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Prediction of Crack Growth in Steam Generator Tubes Using Monte Carlo Simulation

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Abstract: The growth of stress corrosion cracks in steam generator tubes is predicted using the Monte Carlo simulation and statistical approaches. The statistical parameters that represent the characteristics of crack growth and crack initiation are derived from in-service inspection (ISI) non-destructive evaluation (NDE) data. Based on the statistical approaches, crack growth models are proposed and applied to predict crack distribution at the end of cycle (EOC). Because in-service inspection (ISI) crack data is different from physical crack data, a simple method for predicting the physical number of cracks from periodic in-service inspection data is proposed in this study. Actual number of cracks is easily estimated using the method, and the statistical crack growth is simulated using the Monte Carlo method. Probabilistic distributions of the number of cracks and maximum crack size at EOC are obtained from the simulation. Comparing the predicted EOC crack data with the known EOC data the usefulness of the proposed method is examined and satisfactory results are obtained.

keyword: Statistical Assessment, POD (probability of detection), Effective POD, Steam Generator Tube, Structural Integrity, Monte Carlo method.

1 Introduction

Cracking is one of the main degradation mechanisms of steam generators in nuclear power plants. And analysis of its potential damage is essential to assessment of structural integrity. Usually crack analysis has been performed using analytical methods or numerical methods such as finite element method, boundary element method and finite element alternating method [Nikishkov, Park, and Atluri (2001)]. However their results were often regarded as conservative in industrial field. And sometimes it was necessary to do the analysis considering the uncertainty of variables (load, material properties, crack size, and circumstance etc.).

Therefore statistical approaches as well as theoretical (deterministic) ones have been widely used for assessment of structural integrity [EPRI NP-7493 (1991); Chung, Kim and Kim (2000); Maeda, Nakagawa, Yagawa, and Yoshimura (2002); Kurihara, Ueda, and Sturm (1988); Becher and Pederson (1974); Leemans, Leger and Byrne (1993); Wu and Syau (1995); Stroud, Krishnamurthy and Smith (2002)].

Besuner (1987) stated that there are three basic approaches in statistical assessment of structural reliability. One approach is extrapolation from the past data. It can be a very accurate approach, if sufficient relevant data and experiences exist. Another approach is probabilistic fracture mechanics (PFM), where probabilistic contents are introduced into deterministic fracture mechanics theories. The other approach is calibrated probabilistic fracture mechanics (CPFM). The approach intends to combine the former two approaches, and provides reliability prediction when neither the database nor the pure PFM approaches can give appropriate results.

In the extrapolation method, statistical distributions are usually used for interesting variables. The parameters of distribution functions can be obtained from curve fitting of field data for several interesting variables (crack size, crack initiation time, crack growth rate, etc.). It is a shortcoming of this method that we cannot derive the influence of the change in parameters from field data. For example, we cannot know the influence of using more advanced non-destructive inspection system from this approach.

The PFM method is useful for analyzing local and instantaneous problems. But it has difficulties in treating large structures, which have been operating for decades, because of a lot of variables (load, material properties, crack size, and position etc.) and changes of their circumstances and characteristics.

CPFM is a statistical approach combining the advantages of above two methods. In this study we used CPFM

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approach for assessment of integrity of steam generator tubes. Crack growth rate, crack initiation and crack distribution are expressed with statistical distribution functions and a simple crack growth model is adopted.

It is necessary to know the number of physical cracks and their size distribution in assessment of integrity. Here the physical cracks are defined as the cracks that are expected to exist actually in steam generator tubes. But in-service inspection (ISI) crack data is different from the physical crack data, because inspection systems have imperfection in detecting cracks and measuring their sizes. For estimating the physical number of cracks from periodic in-service inspection data a simple method is proposed.

The statistical crack growth is simulated using the Monte Carlo method in order to estimate the number of cracks and the maximum size distribution at interesting time. The necessary statistical characteristics for crack growth rate and crack initiation are calculated from real NDE data.

