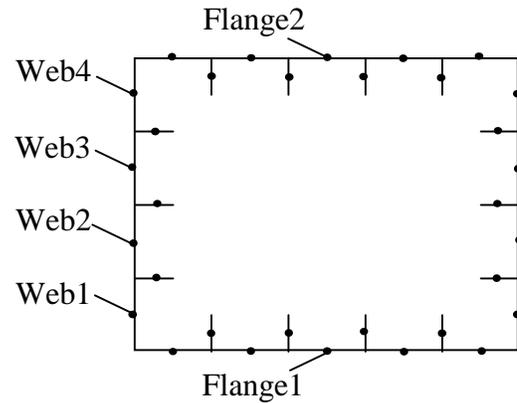


Figure 10 : Cross section of the box section

time will be reduced to less than 8 minutes of the beam model from 120 hours of the shell model, if a workstation with sufficient memory is available. These results show the effectiveness of the proposed beam element model for evaluating the maximum lateral displacements and the lateral residual displacements of box section steel rigid frames.

### 5 Conclusions

The average stress-strain relation of the beam element model, which considers the effect of local buckling, can be reasonably formulated for transverse stiffener spacing. Timoshenko beam elements or equivalent beam elements that can model shear deformation are necessary for constructing beam element models, because bending of thin steel members produces substantial shear stress in their webs.

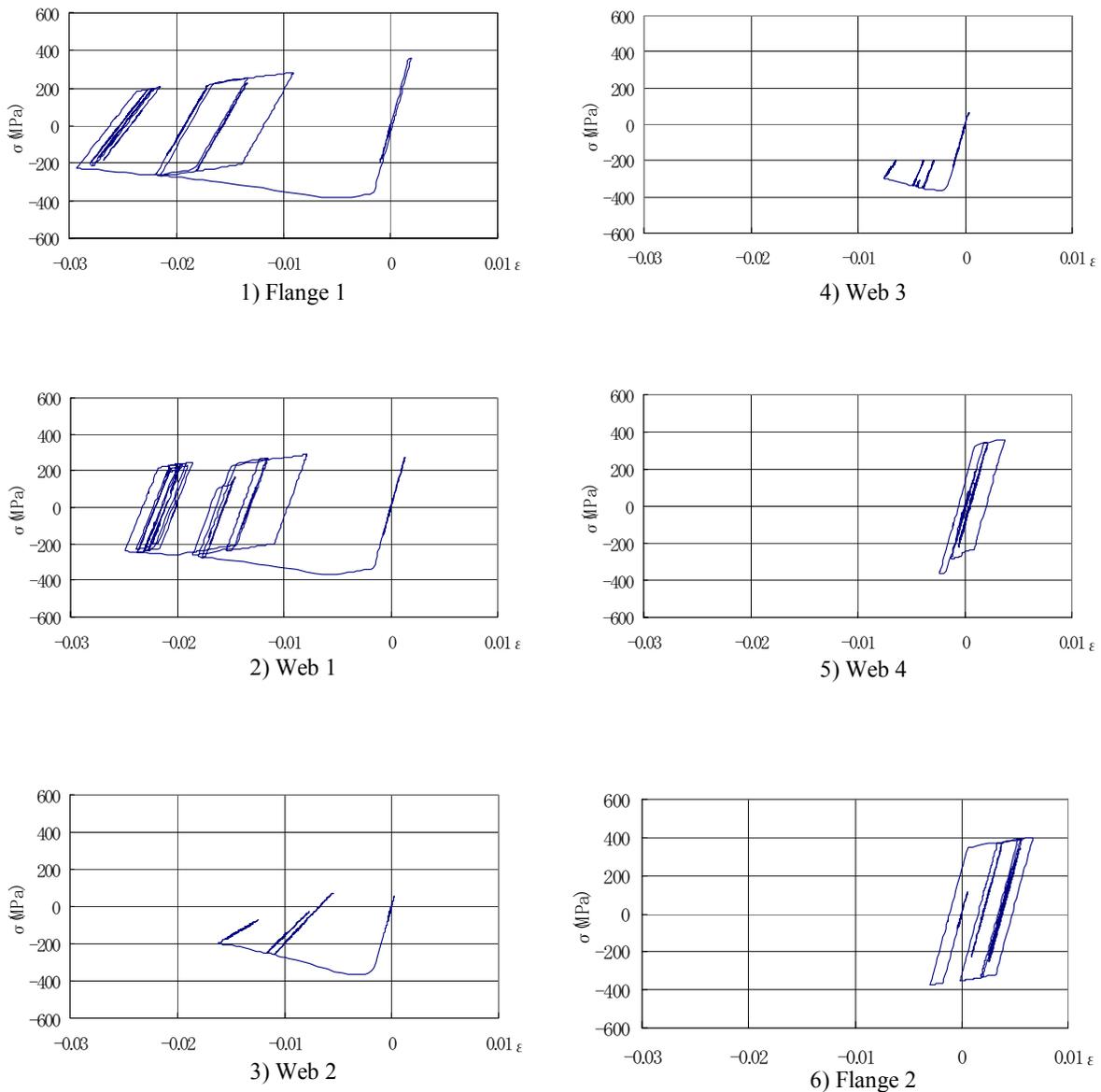
The maximum displacements and the residual displacements of existing box section steel rigid frames may be well estimated by the proposed beam element model that considers the deterioration effect due to local buckling. The proposed beam model can be applicable to box section steel rigid frames that have complicated shapes.

