

# Multi-Disciplinary Optimization for the Conceptual Design of Innovative Aircraft Configurations

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**Abstract:** The paper presents an overview of recent work by the authors and their collaborators on multi-disciplinary optimization for conceptual design, based on the integrated modeling of structures, aerodynamics, and aeroelasticity. The motivation for the work is the design of innovative aircraft configurations, and is therefore first-principles based, since in this case the designer cannot rely upon past experience. The algorithms used and the philosophy behind the choices are discussed.

**keyword:** Multi-Disciplinary Optimization, Conceptual Design, Integrated Aircraft Modelling, Boundary Elements, Reduced Order Models

## 1 Introduction

The aim of this paper is to present an overview of the work of the authors and their collaborators in the field of MDO/CD (*Multi-Disciplinary Optimization for Conceptual Design*), for innovative aircraft configurations, specifically, for civil aviation applications.

In this paper, we emphasize the philosophy used in developing the methodology, what are the criteria used to choose the numerical algorithms utilized, what are the current limitations and what needs to be done. Within this context (that is, multi-disciplinary optimization for the conceptual design in civil aviation), the authors have developed (and are still in the process of developing) a computer code called MAGIC (Multidisciplinary Aircraft design of Innovative Configurations). It should be emphasized that the code MAGIC is not to be considered as a completed piece of software, but as an ever evolving code. Therefore, this paper presents the state of the art on the project – current developments and future objective are discussed in the concluding remarks.

MAGIC is an evolution of the code FLOPS (Mc Cullers

(1984)), which is essentially based on elementary and/or empirical algorithms. Such an approach is not possible for innovative configurations, for which the designer cannot rely upon past experience. Thus, the first and foremost criterion used in developing MAGIC is that the algorithm be based on first principles, whenever possible. This is the approach used in particular in developing the MAGIC modules for structures, aerodynamics, and aeroelasticity (see, *e.g.*, Morino (1974), Morino (1993), Morino, Mastroddi, De Troia, Ghiringhelli, and Mantegazza (1995), Morino and Bernardini (2001), and Morino (2003)), which are all first-principles based and are described in this paper.

In order to validate the code, as it was being developed, MAGIC has been used, by the authors and their collaborators, to perform numerical studies. It may be noted that the original motivation and source of inspiration for our work has been a specific innovative aircraft configuration, which has, as a distinguishing feature, a low induced drag. This was proposed by Frediani (1999) and by him denoted as the *Prandtl-Plane*, in honor of the Prandtl (1924) work on unswept box wings, which have an induced drag considerably lower than the standard configuration.<sup>3</sup> Thus, most of the studies performed with MAGIC deal with the wing design of the Prandtl-Plane, with the fuselage assumed as given (see, for instance, Bernardini, Frediani, and Morino (1999), Mastroddi, Bonelli, Morino, and Bernardini (2002), Morino, Bernardini, Da Riz, and Del Rio (2002), and Morino, Bernardini, and Mastroddi (2003)). These studies have confirmed that the induced drag of this configuration is considerably lower than the induced drag of an equivalent monoplane. This fact allows one to reduce the wingspan of this configuration without major drag penalties, thereby enhancing the capability of respecting the maximum spanwise dimensions (critical for the NLA – New

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<sup>3</sup> The Prandtl-Plane has a counter-swept box-wing, *i.e.*, a biplane with a backward-swept low front wing and forward-swept high back wing (which acts as a horizontal stabilizer as well); these are connected to each other by vertical streamlined connections.

Large Airplanes – with classical wing configuration), as required by existing airport regulations. Moreover, as shown by Morino, Iemma, Bernardini, and Diez (2004), the low induced drag could allow one to reduce community noise at take-off (because of the lower power required) and possibly chemical pollution. Other advantages of the Prandtl–Plane are discussed in details in Frediani (2004), to which the reader is referred for a discussion of different types of Prandtl–Plane configurations. It may be noted that the Prandtl–Plane is closely related to the joined–wing concept; comprehensive overviews on joined–wing configurations are presented in Wolkovitch (1986) and Livne (2001).

It should be noted that, the proposed methodology (as well as the code MAGIC) is by no means limited to the Prandtl–Plane; for instance, in Carpentieri, van Tooren, Bernardini, and Morino (2004), it has been successfully applied also to the optimal design of a Blended–Wing–Body configuration.

A crucial aspect of the philosophy used is related to the fact that we are currently emphasizing the conceptual design phase (MDO/CD). Hence, it is highly desirable to use algorithms that produce accurate predictions with a relatively small computational effort. In other words, the numerical algorithms used should be very efficient and at the same time adequately accurate and apt to be refined as much as necessary. In this paper, we use the term “modeling” to refer to algorithms (such as boundary elements in aerodynamics) that capture the essence of the phenomenon (at a level quantitatively acceptable during the conceptual design phase), and the term “simulation” to refer to those algorithms (such as computational fluid dynamics) that provide a more accurate description of the phenomenon (beyond the level required during the conceptual design phase) at the expense on much higher computational cost. Accordingly, in this work we address the advantages of modeling over simulation in the context of MDO/CD.

A second crucial aspect is that the code MAGIC is at the moment geared specifically towards for civil aviation applications; hence, advantage is taken of this aspect whenever possible, for instance in the use of potential flows (*i.e.*, flows that are potential everywhere except for a zero–thickness wake surface emanating from the trailing edge), which are combined with integral boundary layers for the analysis of viscous effects. In summary, the physical models chosen must be able to capture the

essence of the phenomenon within the specific application of interest, thereby avoiding any unnecessary sophistication.

The last crucial aspect in the philosophy we are following is that strong emphasis be given to the integration of the various disciplines. This implies not only that special care be given to the interfaces, but also that the concurrency of certain types of analysis be exploited whenever possible. For instance, the fact that the natural modes of vibration must be evaluated for the dynamic aeroelastic analysis implies that a modal analysis may be used for the stress analysis as well. Similar considerations hold for steady and unsteady aerodynamics algorithms. What we are saying here is that the final objective is to develop a code that is not a collection of some codes used for the individual disciplines – in our view, it is necessary to start from scratch. Indeed, the methods that are the most convenient for the individual disciplines are not necessarily the most convenient in the global context. Therefore, our work is based upon a critical analysis of the methodologies that are best suited for the stated goal.

On the basis of all these considerations, our choices have been towards the following methodologies: (*i*) a linear elastic finite–element beam model for the wing structure (statics and mode evaluation), (*ii*) quasi–potential flows for the aerodynamic analysis, with an integral boundary–layer analysis for the viscous effects, and (*iii*) modal analysis and reduced order model (ROM) for aeroelasticity. All three of them assure the high efficiency required, with an accuracy that is quite adequate for the conceptual design, within civil aviation applications. We will address these issues in some details in the remainder of the paper. We then critique our choices and provide an overview of our current research activity that would allow us to overcome the present limitations and move towards a more general, but still efficient, formulation, possibly of interest for preliminary design.

## 2 A Brief Survey on MDO

In order to put the present paper in the proper context, a brief review of the current trends in MDO (for Multi–Disciplinary Optimization, or Multidisciplinary Design Optimization) is presented in this section. This review has no pretense of being exhaustive – it has simply the limited objective of making the reader aware of the current interest in MDO and of the complexity of the problems. It may be noted that much of the work reviewed

in this section is not geared toward conceptual design. Nonetheless, it appears appropriate to present the review of such a work, in order to emphasize the difference between the current main trends in MDO and our approach to the problem (*i.e.*, preliminary design). For the same reason, when we discuss our work we use the acronym MDO/CD (Multi-Disciplinary Optimization for Conceptual Design) and not just MDO.