2 Crack growth estimation

2.1 Models for Crack Growth Estimation

Figure 1 illustrates the procedure used in the proposed statistical model for crack growth prediction. The model predicts the number of cracks and their size distribution at the end of ith operation cycle using the NDE data at (i-1)th ISI. Calculation process of crack growth is subdivided into three categories of crack initiation, undetected crack growth and detected crack growth. For the calculation, the information on the crack initiation and the crack growth rate are necessary and the number of physical cracks also must be known. The statistical characteristics of crack initiation and crack growth rate are represented with statistical distribution functions. The parameters of the distribution functions are obtained from statistical analyses of ISI NDE data. And statistical variables of each distribution function are generated in the Monte Carlo simulation. The inverse transform method and the acceptance-rejection method are used as the generation algorithms. The details can be found in the reference [Rubinstein (1981)].

The number of physical cracks is estimated from the number of NDE cracks using the detection uncertainty, i.e. POD (probability of detection), of the used inspection method.

It is easy to obtain the physical sizes from the NDE crack size data once the size uncertainty of the inspection method is known. However, NDE size is used in the simulation because duplicate application of the uncertainty makes the estimation results more confused. Thereby, the number of physical cracks and NDE size data are used respectively for crack growth calculation. From the simulation, the numbers of NDE cracks and physical cracks are predicted at the end of ith operation cycle using the NDE crack data at the beginning of ith operation cycle, which is obtained at (i-1)th ISI after excluding cracks in repaired or plugged tubes. The predicted number of NDE cracks is compared with the inspection results at ith ISI.

2.2 Size and detection uncertainty

2.2.1 Probability of Detection (POD)

Non-destructive inspection systems have been operated in their extreme capability for finding small cracks. In fact, the cracks of the same size may give different inspection results and it is called the uncertainty or imperfection of NDE systems.

Berens (1989) subdivided the uncertainty of NDE into the uncertainty of size and the uncertainty of detection. The uncertainty of size is often characterized in the simplest form of a statistical model of a linear or linearized regression to the measured versus true size data. The uncertainty of detection is characterized in terms of the probability of detection (POD) as a function of crack size. The uncertainty of size is not difficult to handle but the POD is sophisticated and complicated.

It is easy to obtain the physical sizes from the NDE crack size data once the size uncertainty of the inspection method is known. And in case of a single inspection, it is also easy to estimate the number of physical cracks using the POD curve. However, we may be faced with some difficult problems in obtaining the physical number of cracks from the periodic in-service non-destructive inspection data. Cracks can be detected after they reach the minimum detectable crack size. Some cracks are detected immediately after reaching the size. And some cracks are detected after several inspections. In order to calculate the number of physical cracks, the inspection history of each crack must be known. But it is nearly impossible to know the individual inspection history, so an estimation procedure is necessary. As a previous work, Davis (2001) has estimated the physical crack distribution from the measured initial crack distribution using

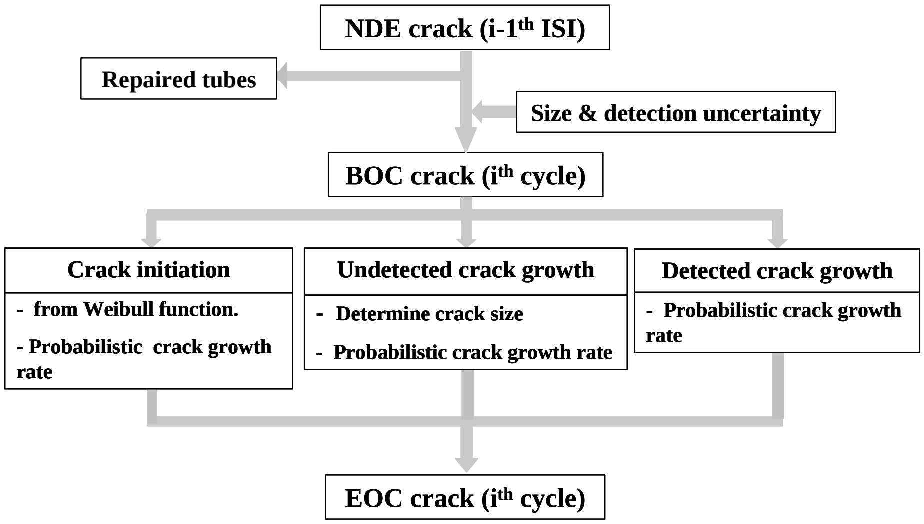

Figure 1 : Procedure of statistical analysis model for crack growth.

