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Genetic Programming Metamodel for Rotating Beams

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Abstract: This paper investigates the use of Genetic Programming (GP) to create an approximate model for the non-linear relationship between flexural stiffness, length, mass per unit length and rotation speed associated with rotating beams and their natural frequencies. GP, a relatively new form of artificial intelligence, is derived from the Darwinian concept of evolution and genetics and it creates computer programs to solve problems by manipulating their tree structures. GP predicts the size and structural complexity of the empirical model by minimizing the mean square error at the specified points of input-output relationship dataset. This dataset is generated using a finite element model. The validity of the GP-generated model is tested by comparing the natural frequencies at training and at additional input data points. It is found that by using a non-dimensional stiffness, it is possible to get simple and accurate function approximation for the natural frequency. This function approximation model is then used to study the relationships between natural frequency and various influencing parameters for uniform and tapered beams. The relations obtained with GP model agree well with FEM results and can be used for preliminary design and structural optimization studies.

Keyword: Rotating beams, Genetic Programming, Empirical modeling

1 Introduction

Rotating beams are important structural members of wind turbines, steam and gas turbines, helicopter rotors and propellers. The governing partial differential equation for vibration of an Euler-Bernoulli rotating beam is given by,

$$\left(EI(x)\,w''\right)'' + m(x)\,\ddot{w} - \left(T(x)\,w'\right)' = f(x,t) \quad (1)$$

where,

$$T(x) = \int_x^R m(x)\,\Omega^2\,x\,dx \quad (2)$$

is the centrifugal tensile load at a distance x from the axis of rotation, and $EI(x)$, $m(x)$ are flexural stiffness and mass per unit length at a distance x from rotation axis and $w(x,\,t)$ and $f(x,\,t)$ are the displacement and force per unit length, respectively.

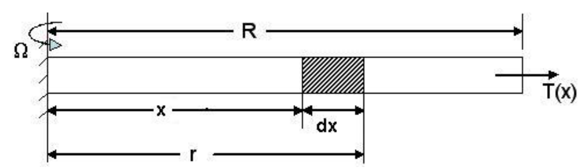

Figure 1: Rotating beam element geometry

A schematic of a rotating beam is shown in Figure 1. Such beams are good models for long slender structures such as helicopter rotor blades and wind turbine rotor blades. Due to centrifugal stiffening, the analysis of rotating beams becomes a challenge. Prediction of the natural frequency of such blades is important because of the design requirement of keeping the frequency away from multiples of the rotor speed. Typically, finite element analysis is required to find the frequencies of even uniform rotating blades and many researchers have worked in this area [Cai, Hong, and Yang (2004)]. The finite element method is widely used in structural analysis as evident from recent literature [Fedelinski and Gorski (2006)]. While Galerkin and Ritz methods can be used to obtain the frequencies of rotating beams, different admissible functions are needed

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for different boundary conditions. However, the finite element method easily accomodates different boundary conditions but has the disadvantage of large sized eigenvalue problem [Wang and Wereley (2004)]. Some studies have used semi-analytical methods such as the Frobenius method to find the frequencies of such beams as reported in Wright, Smith, Thresher, and Wang (1982) and Harris (1992). However, such methods involve retaining a large number of terms in a power series expansion and are difficult to analyze.

Optimization methods are now widely used in engineering problems [de Lacerda and da Silva (2006), Aymerich and Serra (2006), Wang and Wang (2006) and Lian and Liou (2005)]. Designers also use optimization methods to tailor the blade mass and stiffness properties to ensure that the natural frequencies are away from multiples of rotor speed. In such optimization problems, as well as for use for preliminary design, it is very useful to have a high quality approximation or metamodel (model of a model [Crary, Cousseau, Armstrong, and et. al. (2000)]) for the natural frequency of rotating beams.

The development of metamodels for engineering problems has received considerable attention in the past decade. A schematic of a metamodel for a rotating beam is shown in Figure 2. The output $\omega(\underline{x})$ is scalar function of input parameters $\underline{x}(EI,m,R,\Omega)$. For metamodel development, a multivariate function approximation needs to be developed. A model is an abstract mapping between the input variables and the scalar system parameters.

$$F : \underline{x}(EI,m,R,\Omega) \rightarrow \omega \tag{3}$$

We want to represent the model in Eq. 3 by a

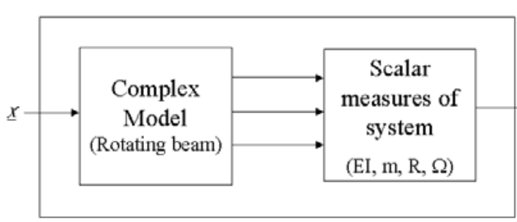


Figure 2: Schematic of a metamodel

functional form which is more efficient to compute.

$$\omega = G(\underline{x}) \tag{4}$$

Several studies in the past literature have investigated methods of approximating finite element models about a given baseline design. For example, neural-networks can be used to develop accurate approximations from input–output relationships [Alam, McNaught, and Ringrose (2004)-Hussain, Barton, and Joshi (2002)]. However, such methods give a black–box representation. In the neural-network approach, a multilayered neural network with i inputs and j outputs is trained with a training set. Subsequently, the given input vector is applied to the network, and a j-dimensional output vector is obtained. In the neural-network approach, the basic drawback is that the optimal configuration of the network is not known *a priori*. Moreover, the training times can be quite large, and the knowledge that is represented internally in the network weights is often opaque. Other methods such as multivariate polynomials tend to have too many terms (a general fifth order polynomial in 20 variables has over 50000 terms) and do not extrapolate well and are difficult to interpret [Meisel and Collins (1973)]. A new technique called Genetic programming (GP) appears to be able to give functional relationships based on input–output relations. Such analytical functions are very useful for approximations of finite element models.

Genetic programming (GP) is gaining attention due to its ability to discover the underlying relationships and expressing them mathematically. GP has been used as a powerful methodology for obtaining solutions of a large number of difficult problems like automatic design, pattern recognition, control, synthesis of neural architectures, symbolic regression, factory job scheduling, electronic circuit design, signal processing, music and picture generation, etc. [KinnearJr. (1994)]. Recent applications of GP also include fault classification [Zhang, Jack, and Nandi (2005)], optimization [Yeun, Kim, Yang, and et. al. (2005), Yeun, Yang, Ruy, and et. al. (2005)] chemical process modeling [Grosman and Lewin (2004)] and image processing. A recent review of GP highlighting its strengths and limitations is given in Bhattacharya and Nath (2001). However, very few works have looked at using GP for structural optimization. Yang and Soh (2005) have discussed the use of GP for automated optimum design of trusses. Ashour, Alvarez, and Toropov (2003) have used GP for predicting the empirical model of shear strength of deep reinforced-concrete (RC) beams. Both of these works discuss the static structural problems. In this paper, GP is used to generate the empirical model of a finite element model for finding natural frequency of a rotating beam. To the best of the authors' knowledge, this is the first application of GP to this important practical problem in the literature.

2 Overview of Genetic Programming

Genetic programming (GP) [Koza (1992)] is an extension of genetic algorithms (GA). Genetic algorithms are now widely used to solve optimization problems [Akula and Ganguli (2003), Pawar and Ganguli (2003)]. GP is a systematic method of using computers to automatically develop optimal programs. Its basis is the Darwinian concept of evolution. While GA uses a string of numbers to represent the solution, the GP creates a large initial population of computer programs with a tree structure. A typical program, representing the expression $(x_1x_2 - x_3)^2$ is shown in Figure 3 as a typical tree structure. These programs are random combinations of elements from the problem–specific function and terminal sets. GP provides a way to successfully conduct the search for a computer program in the space of computer programs and to assess its fitness for the problem. This fitness assessment is usually accomplished by running each program on an input dataset showing the relationship between different variables of the problem. A fitness value is assigned to each program that shows how well it solves the problem.

