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An Ant Colony Optimization Algorithm for Stacking Sequence Design of Composite Laminates

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Abstract: The study reported in this paper explores the potential of Ant Colony Optimization (ACO) metaheuristic for stacking sequence optimization of composite laminates. ACO is a recently proposed population-based search approach able to deal with a wide range of optimization problems, especially of a combinatorial nature, and inspired by the natural foraging behavior of ant colonies. ACO search processes, in which the activities of real ants are simulated by means of artificial agents that communicate and cooperate through the modification of the local environment, were implemented in a specifically developed numerical algorithm aimed at the lay-up optimization (based on a strain energy criterion) of laminated plates subject to in-plane and out-of-plane loads. Numerical analyses were conducted to investigate the quality and reliability of the metaheuristic search procedure under various load cases, geometry configurations and constraint conditions. The analyses indicated that the proposed ACO algorithm is able to achieve reasonably good solutions within very few iterations, and extremely high-quality solutions within a limited number of runs, with respect to the total number of possible solutions, for both uncostrained and constrained optimization lay-up problems. The results obtained during the investigation point out the robustness and effectiveness of the procedure and suggest the use of ACO-based search techniques as practical design tools for laminate lay-up configuration.

keyword: Ant Colony Optimization (ACO), Metaheuristic, Composite laminates, Lay-up optimization.

1 Introduction

Laminated composite materials have been increasingly adopted during the last decades in aerospace, transport and marine applications because of their high specific strength and stiffness and of the possibility of tailoring the mechanical properties by properly selecting the orientation of individual plies. However, the stiffness and strength anisotropy of the material and the large number of variables (such as laminae orientations and stacking arrangement) and constraints usually involved makes the design of composite structures much more complicated than that of conventional structures made of homogeneous isotropic materials [Tsai (1992)].

Several procedures have been developed to help in the design of efficient laminate configurations, which, as opposed to distributed-parameter problems [Cherkaev (2000); Wang and Zhou (2004)], typically requires a discrete-parameter analysis [Haftka and Gurdal (1991)]. Simplified approaches range from the use of preliminary design tools (netting analysis, carpet plots [Fukunaga (1988); Barbero (1999)]) to the adoption of graphical methods in which special lamination parameters are used as continuous design parameters to select the best ply distribution among typical laminate configurations [Miki and Sugiyamat (1993); Fukunaga, Ishikawa, Sato and Sekine (1997)]). In more advanced analyses, the design of the stacking sequence of fiber reinforced composite laminates has been formulated and attacked as either a continuous optimization problem [Tauchert and Adibhatla (1984), Avalle and Belingardi (1995)], where ply thicknesses and ply orientations are represented by continuous real-value design variables, or as a combinatorial optimization problem [Gürdal, Haftka and Hajela (1999)], where, more realistically, because of manufacturing requirement, ply thicknesses are fixed and ply orientation angles are limited to a finite set of predefined values (such as 0°, 90° and ±45°).

Recent investigations have demonstrated the great potential of a new class of methods, called metaheuristics [Glover and Kochenberger (2003)], in dealing with many optimization problems, especially those of a combinatorial nature. Metaheuristic algorithms are defined as

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sets of high-level procedures and concepts that employ low-level heuristics (i.e. approximate procedures able to obtain sufficiently accurate solutions in reasonably short times) in such a way as to both improve the quality of the search process and broaden the application fields of heuristic algorithms.

Examples of metaheuristics include simulated annealing, tabu search, iterated local search, evolutionary computation, particle swarm optimization, ant colony optimization [Glover and Kochenberger (2003); Corne, Dorigo and Glover (1999)]. While all these algorithms are stochastic in nature, they differ in the strategies and techniques adopted for escaping from local optimum regions and favouring a complete exploration of the solution space. In particular, population-based metaheuristics such as evolutionary computation (EC) and ant colony optimization (ACO), that, in every iteration, can deal with a set of solutions rather than with a single solution, provide a convenient way for an efficient exploration of complex search spaces [Blum and Roli (2003)].

Not surprisingly, in view of the suitability of population-based metaheuristics for dealing with discrete optimization problems, various investigations have been conducted on the use of these algorithms for the stacking sequence optimization of composite laminates subject to different loading conditions and constraint sets. Most of the studies published in the open literature adopt EC genetic algorithms or path relinking optimization procedures [Todoroki and Haftka (1998); Potgeiter and Stander (1998); Nam, Hwang and Han (2001); Rama Mohan Rao and Arvind (2005)], in which new solutions are explicitly obtained by the use of one or more combination operators (such as *recombination* or *crossover* in genetic algorithms). On the other hand, to the authors' knowledge, no investigation has been reported on the use of ant colony optimization algorithms, which iteratively construct new solutions by using a statistical distribution function exploiting the experience accumulated by earlier populations over the entire solution space.

ACO is a metaheuristic approach proposed in 1992 by Dorigo [Dorigo (1992); Dorigo, Maniezzo and Colorni (1996)] and inspired by observation of the foraging behavior of real ants, which are able to find the shortest path between the nest and the food source by depositing a chemical substance (pheromone) along their trails and by choosing with higher probability paths marked by stronger pheromone concentrations. This method of indirect communication, in which individuals of a natural system interact with one another by modifying their local environment (pheromone trails are used as medium in ACO procedures) is called stigmergy and was first introduced by Grassé in 1959 [Grassé (1959)]. In analogy with biological systems, ACO simulates the behavior of ant colonies by introducing a population of agents (artificial ants) which cooperate and communicate by stigmergy and which direct the search towards the best solution by probabilistic processing of cumulated information.

Experimental investigations have indicated robustness and versatility as the peculiar strengths of ACO algorithms, which have been successfully implemented to produce optimum or near-optimum solutions in a wide range of different combinatorial problems, such as routing and ordering problems, machine learning, assignment and timetabling, etc. [Dorigo, Di Caro and Gambardella (1999); Maniezzo and Colorni (1999); Dorigo and Stutzle (2004)]

In this study, an ACO algorithm for optimum or near-optimum stacking sequence selection of laminated plates has been developed to investigate the feasibility of ACO metaheuristic concepts for composite laminate design. Numerical analyses, based on the minimum strain energy criterion, have been carried out on laminates subject to various in-plane and out-of-plane loadings and to different constraint conditions, with the aim of exploring the robustness and the effectiveness, with respect to solution quality, of the proposed ACO procedure.

2 Overview of ACO metaheuristic

ACO metaheuristic is a stochastic search method based on the indirect communication of a colony of artificial ants (agents) mediated by artificial pheromone trails [Dorigo and Stutzle (2004)]. In ACO algorithms, artificial ants are probabilistic procedures that incrementally build a new solution by adding solution components to a partial solution under construction. Artificial ants iteratively construct new solutions by moving on a connected graph whose vertices represent the solution components, following movement decisions which take into account both the concentration of pheromone trails and heuristic information on the problem being solved. While moving on this graph, ants lay down pheromone trails on the components or connection used and the released pheromone concentration guides the subsequent search movements of following artificial ants.

The ACO metaheuristic can therefore be applied to any combinatorial optimization problem on condition that a representation may be defined which maps the considered problem to an appropriate construction graph.

In general terms, a minimization (maximization) problem can be represented by the three-element set (S, J, Ω), where S is the set of candidate solutions, J is the objective (or cost) function to be minimized (maximized) and Ω is the set of constraints. This optimization problem may be mapped to a problem characterized by the following items:

- A finite set C of *components* c_i : $C = \{c_1, c_2, \ldots, c_m\}$.
- A set S of candidate solutions, which are defined by sequences $s = [c_i, c_j, \ldots, c_h, \ldots]$ built with elements of C.
- A set of feasible states $\underline{S}$, which is defined as the subset of S which satisfies the boundary conditions Ω.
- An objective (cost) function $J(s)$, associated with each candidate solution s.