POD without consideration of repeated inspections.

2.2.2 *Calculation of the Number of Physical Cracks*

POD is defined as the following equation:

$$\text{POD(a)} = \frac{\text{The number of detected cracks}}{\text{The number of physical cracks}} \tag{1}$$

And it depends on the crack size *a*. Here the physical cracks are defined as the estimated cracks, which actually exist in tubes including undetected cracks. Because the capability of NDE system is not perfect, we can assume that other cracks exist besides detected NDE cracks.

The number of physical cracks is easily derived from the definition of POD as follows.

$$\text{The number of physical cracks} = \frac{\text{The number of detected cracks}}{\text{POD(a)}} \tag{2}$$

The number of physical cracks is easily calculated from Eq. 2 theoretically. But Eq. 2 cannot be applied to periodic ISI NDE data, because the equation does not consider repeated NDE inspections at each end of cycle (EOC). In case of periodic inspection, more sophisticated method is necessary for calculating the number of physical cracks.

In order to calculate the number of physical cracks with periodic NDE data, both probabilities of detecting and missing cracks should be considered. The probability of detecting a crack through *n* times of inspections is calculated using the probability of missing the crack.

The probability of *x* detections through *n* times of inspections is calculated using a binomial function as follows:

$$\binom{n}{x} p^x q^{n-x} = \frac{n!}{x!(n-x)!} p^x q^{n-x} \tag{3}$$

Here p is the POD and q (=1- p) is the probability of miss. And the possibility of detecting a crack at least once through *n* times of inspections becomes:

$$P = 1-(1-p)^n \tag{4}$$

But we confront essential problems in applying Eq. 4 to real periodic ISI data. In order to calculate the possibility of detection through *n* times of inspections, we have to know the exact crack growth history and the number of inspections for each crack. But it is nearly impossible to know them.

2.2.3 *Effective Probability of Detection (POD_{Eff})*

For periodic inspections, the possibility of detecting a crack is different from POD because each crack can be examined more than once. Let the total possibility of detecting a crack be "effective POD". Then the number of physical cracks is expressed as the following:

$$\text{The number of physical cracks} = \frac{\text{The number of detected cracks}}{\text{Effective POD(a)}} \tag{5}$$

And in order to know the crack growth history and the number of inspections for each crack, a statistical simulation approach is used.

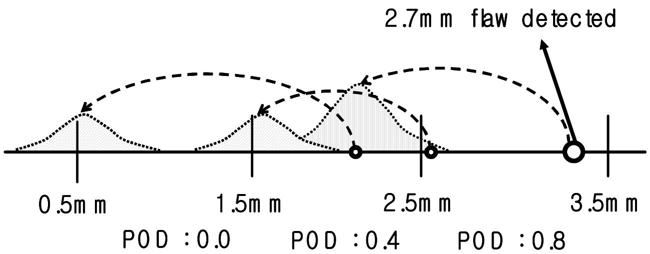

Figure 2 : Schematic diagram of a statistical simulation for estimating crack growth history.

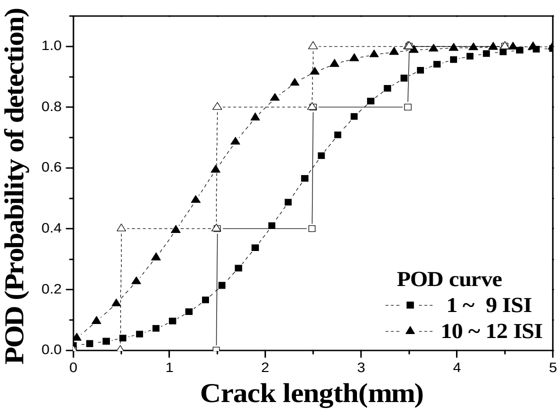

Figure 3 : Continuous and discrete POD curves.