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Table 2 : Dimensions of the steel rigid frame

Element No	Flange			Web			Longitudinal Stiffener		Lateral stiffener	Material type
	Width $b_f$ (mm)	Thickness $t_f$ (mm)	Panel No $n_f$	Width $b_w$ (mm)	Thickness $t_w$ (mm)	Panel No $n_w$	Width $b_r$ (mm)	Thickness $t_r$ (mm)	Spacing $a$ (mm)	
1	1700	19	4	1700	15	4	140	14	1447	SM490YB
2	1700	19	4	1700	15	4	140	14	1500	SM490YB
3	1700	15	4	1700	14	4	140	14	1500	SM490YA
4	1700	15	4	1700	14	4	140	14	1500	SM490YA
5	1700	16	4	1700	15	4	140	14	1800	SM490YA
6	1700	19	4	1700	19	4	140	14	1550	SM490YA
7	1700	22	4	1700	15	4	140	14	1567	SM490YB
8	1700	22	4	1700	15	4	140	14	1500	SM490YB
9	1700	15	4	1700	15	4	140	14	1500	SS400
10	1700	15	4	1700	15	4	140	14	1500	SS400
11	1700	19	4	1700	15	4	140	14	1550	SM490YB
12	1700	28	4	1700	19	4	140	14	1800	SM520
13	1700	15	4	1000	12	1	140	14	1000	SS400
14	1700	15	4	1229	12	1	140	14	909	SS400
15	1700	25	4	1437	19	1	140	14	2000	SM570
16	1700	25	4	1700	19	1	140	14	1800	SM570
17	1700	22	4	1700	15	1	140	14	1200	SM490YB
18	1700	22	4	1700	15	1	140	14	1091	SM490YB
19	1700	22	4	1700	15	1	140	14	1207	SM490YB
20	1700	22	4	1700	15	1	140	14	1207	SM490YB
21	1700	28	4	1700	19	1	140	14	2057	SM570
22	1700	28	4	1700	19	1	140	14	1938	SM570
23	1700	28	4	1700	19	1	140	14	1500	SM520
24	1700	28	4	1700	19	4	140	14	2450	SM570
25	1700	22	4	1700	16	4	140	14	1500	SM490YA
26	1700	22	4	1700	16	4	140	14	1500	SM490YA
27	1700	22	4	1700	16	4	140	14	1500	SM490YA
28	1700	22	4	1700	16	4	140	14	1879	SM490YA
29	1700	28	4	1700	22	4	140	14	1568	SM520
30	1700	19	4	1000	15	1	140	14	1002	SS400
31	1700	19	4	1159	15	1	140	14	1130	SS400
32	1700	19	4	1338	15	1	140	14	1130	SS400
33	1700	28	4	1700	22	1	140	14	2000	SM520
34	1700	28	4	1700	22	1	140	14	2004.7	SM520
35	1700	22	4	1331	15	1	140	14	1254.7	SS400
36	1700	22	4	1147	15	1	140	14	1254.7	SS400
37	1700	22	4	1000	15	1	140	14	1000	SS400



**Figure 11** : Axial stress-strain curves of the rigid frame at the element 25

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