The fitness criteria of programs is used in producing a population of the next generation of programs using the various genetic operators including reproducton, crossover, and mutation. GP provides a way to automatically discover and reuse existing subprograms in the course of automatically creating new computer programs. GP does this search probabilistically without much use of domain knowledge of the problem. The programs are randomly selected to participate in these genetic operations, but the selection function is biased towards highly fit programs. The reproduction operator simply selects an individual program and copies it to next generation. The crossover operator incorporates the variation by selecting two parents and by generating two offsprings from them. The offsprings are produced by swapping randomly selected sub-trees of the parents. The mutation operator produces one offspring from a single parent by replacing a randomly selected sub-tree by a randomly generated sub-tree.

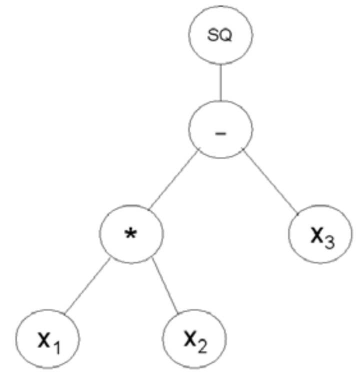


Figure 3: Tree structure of an example program $(x_1x_2 - x_3)^2$

The size and shape of computer programs are determined dynamically during the course of generations. After many generations of GP, the average fitness of the population as well as fitness of each best-of-generation individual program may tend to improve. After a predetermined number of generations, or after achieving the predetermined level of fitness, the best-so-far individual program is designated as the resultant output of GP operation. This output is suggested as the approximate empirical model for the given problem. The GP methodology has advantages of being less problem-dependent, more flexible and having high search efficiency. A typical program consists of several nodes (Figure 3). The terminal nodes contain n variables $x_1, x_2, \ldots, x_n$. The functional nodes contain mathematial operators {e.g. +, -,

*, /, power, square, square root, negation, ...}. There are two types of functional nodes. The binary nodes take two arguments such as addition, multiplication etc. and the unary nodes take one argument such as square, negation etc. Finally, all the functions and terminals should be compatible and be able to pass information to each other. This is called the *closure* property of the GP.

The advantages of using GP are that it can directly operate on the data in their original form, no prior knowledge about data distribution is needed, and it can detect the underlying but unknown relationship that exists among data and express it as a mathematical LISP s-expressions. The LISP s-expressions are computer programs, which can be directly used in the application environment. In this paper, a computer program represents an empirical model of a uniform and a linearly tapered rotating beam. A freely available genetic programming software, GPQUICK, is used for finding the underlying relationships. Before using GP for the rotating beam problem, we illustrate it with a non-rotating beam problem for which closed form solution is known.

3 Non-rotating cantilever beams

Consider the expression for the first natural frequency of a non-rotating uniform cantilever beam whose exact solution is known as [Thomson and Dahleh (2005)],

$$\omega = 3.516\sqrt{\frac{EI}{mR^4}} = C_1\sqrt{\frac{EI}{mR^4}} \tag{5}$$

where, EI is flexural rigidity of beam, m is mass per unit length of beam, R is length of beam.

To obtain the GP model, baseline values of $EI = 1.2\times10^5 Nm^2$, $m = 6.4kg/m$ and $R = 5m$ are considered and input–output relationship data is generated by perturbing variables in the range of $\pm$ 25%. GP is then used to generate relation between the frequency and EI, R and m. The expressions for squared frequency obtained in four different runs of the GP are listed in Table 1. The function with least RMS error is selected as the empirical model for the beam frequency and is shown in Eq. 6. For any values of EI, m and R, the natural frequency can be calculated using this function.

$$\omega = \sqrt{\frac{13E^2I^2}{R^2(EIR + 15m^3R^3 + m(EIR^2 - 19228))}} \tag{6}$$

Figure 4(a) shows the comparison between frequency vs beam-length given by the exact solution and GP function. Similarly, Figure 4(b) shows comparison of frequency vs mass per unit length and Figure 4(c) shows the comparison between frequency vs flexural rigidity for exact solution and GP approximation. Though these figures show that the GP prediction agrees well with the exact solution, but the functions obtained from GP are dimensionally not correct. In general, using dimensional variables in the GP is only effective when the number of variables are less, as mentioned by Keijzer and Babovic (1999) . For a large number of variables, it is useful to transform the physical variables into non-dimensional variables. This allows a reduction in the number of variables and also leads to more compact and physically meaningful expressions using the GP.

The Eq. 5 can be rewritten as follows:

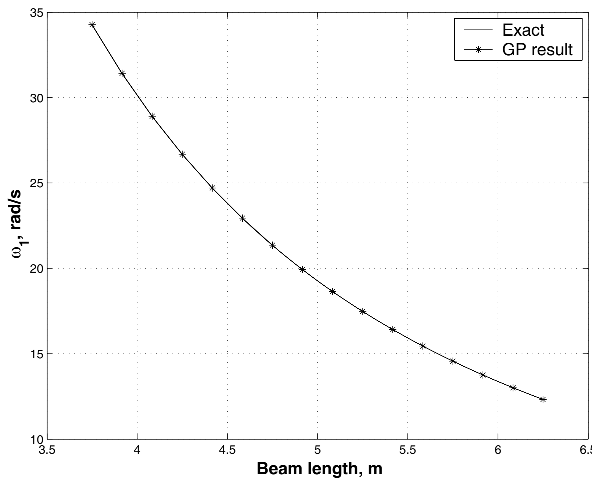
$$\omega = C_1\sqrt{\frac{EI}{mR^4}} = C_1\,\omega_0 \tag{7}$$

To non-dimensionalize the above relation, we consider a reference frequency Ω_R = 1 Hz, and divide Eq. 7 to define,

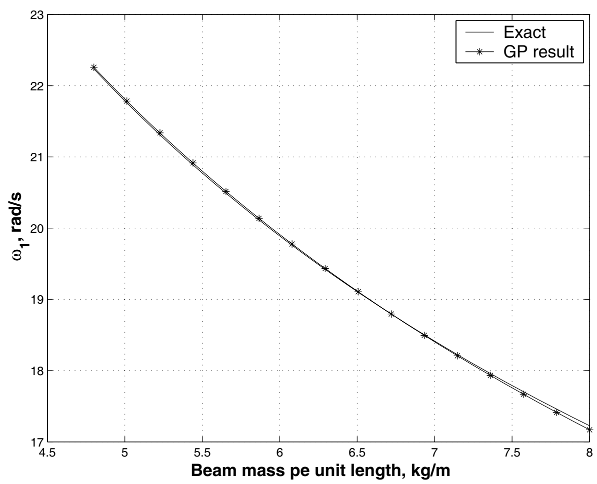
$$\eta = \frac{\omega}{\Omega_R} = C_1\frac{\omega_0}{\Omega_R} = C_1\,\eta_0 \tag{8}$$

Here in Eq. 8, both η and η_0 are non-dimensional variables. For this non-dimensional relationship, we get different approximate relationships using GP in various runs (Table 2). This table shows the functions generated for squared natural frequency. The following function is selected as the GP result on the basis of simplicity and least RMS error. For any given values of EI, m and R, the non-dimensional parameter η_0 is calculated and the natural frequency is obtained using following GP result.

$$\eta = \sqrt{\frac{1}{19136} + \frac{69856489\,\eta_0^2}{5650788}} \tag{9}$$



(a)