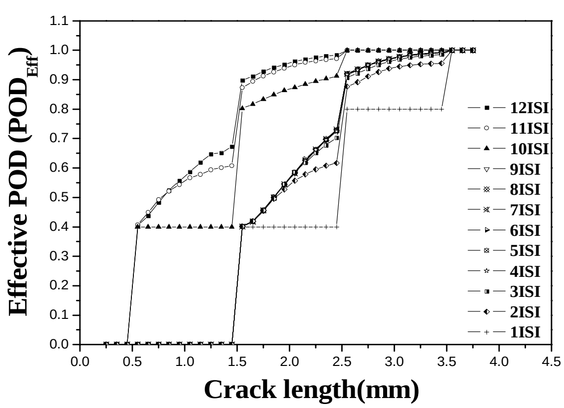

Figure 4 : POD$_{Eff}$ curves obtained from the simulation.

Figure 2 shows the statistical simulation schematically. Let a crack with 2.7mm length be detected at ith ISI for the first time. What is the size of the crack at (i-1)th ISI? If crack growth rate is represented as a probabilistic variable, the size of the crack at (i-1)th ISI becomes also a probabilistic variable with a certain probabilistic distribution as shown in Figure 2.

The crack size distribution at (i-1)th ISI can be obtained from the backward crack growth simulation using probabilistic crack growth rate. For a crack with a certain size at ith ISI, the backward crack growth simulation is made until the crack becomes smaller than the minimum detectable crack size. And the total crack growth history is obtained for the crack. Since POD is a function of crack size, the POD values also can be calculated at each ISI. From the obtained POD values the total POD, i.e. the effective POD can be calculated. By repeating the simulation many times, the mean value of the effective POD is obtained as a function of crack size.

Figure 3 shows the discrete and continuous POD curves used in the simulation at each ISI. In the simulation, discrete POD values are used, because it is more convenient to count the number of flaws and calculate the number of physical flaws. Since NDE system has been changed after the 9^{th} ISI, the POD curve is also changed after the 9^{th} ISI.

Figure 4 shows the effective POD curves at each ISI obtained from the simulation. The effective POD values in Figure 4 are mean values of 1000 simulation results.

As repeating the inspection, effective POD values become lager and show large disparity comparing with the initial POD value. And we can find that effective POD curve reflects the effect of the change of NDE system at the 10^{th} ISI. The number of physical cracks for the given length or length interval can be easily obtained from Eq. 5 using the effective POD.

2.3 Crack initiation

The number of initiated cracks during ith operation cycle is predicted from statistical analysis of NDE data obtained until (i-1)th ISI. Using the Weibull distribution function, the number of initiated cracks in ith ISI is predicted.

Figure 5 illustrates cumulative numbers of initiated cracks at each ISI and their fitting curves with 2-parameter and 3-parameter Weibull distribution functions. The number of initiated cracks is calculated in physical domain, i.e. the number of initiated physical cracks is calculated. It is noted that the 3-parameter Weibull curve is more coincident with data points than

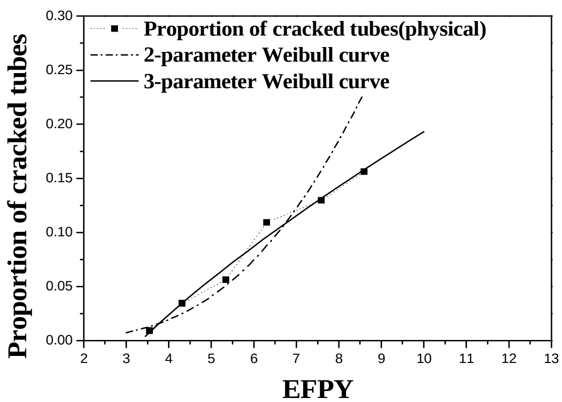

Figure 5 : The 2-parameter and 3-parameter Weibull curves of new crack initiation.