The main activities of an ACO algorithm are illustrated in the flowchart of fig. 1. As mentioned before, artificial ants of ACO algorithms iteratively construct new solutions s by performing randomized walks on a construction graph $G = (C, L)$ in which the nodes are the components $c_i \in C$ while the connections between the nodes are defined by $l_{ij} \in L$. Components c_i and connections l_{ij} can be associated respectively with pheromone parameters τ_i and τ_{ij}, which incorporate information on the past search experience. Similarly, components c_i and connections l_{ij} can have associated heuristic values η_i and η_{ij}, which encode preliminary knowledge about the problem being solved. Both τ and η values are used by artificial agents to select the moves on the construction graph by applying decision schemes based on probability functions usually called *state transition rules* [Blum and Roli (2003)].

The update of pheromone trails can be performed either after an artificial ant has built a complete solution (*online delayed pheromone update*) or at the end of each construction step, with only a partial solution available (*online step-by-step pheromone update*). Moreover, pheromone evaporation can be simulated during the search process, in order to promote exploration of new regions of the solution space and to avoid premature convergence to local minima. Furthermore, optional *daemon actions* can be introduced to perform specific operations which cannot be carried out, or activated by, artificial agents. In fact, the main differences between the various algorithms developed within the ACO metaheuristic framework (ant colony system, ACS; rank-based ant system, AS_{rank}; *MAX-MIN* ant system, MMAS; global-best tour, T^{gb}, to cite a few examples [Glover and Kochenberger (2003), Dorigo and Stutzle (2004)]) concern the mechanisms and techniques specifically introduced to avoid search stagnation to strongly suboptimal paths.

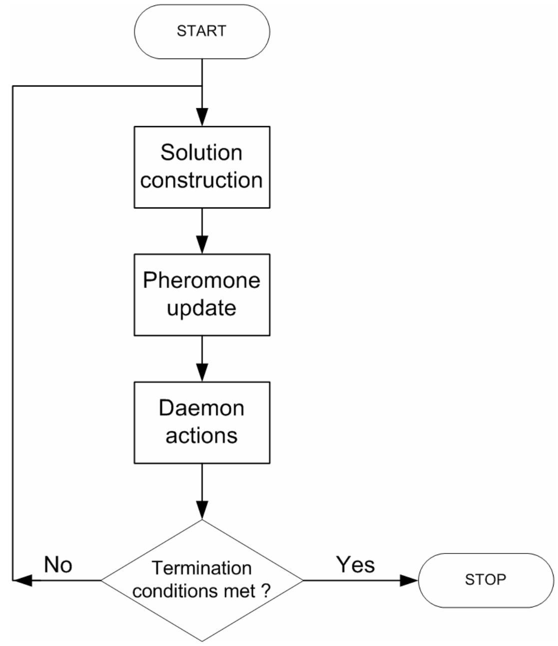

Figure 1 : Basic flowchart of ant colony optimization (ACO) algorithm

The satisfaction of constraints may be implemented in ACO procedures in several ways, such as, for example, by forcing artificial agents to build only feasible solutions (hard penalty), or by allowing them to construct infeasible solutions which will be penalized by artificially augmenting the associated cost function (soft penalty).

Finally, possible ACO termination conditions include limits on the maximum CPU time, the total number of iterations or objective function evaluations, the number or consecutive iterations without improvement in the objective function.

3 Laminate analysis

The classical lamination theory [Jones (1999)] was adopted to define the in-plane and out-of-plane elastic properties of thin laminated plates. The constitutive equations of an orthotropic unidirectional layer in the material (local) coordinate system $1-2$ (1 = fiber direction, 2 = transverse direction; fig. 2) can be expressed as follows:

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \tau_{12} \end{Bmatrix} \tag{1}$$

where the elements of the stiffness matrix $[Q]_{12}$ are related to the engineering properties by the following equations:

$$\begin{aligned} Q_{11} &= \frac{E_1}{1-\nu_{12}\nu_{21}} \\ Q_{22} &= \frac{E_2}{1-\nu_{12}\nu_{21}} \\ Q_{12} &= \frac{\nu_{12}E_2}{1-\nu_{12}\nu_{21}} \\ Q_{66} &= G_{12} \end{aligned} \tag{2}$$

After simple strain and stress transformations, the stress-strain relations in a global $x-y$ coordinate system (fig. 2) may be written as

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \begin{bmatrix} \overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{16} \\ \overline{Q}_{12} & \overline{Q}_{22} & \overline{Q}_{26} \\ \overline{Q}_{16} & \overline{Q}_{26} & \overline{Q}_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} \tag{3}$$

with

$$\overline{Q}_{11} = Q_{11}\cos^4\vartheta + Q_{22}\sin^4\vartheta + 2\,(Q_{12}+2Q_{66})\,\cos^2\vartheta\,\sin^2\vartheta$$

$$\overline{Q}_{22} = Q_{11}\sin^4\vartheta + Q_{22}\cos^4\vartheta + 2\,(Q_{12}+2Q_{66})\,\cos^2\vartheta\,\sin^2\vartheta$$

$$\overline{Q}_{12} = (Q_{11}+Q_{22}-4\,Q_{66})\cos^2\vartheta\sin^2\vartheta + Q_{12}\left(\cos^4\vartheta+\sin^4\vartheta\right)$$

$$\overline{Q}_{16} = (Q_{11}-Q_{12}-2\,Q_{66})\cos^3\vartheta\,\sin\vartheta - (Q_{22}-Q_{12}-2\,Q_{66})\cos\vartheta\,\sin^3\vartheta$$

$$\overline{Q}_{26} = (Q_{11}-Q_{12}-2\,Q_{66})\cos\vartheta\,\sin^3\vartheta - (Q_{22}-Q_{12}-2\,Q_{66})\cos^3\vartheta\,\sin\vartheta$$

$$\overline{Q}_{66} = (Q_{11}+Q_{22}-2\,Q_{12}-2\,Q_{66})\cos^2\vartheta\,\sin^2\vartheta + Q_{66}\left(\cos^4\vartheta+\sin^4\vartheta\right) \tag{4}$$

where ϑ is the angle between the material and global reference systems (fig. 2).

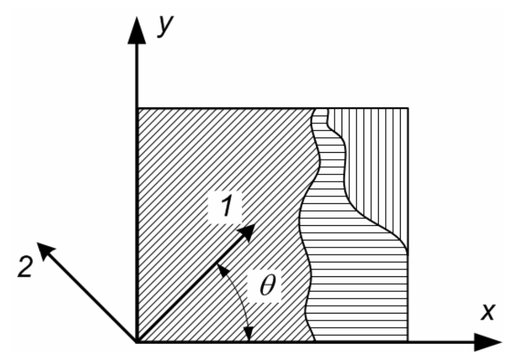

Figure 2 : Global $(x-y)$ and local $(1-2)$ coordinate systems of laminated plates

Forces $\{N\}$ and moments $\{M\}$ per unit length of the laminate cross section are associated with middle plane strains $\{\varepsilon^0\}$ and curvatures $\{\kappa\}$ in the global coordinate system by the following relations:

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} & B_{11} & B_{12} & B_{16} \\ A_{12} & A_{22} & A_{26} & B_{12} & B_{22} & B_{26} \\ A_{16} & A_{26} & A_{66} & B_{16} & B_{26} & B_{66} \\ B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & D_{16} \\ B_{12} & B_{22} & B_{26} & D_{12} & D_{22} & D_{26} \\ B_{16} & B_{26} & B_{66} & D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \\ \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix} \tag{5}$$

where the laminate extensional, coupling and flexural stiffnesses (A_{ij}, B_{ij} and $D_{ij}; i,j$=1, 2, 6) can be written as follows:

$$A_{ij} = \sum_{k=1}^{N} \left(\overline{Q}_{ij}\right)_k (z_k - z_{k-1})$$
$$B_{ij} = \frac{1}{2}\sum_{k=1}^{N} \left(\overline{Q}_{ij}\right)_k \left(z_k^2 - z_{k-1}^2\right)$$
$$D_{ij} = \frac{1}{3}\sum_{k=1}^{N} \left(\overline{Q}_{ij}\right)_k \left(z_k^3 - z_{k-1}^3\right) \tag{6}$$

where N is the total number of layers and z is the distance from the middle plane.