The relevant scientific and technological developments on aerospace-engineering optimization during the last decades have demonstrated the growing need of Concurrent Engineering (CE) teams (Technical Committee (1991)), or of a Collaborative Engineering environment (Monell and Piland (1999)) able to move forward advanced aircraft design. Indeed, these kinds of design methodologies are intrinsically multidisciplinary, since they involve at least aerodynamic and structural tasks (for a more detailed design, the flight stability, propulsion and control systems must also be added).

From these analyses, it becomes more and more apparent how complex aircraft design can be, and how a new figure of engineer is emerging for connecting all the specialized tasks coming out from the updating of whichever other discipline and hence from any specialized engineering branch. Also, low computing time, design reliability, overall life-cycle cost effectiveness and manufacturability, are additional crucial issues that should be taken into account in this framework.

The concept of MDO typically comprises all these issues in the design process, since it is a formal methodology with a mathematical description that takes into account the synergistic and intrinsic coupling that exist in physical complex systems, enabling the optimal design of multidisciplinary complex engineering systems.

It is worth to point out that optimal solutions to problems have very old roots in aerospace engineering – as pointed out by Ashley (1982), it is inherent to the human nature to find the best solutions to the problem – but it is only in the last few decades that MDO problems and concepts have come to the forefront, in a clearly defined manner (Haftka and Gürdal (1992)).

Many studies have been dedicated to quantitative analysis of the coupling between systems (Arian (1997)), and to approaches able to unify the interaction among the various disciplines. Moreover, a very important matter pertains sensitivity analyses, already widely discussed

by Sobieszczanski-Sobieski (1986) (where the problem of extending sensitivity analysis to Computational Fluid Dynamics codes is analyzed), as well as in Yates (1987), Bergen and Kapania (1988), Barthelemy and Bergen (1989), Baysal and Eleshaky (1991), Taylor, Hou, and Korivi (1992), Giunta (1999), Park, Green, Montgomery, and Raney (1999), and Taylor, Green, Newman, and Putko (2001).

Sobieszczanski-Sobieski and Haftka (1996) underlines how the two main challenges in MDO are computational costs and organizational complexity. Since then, this position has evolved to include the more recent challenge of MDO: solution methodology and its software framework. In this sense, a remarkable contribution and an interesting research direction is given in Alexandrov and Lewis (2000a), Alexandrov and Lewis (2000b), Alexandrov and Lewis (2000c), and Alexandrov and Lewis (2002), which analyze the concepts of structural and algorithmic perspectives by studying the optimization by Linear Decomposition and Collaborative Optimization (bi-level optimization) and Distributed Analysis Optimization (single level optimization) approaches (the concept of Collaborative Optimization is also widely described in Kroo and Manning (2000)).

Regarding the issue of obtaining an MDO architecture enabling one to reduce the computational cost, the concepts of Nested Analysis and Design (NAND) approach or the Simultaneous (SAND) one, are widely described in Newman, Hou, and Taylor (1996), where it is also shown how NAND and SAND approaches “*differ only in the frequency at which the iterative analysis and optimization interact*”. A contribution to this kind of problems is given, for example, in Gumbert, Hou, and Newman (2005).

Since analytical and numerical results have shown the deep relationship between the MDO formulation and the solution feasibility and since, in most problems, it is not possible to decide *a-priori* a certain formulation, the capability of *reconfiguration* of the system has been described as necessary by MDO requirement (see Alexandrov and Lewis (2003)). The previous considerations constitute the basis for the development of an innovative approach defined as *reconfigurable multidisciplinary synthesis* (REMS), which is analyzed in Alexandrov and Lewis (2004a) and Alexandrov and Lewis (2004b). In Alexandrov and Lewis (2004a) the REMS approach is defined as a “*conceptual framework that comprises an*

*abstract language and a collection of processes that provide a means for dynamic reasoning about MDO problems in a range of contexts, with assistance from computer science techniques*". Hence, the REMS introduction is useful in order to understand the internal structure of MDO problems and constitutes also a way to better understand solution methodologies.

As mentioned above, the purpose of this short introduction is not to review all the MDO research fields in an exhaustive way, but simply to remind the reader of the complexity and current relevance of MDO problems. Nonetheless, it is appropriate at least to mention the effort done in optimization with variable-fidelity models, *i.e.*, in giving some approximation models in optimization (Alexandrov, Lewis, Gumbert, Green, and Newman (2000) and Alexandrov, Nielsen, Lewis, and Anderson (2000)), the need for comparison between the optimization methods (Kodiyalam and Yuan (2000)), as well as Knowledge Based Engineering discussed in van Tooren (2004), which illustrates modelling techniques that allow one to address different configurations within the same model.

In summary, the field of MDO is quite complicated and in a state of evolution. Hence, the optimization model for the preliminary design of a complete aircraft presented in this paper is to be considered as a small contribution that allows one to appreciate the complexity and the multi-tasks of the MDO approach – indeed, the analysis of a complete aircraft is one of the main issues of MDO research. It should be emphasized again that MAGIC is an evolving piece of software and that, contrary to the literature reviewed above, the emphasis here is on conceptual design for civil aviation applications. Thus, the presentation of the mathematical models reflect the current state of the code development. The paper is written for a broad audience, since the typical reader is not necessarily familiar with the literature in all the fields included here. Thus, the material presented is as much as possible self-contained. Hence, for the sake of completeness, we include some mathematical issues (*e.g.*, completeness of the modes of vibration and relationship with finite-element models), which we consider relevant to understanding the validity of the formulation, and which are dealt with only in the most specialized literature, while typically ignored even in the aeroelastic literature. Current research activity and projected future developments are dealt within the concluding remarks.

### 3 Modeling vs. simulation in structural dynamics

As mentioned above, here we want to emphasize the advantages of modeling over simulation, of course within the context of conceptual design. Thus, we begin with structural dynamics, which presents a clear exemplification of what we mean by “modeling” and “simulation,” and for which the advantages of modeling over simulation are apparent (aerodynamics and aeroelasticity are examined in the following sections). In the following, we discuss the linear formulation, which is standard in conceptual aircraft design. Combining the linearized momentum equation with the constitutive equations for linear elastic material and the expression for the linear strain tensor yields  $\rho \ddot{\mathbf{u}} + \mathbf{L}\mathbf{u} = \mathbf{f}$ , where  $\mathbf{L}$  denotes a self-adjoint tensor operator with Cartesian components  $L_{ik}(\dots) = -C_{ijkl}(\dots)/l_j$  (where  $(\dots)/j := \partial(\dots)/\partial x_j$ ).<sup>4</sup> Using the Galerkin method (more precisely, the Bubnov-Galerkin method, Reddy (1986)), one seeks an approximate solution of the type  $\mathbf{u}(\mathbf{x}, t) = \sum_{n=1}^N u_n(t) \Psi_n(\mathbf{x})$ , where  $\{\Psi_n\}$  is a set of linearly independent vector functions which satisfy suitable homogeneous boundary conditions.<sup>5</sup> In the Galerkin method, the approximate equations are obtained by taking the inner product between the resulting expression and the function  $\Psi_n$  ( $n = 1, \dots, N$ ), to obtain a system of linear second-order differential equations, in the unknown  $u = \{u_n\}$ , given by

$$M\ddot{u} + Ku = f, \quad (1)$$

where the elements of  $M = [M_{kn}]$ ,  $K = [K_{kn}]$ , and  $f = \{f_k\}$  are given by (disregarding the volume – *e.g.*, gravity – forces,  $\mathbf{f}$ , which are negligible in typical aeronautical applications)<sup>6</sup>

$$M_{kn} = \int_{V_S} \rho \Psi_k \cdot \Psi_n dV = M_{nk} \quad (2)$$

<sup>4</sup> We prefer to derive the structural dynamics equations from the differential approach to emphasize the relationship between finite-element (simulation) and modal (modeling) methods, as well as the commonality between solids and fluids. The same results would be obtained using the Lagrange equations of motion.

