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Interaction of Two Parallel Short Fibers in the Matrix at Loss of Stability

A.N.Guz and V.A.Dekret[1]

Abstract: Stability problem of composite material reinforced by two parallel short fibers is solved. The problem is formulated with application of equations of linearized three-dimensional theory of stability. The composite is modeled as piecewise-homogeneous medium. The influence of geometrical and mechanical parameters of the material on critical strain is investigated.

keyword: Stability problem, Composite material, Nanomaterial, Nanocomposite, Short fibers, Three-dimensional linearized theory of stability, Non-uniform subcritical state, Finite difference method.

1 Introduction

Loss of stability in structure of fibrous and layered materials is the basic destruction mechanism of composite materials at compression. It is quite obvious, that the adequate description of such complex phenomenon as loss of stability in structure of composite materials cannot be reliably enough realized within the framework of two-dimensional applied theory of stability of thin-walled elements (cores, plates and membranes). It is necessary to apply the three-dimensional theory of stability of deformable bodies to describe these phenomena.

Now wide classes of problems about loss of stability in structure of layered and fibrous materials with polymeric and metal matrixes are investigated with application of three-dimensional linearized theory of stability of deformable bodies (see, for example, [Guz (1971), (1990), (1999) , (2004)]). The results for fibrous and layered composite materials are discussed in monographies [Guz (1971), (1973), (1990)]. The modern analysis of the results is discussed in a reviews [Babich, Guz , and Chekhov (2001)] and [Guz A.N., and Guz I.A. (2004)]. All results are obtained for unidirectional fibrous and layered composites which are infinite in a direction of the compression.

For a composite materials reinforced by short fibers (fibers of the final sizes in a longitudinal direction) the concrete results is submitted in [Guz, Dekret, Kokhanenko (2001), (2003), (2004)]. In [Guz, Dekret, Kokhanenko (2001)] the investigation of stability of composite material weakly reinforced by short fibers is presented and interaction between fibers is not taken into account. In [Guz, Dekret, Kokhanenko (2003), (2004)] interaction between two serially placement fibers in a longitudinal direction (in direction of the compression) is considered.

In this research of a stability problem of composite material reinforced by two parallel short fibers is carried out. It can be considered as continuation of previous researches for a case, when it is possible to allocate nearby two parallel fibers (along a direction of the compression), for which (on the basis of their close accommodation) it is necessary to take into account interaction of two fibers in a plane of cross-section at loss of stability.

Significant interest represents a question on applicability of the obtained results to research of nanomaterials and nanocomposites.

It should be emphasized that the term "nanocomposite", when used in nanomechanics, may imply one of two different types of nanomaterials. The first type is a material consisting of a matrix filled with nanopartcles. Such a structure fully complies with the terminology adopted in the micromechanics of polymer- and metal-based composites. Such composites can be studied with approaches and methods of the micromechanics of composites when conditions for continuum solid mechanics, formulated in [Guz and Rushchitskii (2003)], are satisfied. The second type of nanoformation has a rather complex internal structure where each particle can be considered as a nanocomposite with its own internal structure. In this context the term "nanocomposite" cannot be applied in the micromechanics of polymer- and metal-based composites, because here each element of the filler is always modeled within the framework of solid mechanics (as a division of continuum mechanics).

[1] The Institute of Mechanics National Academy of Sciences of Ukraine, Nesterov str., 3, 03680, Kiev, Ukraine. Tel.: (38044) 4569351, Fax: (38044) 4560319, E-mail: guz@carrier.kiev.ua, dekret@inmech.kiev.ua

Thus this investigation can be applied for mechanics of the first type nanocomposites in accordance with theory of compressive failure of nanocomposites [Guz A.N., Roger, Guz I.A. (2005)].

The basic attention, in present article, is given to the analysis of the obtained laws the interference of two short fibers in a matrix at loss of stability depending on distance between these fibers. The stability problem was formulated as plane problem by using a model of a piecewise-homogeneous medium and the linearized three-dimensional theory of stability. The matrix and fibers was considered as linear-elastic compressed body.