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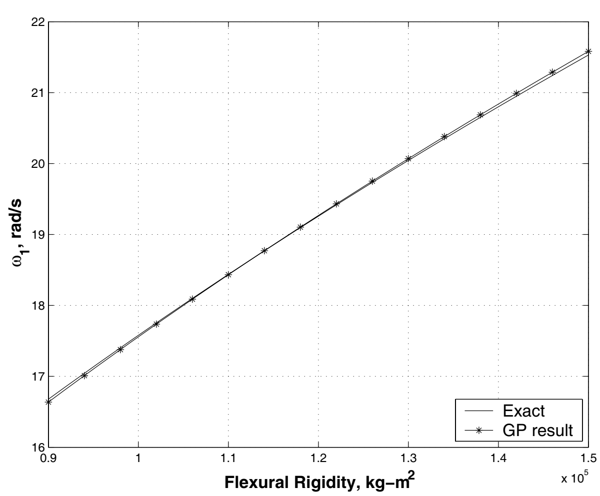
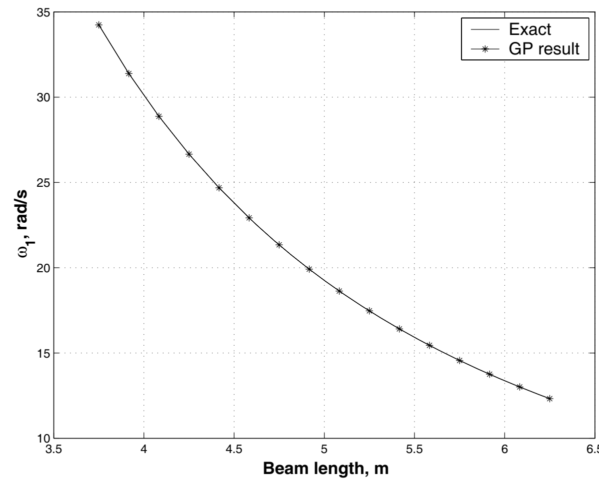
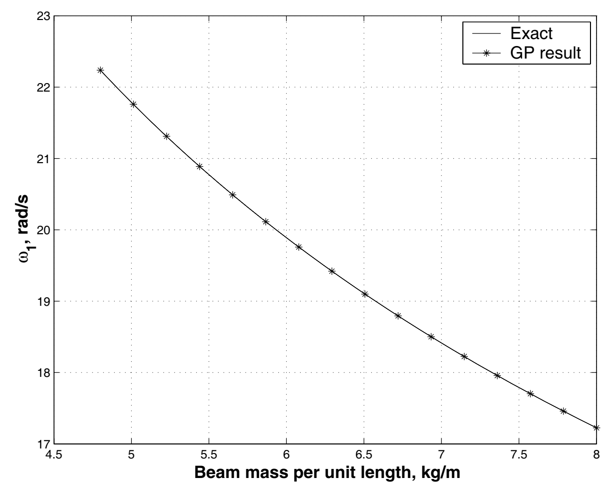


(c)

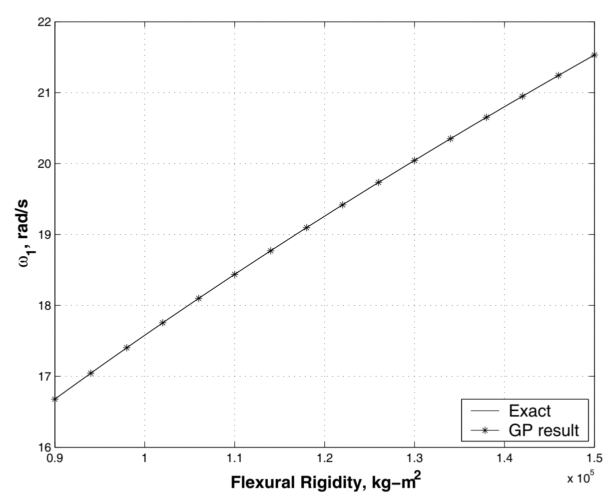
Figure 4: Effect of (a)beam length, (b)beam mass per unit length and (c)flexural rigidity on the first natural frequency of non-rotating cantilever beam (dimensional parameters)



(a)



(b)



(c)

Figure 5: Effect of (a)beam length, (b)beam mass per unit length and (c)flexural rigidity on the first natural frequency of non-rotating cantilever beam (non-dimensional parameters)

Table 1: Results of multiple GP runs for dimensional variables for non-rotating cantilever beam

Generations	RMS error	Expression	Comments
1000000	31.6201	$-39R+\frac{128778+5EI+3310R-m^3R^2}{7(m^2+78R-254)}$	Complex functon, High error
500000	1.89913	$\frac{13E^2I^2}{R^2(EIR+15m^3R^3+m(EIR^2-19228))}$	Least error, Selected GP result
500000	47.2609	$\frac{5\left(17+R+\frac{2E^2I^2+29m-6EI(9m^3R-2449)}{EI(m+25R^2-136)}\right)}{R}$	Complex function, High error
500000	38.8107	$-47+2R+R^2+\frac{1}{m}\Big(-104+EI\left(-\frac{1}{30}+\frac{65}{49R^2}\right)+3R^2\Big)$	Complex function, High error

Table 2: Results of multiple GP runs for non-dimensional variables for non-rotating cantilever beam

Generations	RMS error	Expression	Comments
100000	0.001802	$\frac{445\eta_0^2}{36}+\frac{13}{46+\frac{803088\left(\eta_0^2-3\right)}{\left(30+\eta_0^2\right)\left(\eta_0^4+3\eta_0^2-5\right)}}$	Complex function
36469	2.95e-05	$\frac{1}{19136}+\frac{69856489\,\eta_0^2}{5650788}$	Selected GP result
30419	3.356e-05	$12\left(\eta_0^2+\frac{5313\eta_0^2(92+3\eta_0^2)}{16191678+527989\eta_0^2}\right)$	Complex function

Figure 5(a) shows the comparison between frequency vs beam-length given by the exact solution and non-dimensional GP function in Eq. 9. Similarly, Figure 5(b) shows comparison of frequency vs mass per unit length and Figure 5(c) shows the comparison between frequency vs flexural rigidity given by the exact solution and non-dimensional GP function. These figures show that the GP can predict the underlying input–output relationship that agrees well with the exact solution, and these relationships are physically more meaningful when the input dataset is generated using non-dimensional variables. The insight developed during this non-dimensionalization is used in solving the rotating cantilever beam problem in the next section.

Note that it follows from the physics that $\eta = 0$ when $\eta_0 = 0$, which happens when $EI = 0$. Therefore, the expression in Eq. 9 can be modified as,

$$\eta = \sqrt{\frac{69856489\,\eta_0^2}{5650788}} = 3.516\,\eta_0 \tag{10}$$

which matches the exact solution in Eq. 5. In general, since GP is a computational tool, it is a good idea to slightly modify the GP-generated functions using physical knowledge of the problem [Ashour, Alvarez, and Toropov (2003)].

The above case shows that the GP can give a functional approximation to any complicated relationship, which can match with the exact relation to a reasonable extent. It can even give the exact closed form solution in some cases. The power of GP becomes much more clearer when it is used for problems with no closed form solution, such as rotating beams.

4 Rotating cantilever beams

As discussed before, the natural frequencies of the rotating beams cannot be obtained from a closed form solution and needed to be obtained using the finite element method. Here we consider two cases of rotating beams, first the uniform and then linearly tapered cantilever beam. *h*-version FEM is used to model both types of beams. Using the relationship data obtained form FEM, we develop the GP approximation for first natural frequency.

4.1 Uniform beam

For the simplest case, we consider uniform cantilever beams as the idealization of helicopter rotor blades.

4.1.1 Parameters affecting the natural frequency

In this case, the output variable, ω_1(first natural frequency) is a function of EI(flexural rigidity), R(radius of rotor), m(mass per unit length of rotor blade) and Ω(rotation speed), i.e.,

$$\omega_1 = f(m, EI, R, \Omega) \tag{11}$$

Since the above relation is a nonlinear one, we transform the physical variables into dimensionless parameters to reduce the complexity of relation and to obtain a better fit of GP model. Though if the number of independent variables is small and the relation is simple algebraic, the dimensional GP response function can be generated. This transformation has reduced the number of independent parameters in the dataset to one, which is defined as non-dimensional stiffness (k) such that,

$$k = \frac{EI}{mR^4\Omega^2} \tag{12}$$

and the non-dimensional natural frequency can be defined as,

$$\gamma = \frac{\omega_1}{\Omega} = f(k) \tag{13}$$

Note that k is similar to η_0 used for the non-rotating beam in Eq. 8 where a polynomial expansion of η^2 in Eq. 9 gave result matching the exact solution closely.