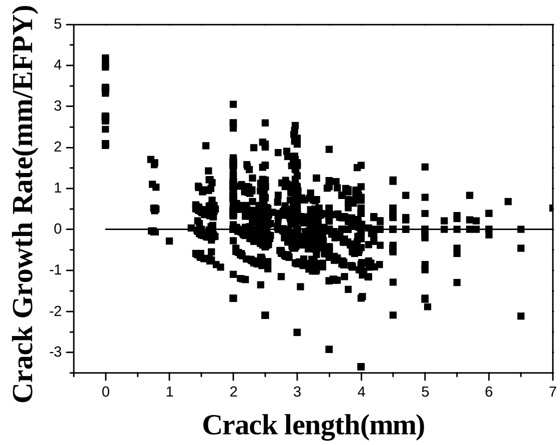

Figure 6 : NDE crack growth rate data.

the 2-parameter Weibull curve as shown in Figure 5. Therefore the 3- parameter Weibull function is used for estimating the number of initiated cracks in the simulation.

2.4 Probabilistic Crack Growth Rate

Figure 6 shows crack growth rate data for axial cracks in steam generator tubes in a Korean plant. The steam generators have been operated for 12.38 EFPY (effective full power year) and 12 in-service inspections were made. The crack growth rate at ith inspection is obtained by dividing the crack length increment by the time interval between ith and (i+1)th inspections. The units of crack length and time are mm and EFPY respectively. Many data points in Figure 6 show negative crack growth rates, which are inconsistent values. The negative growth rate is due to the uncertainty of NDE.

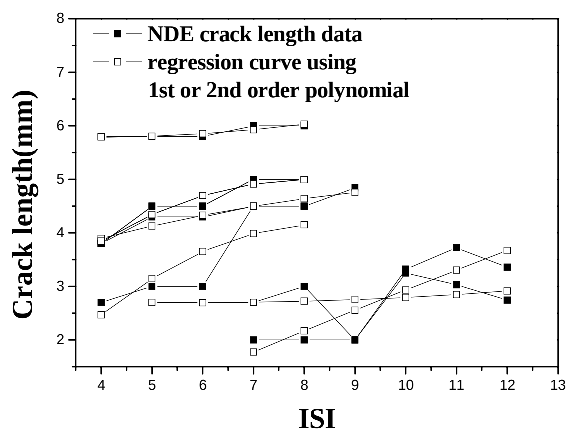

Figure 7 : Regression of crack length data with polynomials.

Figure 7 shows the crack length data measured at each ISI. It can be noted that the crack length does not increase always. In order to obtain nonnegative and more consistent crack growth rate values, the regression analysis is done for the crack length data at each ISI using a polynomial. Several results of the regression analysis are given in Figure 7. When the number of data points is less than 4, the linear regression is used.

However when the number of data points is equal to or more than 4, the polynomial of the second degree is used for the regression. The crack growth rate is the gradient of the regression curve at the time. Figure 8 represents the crack growth rate obtained from the regression analysis. Nearly all the data points show positive crack growth rate after the regression. Few data points that show negative or zero crack growth rates are excluded. If there is a relationship between the crack length and the crack growth rate, crack growth rate may be expressed as a function of crack size.

In order to find a relation between crack length and crack growth rate in Figure 8, a regression analysis is made with the 3rd and 4th order polynomials. From the regression results, it is noted that the crack growth rate decreases as the crack length increases. This result is

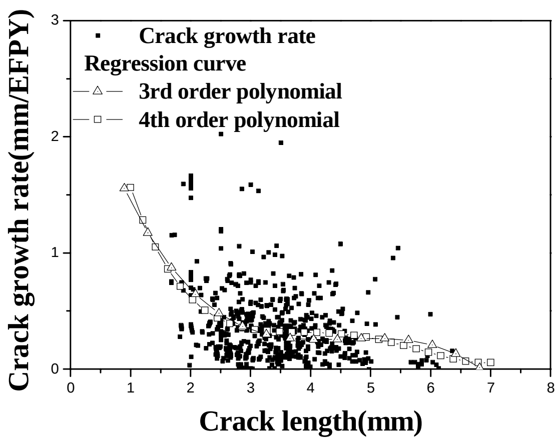

Figure 8 : Crack growth rate data and regression with a polynomial.