<sup>5</sup> For the issues related to the distinction between essential and natural boundary conditions, see, *e.g.*, Reddy (1986).

<sup>6</sup> The symmetry of the matrix  $M$  is self-evident from Eq. 2. The symmetry of the matrix  $K$  stems from the facts that  $\mathbf{L}$  is self-adjoint and that  $\Psi_n$  are assumed to satisfy the corresponding homogeneous boundary conditions. Finally, the expression for  $f_n$  has been obtained by treating the forces  $\mathbf{t}$  (which appear in the boundary conditions) as volume forces (of the type  $\mathbf{t}d\eta$ ), where  $\eta$  is the arclength along the normal to  $S$ ) that appear in the differential equation, thereby yielding homogeneous boundary conditions.

$$K_{kn} = \int_{\mathcal{V}_s} \Psi_k \cdot \mathbf{L} \Psi_n d\mathcal{V} = K_{nk} \quad (3)$$

$$f_k = \oint_S \mathbf{t} \cdot \Psi_k dS \quad (4)$$

Next, we discuss the choice for the functions  $\Psi_n$ . In the *finite–element method*,  $u_n$  typically denotes nodal values of the displacement components, whereas  $\Psi_n(\mathbf{x})$  are suitable interpolation functions. We refer to this approach as *simulation*. Generally speaking, in order to have a good approximation of the solution, the number  $N$  of the unknowns  $u_n$  is very high (for a wing treated as a beam,  $N = 10 \div 30$ ; for the complete aircraft configuration,  $N = 10^4 \div 10^6$ ). Thus, the numerical solution of the above equations is highly computer intensive.

As mentioned above, as an alternative, one may use a modal approach (spectral method), *i.e.*, set

$$\mathbf{u}(\mathbf{x}, t) = \sum_{m=1}^M q_m(t) \Phi_m(\mathbf{x}), \quad (5)$$

where  $q_n(t)$  are known as generalized (Lagrangian) coordinates, whereas  $\{\Phi_n\}$  are the normalized natural modes of vibration of the structure (eigenfunction of the operator  $\mathbf{L}$ ), which satisfy the equation

$$\mathbf{L}\Phi = \rho\lambda\Phi \quad (6)$$

with homogeneous boundary conditions (for a complete aircraft, these are  $\mathbf{t} = \mathbf{T}\mathbf{n} = \mathbf{0}$ ). Note that the expansion in Eq. 5 is legitimate, because the  $\Phi_n$ 's form a complete set of orthogonal functions. This may be shown as follows. Consider first a structure that is fully constrained (*i.e.*, not free to move in rigid body motion), such as a cantilevered wing. In this case, we have: (1)  $\mathbf{L}$  is invertible; (2)  $\mathbf{L}^{-1}$  has the same eigenfunctions as  $\mathbf{L}$ , whereas the corresponding eigenvalues are the reciprocal of those of  $\mathbf{L}$ ; (3) the operator  $\mathbf{L}^{-1}$  is an integral operator, over a Jordan domain, with a kernel (influence function, *i.e.*, Green function) which is at most weakly singular (*i.e.*, integrable response to any unit load); (4) these types of operators are compact (Kress (1989), theorem 2.21); (5) the eigenfunctions of a self–adjoint operator corresponding to different eigenvalues are orthogonal; those corresponding to the same eigenvalue span an  $N$ –dimensional space, from which one may extract an orthogonal basis

(via Gram–Schmidt process); (6) the set of all the orthogonal eigenfunctions of an invertible, compact, self–adjoint operator is complete (Hochstadt (1989), theorem 13, p. 61, where now  $f_0 = 0$ , since the null space of an invertible operator consists only of the element 0). In the case of an aircraft, the proof is a bit more complicated because  $\mathbf{L}$  is not invertible, due to the rigid–body degrees of freedom which span the null space of  $\mathbf{L}$ ; the generalization of the above results to the case in which  $\mathbf{L}$  is singular may be obtained using the approach of Hochstadt (1989), p. 76.

Using the Galerkin method and Eq. 5 yields  $M = 1$  (the expression for  $M$  stems from the well known property of orthogonality of the natural modes; the modes are assumed to be normalized, so as to have the value one along the diagonal of  $M$ ). In addition,  $\mathbf{K} = \Omega^2$ , where  $\Omega^2 = [\omega_n^2 \delta_{kn}]$  (with  $\omega_n^2 = \lambda_n$ ) (the expression for  $\mathbf{K}$  follows from the definition of natural modes of vibration, Eq. 6; also, recall that the eigenvalues of a self–adjoint operator are all real, in our case, positive,  $\lambda = \omega^2 > 0$ , because  $\mathbf{L}$  is a positive definite operator, *i.e.*,  $\langle \mathbf{a}, \mathbf{L}\mathbf{a} \rangle > 0$ , for all  $\mathbf{a} \neq \mathbf{0}$ ). Therefore, Eq. 1 reduces to

$$\ddot{\mathbf{q}} + \Omega^2 \mathbf{q} = \mathbf{e}, \quad (7)$$

where  $\mathbf{e} = \{e_k\}$ , with  $e_k = \oint_S \mathbf{t} \cdot \Phi_k dS$ .

It is a rather common belief that the finite–element formulation (Eq. 1) is more accurate than the modal approach (Eq. 7). This is not necessarily true. In fact, it is easy to show that using the Galerkin method with the base functions given by the approximate eigenfunctions (*i.e.*, Eq. 7, with  $\omega_k$  and  $e_k$  obtained from the approximate finite–element model,  $\mathbf{K}\mathbf{u} = \omega^2 \mathbf{M}\mathbf{u}$ ) is fully equivalent to diagonalizing the finite–element equations. From this observation, we gather that – if the number of modes  $M$  equals the number of finite element unknowns  $N$  – the approximate–mode equations are fully equivalent to the finite–element equations (Eq. 1). On the other hand, in the modal model one may drastically reduce the number of unknowns (truncation of the system to the first  $M$  modes), typically with no significant loss in accuracy. Indeed, in aeroelastic analysis, we always have  $M \ll N$  (the modes corresponding to the lowest frequencies are used; for a complete configuration,  $M = 10 \div 30$ , a considerable reduction with respect to the finite–element approach, where for a full configuration  $N = 10^5 \div 10^6$ ). Consequently, it is apparent that, within a MDO/CD context, structural–dynamics *model-*

ing (*i.e.*, approximate–mode approach) is preferable to structural–dynamics *simulation* (*i.e.*, finite elements).

Another rather common belief is that an advantage of the modal approach (Eq. 7), over the finite–element one (Eq. 1), is due to the fact that, in Eq. 7, the equations are uncoupled. Whereas this fact is true within the field of structural dynamics, this property does not apply in aeroelasticity and aircraft dynamics, because in this case coupling appears through the aerodynamic forces, which are functions of the unknown  $\mathbf{u}$ . Nonetheless, the modal approach is still the most advantageous in aeroelasticity and aircraft dynamics as well – the reason is that, for smooth functions, the convergence rate of an expansion in terms of orthogonal functions, such as the natural modes of vibration, is very high.

This implies that even approximate natural modes (obtained by using the finite–element method described above, as applied to the solution of the eigenvalue problem) are adequate, because the relevant aspect is the orthogonality of the base functions, not the decoupling of the equations. Indeed, it is easy to show that the approximate finite–element modes of vibration satisfy the same orthogonality conditions as the exact ones.