A major simplification of the laminate stiffness matrix is achieved if we restrict the attention to symmetric laminates (i.e. laminates having both geometric and material property symmetry about the middle surface) for which B_{ij} = 0 (i, j=1, 2, 6). Symmetric laminates are often required in design procedures to eliminate bending-extension coupling under mechanical loading and warping (upon release from the mold) after curing.

Another special class of laminates of practical interest is that of balanced laminates (i.e. laminates characterized by an equal number of plies in the +ϑ and -ϑ orientations), for which A_{12} = A_{16} =0, and therefore no shear/extension coupling exists.

The strain energies per unit area u of a symmetric laminated plate subject to in-plane (N_x, N_y, N_{xy}) or out-of-plane (M_x, M_y, M_{xy}) loads can thus be finally expressed respectively as

$$u = \frac{1}{2}\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \end{Bmatrix}^T \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix}^{-1} \begin{Bmatrix} N_x \\ N_y \\ N_{xy} \end{Bmatrix} \tag{7}$$

or

$$u = \frac{1}{2}\begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix}^T \begin{bmatrix} D_{11} & D_{12} & D_{16} \\ D_{12} & D_{22} & D_{26} \\ D_{16} & D_{26} & D_{66} \end{bmatrix}^{-1} \begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix} \tag{8}$$

4 Optimization problem formulation and numerical examples

The design objective of this study is the selection of the laminate configuration which minimizes the strain energy of the plate (i.e. which maximizes the average stiffness) when subjected to in-plane or out-of-plane loads. The layer orientation angles are taken as the design variables, and both the layer thickness and the total number of layers are assigned, as frequently required in practical design problems. The optimization problem can be therefore stated as follows:

- given a set of m possible orientation choices for each of the N layers,
- minimize the strain energy U of the laminated plate subject to assigned loads,
- while (optionally) satisfying additional constraints (such as, for example, the requirement of special laminate configurations or specification of desired stiffness properties along selected directions).

The application of the ACO metaheuristic to the solution of this design problem will be illustrated by means of numerical examples dealing with the case of rectangular laminated plates made of orthotropic layers. Since metaheuristic procedures can not guarantee the optimality of the solution found, the quality of the ACO solutions was evaluated, for each problem, by comparison with either the optimal solution of the equivalent problem with continuous design variables or, in selected cases, with the optimal solution of the discrete problem as obtained by enumerative analyses.

In all cases examined, T300/5208 graphite/epoxy layers of thickness t= 0.125 mm and with the following properties

E_1 = 181 GPa E_2 = 10.3 GPa
G_{12} = 7.17 GPa ν_{12} = 0.28

were selected for the calculations.

4.1 Numerical example I : in-plane loads

A single-element laminated panel subject to in-plane normal (N_x and N_y) and shear (N_{xy}) loads, as illustrated in fig. 3, was examined with the aim to determine the optimum or near-optimum ply distribution.

A numeric algorithm based on ACO metaheuristic was specifically developed on the basis of the following main assumptions:

1. the laminate is symmetric and composed of a total number of N (= $2\cdot n$) plies of constant thickness t;

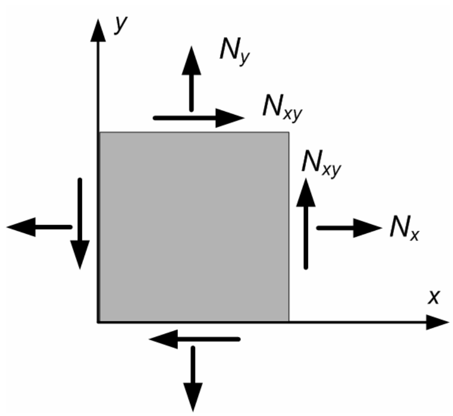

Figure 3 : Laminated plate subject to in-plane loads

2. the set S of candidate solutions is defined by laminates with stacking sequences of the form $[\vartheta_1/\vartheta_2/\ldots/\vartheta_n]_S$, where the orientation ϑ_ι of each layer may only assume values belonging to a set C of pre-assigned angle orientations $\{\alpha_1, \alpha_2, \ldots, \alpha_m\}$ (components).

To investigate the efficiency and robustness of the proposed optimization algorithm under different geometry and loading conditions, the optimization code was run for different numbers of layers and pre-assigned ply orientations, and for various combinations of applied loads.

In particular, the search of (near-)optimum stacking sequences was performed for laminates up to 32-layer thick and for sets of admissible orientation angles having from 4 up to 36 components. The ply orientations were always selected so as to realize, as frequently required in laminate manufacturing, an equally spaced set of angles ranging from 0 ° to 90 °; the generic set of orientation components can therefore be expressed in the form (0, $\pm l \cdot \pi/m$, 90; l= 1,..., m/2-1) . As an example, the choice of m = 12 corresponds to the following set of ply orientation components : {-75 °,-60 °, -45 °, -30 °, -15 °, 0 °, 15 °, 30 °, 45 °, 60 °, 75 °, 90 °}.

Various combinations of applied in-plane loads, representative of uniaxial, biaxial, pure shear and combined loading, were adopted for analyzing the performance of the search procedure. Typical results obtained for selected configurations are reported and discussed further on in this section.

4.1.1 *Implementation of ACO metaheuristic*

As briefly introduced in paragraph 2, the implementation of the ACO metaheuristic is based on the analysis of the information incorporated in the two vectors/matrices τ and η.

Since the in-plane stiffness matrix of laminates is affected only by the orientation of layers, and not by the through-thickness ply location, τ may be defined as an m-dimensional vector, whose generic terms τ_i represent the pheromone concentration associated to ply orientation ϑ_i. The developed ACO procedure starts with the initialization of elements τ_i to a constant value τ_0 for the first iteration of the search process. At the end of k^{th} iteration, after a new solution has been built, the algorithm calculates the performance index $\Delta\tau_k$ as a function of the quality of the current candidate solution by the expression:

$$\Delta\tau_k = \frac{1}{U_k} \tag{9}$$

where $U_k$is the strain energy of the laminate at the present iteration. Following an *online delayed update* procedure, the performance index, which represents the amount of pheromone to be released, is then added to each element of vector τ to modify the pheromone values accumulated up to the previous iteration.

On the other hand, vector $\boldsymbol{\eta}$ contains heuristic values directly related to the specific search problem. In order to promote the selection of the most promising (with respect to minimization of elastic energy) layer orientations, its generic terms η_i are set to the constant values

$$\eta_i = \frac{U^*_{\min}}{U_i} \tag{10}$$

where U_i is the strain energy stored in a laminate with all plies oriented in the direction ϑ_i and $U^*_{\min} = \min U_i \;\; (i = 1,\ldots,m)$.

Vectors $\boldsymbol{\tau}$ and $\boldsymbol{\eta}$ are then used to build the *ant decision table* $\boldsymbol{a}$, whose generic element, which represents the probability for the code of choosing ϑ_i as the next orientation angle, is defined [Dorigo and Stutzle (2004)] as:

$$a_i = \frac{\tau_i^\alpha \eta_i^\beta}{\sum\limits_{j=1}^{m} \tau_j^\alpha \eta_j^\beta} \tag{11}$$

where α and β are user-defined non-negative weight parameters. It is immediately seen that increasing the values of α enhances the importance of probabilistic information, while increasing the values of β tends to amplify the effect of heuristic parameters.