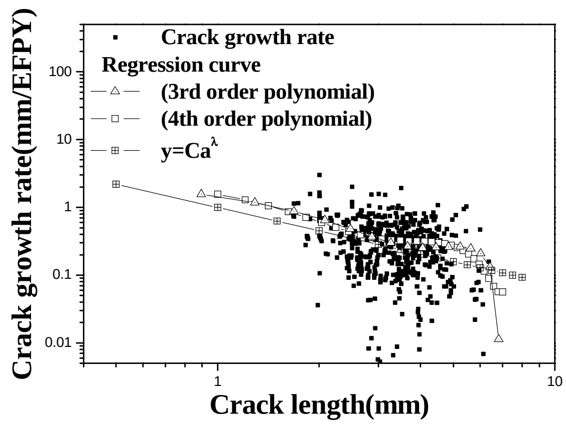

Figure 9 : The regression analysis of crack growth rate.

consistent with the work done by Chung, Kim and Kim (2000). It is well known that stress corrosion crack growth rate is highly affected by plastic deformation and residual stresses. The crack growth rate decreases as the crack length increases because crack front is moving from the highly susceptible region to the less susceptible region.

The crack growth rate is assumed to satisfy the following equation:

$$\frac{da}{dt} = f(a)z \tag{6}$$

where a is crack length, t is time and $f(a)$is a function of the crack length a. And z is a statistical random variable and used to represent random error. One simple form of Eq. 6 is the following:

$$\frac{da}{dt} = Ca^{\lambda}z \tag{7}$$

Where C and λ are constants. Taking logarithm to both sides of Eq. 7 leads to:

$$log\left(\frac{da}{dt}\right) = logC + \lambda loga + logz \tag{8}$$

The constants log C and λ can be obtained from the linear regression between the variables *log a* and *log(da/dt)*. Figure 9 represents this procedure. In Figure 9, the logarithmic scale is used for both horizontal and vertical axes. The linear regression line is plotted in the Figure. For comparison, the regression curves of the 3rd and 4th order polynomial are also plotted. It can be noted that the differences between the linear regression line and the lines of polynomials are small.

According to assumption of linear regression, *log z* is an error, which consists of two parts [Mendenhall, Beaver and Beaver (2003)]. One is pure experimental error and the other is error due to lack of fit. If the model adequately fit the data, then *log z* is calculated by pure experimental error. F-test results of errors in Figure 9 are represented in Table 1. In Table 1, F-value is larger than $F_{0.001}(1, 698)$. Therefore Eq. 8 is an appropriate model in the 0.001 significance level.

Table 1 : Statistical analysis of regression errors in Figure 9 using F-distribution (F-test).

Source	DF	Sum of Squares	Mean Squares	F-value	$F_{0.001}(1, 698)$
Model	1	66.333	66.333		
Error	698	540.941	0.775	85.592	10.830
Total	699	607.274			

And *log z* is obtained with the sum of squares for error (SSE) or standard error of estimate (root mean of squares for error, RMSE), which is calculated from:

$$RMSE = \sqrt{\frac{\sum\left(Y_i - \hat{Y}\right)^2}{n-2}} = \sqrt{\frac{SSE}{n-2}} \tag{9}$$

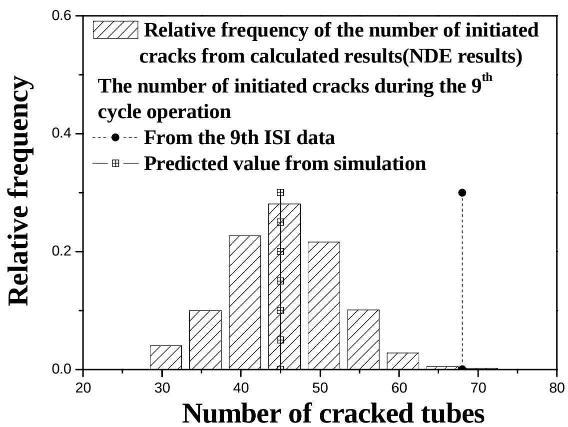

Figure 10 : Relative frequency distribution of the predicted number of newly detected cracks at the 9th ISI from the simulation.