Finally, a few comments on the convergence rate are in order. For  $M \ll N$ , the  $M$ th approximate mode is virtually identical to the exact one. Hence, the convergence rate of the approximate–mode expansion is initially similar to that of spectral methods (*i.e.*, much higher than that for finite–elements; for a discussion of the convergence properties of *spectral methods* – of which the expansion in terms of the natural mode of vibration is a particular case – the reader is referred to Gottlieb and Orszag (1977)). On the other hand, as  $M$  increases to its maximum value,  $N$ , the convergence rate becomes gradually poorer, since for  $M = N$ , the modal expansion is fully equivalent to the finite–element one, as the two span exactly the same space. Moreover, it should be noted that the value of  $N$  is not dictated by the convergence of the approximate modes of vibration to the exact ones, but simply by the convergence of the flutter speed (the only parameter relevant in MDO), as both  $M$  and  $N$  go to infinity (with considerable reduction of  $N$ ).

#### 4 Modeling vs. simulation in aerodynamics

Next, consider aerodynamics. Again, we begin with simulation methodologies, as useful background for dis-

cussing the modeling methodology proposed here. In our definition, aerodynamics simulation is based upon the solution of the conservation equations of mass (continuity), momentum (Euler or Navier–Stokes), and energy, by a methodology broadly known as CFD (Computational Fluid Dynamics). The CFD method most commonly used is the finite–volume technique, which consists of writing a discretized form of the conservation principles for a small volume. This may be considered as a special approach to obtain finite–difference expressions, and also as a very crude finite–element formulation for the above equations (with weight functions equal to one within the element, and to zero otherwise – partition of unity). Again, the number of degrees of freedom for a complete aircraft configuration is very high (*e.g.*,  $10^5 \div 10^7$ , the lower numbers being obtained in the inviscid case, or when an inviscid–viscous coupling is used, *e.g.*, with a finite–volume Euler external–flow analysis coupled with a boundary–layer or thin–Navier–Stokes analysis). Thus, these techniques are highly computer intensive; while fundamental in a simulation environment, they are not suitable in an MDO/CD context, in which it is desirable to utilize simpler methods, able to yield accurate solutions with computational efforts reduced as much as possible.

Indeed, in the case of interest here – civil aviation – we are dealing primarily with high–Reynolds–number attached flows, and traditional numerical methods in aerodynamics (where a boundary element code is coupled with an integral boundary–layer analysis) are tools more convenient than CFD.<sup>7</sup> Specifically, the method we propose for MDO/CD is a boundary–element analysis for compressible (subsonic) quasi–potential flows (*i.e.*, flows that are potential everywhere except for the wake surface, which is the locus of the points emanating from the trailing–edge, Morino (2003)), coupled with an integral boundary–layer analysis; the potential–viscous cou-

<sup>7</sup> Here, at the risk of oversimplifying the situation, we think of a fluid dynamicist as someone starting with very low Reynolds numbers, in the limit  $Re = 0$ , and working his way up; indeed, much of the work in CFD started with low Reynolds number flows. On the contrary, an aerodynamicist starts from attached flows with very high Reynolds number, in the limit  $Re = \infty$ , and works his way down. Indeed, classical aerodynamic formulations are based upon Prandtl’s work on viscous/inviscid interaction, with thin attached boundary layers, which imply very high Reynolds numbers. For the attached high–Reynolds–number flows of interest here, the aerodynamicist’s approach is at least as accurate as that of the computational fluid dynamicist (see, *e.g.*, Cebeci and Cousteix (1998)).

pling is based upon the Lighthill (1958) transpiration–velocity approach (see Section 6). The reason for this choice is that the proposed boundary element method for quasi–potential subsonic flows requires the same order of magnitude of computational effort as the methods typically used in industry for conceptual design (e.g., vortex–method analysis for incompressible potential flows), while at the same time is obviously more sophisticated than those methods in terms of physical/geometrical representation and considerably more accurate.

In this section, for simplicity, we present the formulation for the limited case of incompressible quasi–potential flows (the extension of the formulation to compressible flows is treated extensively in Morino (1993), Morino and Bernardini (2001), and Morino (2003), to which the reader is referred for details). An inviscid, incompressible, initially–irrotational flow remains, at all times, quasi–potential. In this case, the velocity field,  $\mathbf{v}$ , may be expressed as  $\mathbf{v} = \nabla\varphi$  (where  $\varphi$  is the velocity potential). Combining with the continuity equation for incompressible flows,  $\nabla \cdot \mathbf{v} = 0$ , yields  $\nabla^2\varphi = 0$ . The boundary conditions for this equation are as follows. The surface of the body,  $\mathcal{S}_B$ , is assumed to be impermeable; this yields  $(\mathbf{v} - \mathbf{v}_B) \cdot \mathbf{n} = 0$ , i.e.,  $\partial\varphi/\partial n = \chi := \mathbf{v}_B \cdot \mathbf{n}$  (where  $\partial/\partial n = \mathbf{n} \cdot \nabla$ , whereas  $\mathbf{v}_B$  is the velocity of a point  $\mathbf{x} \in \mathcal{S}_B$ , and  $\mathbf{n}$  is the outward unit normal to  $\mathcal{S}_B$ ). At infinity, in a frame of reference fixed with the undisturbed air, we have  $\varphi = 0$ . The boundary conditions on the wake surface,  $\mathcal{S}_W$ , are obtained from the balance principles of mass and momentum across a surface of discontinuity and are given by: (i) the wake surface is impermeable, and (ii) the pressure,  $p$ , is continuous across it. These imply that, for  $\mathbf{x}$  on  $\mathcal{S}_W$ , (i)  $\Delta(\partial\varphi/\partial n) = 0$ , where  $\Delta$  denotes discontinuity across  $\mathcal{S}_W$ , and (ii)  $\Delta\varphi = \text{constant}$  in time following a wake point  $\mathbf{x}_w$  (whose velocity is the average of the fluid velocity on the two sides of the wake), i.e.,  $\Delta\varphi(\mathbf{x}_w, t) = \Delta\varphi(\mathbf{x}_{TE}, t - \tau)$ , where  $\tau$  is the time required to the material point to move from the trailing edge point  $\mathbf{x}_{TE}$  to the wake point  $\mathbf{x}_w$ . Hence,  $\Delta\varphi$  on the wake equals the value it had when  $\mathbf{x}_w$  left the trailing edge. Finally, the trailing–edge condition states that, at the trailing edge,  $\Delta\varphi$  on the wake equals  $\varphi_2 - \varphi_1$  on the body, where the subscripts 1 and 2 denote the two sides of the wing surface (for a detailed analysis of this issue, see Morino and Bernardini (2001)). Once the above problem has been solved, the pressure is obtained from the Bernoulli theo-

rem.

In the methodology used in the code MAGIC, the above problem for the velocity potential is solved by a boundary–element formulation. The boundary integral representation for this problem, using the above wake boundary conditions, is given by (see Morino (1993) and Morino (2003))

$$\begin{aligned} \varphi(\mathbf{x}, t) = & \oint_{\mathcal{S}_B} \left( G\chi - \varphi \frac{\partial G}{\partial n} \right) dS(\mathbf{y}) \\ & - \int_{\mathcal{S}_W} \Delta\varphi_{TE}(t - \tau) \frac{\partial G}{\partial n} dS(\mathbf{y}), \end{aligned} \quad (8)$$

with  $G = -1/4\pi\|\mathbf{y} - \mathbf{x}\|$ , whereas  $\chi$  is prescribed from the above impermeability boundary condition. Note that, in the absence of the wake, Eq. 8, in the limit as  $\mathbf{x}$  tends to  $\mathcal{S}_B$ , yields a boundary integral equation for  $\varphi$  on  $\mathcal{S}_B$ , with  $\chi$  on  $\mathcal{S}_B$  known from the boundary condition. Once  $\varphi$  on the body is known,  $\varphi$  (and hence  $\mathbf{v}$  and, by using Bernoulli’s theorem,  $p$ ) may be evaluated everywhere in the field. The situation is similar in the presence of the wake, since, by applying the wake and trailing–edge conditions,  $\Delta\varphi$  on the wake may be expressed in terms of  $\varphi$  over the body at preceding time steps. It should be noted that the geometry of the wake is not known *a priori*. However, in the case of airplanes one may assume, with virtually no loss in accuracy, the wake to be parallel to the undisturbed flow (small–disturbance assumption), which we take to be the direction of the  $x$ –axis ( $\mathbf{v}_\infty = U_\infty\mathbf{i}$ ; for a free wake analysis, see Morino (1993)). Consistently, we have that  $\tau$  is given by  $\tau = (x_w - x_{TE})/U_\infty$ . Note that now the integral operator is linear. Then, taking the Laplace transform of Eq. 8, with zero initial conditions, one obtains<sup>8</sup>