Escape mechanisms have been also introduced in order to avoid premature convergence to suboptimal paths and local minima and to promote the exploration of different regions of the solution domain during the search activities. Pheromone evaporation is simulated by applying the following reduction to the elements of vector τ at the end of every iteration:

$$\tau_i \leftarrow \tau_i\,(1-\rho) \qquad 0 \leq \rho < 1$$

In addition, following the *MAX-MIN* approach [Blum and Roli (2003), Dorigo and Stutzle (2004)], all τ_i values are constrained between limit values τ_{min} and τ_{max}, so as to limit the influence of too small or too high probability values on the search process, while the initial value τ_0 is set to τ_{max}. Moreover, a daemon action is introduced which totally resets the memory accumulated in vector τ every s iterations.

It has been clearly demonstrated [Dorigo, Maniezzo and Colorni (1996), Dorigo and Stutzle (2004)] that the choice of the values of the parameters controlling the algorithm behavior (α and β, in particular) may significantly affect both the rate of convergence of the search process and the quality of solution; however, the nature of the influence exerted by these parameters is usually problem-specific and no fully general rules are yet available for their selection.

For this reason, a preliminary exploratory phase was devoted to identify a set of algorithm parameters and stopping conditions able to ensure convergence to good solutions in reasonable time for a set of representative test-cases. This analysis, based on the results of the optimization process of a laminate subject to pure uniaxial, uniform biaxial and pure shear load configurations, showed in particular that the robustness and versatility of the algorithm could be greatly improved by dynamically modifying the value of the parameter β during the search process, so as to exploit the potential of different balances between acquired (population-based) experience and a priori heuristic information. An additional daemon action which, every s iterations, modifies the value of β in a circular way between the bottom and upper limits β_{min} and β_{max} was therefore introduced in the ACO search process.

A limit on the number of consecutive iterations without any reduction in the objective function was chosen as condition for algorithm termination. Since the complexity of the problem depends on the number of design variables, the limit value *NI* was expressed as a function of the number D of all acceptable solutions by the empirically determined equations:

$$\begin{aligned} &NI = 250 && \left(D \leq 2\cdot 10^4\right) \\ &NI = 3000\,\log(D) && \left(D > 2\cdot 10^4\right) \end{aligned} \tag{12}$$

In the case of in-plane stiffness optimization, the number of candidate solutions for unconstrained problems reduces to the number of combinations with repetitions, which is defined by the equation [Cameron (1994)]:

$$D = \frac{(m+n-1)\,!}{(n)\,!\,\,(m-1)\,!} \tag{13}$$

The following set of constants was selected after the exploratory tuning stage and adopted for all the analyses throughout the study:

$$\begin{aligned} &\alpha = 1 \\ &\beta_{\text{min}} = 0.1\,; \quad \beta_{\text{max}} = 30 \\ &s = NI/10 \\ &\rho = 0.05 \\ &\tau_{\text{min}} = 10\,\eta_{\text{max}} \\ &\tau_{\text{max}} = 100\,\eta_{\text{max}} \end{aligned} \tag{14}$$

It is worth noting that the proposed values do not, by any means, represent the optimal parameter setting in each situation; rather, they were observed to correspond to search processes characterized, for the specific problems investigated, by a reasonable trade-off between the two peculiar inspection strategies of ACO algorithms (exploration of unvisited regions and exploitation of past search experience), thus ensuring, in the set of selected test-cases, quick convergence to high-quality solutions.

4.1.2 Optimization results for in-plane loads

Tables 1a to 1d present the laminate lay-ups which minimize the strain energy as obtained by typical runs of the developed ACO algorithm for various values of N and

Table 1 : Optimum lay-ups of laminates subject to in-plane loads as obtained by typical ACO runs

(a)

Load case a) : N_x = 1, N_y = 0.5, N_{xy} = 0.5				
N	*m*	ACO optimal stacking sequence	Quality ratio	Number of iterations at stopping conditions
8	4	$[0/45_3]$s	1.066	322
8	12	$[15/30_2/60]$s	1.004	259
8	36	[10/30/35/55]s	1.002	16613
16	4	$[0_3/45_5]$s	1.065	431
16	12	$[15_3/30_2/45_2/60]$s	1.000	$1.951 \cdot 10^4$
16	36	$[-10/30_3/35/40_2/45]$s	1.000	$3.420 \cdot 10^4$
32	4	$[0_5/45_{11}]$s	1.054	431
32	12	$[15_3/30_9/45_3/90]$s	1.000	$2.940 \cdot 10^4$
32	36	$[10/25/-30/30_2/35_{10}/55]$s	1.000	$5.545 \cdot 10^4$
Optimal lay-up for continuous design variables → $(31.72_{0.930}/-58.28_{0.070})$				

(b)

Load case b) : N_x = 1, N_y = 0, N_{xy} = 0.5				
N	*m*	ACO optimal stacking sequence	Quality ratio	Number of iterations at stopping conditions
8	4	$[0_2/-45/45]$s	1.763	264
8	12	$[15/30_2/-75]$s	1.131	653
8	36	$[20_2/25/-65]$s	1.078	$1.584 \cdot 10^4$
16	4	$[0_3/45_3/-45/-90]$s	1.757	261
16	12	$[15_3/30_4/-75]$s	1.060	$1.945 \cdot 10^4$
16	36	$[20_5/25_2/-65]$s	1.009	$5.975 \cdot 10^4$
32	4	$[0_7/-45_2/45_6/90]$s	1.729	663
32	12	$[15_7/30_7/-60/-75]$s	1.057	$3.017 \cdot 10^4$
32	36	$[20_6/25_8/-65/-70]$s	1.005	$5.489 \cdot 10^4$
Optimal lay-up for continuous design variables → $(22.5_{0.884}/-67.5_{0.116})$				

(c)

Load case c) : N_x = 0, N_y = 0, N_{xy} = 1				
N	*m*	ACO optimal stacking sequence	Quality ratio	Number of iterations at stopping conditions
8	4	$[-45_2/45_2]$s	1	261
8	12	$[-45_2/45_2]$s	1	536
8	36	$[-45_2/45_2]$s	1	$1.629 \cdot 10^4$
16	4	$[-45_4/45_4]$s	1	411
16	12	$[-45_4/45_4]$s	1	$1.961 \cdot 10^4$
16	36	$[-45_4/45_4]$s	1	$4.700 \cdot 10^4$
32	4	$[-45_8/45_8]$s	1	479
32	12	$[-45_8/45_8]$s	1	$4.974 \cdot 10^4$
32	36	$[40/-50/+50/-45_7/45_6]$s	1.005	$9.195 \cdot 10^4$
Optimal lay-up for continuous design variables → $(-45_{0.5}/45_{0.5})$				

(d)

Load case d) : $N_x = 0$, $N_y = 1$, $N_{xy} = 1$				
N	*m*	ACO optimal stacking sequence	Quality ratio	Number of iterations at stopping conditions
8	4	$[0/45_3]$s	1.572	328
8	12	$[-30/60_3]$s	1.020	709
8	36	$[-30/55/60_2]$s	1.007	$1.734 \cdot 10^4$
16	4	$[0/-45/45_5/90]$s	1.569	404
16	12	$[-30_2/45/60_5]$	1.025	$1.436 \cdot 10^4$
16	36	$[-35_2/55/60_5]$s	1.007	$3.995 \cdot 10^4$
32	4	$[-45_5/45_7/90_4]$s	1.564	321
32	12	$[-30_4/45_4/60_8]$s	1.059	$3.103 \cdot 10^4$
32	36	$[-20/-30_2/-40/55_5/60_7]$s	1.019	$4.515 \cdot 10^5$
Optimal lay-up for continuous design variables → $(-31.72_{0.257}/58.28_{0.743})$				

m under four representative loading cases. The numbers of candidate solutions associated with the different laminate configurations analyzed are shown in Table 2. The analysis of data reported in these tables clearly indicate that, as expected, because of the peculiar nature of population-based methods, exhaustive enumerative approaches (whereby all design configurations are examined and tested) may result much more efficient than ACO search processes in problems characterized by a small number of possible solutions. On the other hand, the huge number of candidate solutions associated to the most complex cases analyzed made it impractical, for time reasons, to even determine, by enumerative analysis, the real global optimum which was needed to assess the performance of the ACO procedure. The quality of ACO solutions was therefore evaluated by a parameter (called quality ratio) defined as the ratio between the strain energy associated with the best stacking sequence selected by the ACO procedure (within a discrete-variable search domain) and the minimum strain energy of the equivalent continuous (i.e. with real-valued design variables) problem. In the latter case, explicit closed form relations are available [Fukunaga and Sekine (1993)] to identify the global optimal stacking sequences, which are reported in the tables in terms of layer orientations and ply thickness ratios. The strain energy associated to the optimum stacking sequence obtained as a solution of a continuous minimization problem obviously represents a lower bound to the solution of the equivalent discrete-valued variables optimization problem.