2 Statement and technique of the solution of problems

We shall consider a composite material reinforced by fibers with the final sizes in a longitudinal direction. At that small concentration of the fibers owing to irregular structure there are "pairs" of close placed parallel fibers which cooperate with each other at loss of stability. Within the framework of model of plane deformation we shall investigate of the internal loss of stability of the composite which is affected by the influence of boundary surfaces, and are completely determined only by the material properties.

In this connection, in the Cartesian coordinates x_1Ox_2, the composite material is modeled by the infinite matrix, filled with two identical parallel cylindrical fibers directed along the Ox_1 axis. In a direction Ox_1 the composite in compression of constant intensity P (Fig.1).

Let's consider forms of stability loss of fibres in structure of a material. From physical point of view it is assumed that the considered mechanism of stability loss of fibres is feasible by the following schemes (Fig.1). First two schemes (Fig.1a,b) represent forms of stability loss when inclusions lose stability on the buckling form. Thus the first scheme (Fig.1a) corresponds to stability loss on "the buckling form in one phase", and the second (Fig.1b) – to stability on "the buckling form in an antiphase". The third scheme (Fig.1c) shows assumed variants of "almost rigid turn" when bonding material near the contact does not exert an adequate restraining effect and, in stability loss a joint ("nearly plastic joint") formed at the contact.

Under a given mode of loading, the material in question will apparently buckle by one scheme or another, depending on the geometrical and the mechanical characteristics of the composite components. The foregoing assuming is applied only to the analysis of physical character phenomenon. Below in present article the values of critical parameters and corresponding forms of stability loss are determined as a result of the numerical solution of the strict three-dimensional equations of linearized theory of stability of deformable bodies.

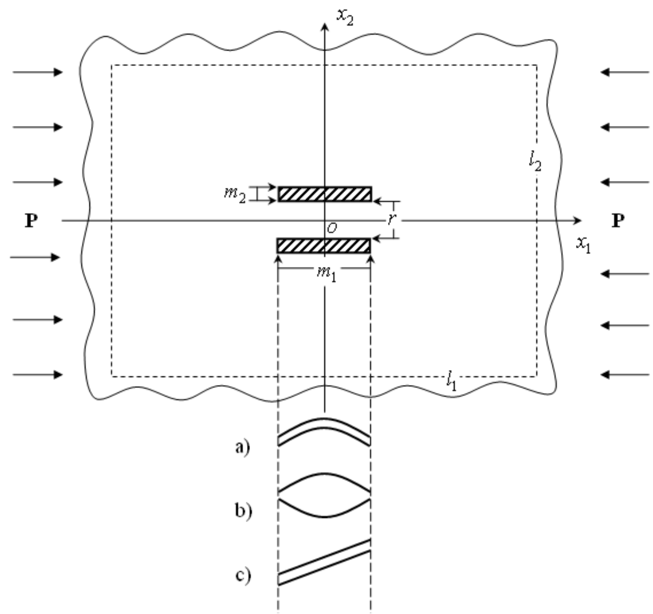

Figure 1 : Plane model of composite

Investigation of problems within the framework of three-dimensional linearized theory of stability of deformable bodies becomes more complicated because within the composite, under a given mode of loading, we have non-uniform subcritical state. It is quite obvious, that in considered problems any results can be obtained only by means of numerical methods.

Subcritical state is defined from the equations of the linear theory of elasticity. In the Cartesian system of coordinates $x_1Ox_2 \equiv xOy$ we shall formulate a problem of determination subcritical state which components we shall mark with 0 upper index. We will derive the equations in a dimensionless form.