$$\begin{aligned} \tilde{\varphi}(\mathbf{x}) = & \oint_{\mathcal{S}_B} \left( G\tilde{\chi} - \tilde{\varphi} \frac{\partial G}{\partial n} \right) dS(\mathbf{y}) \\ & - \int_{\mathcal{S}_W} \Delta\tilde{\varphi}_{TE} e^{-s\tau} \frac{\partial G}{\partial n} dS(\mathbf{y}), \end{aligned} \quad (9)$$

where  $\tilde{\phantom{x}}$  denotes Laplace–transformed functions. Equation 9 may be discretized by dividing the surfaces  $\mathcal{S}_B$  and  $\mathcal{S}_W$  into small elements,  $\mathcal{S}_j$  ( $j = 1 \cdots N_B$ ), and  $\mathcal{S}_n$  ( $n = 1, \cdots, N_W$ ) respectively, and assuming  $\tilde{\varphi}$ ,  $\tilde{\chi}$ , and

<sup>8</sup> The role of the initial conditions in aerodynamics is addressed in Morino (1974). However, this is only of theoretical interest, since its implementation is not feasible, as well as inconsequential.

$\Delta\tilde{\phi}$  to be constant within each element.<sup>9</sup> This yields the matrix  $E_{IE}$  (used in the next section), which relates the vector of the values of the velocity potential (evaluated at the element centers), to the vector of the normal–wash,  $\chi = \partial\phi/\partial n$  (also evaluated at the element centers). Note that the dependence of  $E_{IE}$  upon  $s$  is transcendental in nature, because of the exponentials arising from the Laplace transform of the delay.

## 5 Aeroelastic modeling

In this section, we show how the structural and the aerodynamic modeling may be coupled to obtain the formulation for aeroelasticity. This is accomplished by noting that the vector of the generalized aerodynamic forces is given by

$$\tilde{e} = q_D E(\check{s}) \tilde{q}, \quad (10)$$

where  $q_D = \frac{1}{2}\rho_\infty U_\infty^2$  is the dynamic pressure, whereas  $\check{s} = s\ell/U_\infty$  is the dimensionless Laplace parameter (also known as complex reduced frequency; note that the reduced frequency,  $k = \omega\ell/U_\infty$ , is given by  $k = \text{Imag}(\check{s})$ ). The matrix  $E$  is given (in the Laplace domain) by  $E(\check{s}) = E_{GF} E_{BT}(\check{s}) E_{IE}(\check{s}) E_{BC}(\check{s})$ , where:

- (i) the matrix  $E_{BC}$  (obtained from the boundary condition,  $\chi = (U_\infty \mathbf{i} + \sum_n \dot{u}_n \Phi_n) \cdot \mathbf{n}$ ) relates the vector  $\tilde{f}_\chi$  of the dimensionless normal–wash at the element centers, to the generalized coordinates vector  $\tilde{q}$ , as  $\tilde{f}_\chi = E_{BC} \tilde{q}$ ;
- (ii)  $E_{IE}$  (obtained from the integral equation, see paragraph that follows Eq. 9) relates the vector  $\tilde{f}_\phi$  of the dimensionless velocity potential at the element centers, to  $\tilde{f}_\chi$ , as  $\tilde{f}_\phi = E_{IE} \tilde{f}_\chi$ ;
- (iii) the matrix  $E_{BT}$  (obtained from the linearized Bernoulli theorem,  $c_p = -2(\phi + U_\infty \partial\phi/\partial x)/U_\infty^2$ ) relates the vector  $\tilde{c}_p$  of the pressure coefficient at the element centers, to  $\tilde{f}_\phi$ , as  $\tilde{c}_p = E_{BT} \tilde{f}_\phi$ ;
- (iv) the matrix  $E_{GF}$  (obtained from the definition of  $e_k$ ) relates the vector  $\tilde{e}$  of the generalized aerodynamic forces, to  $\tilde{c}_p$ , as  $\tilde{e} = q_D E_{GF} \tilde{c}_p$ .

Combining the structural dynamics equations, Eq. 7, one obtains, in the Laplace domain,

$$s^2 \tilde{q} + \Omega^2 \tilde{q} = q_D E(\check{s}) \tilde{q} + I.C. \quad (11)$$

where  $I.C.$  comprises the structural initial condition terms (as mentioned above, it is convenient to assume zero initial conditions for aerodynamics).

In order to find the time–domain solution of this equation one should solve for  $\tilde{q}$ , and take the inverse Laplace transform. This in turn involves finding the infinite roots of the (transcendental) equation  $\text{Det}[s^2 I + \Omega^2 - q_D E(\check{s})]$ . In order to avoid this, some traditional methods, such as the  $V$ – $g$  and the  $p$ – $k$  methods, have been used in the past. These are briefly illustrated below in order to make the reader appreciate the advantage of using reduced order models. Consider first the  $V$ – $g$  method.<sup>10</sup> Setting  $q(t) = \tilde{q} e^{i\omega t}$ , and combining Eqs. 7 and 10 yields

$$-\omega^2 \tilde{q} + \Omega^2 \tilde{q} = \frac{1}{2} \rho_\infty U_\infty^2 E(k) \tilde{q} \quad (12)$$

In the modified  $V$ – $g$  method (see the qualifications in the footnote above), Eq. 12 is left–multiplied by  $\Omega^{-2}/\omega^2$  to yield

$$[A(k) - \lambda I] \tilde{q} = 0 \quad (13)$$

with  $\lambda = 1/\omega^2$  and  $A(k) = \Omega^{-2} (\frac{1}{2} \rho_\infty E(k) \ell^2 / k^2 + I)$ . Then, one performs a sweep over  $k$  and identifies all the values,  $k_m$ , for which there exists a real positive eigenvalue,  $\lambda_m$ . In this case, the solution is physically meaningful and corresponds to sinusoidal motion. From  $k_m$  and  $\lambda_m$ , one obtains  $U_m = 1/\sqrt{\lambda_m}$  and  $\omega_m = k_m U_m / \ell$ .

<sup>10</sup>The name arises from the fact that originally  $V$  was used to denote the undisturbed velocity,  $U_\infty$ , whereas  $g$  was used to denote an artificial structural–damping coefficient that was introduced in order to assure that the solution would be indeed on the imaginary axis (*i.e.*, of the  $e^{i\omega t}$ –type), which was the only type of aerodynamics available in those days. While the essence of the method may be illustrated without discussing the role of the artificial damping  $g$ , as outlined in the text, nonetheless, to be specific, in the traditional  $V$ – $g$  method, an unknown structural damping term of the type  $ig\Omega^2$  is added (see Fung (1955), p. 239, Bisplinghoff, Ashley, and Halfman (1955), pp. 566 and 611, and Bisplinghoff and Ashley (1962), p. 384); then, one still has Eq. 13, with now  $\lambda = (1 + ig)/\omega^2$ . Then,  $g$  is interpreted as the amount of structural damping to be added in order to have a sinusoidal solution (*i.e.*, positive  $g$  corresponds to an unstable solution). When  $g = 0$  one obtains the same results as discussed in the text. Note that, for each root, there corresponds a different value of  $g$ . Thus,  $g$  is understood as that corresponding to the least damped root.