4.1.2 Finite Element Model

The FEM modelling of a uniform rotating cantilever beam is done and its natural frequencies were obtained. In h-version FEM, it is an accepted fact that $10n$ elements can give only up to n natural frequencies to an acceptable accuracy. So, 20 elements of equal length were selected to achieve a good accuracy. The classical 4–noded uniform beam element, with displacement and slope continuity (C^1 continuity) using cubic Hermite polynomials, is used. The mass and stiffness matrices for such a beam element can be obtained from energy expressions and are shown in Appendix A. The results of FEM model is validated with the earlier works by Hodges and Rutkowsky (1981), Wang and Wereley (2004), and Wright, Smith, Thresher, and Wang (1982). Table 3 shows the natural frequencies obtained for uniform cantilever beam using h-version FEM with two non-dimensional rotating speeds, $\lambda = 0$ and $\lambda = 12$. It can be observed that the frequencies compare very well with the other published results.

4.1.3 Empirical model obtained by GP

For the numerical results in the study using GP, we consider a hingeless helicopter rotor which can be represented as a uniform rotating cantilever beam for modelling purposes. The range of independent variables (m, EI, R, Ω) is chosen $\pm$ 25% from the typical values for a helicopter rotor blade. The typical values for helicopter rotor are $EI = 1.2 \times 10^5 kg\,m^2$, $m = 6.4\,kg/m$, $\Omega = 40\,rad/s$ and $R = 5\,m$, which are similar to the values used in Pawar and Ganguli (2003). The non-dimensional stiffness is calculated for this range of variables and used as the input parameter. The natural frequency calculated using FEM model is used as the output parameter.

This data is then used to train the GP code. The mathematical operators addition, subtraction, multiplication, division and negation were used for GP run. The advantage of using GP for function approximation is due to its ability to discover the underlying input–output relationship in the given data and express it mathematically. This relationship is expressed as a LISP s–expression, which can be easily converted into a mathematical relation. Multiple runs were performed and solutions were analysed on the basis of the simplest generated model that confirms the training dataset as good as possible. The best five runs are shown in Table 4. For the first mode, the expression given by GP after 500,000 generations in fifth run shows an RMS error of 0.0074 as compared to FEM results. The coefficients of this expression are adjusted in order to minimize the error. The

Table 3: Validation of Non-dimensional natural frequencies of cantilever uniform beam

Mode	hFEM	Wang and Wereley (2004)	Wright et. al. (1982)	Hodges and Rutkowsky (1981)
		$\lambda = 0$		
1	3.5160	3.5160	3.5160	3.5160
2	22.0345	22.0345	22.0345	22.0345
3	61.6972	61.6972	61.6972	61.6972
4	120.902	120.902	120.902	N/A
5	199.8616	199.860	199.860	N/A
		$\lambda = 12$		
1	13.1702	13.1702	13.1702	13.1702
2	37.6031	37.6031	37.6031	37.6031
3	79.6145	79.6145	79.6145	79.6145
4	140.5348	140.534	140.534	N/A
5	220.5383	220.536	220.536	N/A

Table 4: Results of multiple GP runs for rotating uniform cantilever beam

Generations	RMS error	Expression	Comments
100000	0.02981	$1+\frac{24k}{1-\frac{3k(-133+960k^2)}{2(1+k)(19+384k^2)}}$	Complex function, High error
100000	0.00759	$\frac{19}{17}+14k-k^2\left(8+\frac{1}{5k+0.5}\right)$	Complex function, High error
100000	0.0100481	$2k+\frac{35+352k}{31+k+38k^2}$	Complex function, High error
100000	0.009158	$\frac{3}{80}\left(30+361k+126k^2\right)$	High RMS error
500000	0.007411	$\frac{1}{77}\left(86+1078k-755k^2+624k^3\right)$	Selected GP result

Table 5: Comparison of natural frequency calculated using FEM and GP expression

% change					
EI	m	R	FEM frequency (γ)	GP frequency (γ)	% error
0	0	0	1.1760	1.1758	-0.017
10	10	10	1.1392	1.1406	0.123
-10	10	10	1.1236	1.1265	0.258
10	-10	10	1.1573	1.1576	0.026
10	10	-10	1.2310	1.2311	0.008
-10	-10	10	1.1392	1.1406	0.123
10	-10	-10	1.2643	1.2652	0.071
-10	10	-10	1.2026	1.2023	-0.025
-10	-10	-10	1.2310	1.2311	0.008
20	20	20	1.1134	1.1177	0.386
-20	20	20	1.0901	1.0988	0.798
20	-20	20	1.1441	1.1452	0.096
20	20	-20	1.3183	1.3208	0.190
-20	-20	20	1.1134	1.1177	0.386
20	-20	-20	1.4237	1.4287	0.351
-20	20	-20	1.2413	1.2417	0.032
-20	-20	-20	1.3183	1.3208	0.190

resultant GP expression becomes,

$$\gamma^2 = \frac{1}{77}\left(86.5 + 1078\,k - 755\,k^2 + 624\,k^3\right) \quad (14)$$

The constant term in the above equation accounts for centrifugal stiffening and the terms dependent on k are due to the structural effect of EI. For any values of EI, m, R and Ω, the non-dimensional stiffness k is calculated from Eq. 12 and then natural frequency can be obtained using above GP result.

Now we consider some example values for the input parameters and calculate the natural frequency using GP expression, and compare with the FEM results. For a fixed rotation speed $\Omega = 40\,rad/sec$, the other three variables, namely, flexural stiffness, mass per unit length and blade length are varied to $\pm 10\%$ and $\pm 20\%$, and the natural frequency is calculated using GP-generated function. These results are compared with FEM results in Table 5. This is a typical method of validating a new metamodel about a baseline design. Thus for the $\pm 10\%$ and $\pm 20\%$ perturbation we have 2^j points, where $j = 3$ for the three variables EI, m and R. These points are the vertices of a cube with the baseline point as the centre point. The percentage error shown in the table is very less, which proves the validity of this model for calculating natural frequency of rotating blades.

As said earlier, prediction of the natural frequency of rotor blades is important because of the design requirement of keeping the frequency away from multiples of the rotor speed. If the natural frequency obtained from a chosen set of parameters falls near to the integral multiples of rotating speed, the input parameters need to be modified. If FEM is used to calculate the frequency in every iteration, it would be expensive in terms of computational efforts and time. Whereas, analytical expression obtained from the GP can give the frequency in a straight forward manner.

Next, the fundamental frequency obtained by FEM and the approximate function given by GP are compared at points other than the training points and a good resemblance is seen. Figure 6(a) shows the comparison between FEM and GP fundamental frequency changing with mass per unit length m, Figure 6(b) shows the variation of frequency with flexural rigidity EI, Figure 6(c) shows the variation of frequency with blade length R, and Figure 6(d) shows the variation of frequency with rotation speed Ω. These graphs are plotted against one variable at a time keeping others constant at the baseline values of the helicopter rotor blade. Figures 7 - 9 show the comparision of FEM and GP fundamental frequency by varying two variables at a time. Again we can see that the overall trends as well as the magnitudes of the curves are well captured by the GP.