To this end, the linear dimensions are referred to the fiber length, and the surface load P and the stresses σ_{ij}^0 to the quantity $(1-\nu)/2G$ (G, ν- shear modulus and Poisson's ratio). At the considered way of transition to dimensionless variables, the dimensionless external loading $p = P(1-\nu)/2G$ is proportional to value of longitudinal deformation which can be accepted as loading parameter. The perturbations caused by presence of fibers, are distributed to final distance, therefore components of a subcritical can be present as the sum of unperturbed (with an index$u$) and perturbed (with an index$p$) states

$$\sigma_{ij}^{0} = \sigma_{ij}^{0\,u} + \sigma_{ij}^{0\,p},\quad \varepsilon_{ij}^{0} = \varepsilon_{ij}^{0\,u} + \varepsilon_{ij}^{0\,p},\quad u_i^0 = u_i^{0u} + u_i^{0p}. \tag{1}$$

Thus, components of the unperturbed strain-stress states, for a considered case, are defined from the relationships

$$\sigma_{11}^{0\,u} = p,\quad \sigma_{22}^{0\,u} = 0,\quad \sigma_{12}^{0\,u} = 0;\quad \varepsilon_{11}^{0\,u} = p,\quad \varepsilon_{22}^{0\,u} = -\frac{\nu}{1-\nu}p,\quad \varepsilon_{12}^{0\,u} = 0. \tag{2}$$

For definition of the basic strain-stress states it is necessary to find the displacements $u_i^0(x)$satisfying within the composite components to the equilibrium equations

$$\frac{\partial \sigma_{ij}^0}{\partial x_i} = 0, \tag{3}$$

the attenuation conditions of perturbations

$$\sigma_{ij}^{0\,p} \to 0,\ u_i^{0p} \to 0,\ x_i \to \pm\infty \tag{4}$$

and conditions of adhesion on a contact of the composite components

$$[n_i\sigma_{ij}^0] = 0,\ \ [u_i^0] = 0. \tag{5}$$

Here: $[f(x)]$ =f(x-0)-f(x+0) - jump of function$f(x)$; n_i - is a component of the external-normal vector. Within the composite component, Hooke's law has the form

$$\sigma_{ij}^0 = \frac{1-\nu}{1-2\nu}\left((1-\nu)\varepsilon_{ii}^0 + \nu\varepsilon_{jj}^0\right),\quad \sigma_{12}^0 = (1-\nu)\varepsilon_{12}^0,\ j = 3-i \tag{6}$$

In relationships (1) - (6) and further indexes change from 1 up to 2, and the standard agreement on summation is adopted.

Let's consider the problem of stability loss of a composite material in its structure. As mathematical model we use the equations of three-dimensional linearized theory of stability for small initial deformations when the initial state is defined from the equations of the linear theory of elasticity (the second variant of the theory of small initial deformations) [Guz (1999)]. The equations of stability we are written in the dimensionless form. We note, that in a subcritical state, according to (2), dimensionless external loading is proportional to value of longitudinal deformation p, which we accept as loading parameter. With the use of the concept of simple loading, we reduce a problem of stability to a two-dimensional spectral problem. For this purpose we shall allocate loading parameter by means of replacement $\sigma_{ij}^0 = p\sigma'^0_{ij}$. For the solution of the stability problem it is necessary to determine eigensolution of the generalized problem on eigenvalues, described by the following equations

$$\frac{\partial}{\partial x_i}\left(\sigma_{im} + p\sigma'^0_{ij}\frac{\partial u_m}{\partial x_j}\right) = 0, \tag{7}$$

the conditions for attenuation of perturbations

$$n_i\left(\sigma_{im} + p\sigma'^0_{ij}\frac{\partial u_m}{\partial x_j}\right) \to 0,\quad u_m \to 0,\ x_i \to \pm\infty \tag{8}$$

and the ideal-contact conditions

$$\left[n_i\left(\sigma_{im}^{+} p\sigma'^0_{ij}\frac{\partial u_m}{\partial x_j}\right)\right] = 0,\ \ [u_m] = 0. \tag{9}$$

Within a composite component, the perturbations satisfy Hooke's law, which has the form (6) for the corresponding parameters of the perturbed state.