<sup>9</sup>We refer to this as the zeroth–order boundary–element formulation, see Morino (2003); for a third–order formulation, see Morino and Bernardini (2001).



The flutter speed is given by the lowest  $U_m$ . Next, consider the  $p$ - $k$  method (Hassig (1971)). This consists of writing Eq. 12, with  $\omega$  replaced with the Laplace parameter (the name stems from the fact that the complex reduced frequency  $\check{s} = s\ell/U_\infty$  is traditionally denoted by the symbol  $p$ , here used to denote pressure). Then, one iterates with  $k^{(n+1)} = \text{Im}g(\check{s}^{(n)})$ . As in the  $V$ - $g$  method, only one root (the critical one) is meaningful.

It is apparent from the brief outline above, that these methods are cumbersome and not apt for use in MDO/CD. In recent years, a new trend has emerged that consists of rational matrix approximations of the function  $E = E(k)$ , such that the resulting equations form a system of first-order ordinary differential equations, whose stability analysis requires simply the use of a root locus of the eigenvalues of a matrix by varying  $U_\infty$  (finite-state aeroelasticity, or reduced-order models). Probably, the earliest example of this approach is the work of Jones (1940) who gives a rational approximation for the Theodorsen function and the corresponding time-domain approximation for the Wagner function. In the matrix formulation, the concept was introduced by Roger (1977). A widely used scheme is that by Karpel (1982). An interesting approach to the problem is presented by Venkatesan and Friedmann (1986). The specific reduced-order model of interest here is based on the model presented in Morino, Mastroddi, De Troia, Ghiringhelli, and Mantegazza (1995). This consists of expressing the aerodynamic matrix  $E(\check{s})$  as<sup>11</sup>

$$E(\check{s}) \simeq \hat{E}(\check{s}) = E_2\check{s}^2 + E_1\check{s} + E_0 + (\check{s}\mathbb{I} + F)^{-1}G \quad (14)$$

where  $E_k$ ,  $G$ , and  $F$  are fully populated square matrices, which are independent of  $\check{s}$ . These matrices are evaluated by a least square procedure on a set of numerical data for the matrix of the aerodynamic forces  $E(\check{s})$ . The aeroelastic system resulting from Eqs. 11 and 14 is equivalent to

$$\begin{aligned} s^2\check{q} + \Omega^2\check{q} &= \frac{1}{2}\rho_\infty U_\infty^2 (\check{s}^2 E_2\check{q} + \check{s}E_1\check{q} + E_0\check{q} + \check{r}) \\ (\check{s}\mathbb{I} + F)\check{r} &= G\check{q}, \end{aligned} \quad (15)$$

which may be easily transformed into the time domain to yield a system of linear homogeneous first-order differential equations of the type  $\dot{x} = A(U_\infty)x$ , where  $x^T =$

$[q^T, \dot{q}^T, r^T]$ . This approach allows one to perform the flutter analysis through a root locus of the eigenvalues of the matrix  $A(U_\infty)$ , thereby avoiding the above mentioned traditional methods, which unnecessarily complicate the optimization procedure.

The same reduced-order model is used for the gust response analysis (see Morino, Bernardini, and Mastroddi (2003) for details).

## 6 Viscous flow modeling

In this section, we consider the modeling for the viscous-flow correction. The analysis is limited to steady attached high-Reynolds-number flows, where the vortical region (*i.e.*, boundary layer and wake) has a small thickness (as mentioned above, viscosity effects are usually not included in conceptual design for unsteady aerodynamics, which is only needed for the linear analysis of flutter and gust response). Outside boundary layer and wake, the flow is irrotational and is solved by using a quasi-potential-flow model obtained by introducing, in the boundary integral formulation described above, a viscous-flow correction based on Lighthill's equivalent sources approach (Lighthill (1958)). This consists of modifying the impermeability boundary conditions  $\partial\varphi/\partial n = \mathbf{v}_B \cdot \mathbf{n}$ , into  $\partial\varphi/\partial n = \mathbf{v}_B \cdot \mathbf{n} + \chi_v$ , where the transpiration velocity  $\chi_v$  is given by

$$\chi_v = \frac{\partial}{\partial s_1} \int_0^\delta (u_e - u) d\eta + \frac{\partial}{\partial s_2} \int_0^\delta (v_e - v) d\eta \quad (16)$$

where  $s_1$  and  $s_2$  are local orthogonal arclengths over the wing surface,  $\delta$  is the boundary-layer thickness, and  $u_e$  and  $v_e$  the velocities at the external edge of the boundary layer respectively in  $s_1$  and  $s_2$  directions; a similar correction is used on the wake surface, with  $\Delta(\partial\varphi/\partial n) = (\chi_v)_2 + (\chi_v)_1$  (see Morino, Salvatore, and Gennaretti (1999) for an in-depth analysis of this point). An integral formulation is used for the boundary layer (for attached flows, this approach yields results as accurate as those obtained by differential methods, with considerably reduced computational effort).

We have considered three models with different levels of sophistication: (1) a very simple model based upon the classical Blasius theory used as strip theory, (2) a two-dimensional integral boundary-layer formulation used as strip theory (see below), and (3) a three-dimensional integral boundary layer formulation (see also below). Note

<sup>11</sup> The leading term being of  $O(\check{s}^2)$  is motivated by the fact that we want  $\hat{E}(\check{s})$  to have the same order as  $E(\check{s})$  (*i.e.*,  $O(\check{s}^2)$ ), which stems from  $E_{BC}(\check{s}) = O(\check{s})$ ,  $E_{IE}(\check{s}) = O(1)$ ,  $E_{BT}(\check{s}) = O(\check{s})$ , whereas  $E_{GF}(\check{s})$  is independent of  $\check{s}$ .

that, in general, three-dimensional effects within the boundary layer may be neglected with a minor loss of accuracy for applications to wings with large aspect ratio and reduced sweep angle; this is even more applicable in the case of the Prandtl–Plane wing, where no tip effect exists. In all the models, the viscosity correction to the potential flow is evaluated through the Lighthill (1958) transpiration velocity, as mentioned above (through both  $S_B$  and  $S_W$ ).

The first model, limited to laminar flows, with an empirical correction for turbulent flows, was used initially, just to have an order of magnitude of the correction. In the second one, the laminar portion is computed by the Thwaites (1949) method, the turbulent portion by the Green, Weeks, and Brooman (1973) ‘lag–entrainment’ method; the transition from laminar to turbulent flow is detected by the Michel (1952) method. Matching of the boundary–layer solution with the viscous–flow–corrected potential–flow solution is obtained through classical direct iteration. The viscous drag is evaluated with the Squire and Young (1938) approach. Finally, the three–dimensional integral boundary–layer algorithm uses two equations for momentum (extension of von Kármán equation to three–dimensional flows) coupled by two auxiliary equations: the first is the kinetic energy equation and the second is the transport equation for the maximum shear stress coefficient (‘lag’ equation; see Morino, Bernardini, Da Riz, and Del Rio (2002) for details). In order to complete the problem, following Nishida (1986) and Milewski (1987), we use the streamwise closure relations proposed by Drela (1989), along with the crosswise relations of Johnston (1960). The Mughal (1992) scheme is used for the solution. The matching of the thin–layer solution with the potential–flow solution, is obtained by the simultaneous coupling method by Drela (1989), which solves the viscous and inviscid equations simultaneously; this scheme is stable even for separated flows.

Other contributions to the drag (such as interference drag) are currently evaluated by empirical corrections from Mc Cullers (1984).