4.2 Tapered beams

In practical problems of rotor blades, we have non-uniform blades which can be better idealized by linearly tapered beams for the ease of analysis. So, the natural frequency(ω_1) is a function of taper parameters(α and β) in addition to EI, R, m and Ω. The flexural rigidity(EI) and mass per unit length(m) are taken to vary linearly [Wright, Smith, Thresher, and Wang (1982)] along the beam length as follows:

$$m = m_0\left(1 - \alpha\frac{x}{R}\right) \quad (15)$$

$$EI = EI_0\left(1 - \beta\frac{x}{R}\right) \quad (16)$$

where m_0 and EI_0 correspond to values of the mass per unit length and the flexural rigidity at the root of the beam, respectively.

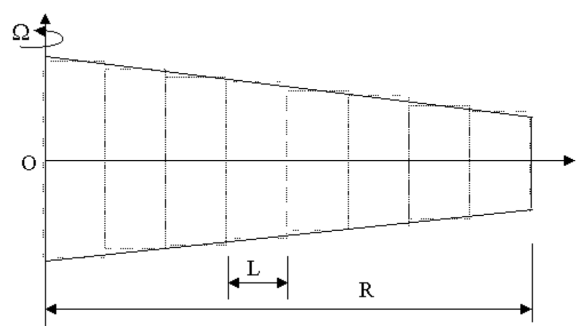
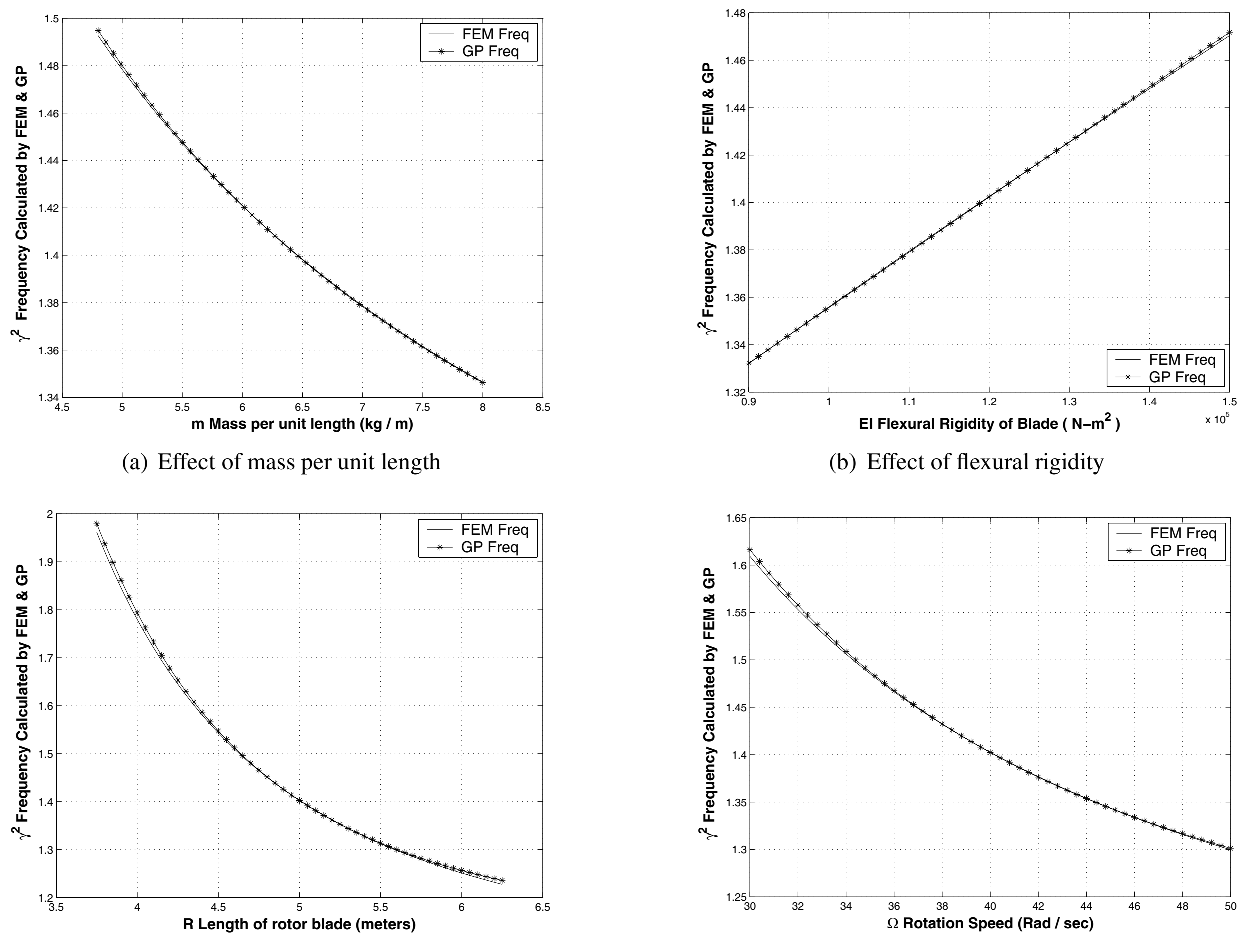


Figure 10: The idealization of a linearly tapered beam using uniform elements

Table 6: Results of multiple GP runs for rotating tapered beam with different taper parameters

Generations	RMS error	Expression	Comments
		$\alpha = 0.1,\ \beta = 0.1$	
370000	0.00163	$\frac{99}{92} + k\left(13 + \frac{1}{\frac{3}{13}+7k}\right)$	Complex function
500000	0.01059	$\frac{77}{68} + 14k - 20k^3 - 2k^4$	High error
475000	0.00731	$\frac{10}{9} + \frac{46}{3}k - \frac{29}{3}k^2 - 80k^3 + 3024k^4$	Selected GP function
		$\alpha = 0.5,\ \beta = 0.5$	
425000	0.00583	$\frac{17}{15} + 21k - 22k^2 + k^4\left(64 + \frac{13376}{15+152k}\right)$	Complex function
500000	0.00819	$\frac{8}{7} + \frac{950}{47}k - \frac{89}{10}k^2$	Higher error
500000	0.00768	$\frac{4}{77}\left(22 + 391k - 227k^2 - 320k^3\right)$	Selected GP function
		$\alpha = 0.8,\ \beta = 0.95$	
500000	0.00251	$\frac{107}{89} + \frac{66856}{2225}k - \frac{18}{5}k^2 - \frac{6630}{5243k(3983+377910k)}$	Complex function
500000	0.01005	$\frac{1+32k}{3040}\left(3467 - 7410k + 27075k^2\right)$	High error
180000	0.00977	$\frac{98}{83} + \frac{145616245}{4776387}k - \frac{5279}{1056}k^2$	Selected GP function