Values of the critical parameters and the form of stability loss are determined from eigensolution (p^*, u^*) corresponding minimal eigenvalue on the module of the problem (7)-(9). As the problem (7)-(9) is self-conjugated and positive defined ([Guz (1999)] and [Kokhanenko (2001)]). The critical parameters it is necessary to find only first eigensolution $(p^*, u^*) \equiv (p_1, u_1)$ of the problem (7)-(9). Thus the value of critical strain, dimensionless and dimensional critical loading are determined as

$$\varepsilon_{11}^{cr} = p^{cr} = p_1,\quad \varepsilon_{22}^{cr} = -\frac{\nu}{1-\nu}p_1,\quad P^{cr} = \frac{2G}{1-\nu}p_1. \tag{10}$$

An approximate solution of the problem is searched by means of finite difference method. For that an infinite area of the composite is replaced with rectangular domain $\overline{\omega}$. The sizes of the domain should be so that their further increase does not affect the results of calculations. These sizes are determined from a computational experiment. Construction of the discrete models is realized with use of the concept of the basic scheme [Kokhanenko (2001)]. By corresponding summation of basic schemes on each unit of net area $\overline{\omega}$ discrete tasks was obtained.

The differential problem on determination of a stress-strain state in the linear theory of elasticity (1)-(6) on the net $\overline{\omega}$ is associated with the difference problem

$$\mathbf{A}\,\mathbf{u} = \mathbf{\Phi},\ \mathbf{x} \in \overline{\omega}\,,\ \text{or}\ A_i\,\mathbf{u} = \Phi_i,\ \mathbf{x} \in \overline{\omega}, \tag{11}$$

where

$$A_i\,\mathbf{u} = \sum_{\xi \in \mathbf{x}} a_i(\xi)\ \mathbf{u},\ \Phi_i = \sum_{\xi \in \mathbf{x}} \varphi_i(\xi)\,,\ \mathbf{x} \in \overline{\omega}\,. \tag{12}$$

Components of the basic scheme look like

$$a_i(\xi)\mathbf{u} = -H\frac{\sigma_{ji} + \sigma_{ji}^{\xi_j}}{\eta_{\xi_j}}, \quad \varphi_i(\xi) = -\frac{2H}{\eta_{\xi_i}}\overset{0}{P_i} \tag{13}$$

The differential problem of the three-dimensional theory of stability (7)-(9) on the net $\overline{\omega}$ is associated with the generalized difference eigenvalue problem

$$\mathbf{A}\,\mathbf{u} = p\,\mathbf{B}\,\mathbf{u},\ \mathbf{x} \in \overline{\omega}\,,\ \text{or}\ A_i\mathbf{u} = p\,B_i\mathbf{u},\ \mathbf{x} \in \overline{\omega}\,, \tag{14}$$

where

$$A_i\,\mathbf{u} = \sum_{\xi \in \mathbf{x}} a_i(\xi)\ \mathbf{u},\ B_i\,\mathbf{u} = \sum_{\xi \in \mathbf{x}} b_i(\xi)\ \mathbf{u},\ \mathbf{x} \in \overline{\omega}\,. \tag{15}$$

Components of the base circuit look like

$$a_i(\xi)\mathbf{u} = -H\frac{\sigma_{ji} + \sigma_{ji}^{\xi_j}}{\eta_{\xi_j}},$$
$$b_i(\xi)\mathbf{u} = -H\frac{\overset{0}{\sigma}_{jk}\,u_{i,\xi_k} + (\overset{0}{\sigma}_{jk}\,u_{i,\xi_k})^{\xi_j}}{\eta_{\xi_j}} \tag{16}$$

Methods of solution of mesh equations were used to solve discrete problems. The Kholetskii method was used to solve the systems of linear equations and the method of subspace iteration was applied to the solution of the eigenvalue problems. Some approaches to the similar problems is submitted in [Ferretti (2004)] and [Yunfa Zhang and Zihui Xia (2004)].

3 The analysis of results of calculations

By the described technique the influence of distance between fibers $r^* = r_/m_1$ (Fig.1) on the value of critical strain ε_{11}^{cr} and the form of stability loss of fibers in structure of a composite material is investigated. Calculations are performed for the following values of parameters of a composite components: the ratio of moduli of elasticity of components $100 \leq E^* \leq 1000$, where $E^* = E_1/E_2$; Poisson's ratio $\nu_1 = \nu_2 = 0.35$; the form factor of fibers $100 \leq k \leq 500$, where $k = m_1/m_2$ (Fig.1). The dimensionless distance between inclusions r^* was changed in an interval $0.01 \leq r^* \leq 10$.