Traces of typical runs of the ACO algorithm for the optimization of a 16-layer laminate with 36 acceptable layer orientations (*N*=16; *m*=36) under different combinations of applied loads are shown in fig. 4.

Table 2 : Numbers of candidate solutions associated with different laminate configurations subject to in-plane loads

N	*m*	Number of candidate solutions
8	4	35
8	12	1365
8	36	$8.225 \cdot 10^4$
16	4	165
16	12	$7.558 \cdot 10^4$
16	36	$1.450 \cdot 10^8$
32	4	969
32	12	$1.304 \cdot 10^7$
32	36	$7.175 \cdot 10^{12}$

The analysis of results such as those illustrated in tables 1a-d and fig. 4 indicates that the proposed ACO algorithm is effective in identifying optimum or near-optimum stacking sequences of laminates under the various in-plane load combinations investigated. In the pure shear loading case (table 1c), for instance, we notice that the optimal (+45°/-45°) angle-ply configuration is exactly captured or closely estimated even for laminates defined by high values of both *N* and *m*, characterized by extremely large numbers of candidate solutions. Very high-quality solutions are easily identified under combined loadings as well, as seen from the results of tables 1a,b,d. As an example, the optimal stacking sequence of

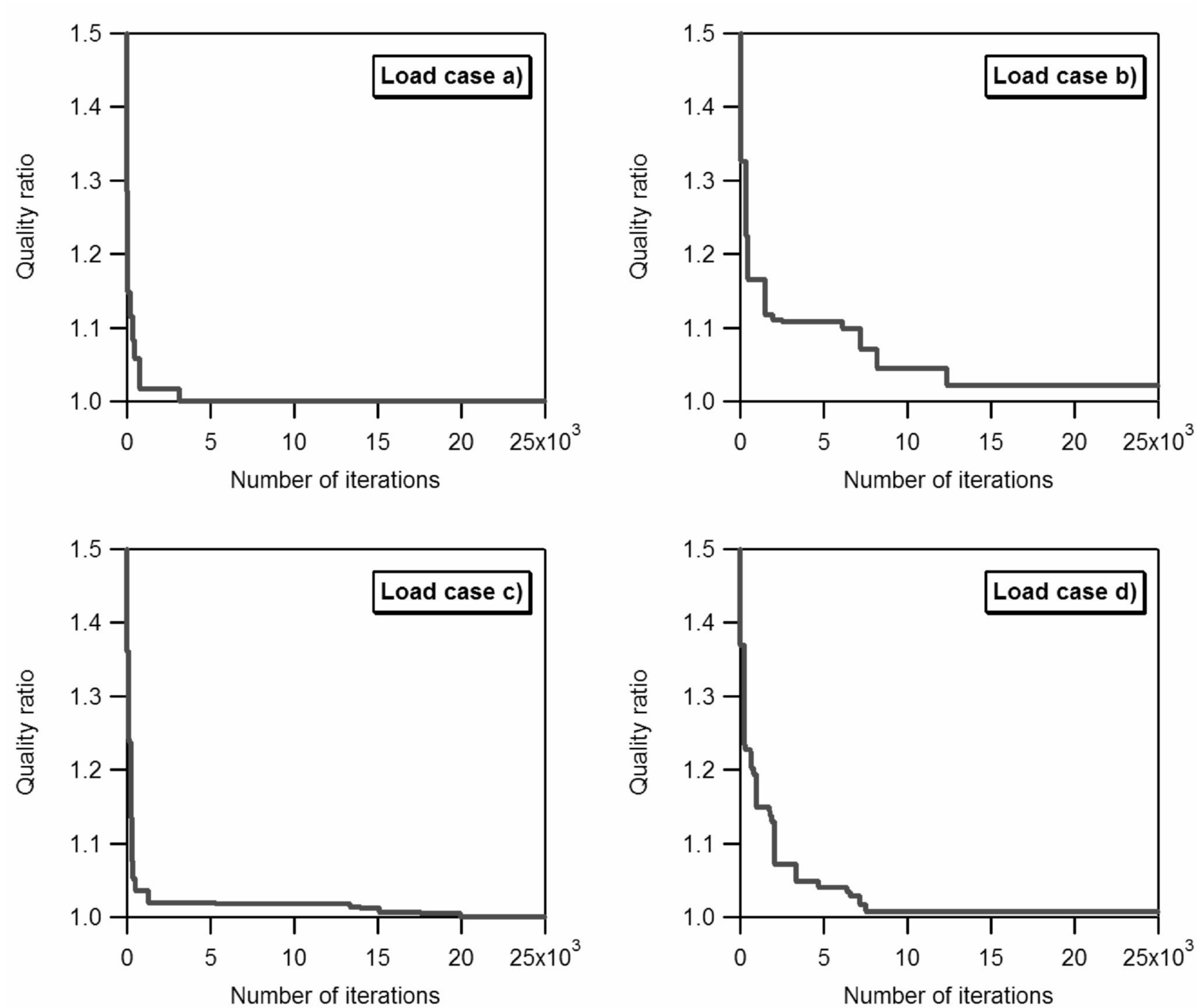

Figure 4 : Typical runs of ACO algorithm for *N*=16/*m*=36 laminates under different in-plane load cases

the continuous minimization problem associated to the load case b) ([$22.5_{0.8843}$/$-67.5_{0.1157}$]) is very close, for *N*=32 and *m*=36, to the sequence identified through the ACO procedure ([20_6/25_8/-65/-70]s), which may store an elastic strain energy only 0.5% higher than the lower bound corresponding to the minimum of the continuous problem.

The results of tables 1a-d also show the effect of the number of layers *N* and of the number of acceptable ply orientations *m* upon the optimum solution. As expected, increasing the values of *N* and *m* results in a decrease of the optimal strain energy, which tends to converge towards the solution of the continuous optimization problem; we notice, in particular, that reducing the number of acceptable ply orientations can significantly increase the optimum value of elastic energy potentially achievable with the imposed angle requirements (as an example, the results reported in table 1d indicate that reducing the number of acceptable orientations from 12 to 4, i.e. decreasing the angular span between orientation components from 15° to 45°, induces an approximate 50% increase in the optimal strain energy absorbed by the laminate.