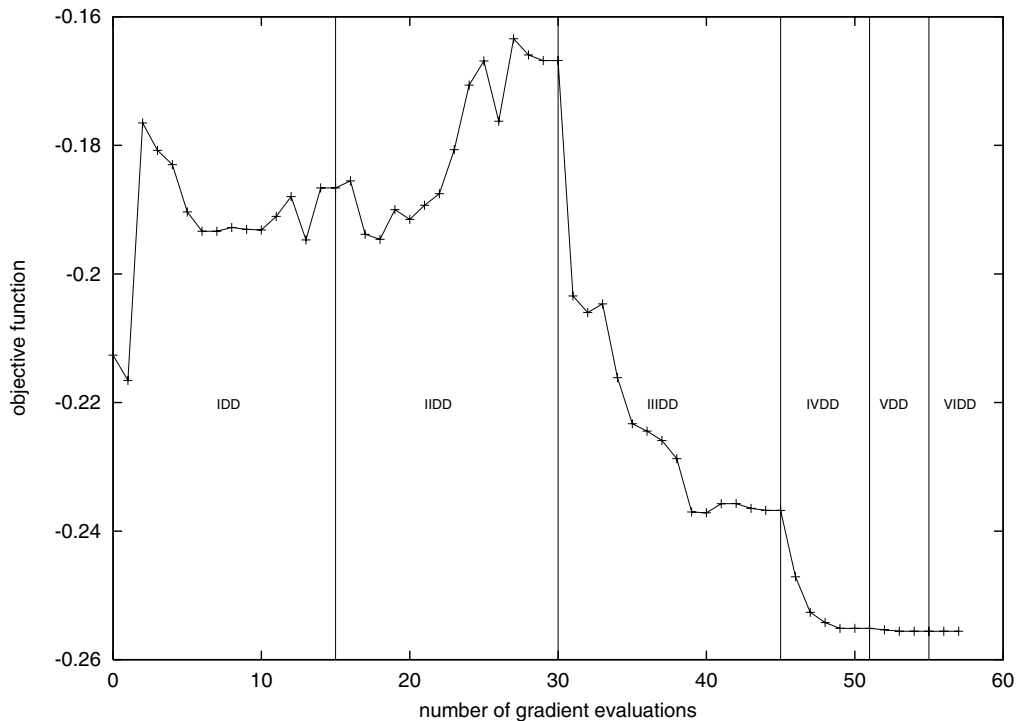
## 7 Validation

As mentioned above, MAGIC is in a state of evolution. Thus, it seems appropriate to present some numerical results (from Morino, Bernardini, and Mastroddi (2003)), simply to clarify the extent of the applicability of the

current version of the code MAGIC, which although not fully developed, is nonetheless already quite a useful tool for conceptual design (see also Morino, Bernardini, Gregorio, Willcox, and Harris (2004) and Carpentieri, van Tooren, Bernardini, and Morino (2004), which present additional results obtained with extensions of MAGIC, pertaining, respectively, life–cycle costs and a Blended–Wing–Body configuration).

The results presented here have been obtained using for the wing a beam model (for the static analysis as well as the evaluation of the natural modes of vibration). The aerodynamic formulation presented above is used for steady as well as unsteady aerodynamics. For the evaluation of the steady–state potential–aerodynamics loads (lift and induced drag), we use the formulation of Gennaretti, Salvatore, and Morino (1996) – an exact extension of the work by Trefftz (1921). The unsteady quasi–potential aerodynamics formulation is used for flutter and gust response (when the viscosity effects are typically negligible). The finite–state reduced order model for the generalized forces is used in the aeroelastic analysis. The two–dimensional integral boundary–layer formulation, used as ‘strip–theory’ in three–dimensional applications, has been validated by comparison with experimental results available in literature, in the case of: (i) isolated wing, (ii) biplane, and (iii) box–wing configuration (see Bernardini, Frediani, and Morino (1999), which presents in particular the polar at  $Re = 5.1 \cdot 10^5$  of a box–wing configuration; the results are in good agreement with the experimental and numerical results by Gall and Smith (1987). On the basis of these results, we used the strip–theory approach, which we believe to be a better candidate for MDO/CD in that yields results comparable to the three–dimensional ones, with less computational effort.

For the sake of conciseness, the results presented are limited to a standard 622–passenger configuration, a cruise altitude of 30.000 ft and a cruise Mach number of  $M_\infty = 0.75$  (the results are from Morino, Bernardini, and Mastroddi (2003), to which the reader is referred for additional results, in particular for an application to the Prandtl–Plane). As mentioned above, in the present version of the code, the fuselage is prescribed (in other words, the optimization pertains solely the wing system). The fuselage is 72.84m (239 ft) long, 8.23m (27 ft) wide, and 7.01m (23 ft) high. Also, the aircraft span is set to be  $b = 80m$  (262.4 ft), *i.e.*, the limit given by air-



**Figure 1** : Objective function for standard configuration.

port constraint. The propulsion system consists of four underwing-mounted turbo-fan engines.

We decided to concentrate on empty weight and efficiency, giving only a lower boundary for the range and prescribing the fuel volume. Hence, the objective function considered for the results presented in the following is a combination of: (i) aerodynamic efficiency,  $E$  (as indicative of operative costs), (ii) empty weight,  $W_e$  (as indicative of manufacturing costs),  $J = E_{ref}/E + W_e/W_{e,ref}$ , where subscript *ref* denotes reference values of the above variables (for a deeper analysis of this point, see the life-cycle cost analysis of Morino, Bernardini, Gregorio, Willcox, and Harris (2004)).

The evaluation of the aircraft empty weight,  $W_e$ , is based on a standard recursive algorithm (see Corning (1977)) for conceptual design.

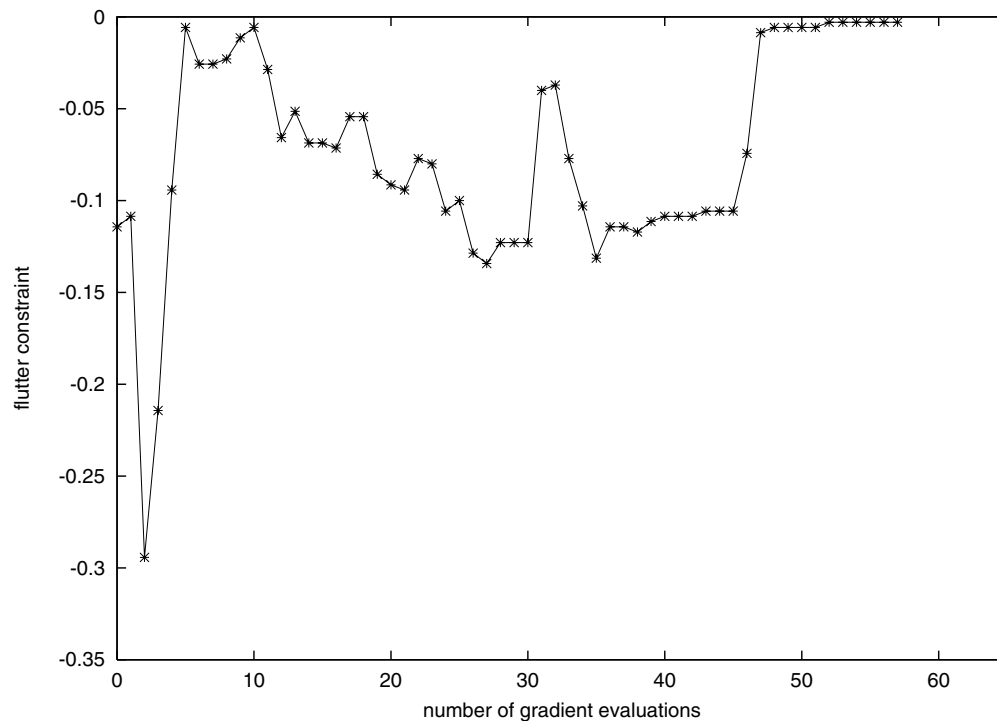
Starting from the reference configuration, a preliminary analysis has been performed to define the constraint limits (see Table 1) and the reference parameters to be introduced in the objective function in order to normalize it. Regarding the limit values of the constraints we have imposed: (i) a minimum range value  $R_{min} = 5000$  nm; (ii) a maximum value of the normal stress in spars and stringers  $\sigma_{max} = 400MPa$ ; (iii) a maximum value of