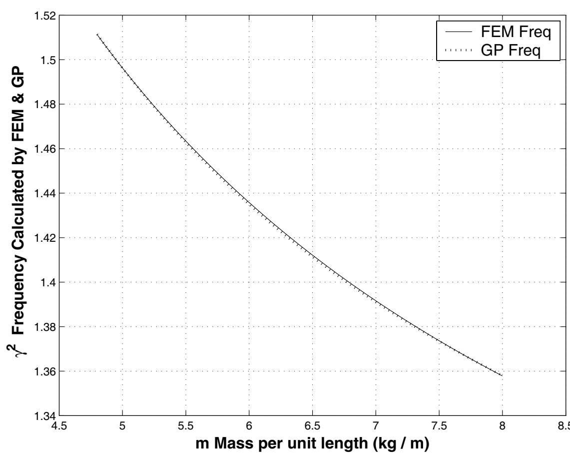
(a) Effect of mass per unit length

(b) Effect of flexural rigidity

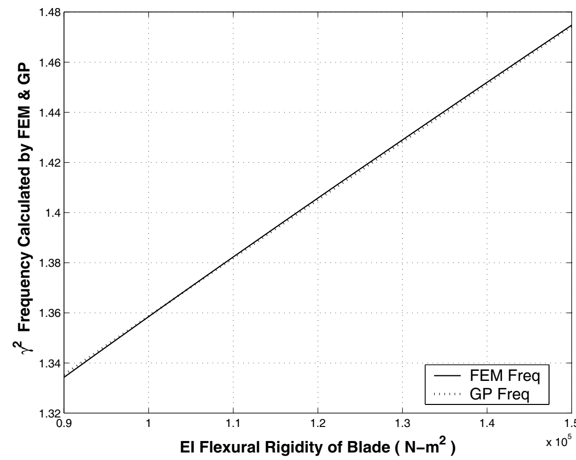
(c) Effect of blade length

(d) Effect of rotating speed

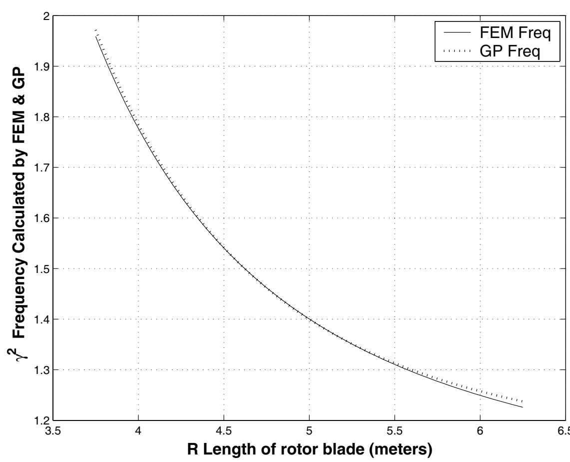
Figure 11: First natural frequency of tapered blade with taper ratios α=0.1, β = 0.1



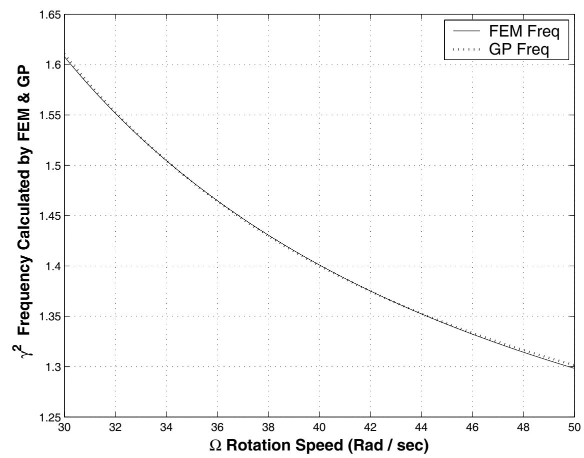
(a) Effect of mass per unit length



(b) Effect of flexural rigidity



(c) Effect of blade length



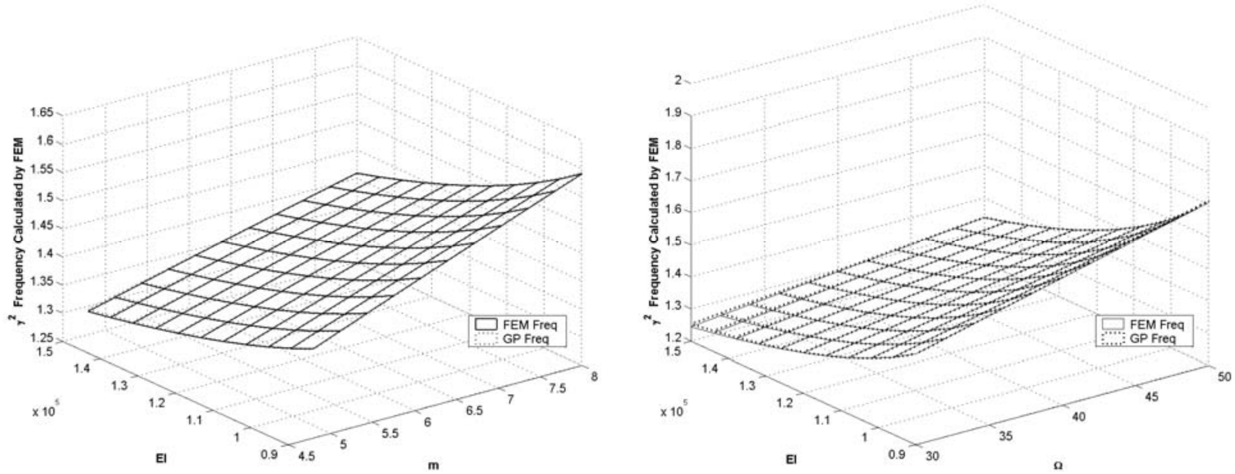
(d) Effect of rotating speed

Figure 6: First natural frequency of uniform hingeless helicopter rotor blade

In this section, we have considered the three different sets of taper parameters and the finite element model is developed for finding the natural frequency of the tapered beam. A sketch showing an idealization of tapered beam with uniform elements is given in Figure 10. 200 uniform elements were used to idealize the tapered beam into a stepped beam and this FEM model was validated with published results for the first mode frequency by Wang and Wereley (2004) and Wright, Smith, Thresher, and Wang (1982). For this stepped beam, the natural frequency is calculated for different values of input variables using h-version FEM. The input-output relationship data generated from FEM is then used to obtain the GP approximations for the natural frequency of rotating tapered beams in terms of non-dimensional stiffness k. The results of multiple GP runs for three different sets of taper parameters are shown in Table 6. In each case, among the three results, the one showing least RMS error and is a polynomial function, is selected as GP approximate function. For any given values of EI, m, R and Ω, the non-dimensional stiffness k is calculated and then GP-generated expression is used to calculate the natural frequency. To further reduce the error, the coefficients of these expressions are adjusted and the resultant expressions are obtained as follows:

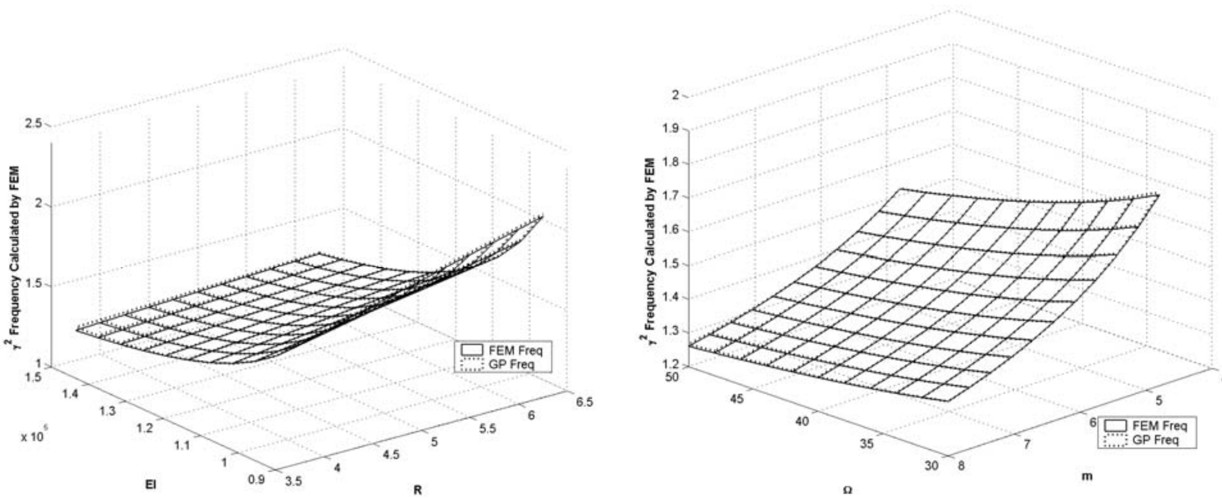
$$\gamma^2 = \frac{179}{160} + \frac{46}{3}k - \frac{29}{3}k^2 - 80k^3 + 3024k^4 \tag{17}$$