Since ACO algorithms are partially random in nature, the performance of the proposed optimization procedure under the various loading conditions was assessed by evaluating the *practical reliability* [Rama Mohan Rao and Arvind (2005); Leriche and Haftka (1993)], which is defined as the percentage of runs that, at a specified iteration, reach a solution within a preset distance from the optimal solution. In this study, the practical reliability was calculated by launching 100 ACO runs and deter-

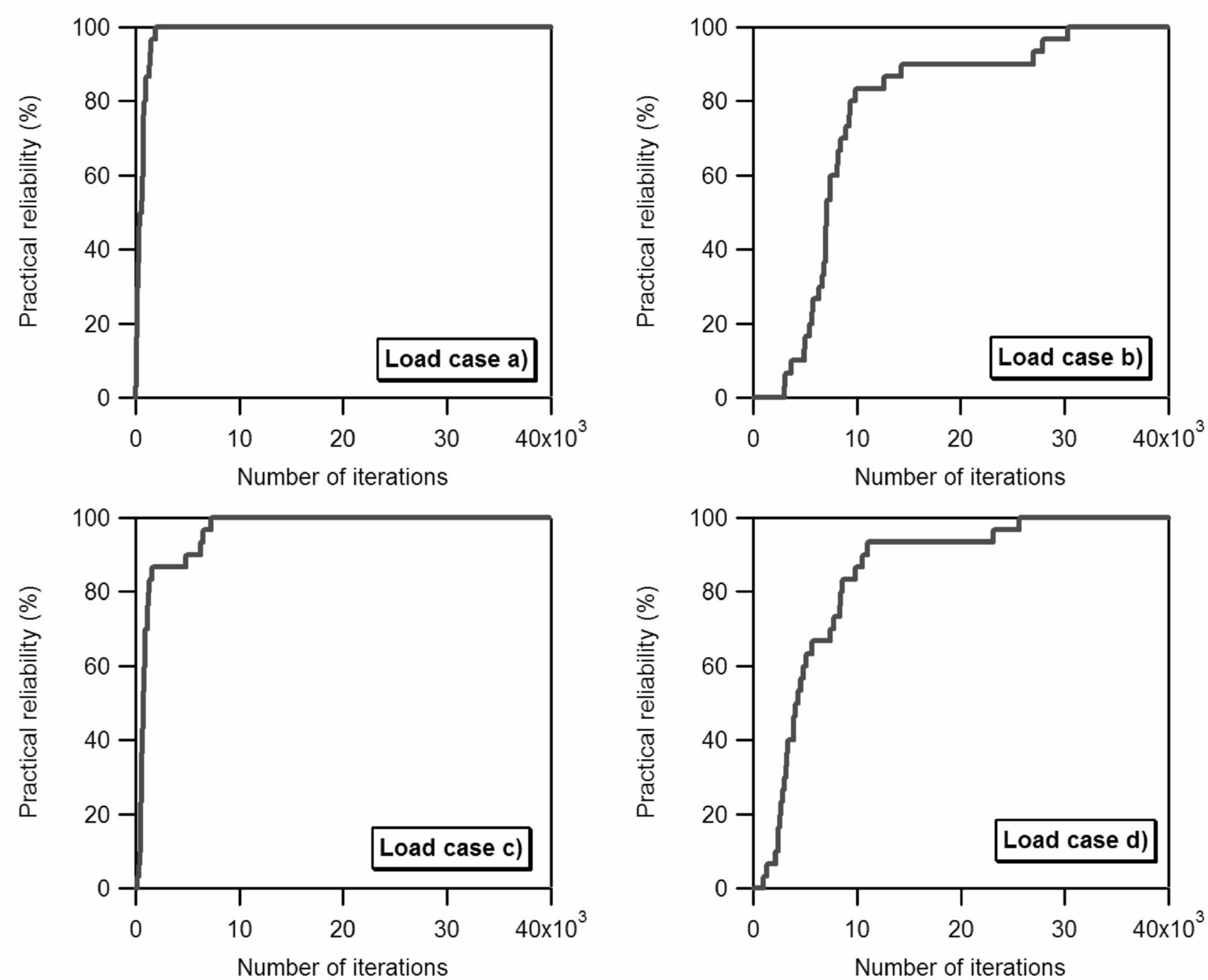

Figure 5 : Practical reliability of ACO algorithm for N=16/m=36 laminates under different in-plane load cases

mining, at each iteration, the number of solutions characterized by an associated strain energy within 1% of the global optimal strain energy of the equivalent continuous problem. Fig. 5 shows the reliability versus number of iterations in N=16/m=36 laminates for the test-cases previously analyzed in tables 1a-d. These plots show that the proposed algorithm produces both reasonably reliable solutions after very few iterations and very high-quality solutions in a small number of iterations as compared to the total number D of feasible solutions (for example, D = $1.450 \cdot 10^8$ for N=16/m=36 laminates). On the other hand, the analysis of these plots and of those reported in fig. 4 indicates a significant influence of the relative values of load components on the convergence rates of the search procedure. This behavior is probably related to the use of the simple law expressed by equation (10) for defining the heuristic terms η_i as a function of orientation angle ϑ_i, which obviously does not take into account the mutual interaction of layers when stacked together into a laminate. When applied during ACO runs under unfavorable load conditions, the heuristic information built up through this rule may lead to search processes strongly directed toward regions of the solution space which do not contain the global optimum of the objective function; in these cases, multiple ACO iterations are required to accumulate, through the interaction of artificial agents, the level of knowledge required to identify and systematically explore the most promising search spaces. It should be noted, however, that high-quality solutions were attained with reasonable convergence rates also under the less favorable, most computationally expensive, loading conditions.

It is easily seen that the requirement of a balanced stacking sequence, necessary to avoid extensional/shear stiff-

Table 3 : Optimum lay-ups of balanced laminates subject to in-plane load (case b; N_x = 1, N_y = 0, N_{xy} = 0.5) as obtained by typical constrained ACO runs.

Constraint set 1) : 0.45 < E_y/E_x < 0.55						
N	m	Optimal lay-up as obtained by enumerative analysis	ACO optimal lay-up	Quality ratio	Number of iterations at stopping conditions	Number of candidate solutions for balanced lay-ups (unconstrained problem)
24	36	$[\pm10/\pm20/\pm25_2/\pm60_2]$s	$[\pm10/\pm20/\pm25_2/\pm60_2]$s	1	$2.439\cdot10^4$	$4.496\cdot10^6$
Constraint set 2) : 0.45 < E_y/E_x < 0.55 and 0.4 < G_{xy}/E_x < 0.5						
N	m	Optimal lay-up as obtained by enumerative analysis	ACO optimal lay-up	Quality ratio	Number of iterations at stopping conditions	Number of candidate solutions for balanced lay-ups (unconstrained problem)
24	36	$[\pm20/\pm25_3/\pm55/\pm60]$s	$[\pm20/\pm25_3/\pm55/\pm60]$s	1	$2.254\cdot10^4$	$4.496\cdot10^6$

ness coupling effects, may be directly enforced in the ACO algorithm by encoding pairs of +ϑ/-ϑ angle layers (ϑ $\neq$ 0, 90) as a single design variable. The satisfaction of more general constraints, which can be expressed by disequalites of the form $g_k(s) \leq 0$, was implemented in the ACO procedure by a penalty method, i.e. by transforming the constrained minimization of objective function $J(s)$ into an unconstrained minimization of the function

$$\overline{J}(s) = J(s) + r\sum_k \max\left(g_k(s), 0\right)$$

where r is a penalty parameter set to 100 in the calculations.

Various constrained optimizations were successfully carried out by this approach and very good solutions were achieved even with stringent or multiple constraints. The results reported in Table 3 show for example that the ACO algorithm correctly identified the global optimum stacking sequences of balanced laminates subject to the following constraints on the axial and tangential stiffnesses:

$$0.45 \leq \frac{E_y}{E_x} \leq 0.55 \text{ constraint case 1)}$$

$$\left.\begin{array}{l} 0.45 \leq \frac{E_y}{E_x} \leq 0.55 \\ 0.4 \leq \frac{G_{xy}}{E_x} \leq 0.5 \end{array}\right\} \text{constraint case 2)}$$

4.2 Numerical example II : out-of-plane loads

As a second test-case, the optimization problem of a simply supported laminated panel subject to out-of-plane loads was considered (fig. 6).