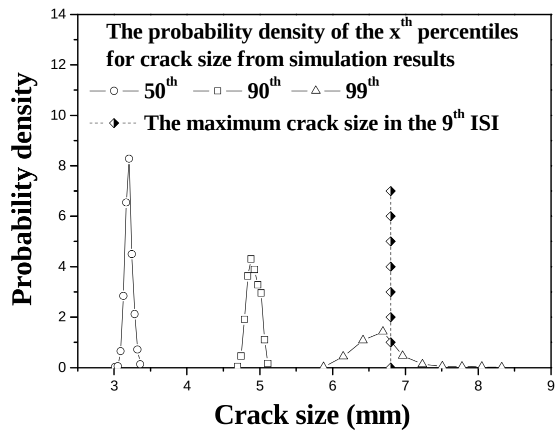

Figure 11 : The probability density of 50th, 90th and 99th percentiles for crack size from simulation results

Where Y_i is crack growth rate data, $\hat{Y}$is the value calculated from regression equation and n is the number of data points. Therefore $log\, z$ is expressed by normal distribution, $N(0,\ RMSE^2)$ and we can obtain statistical crack growth rate using Eq. 7.

3 Simulation Results

The number of cracks and their sizes are predicted statistically using the Monte-Carlo simulation represented in Figure 1. Using NDE data at the 8th ISI, the number of initiated cracks and the maximum crack size at the 9th ISI are predicted. The simulation is repeated 1000 times and the results are statistically analyzed, thereby the simulation results are obtained as probabilistic distributions.

Figure 10 illustrates the relative frequency distribution of the number of newly detected cracks at 9th ISI. Comparing with the NDE data, 68 cracks at the 9th ISI, simulation results show somewhat smaller number of cracks. The reason of this underestimation may be that the used POD is larger than the true POD.

Even if there is discrepancy between the predicted number of cracks and the NDE data it is limited to the cracks with short length. For the long cracks, the predicted number of detected cracks is always nearly the same as the number of NDE cracks because the POD and the effective POD have the values close to 1.

Table 2 : The 50th, 90th, 95th and 99th percentiles for crack size with 50%, 90% and 99% confidence level from simulation results.

	The xth percentile for crack size			
Confidence level	**50th**	**90th**	**95th**	**99th**
50%	**3.2**	**4.9**	**5.4**	**6.6**
90%	**3.3**	**5**	**5.5**	**7**
99%	**3.3**	**5.1**	**5.6**	**7.6**
9th ISI Max.	**6.8**	**6.8**	**6.8**	**6.8**

Figure 11 illustrates probability density distributions of crack size corresponding to the 50th, 90th and 95th percentiles at 9th ISI. The distributions are obtained from 1000 simulation results. The distribution for the maximum crack size also can be obtained, but unusually large maximum crack sizes are often obtained. That is due to the probabilistic crack growth rate and the large random error, z in Eq. 7. Therefore we can get more consistent estimated value for the maximum crack size if we use the percentiles instead of the maximum crack size distribution. The percentiles for crack size corresponding to three confidence levels are given in Table 2. Here the unit of crack size is mm. The maximum crack size measured at the 9th ISI is 6.8mm and the 99% percentile crack size corresponding to 50% confidence level shows the similar value.

4 Conclusion

The growth of stress corrosion cracks in steam generator tubes is estimated using the Monte Carlo method and statistical approaches. The statistical parameters that represent the characteristics of the crack growth and the crack initiation are derived from the In-Service Inspection (ISI) NDE data.

The proposed analysis method is applied to predict the crack distribution at EOC. Comparing the predicted EOC crack data with the known EOC data the usefulness of the proposed method is examined.

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