For studying possible forms of stability loss of fibres, we shall consider character of change of perturbations u_2 of the strip of a composite which contains a fibers material (Fig.2). The dependence of the normalized perturbations $u_2^*(x_1) = u_2/|u_2^{\max}|$, where $|u_2^{\max}|$ - maximum of the module value of perturbations in considered area is shown in Fig.2 for various values of distance between inclusions $r^* = 10$, $r^* = 1$, $r^* = 0.01$, here $E^* = 100$ and $k = 100$.

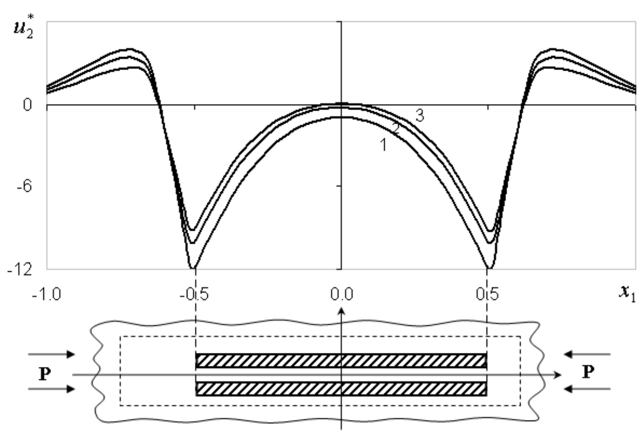

Figure 2 : Stability loss forms of fibers

For all the values of distance between inclusions the form of stability loss of the fibers is similar to "the buckling form in one phase" (Fig.1a). According to [Guz, Dekret, Kokhanenko (2001)], for lower values k can be realized the form of stability loss with "rigid turn" (Fig.1c). Thus, the forms of stability loss were obtained as result of the numerical solution of a stability problem coincides with forms of stability loss, assumed from physical point of view.

Let's consider the influence of interaction of short fibres in structure of a composite material on value of critical strain. The dependence of critical strainε_{11}^{cr} on value from distance between inclusions $r^*$is represented in Fig.3, here curves 1 - $E^* = 100$, $k = 100$; 2 - $E^* = 1000$, $k = 100$; 3 - $E^* = 500$, $k = 500$.

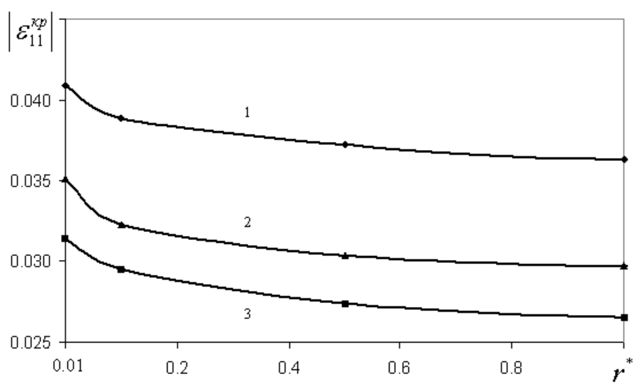

Figure 3 : Dependence of critical strain from value of distance between inclusions

The values of a critical strain were obtained below than limiting deformation, that corresponding to the breaking point of material. It testifies the destruction of the composite material owing to stability loss in structure before breaking point is reached. It is established, that with reduction of distance between inclusions the module of value of critical strain grows. Dependence of value of critical strain on the distance between fibers has monotonous character.

4 Conclusion

The results obtained allow us to conclude that, under plane-strain conditions, a compression at infinity by a load of constant intensity and directed along the fibers may result in fracture of the composite reinforced two parallel short fibers due to stability loss of its structure earlier than the ultimate strength of the material is reached.

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