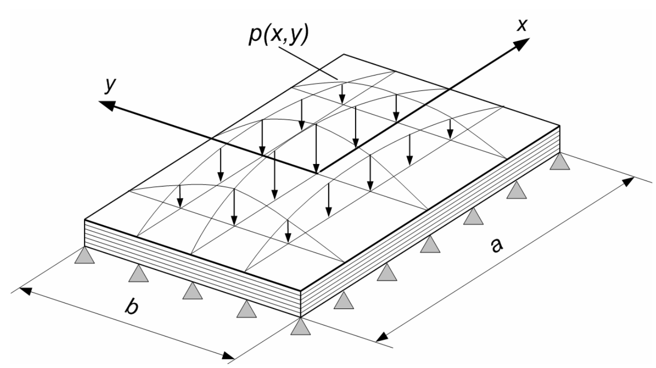

Figure 6 : Laminated plate subject to out-of-plane loads

The system under study consists of a rectangular laminated panel loaded by a normal pressure distribution $p(x,y)$ symmetrical with respect to the geometry axes x,y of the plate. An equivalent system was studied by Avalle and Belingardi (1995), who developed closed-form solutions to the problem of maximizing the average global stiffness of the plate in a continuous solution space where ply thicknesses and ply orientations are assumed as real-valued design variables. In this case, it may be proved that the maximum stiffness of the panel is achieved with

Table 4 : Optimum lay-ups of laminates subject to out-of-plane loads as obtained by typical ACO runs

(a)

Plate aspect ratio *b/a* = 0.75				
N	*m*	ACO optimal stacking sequence	Quality ratio	Number of iterations at stopping conditions
8	4	$[-45/45_3]s$	1.050	326
8	12	$[60/-60_2/-45]s$	1.010	681
8	36	$[-55/55_2/65]s$	1.004	$2.397 \cdot 10^4$
16	4	$[45/-45_2/45/-45/45_2/-45]s$	1.050	$1.856 \cdot 10^4$
16	12	$[60/-60/60/-60_3/75/60]s$	1.008	$4.473 \cdot 10^4$
16	36	$[55/-55/-60/55/-60/-55_2/55]s$	1.003	$6.092 \cdot 10^4$
32	4	$[45/-45/45/-45/45_2/-45_5/45_2/-45_2/90]s$	1.047	$4.307 \cdot 10^4$
32	12	$[60/-60_3/60_2/-60/60/75/-60/60_2/-60_2/-75_2]s$	1.011	$1.364 \cdot 10^4$
32	36	$[60/-55_2/65/-60/55/-65/60/50/-80/-60/-85/-45/-65/55/-75]s$	1.014	$1.013 \cdot 10^4$
Optimal orientation angles for continuous design variables → (-55.58°/55.58°)				

(b)

Plate aspect ratio *b/a* = 1				
N	*m*	ACO optimal stacking sequence	Quality ratio	Number of iterations at stopping conditions
8	4	$[-45/45_3]s$	1.004	274
8	12	$[-45/45_3]s$	1.004	456
8	36	$[-45/45_3]s$	1.004	$2.86 \cdot 10^4$
16	4	$[45/-45_2/45/-45/45_2/-45]s$	1.000	$2.142 \cdot 10^4$
16	12	$[-45/45_2/-45/45/-45_2/45]s$	1.000	$4.506 \cdot 10^4$
16	36	$[45/-45_2/45/-45/45_2/-45]s$	1.000	$7.438 \cdot 10^4$
32	4	$[45/-45/45/-45_2/45/-45/45/-45/45/-45/45_2/-45/45/-45]s$	1.000	$4.755 \cdot 10^4$
32	12	$[-45/45_2/-45_2/45/-45/45_3/-45_3/45_2/-45]s$	1.000	$9.463 \cdot 10^4$
32	36	$[45_3/-45_7/45_2/-50/-40/-50/-45]s$	1.000	$1.446 \cdot 10^5$
Optimal orientation angles for continuous design variables → (-45°/45°)				

(c)

Plate aspect ratio *b/a* = 1.5				
N	*m*	ACO optimal stacking sequence	Quality ratio	Number of iterations at stopping conditions
8	4	$[0_4]s$	1.048	306
8	12	$[-30/30_3]s$	1.006	646
8	36	$[-25/ 30/25_2]s$	1.003	$2.163 \cdot 10^4$
16	4	$[25/-25_2/-20_2/-30/35/30]s$	1.003	$1.727 \cdot 10^4$
16	12	$[30/-30_2/30/-30/15/-30/0]s$	1.004	$3.694 \cdot 10^4$
16	36	$[-30/25/30/-25/-35/25/-25/10]s$	1.004	$8.957 \cdot 10^4$
32	4	$[-30/25/-20/25/-20/-25/30/25/30/5/15_2/-5/-20/-5/5]s$	1.005	$4.131 \cdot 10^4$
32	12	$[-30_2/30_3/-30/30_2/15/-15/30/15/0/15/0/-30]s$	1.006	$8.102 \cdot 10^4$
32	36	$[-30/-20/ 25/-30/20/25_2/-30/-10/20/-15/30/20/25/20/35]s$	1.007	$1.772 \cdot 10^5$
Optimal orientation angles for continuous design variables → (-26.78°/26.78°)				

(d)

Plate aspect ratio *b/a* = 2				
N	*m*	ACO optimal stacking sequence	Quality ratio	Number of iterations at stopping conditions
8	4	$[0_4]$s	1	378
8	12	$[0_4]$s	1	433
8	36	$[0_4]$s	1	$2.815\cdot10^4$
16	4	$[0_8]$s	1	$1.495\cdot10^4$
16	12	$[0_8]$s	1	$4.241\cdot10^4$
16	36	$[0_8]$s	1	$7.159\cdot10^4$
32	4	$[0_{16}]$s	1	$4.342\cdot10^4$
32	12	$[0_{16}]$s	1	$9.346\cdot10^4$
32	36	$[0_9/\text{-}5/0_3/5/0_2]$s	1.008	$2.187\cdot10^5$
Optimal orientation angle for continuous design variables → (0 ˚)				

Table 5 : Optimum lay-ups of laminates subjected to out-of-plane loads as obtained by typical constrained ACO runs

Constraint set 1) : 0.95 < D_{11}/D_{22}< 1.05						
N	*m*	Optimal lay-up as obtained by enumerative analysis	ACO optimal stacking sequence	Quality ratio	Number of iterations at stopping conditions	Number of candidate solutions (unconstrained problem)
14	12	$[\text{-}45/45_2/\text{-}45/45/\text{-}30/30]$s	$[\text{-}45/45_4/\text{-}30/75]$s	1.009	$3.177\cdot10^4$	$3.583\cdot10^7$
Constraint set 2) : 0.95 < D_{11}/D_{22}< 1.05 <u>and</u> D_{66}/D_{11} < 0.10						
N	*m*	Optimal lay-up as obtained by enumerative analysis	ACO optimal stacking sequence	Quality ratio	Number of iterations at stopping conditions	Number of candidate solutions (unconstrained problem)
14	12	$[0/90_2/0/\text{-}60/30_2]$s	$[0/90_2/0/\text{-}60/\text{-}15/30]$s	1.001	$4.621\cdot10^4$	$3.583\cdot10^7$

($\pm\beta$) angle-ply laminates (where β is a function of the plate aspect ratio) with layer thicknesses arranged so as to assure the global orthotropy ($D_{16} = D_{26} = 0$) of the laminate.