**Table 1** : Constraints for the objective function

|                                  |  |
|----------------------------------|--|
| Range constraint                 | $R \geq R_{min}$                           |
| Maximum normal stress constraint | $\sigma_{max} \leq \sigma_{max}^{max}$     |
| Maximum shear stress constraint  | $\tau_{max} \leq \tau_{max}^{max}$         |
| Flutter speed constraint         | $U_F \geq U_{F_{min}}$                     |
| Divergence speed constraint      | $U_D \geq U_{D_{min}}$                     |
| Volume of fuel constraint        | $V_F \leq V_{F_{available}}$               |
| Gust load constraint             | $\Delta n \leq \Delta n_{max}$             |
| Longitudinal static-stability    | $C_{M_{\alpha}} \geq C_{M_{\alpha}}^{max}$ |

the shear stress in the skin  $\tau_{max} = 200MPa$ ; (iv) a minimum flutter/divergence speed  $U_{F_{min}} \equiv U_{D_{min}} = 320m/s$ ; (v) a maximum value of the incremental gust load factor  $\Delta n_{max} = 2.5$ ; (vi) a maximum pitch moment coefficient  $C_{M_{\alpha}}^{max} = -10^{-3}$ .

Table 2 summarizes the design variables and the state space variables in the initial and final configurations, whereas Figures 1 and 2 depict respectively the objective function and the critical constraint (flutter velocity) evolutions. The initial and optimized configuration are also depicted in Figs. 3 and 4.



**Figure 2** : Critical constraint for standard configuration.



**Figure 3** : Wing planform: initial configuration.



**Figure 4** : Wing planform: optimized configuration.

## 8 Concluding remarks

The code MAGIC, in the version reported here, has a series of limitations that are currently being addressed. The items that require improvement and that are under consideration may be grouped in the following categories: (1) applications to other innovative configurations; (2) structures and materials; (3) aerodynamics; (4) aeroelasticity; (5) aeroacoustics; (6) life-cycle costs; (7) optimizer.

Regarding different types of innovative configurations,

as mentioned above, our applications thus far are limited to the 620-passenger Prandtl-Plane and, recently, a Blended-Wing-Body aircraft (Carpentieri, van Tooren, Bernardini, and Morino (2004)). Other applications are being considered, ranging from a 250-passenger commuter airplane to a general aviation airplane.

Regarding the structural model, the current finite-element model, a simple equivalent-beam model, has been used in the results presented above. This is acceptable for the conceptual design of the types of problems

**Table 2** : Initial and optimized parameters

| Design and state space variable       | Initial | Optimized |
|---------------------------------------|---------|-----------|
| Root chord (m)                        | 17.0    | 15.86     |
| Tip chord (m)                         | 7.0     | 4.76      |
| Root spar thickness (m)               | 0.020   | 0.014     |
| Tip spar thickness (m)                | 0.010   | 0.011     |
| Sweep angle (degree)                  | 30.00   | 27.85     |
| Wing root built-in angle (degree)     | 6.0     | 7.0       |
| Wing tip built-in angle (degree)      | 3.0     | 3.6       |
| Wing root skin thickness (m)          | 0.0025  | 0.0021    |
| Wing tip skin thickness (m)           | 0.0015  | 0.0011    |
| Empty weight ( $W_e$ ) (Kg)           | 201,025 | 190,450   |
| Range ( $R$ ) (nm)                    | 6,967   | 7,507     |
| Efficiency ( $E$ )                    | 16.43   | 17.19     |
| Flutter speed $U_F$ (m/s)             | 390     | 351       |
| Max direct stress (MPa)               | 167.3   | 205.0     |
| Max shear stress (MPa)                | 121.8   | 185.5     |
| I wing-system natural frequency (Hz)  | 0.66    | 0.63      |
| II wing-system natural frequency (Hz) | 1.85    | 1.65      |
| Max wing-tip deflection in cruise (m) | 1.95    | 2.53      |

considered thus far (standard and Prandtl-Plane wing, with fuselage prescribed). The sophistication of this model is already marginal for the Blended-Wing-Body application (Carpentieri, van Tooren, Bernardini, and Morino (2004)). Recently, a more sophisticated model (beams plus in-plane loaded plates) has been added to MAGIC (see Bernardini and Mastroddi (2004) for details). A full three-dimensional finite-element method (which is based upon the Hermite interpolation, and has been developed specifically for optimization, but is not yet included in MAGIC), is presented in Morino, Bernardini, Cerulli, and Cetta (2004). In addition, a weight reduction technique based upon the approach used by Wang and Wang (2004) is currently under consideration. Another limitation of MAGIC regards the materials: in the current version of MAGIC (wing optimization), assumes the wing to be of a prescribed homogeneous isotropic material. The extension to non-homogeneous non-isotropic materials is relatively straightforward. On the contrary, an optimization that automatically chooses the most convenient material available is quite complicated and no activity in this direction is currently underway (although highly desirable).

Regarding steady-state aerodynamics, the equations used are linearized: thus, they are valid for subsonic analysis, but not for transonic analysis, which requires the use of volume elements to take into account the non-linear terms. A code for the steady-state transonic analysis has been developed (see, e.g., Iemma and Morino (1997)), but not yet included in MAGIC. On unsteady aerodynamics (aeroelasticity), we have a similar situation: a code for transonic analysis has been developed (Iemma, Gennaretti, and Albanesi (2004)), but not yet included in MAGIC; the formulation used is linear (specifically, it is obtained by linearizing the non-linear volume terms, and as well known this is adequate, since only the stability boundary is of interest here); hence, the formulation for reduced order model of Section 5 applies here as well. The next item, aeroacoustics, has been addressed by Morino, Iemma, Bernardini, and Diez (2004) and Iemma, Diez, and Morino (2005), where preliminary work on the subject is presented (with take-off noise included in the objective function). Major developments in this direction is currently underway and is expected to be incorporated in MAGIC, since community noise is one of the major items of concern in civil aviation. Finally,

as mentioned above the objective function is based upon a linear combination of empty weight, fuel consumption, and efficiency. In Morino, Bernardini, Gregorio, Willcox, and Harris (2004) the objective function is based upon life-cycle costs. Further analysis in this direction is warranted.

In reference to the last item – the optimizer – different optimization strategies are being explored (*e.g.*, conjugate gradient with feasible direction algorithm) and compared to existing optimization codes, such as SNOPT (Gill, Murray, and Saunders (2002)). Also, some of the issues addressed above require that the optimizer be extended to include genetic algorithms (see, for instance, Zhu, Liu, Wang, and Yu (2004)).

If all the items discussed above were to be included, the models in code MAGIC would become much more sophisticated and the code itself might become apt to preliminary design. However, the inclusion of all these items in MAGIC would render the code much too computer intensive and its use be inconceivable for practical preliminary-design applications, within the optimization techniques presently available.

However, these types of applications would be possible using an optimization procedure proposed by Alexandrov and Lewis (1998), where models with different levels of sophistication may be combined. Specifically, in the two-level implementation, a sophisticated model is used to “calibrate” a simple one, through an affine transformation which is kept constant during several iterations of the optimization procedure. This yields the same level of accuracy as the sophisticated model, with an efficiency only slightly lower than that obtained with the simple model. The implementation of the Alexandrov and Lewis (1998) procedure in MAGIC is currently underway. As mentioned above, this would allow us to overcome the present limitations and move towards a more general, but still efficient, formulation, possibly of interest for preliminary design.

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