$$\gamma^2 = \frac{4}{77}\left(22.1 + 391k - 227k^2 - 320k^3\right) \tag{18}$$



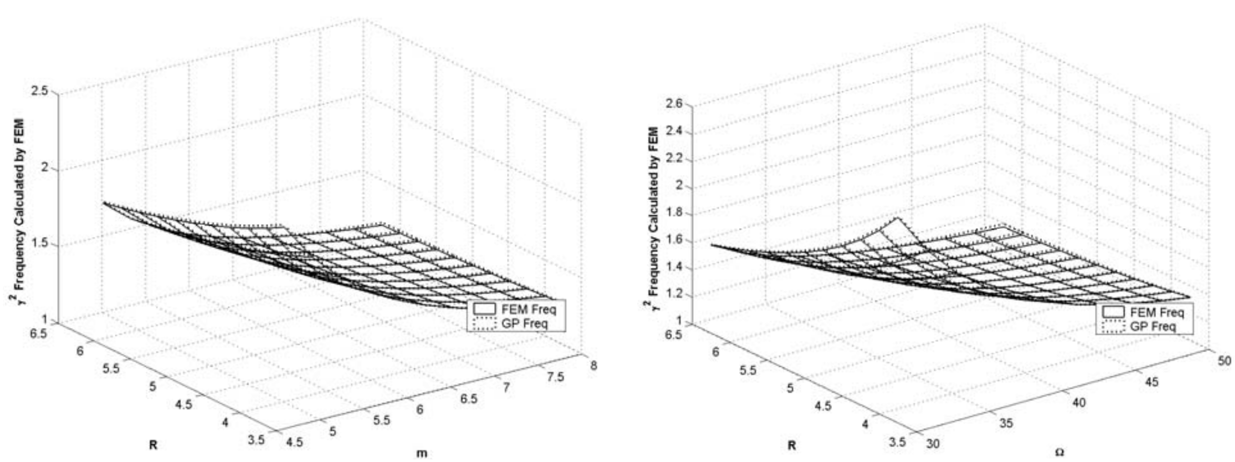
(a) Effect of flexural stiffness and mass per unit length (b) Effect of flexural stiffness and rotating speed

Figure 7: First natural frequency of uniform hingeless helicopter rotor blade



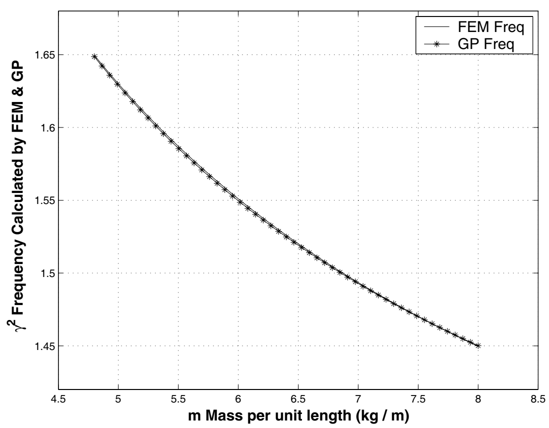
(a) Effect of flexural stiffness and blade length (b) Effect of mass per unit length and rotating speed

Figure 8: First natural frequency of uniform hingeless helicopter rotor blade

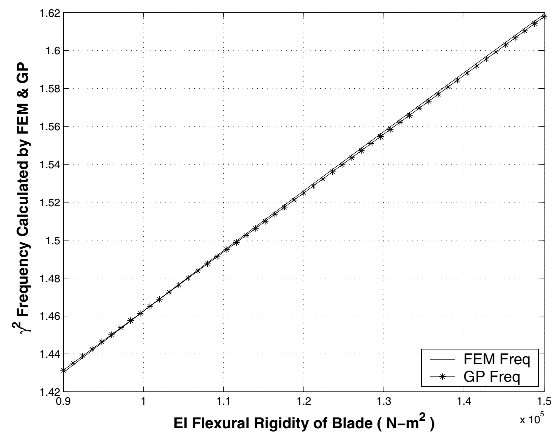


(a) Effect of blade length and mass per unit length (b) Effect of blade length and rotating speed

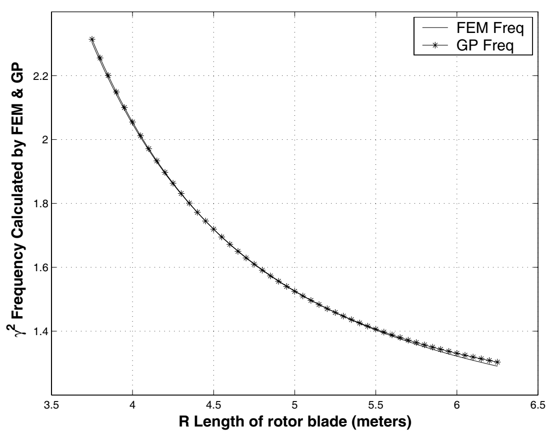
Figure 9: First natural frequency of uniform hingeless helicopter rotor blade



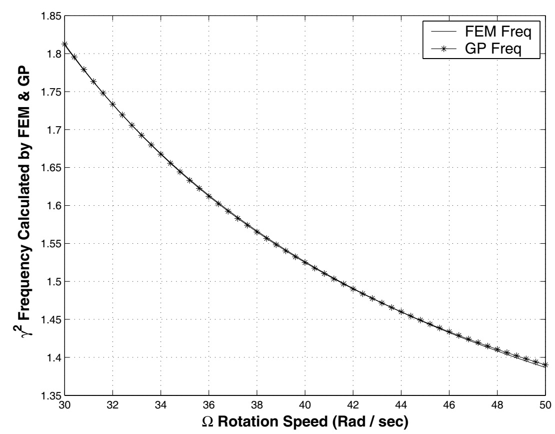
(a) Effect of mass per unit length



(b) Effect of flexural rigidity



(c) Effect of blade length



(d) Effect of rotating speed

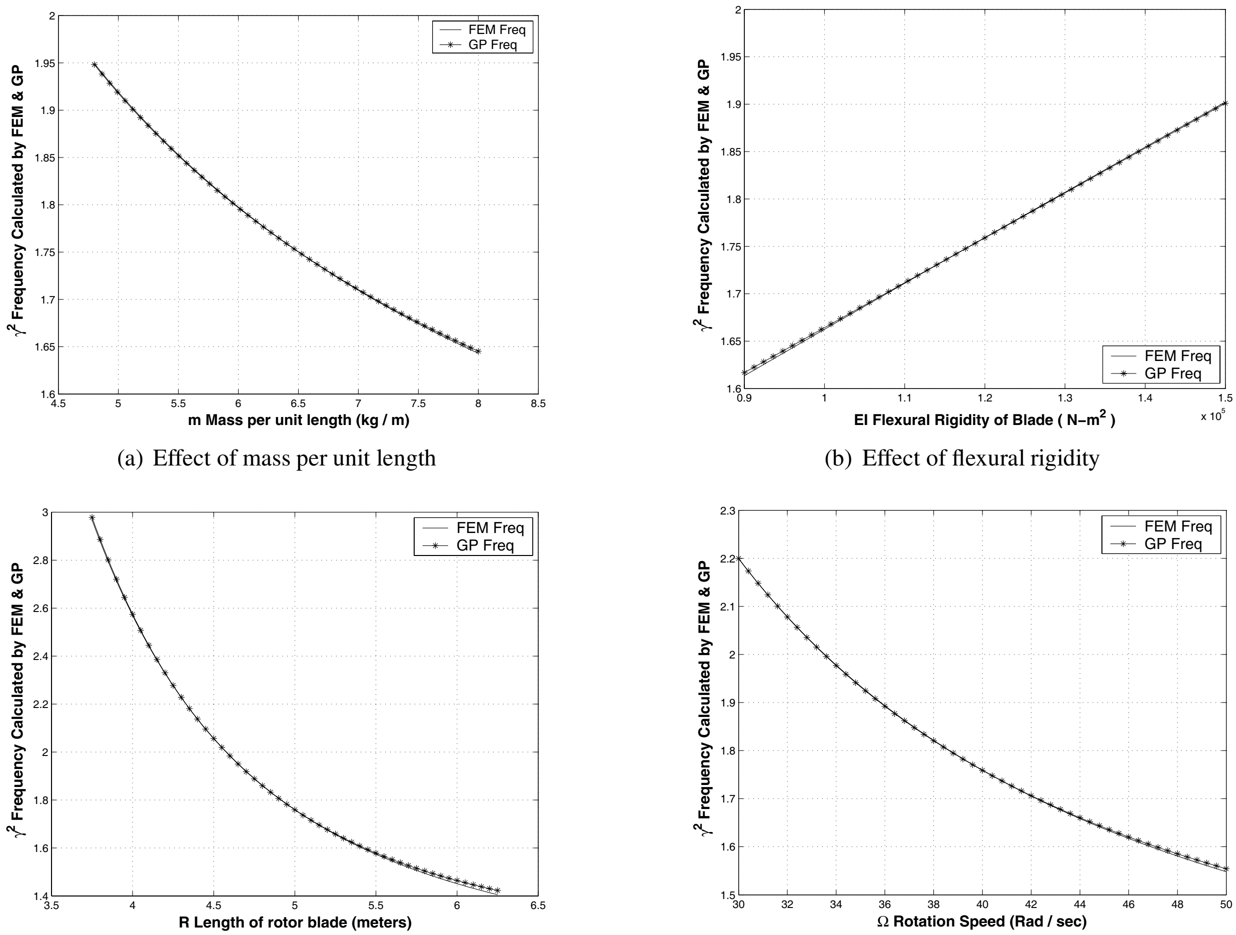
Figure 12: First natural frequency of tapered blade with taper ratios α=0.5, β = 0.5