Following the approach described in Avalle and Belingardi (1995), a good approximation of the plate deflection w may be written as

$$w(x,y) = w_1 \cos\frac{\pi x}{a}\cos\frac{\pi y}{b} + w_2 \sin\frac{2\pi x}{a}\sin\frac{2\pi y}{b} \quad (15)$$

and the application of the Ritz-Rayleigh method leads to the following expression for the elastic energy of the laminate

$$U = \frac{2P^2}{\pi^2 a b}\frac{1}{D - 3.245\frac{D_6^2}{D}} \quad (16)$$

where

$$D = \frac{D_{11}}{a^4} + \frac{2(D_{12}+2D_{66})}{a^2b^2} + \frac{D_{22}}{b^4}$$
$$D_6 = \frac{D_{16}}{a^3 b} + \frac{D_{26}}{ab^3} \quad (17)$$

and

$$P = \int_{-a/2}^{a/2}\int_{-b/2}^{b/2} p(x,y)\,dx\,dy \quad (18)$$

Since both orientation and through-thickness location of layers affect the laminate flexural stiffness, pheromone and heuristic data required by the ACO algorithm are now represented through $m \times n$ τ and η matrices which map the information about orientation and distance from mid-surface of each layer. Columns of matrices τ and

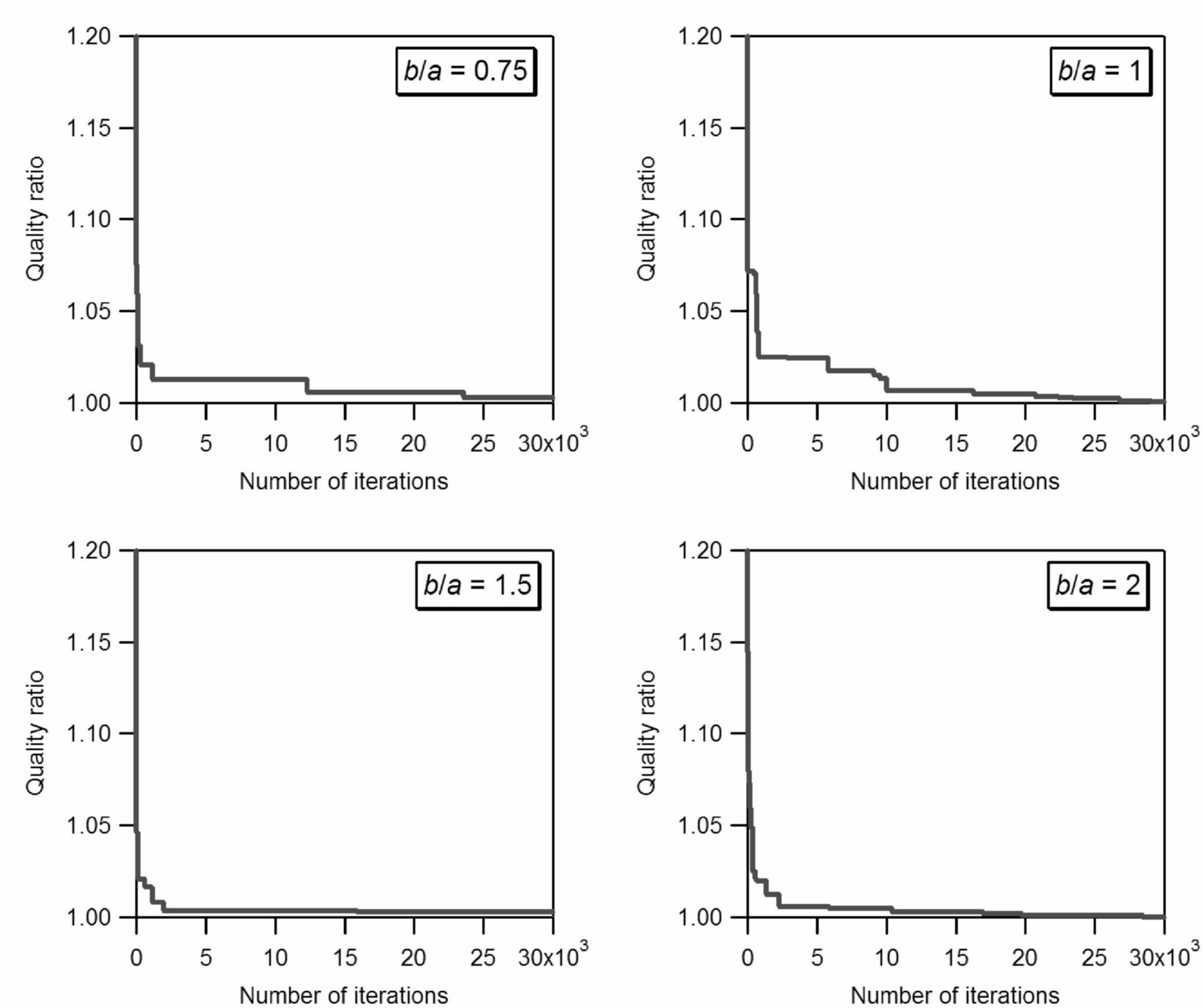

Figure 7 : Typical runs of ACO algorithm for N=16/m=36 laminated plates with different aspect ratios and subject to out-of-plane loads.

η contain pheromone and heuristic information related to the corresponding individual plies of the laminate, and their elements are defined with the same rules adopted for the optimization of laminates under in-plane loadings.

Tables 4a-d and 5 show the optimal stacking sequences obtained by application of the ACO algorithms to both unconstrained and constrained problems for laminates with various N and m values and different aspect ratios b/a. Preset limit values for the relative magnitudes of flexural (D_{11} and D_{22}) and torsional (D_{66}) stiffnesses of the laminate were imposed as constraints as reported in Table 5. Similarly to what was done in the previous section, the quality ratio is defined as the ratio between the ACO solution and the closed-form optimal solution for continuous design variables. The sizes of the solution spaces for the unconstrained optimization problems (which, in the case of out-of-plane loads, correspond with the numbers of permutations with repetitions of m ply orientations arranged in n locations) are reported in Table 6, while traces of typical unconstrained ACO runs for the optimization of a 16-layer laminate with 36 possible layer orientations (N=16, m=36) are shown in fig. 7.

These results show that the proposed ACO algorithm is able to produce high quality solutions to the problem of stacking sequence optimization also in the case of laminates subject to out-of-plane loads, thereby confirming the robustness and versatility of the ant colony optimization metaheuristic. It is worth noting, in addition, that the histories reported in the plots of fig. 7 clearly reveals

Table 6 : Numbers of candidate solutions associated with different laminate configurations subject to out-of-plane loads.

N	*m*	Number of candidate solutions
8	4	256
8	12	$2.074 \cdot 10^4$
8	36	$1.680 \cdot 10^6$
16	4	$6.553 \cdot 10^4$
16	12	$4.300 \cdot 10^8$
16	36	$2.821 \cdot 10^{12}$
32	4	$4.295 \cdot 10^9$
32	12	$1.849 \cdot 10^{17}$
32	36	$7.958 \cdot 10^{24}$

that quite good solutions are again achieved after only very few iterations, thus indicating the effectiveness of the method in finding design solutions of practical interest in reasonably short times.

5 Conclusions

An Ant Colony Optimization algorithm was developed for stacking sequence optimization (with respect to the average stiffness) of laminated plates subject to in-plane and out-of-plane loads. General implementation techniques and details on specific daemon procedures expressly devised for increasing quality and robustness of the ACO search strategy are presented and discussed in the paper. Numerical analyses were conducted to explore the efficiency and reliability of the metaheuristic procedure under various load cases, geometry configurations and constraint conditions. Even though fine tuning of algorithm parameters was observed to significantly improve the performance of the procedure, all numerical experiments were conducted with the same set of parameter values in order to evaluate the robustness and versatility of the developed search algorithm.

The quality of ACO solutions was assessed by comparison with closed-forms solutions available in the literature for optimization of equivalent problems based on continuous real-valued design variables. The results of the analyses indicate that the proposed ACO metaheuristic procedure is able to produce high-quality solutions for both uncostrained and constrained optimization problems in a limited number of iterations, as compared to the number of all possible design solutions. On the other hand, reasonably good solutions of practical interest are usually obtained, in extremely short computation times within the first ACO runs, thereby suggesting the use of the method as a valuable design tool for laminate lay-up selection in a wide range of application problems.

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