$$\gamma^2 = \frac{98.7}{83} + \frac{145616245}{4776387}k - \frac{5279}{1056}k^2 \tag{19}$$

Eq. 17, 18 and 19 give the approximate functions for the natural frequency for beams with taper ratios (α, β) of (0.1, 0.1), (0.5, 0.5) and (0.8, 0.95), respectively. The RMS error with these functions is of the order 10^{-5}. These expressions are physically reasonable with a constant term for centrifugal stiffening and the k-dependent terms for structural stiffening.

Figures 11, 12 and 13 show the variation of frequency with respect to the input parameters(EI, m, R, Ω) given by GP function and FEM model, for three sets of taper ratios. It is clear from these figures that the GP is able to capture the underlying relationship between input-output variables quite accurately. Hence, we can use the GP expressions for finding the natural frequency of a tapered beam with a given fixed taper ratio and for varying values of mass and stiffness. Similar calculations using FEM will cost much in terms of computational efforts and time, whereas the GP expression can directly give the output for any given set of values of input variables. In optimization problems involving many iterations, GP approximations will thus significantly reduce the cost and time of the calculations.

We need to point out that the cost of GP approximations is high and it took about 4 hours of computer time on a Pentium 4 computer to get the results in this study. However, once the GP approximation is developed, it is a very efficient functional expression. In this regard, GP is similar to



(a) Effect of mass per unit length

(b) Effect of flexural rigidity

(c) Effect of blade length

(d) Effect of rotating speed

Figure 13: First natural frequency of tapered blade with taper ratios α=0.8, $\beta = 0.95$

other metamodeling methods such as those based on neural networks where the training time can be very large but once the approximation is obtained, the time required to perform the analysis becomes much less. GP has an advantage over most other methods because it gives analytical expressions as approximations which can be used by analytical optimization methods based on optimality criteria. We should also point out that the use of metamodels is typically recommended only for problems where many evaluations of the computer code are needed such as in optimization and for Monte Carlo simulations.

5 Conclusions

A functional approximation model to calculate the natural frequency of rotating cantilever beam is obtained using genetic programming (GP). A finite element model (FEM) of the rotating beam is developed and frequencies are validated with data from the published literature. FEM results are then used to train the GP and validate the empirical model. It is found that using non-dimensional variables results in a considerable reduction in the number of design variables and in physically meaningful expressions. In particular, a certain combination of variables resulting in a non-dimensional stiffness $k = EI/mR^4\Omega^2$ results in a reduction of the problem dimensionality from four to one. While dimensional GP's work well with very small dimensions, they become less useful as the dimensions of the problem increases. All the results show good agreement between the predicted GP model and FEM results, both for uniform and tapered rotating beams.

The approximate model developed using GP can therefore be used in structural optimization for blade design and any other applications, which typically involve validity of the approximation in a region in the neighborhood of the baseline design. As optimization involves many iterations, using an empirical model will be advantageous in terms of cost and time in comparision to FEM models. Such simple expressions can be developed for a given beam structure and used as an empirical formula for preliminary design calculations and design improvements without using a FEM model.

Appendix

Element mass matrix

$$M = m_i L_i \begin{bmatrix} \frac{13}{35} & \frac{11 L_i}{210} & \frac{9}{70} & \frac{-13 L_i}{420} \\ \frac{11 L_i}{210} & \frac{L_i^2}{105} & \frac{13 L_i}{420} & \frac{-L_i^2}{140} \\ \frac{9}{70} & \frac{13 L}{420} & \frac{13}{35} & \frac{-11 L_i}{210} \\ \frac{-13 L_i}{420} & \frac{-L_i^2}{140} & \frac{-11 L_i}{210} & \frac{L_i^2}{105} \end{bmatrix}$$

where m_i is the mass per unit length and L_i is the length of element i.

Element stiffness matrix

$$K = \frac{(EI)_i}{L_i^3} \begin{bmatrix} 12 & 6 L_i & -12 & 6 L_i \\ 6 L_i & 4 L_i^2 & -6 L_i & 2 L_i^2 \\ -12 & -6 L_i & 12 & -6 L_i \\ 6 L_i & 2 L_i^2 & -6 L_i & 4 L_i^2 \end{bmatrix}$$

where $(EI)_i$ is the flexural rigidity of element i.

Matrices contributing to stiffness due to centrifugal effect

$$T = \Omega^2 \left(A_i \begin{bmatrix} \frac{3}{5 L_i} & \frac{1}{20} & \frac{-3}{5 L_i} & \frac{1}{20} \\ \frac{1}{20} & \frac{L_i}{15} & \frac{-1}{20} & \frac{-L_i}{60} \\ \frac{-3}{5 L_i} & \frac{-1}{20} & \frac{3}{5 L_i} & \frac{-1}{20} \\ \frac{1}{20} & \frac{-L_i}{60} & \frac{-1}{20} & \frac{L_i}{15} \end{bmatrix} - m_i L_i \begin{bmatrix} \frac{6}{35} & \frac{L_i}{28} & \frac{-6}{35} & \frac{-L_i}{70} \\ \frac{L_i}{28} & \frac{L_i^2}{105} & \frac{-L_i}{28} & \frac{-L_i^2}{140} \\ \frac{-6}{35} & \frac{-L_i}{28} & \frac{6}{35} & \frac{L_i}{70} \\ \frac{-L_i}{70} & \frac{-L_i^2}{140} & \frac{L_i}{70} & \frac{3 L_i^2}{70} \end{bmatrix} - m_i X_i \begin{bmatrix} \frac{3}{5} & \frac{L_i}{10} & \frac{-3}{5} & 0 \\ \frac{L_i}{10} & \frac{L_i^2}{30} & \frac{-L_i}{10} & \frac{-L_i^2}{60} \\ \frac{-3}{5} & \frac{-L_i}{10} & \frac{3}{5} & 0 \\ 0 & \frac{-L_i^2}{60} & 0 & \frac{L_i^2}{10} \end{bmatrix} \right)$$

where, for ith element $A_i = A_{i-1} + m_i (2i-1) L_i^2$ and

$X_i = (N-1) L_i$, N = number of elements

Effective element stiffness matrix is given by

$$K_{eff} = K + T$$

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