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Analytical Investigation of Depth Non-homogeneity Effect on the Dynamic Stiffness of Shallow Foundations

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Abstract: The vertical response of a rigid circular foundations resting on a continuously non-homogenous half space is studied analytically. The half space is considered as a liner-elastic media with a shear modulus increasing continuously with depth. The system of governing differential equations, based on the mentioned assumption, consist of two partial differential equations, is converted to ordinary equations' system by employing Hankel Integral transform. Using the method of extended power series (Frobenius Method) led to the general solution for the latter system. The mixed boundary problem is solved by introduction of functional expansion for the stress distribution under the foundation using appropriate base functions. Selected numerical results are presented to demonstrate the effect of depth non-homogeneity on the vertical dynamic stiffness (Impedance) of the foundation.

Keyword: Depth Non-homogeneity, Impedance Function, Circular Footing, Stress Distribution

1 Introduction

Considering a constant depth profile for the shear modulus of soil in different soil-structure interaction problems is a rather poor approximation to the real sub-soil conditions since soil stiffness usually varies with depth.

In this paper, as shown in Fig. 1, a rigid circular foundation, laid on the surface of a compressible linear –elastic media is considered. The mass density and Poisson's ratio of the media are constant but the shear modulus varies solely with depth. The variation of shear modulus is described by an exponential function as follows [Selvadurai (1986)].

$$G^{(}z) = G_\infty - (G_\infty - G_0)e^{-\alpha z} \tag{1}$$

Where G_0 and G_∞ are the shear modulus at the nsurface and infinite depth respectively and α is a constant with the dimension of inverse length, called coefficient of depth non-homogeneity or non-homogeneity parameter. By varying the parameters α, G_0 and G_∞, a wide range of real soil strata can be approximately described by Eq. 1.

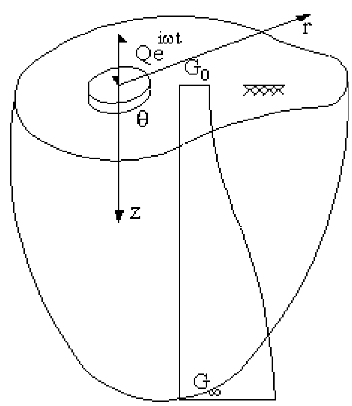

Figure 1: Circular foundation laid on a continuously non-homogenous half space

The importance of considering soil non-homogeneity into geotechnical analysis has been recognized in the past by Gibson (1974).

The fundamental study of Gibson on the response of a linearly non-homogeneous incompressible soil with zero surface modulus to a vertical surface load has cased to start a number of subse-

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quent studies to remove the restrictions such as incompressibility or vanishing surface modulus and also considering the various cases of vertical surface or interior loading of a half-space have been considered [Gibson and Sills (1971)], [Brown and Gibson (1972)], [Rajabkase (1990)].

The other class of non-homogeneous soil models refers to semi analytical and numerical methods using finite element or the other techniques for multilayered soils whereby the continuous modulus variation is approximated by a staircase profile [Gibson et. al. (1971)], [Oner(1990)], [Tadeu et al.(2004)] or implies a very strong non-homogeneity which is not appropriate for the soils [Gazetas(1980)].

A realistic analytical soil model, using the mentioned exponential function for the variation of shear modulus, was adopted by Vrettos to solve the dynamic and static Boussinesq problem [Vrettos (1991),(1988)]. The similar method have been used by the authors to present Green Functions for a continuously non-homogeneous saturated media and investigate the effect of depth non-homogeneity on these Functions [Seyrafian et al. (2006)] and more recently the Green Functions for the Unsaturated soils have been also presented [Jabbari et al. (2007)]. Also Vrettos has used the mentioned fundamental solution for study the settlement of rectangular footing on non-homogeneous soil by subdividing the contact area into a finite number of uniformly loaded quadratic elements and imposing the rigid body translation condition for the determining the stress magnitude at each element[Vrettos (1998)]. For the same model, Selvadurai treated the contact problem of the half-space identation by a rigid circular foundation by numerically solving the associated integral equations [Selvaduri(1986)].

In this paper, firstly the system of governing differential equations, for the mentioned media, obeying linear-elastic constitutive law is derived. The system of equations, formed by two coupled partial differential equations, is converted to ordinary differential equations' system by means of Hankel integral transforms. Then this system of equations is solved by use of generalized power series (Frobenius method) and the general solution is derived. Considering a functional expansion, using appropriate base functions [Noorzad (1994)], led to solve the mixed boundary value problem without any subdivision of the contact area. The coefficients of the functional expansion are determined by imposing a unit body translation and the final solution or expressions for the displacement field in the any interior point of the media are derived by satisfying the boundary conditions of the problem including the radiation condition. Finally the dynamic stiffness of the rigid circular foundation resting on the continuously depth non-homogeneous half space is presented and the effect of non-homogeneity parameter is investigated analytically.

2 Governing Differential Equations

Let (r,θ,z) be a cylindrical coordinate system, owing to the axisymmetric nature of the problem, the motions generated by the load configuration are independent of the angular coordinate θ and only displacements in the r-and z-directions occur. So the equations of motion are:

$$\frac{\partial\sigma_{rr}}{\partial r}+\frac{\partial\sigma_{zr}}{\partial z}+\frac{\sigma_{rr}-\sigma_{\theta\theta}}{r}=\rho\ddot{u}_r \tag{2}$$

$$\frac{\partial\sigma_{rr}}{\partial r}+\frac{\partial\sigma_{zz}}{\partial z}+\frac{\sigma_{rz}}{r}=\rho\ddot{u}_z \tag{3}$$

Where σ_{ij} $(i,j=r,\theta,z)$ are the components of the stress-tensor, ρ denotes mass density of media and over dotes indicates derivatives respect to time. According to the linear elastic constitutive law, the stress-displacement relations can be written as:

$$\begin{aligned}
\sigma_{rr}&=\lambda^*\left(\frac{\partial u_r}{\partial r}+\frac{u_r}{r}+\frac{\partial u_z}{\partial z}\right)+2G^*\frac{\partial u_r}{\partial r}\\
\sigma_{zz}&=\lambda^*\left(\frac{\partial u_r}{\partial r}+\frac{u_r}{r}+\frac{\partial u_z}{\partial z}\right)+2G^*\frac{\partial u_z}{\partial z}\\
\sigma_{\theta\theta}&=\lambda^*\left(\frac{\partial u_r}{\partial r}+\frac{u_r}{r}+\frac{\partial u_z}{\partial z}\right)+2G^*\frac{u_r}{r}\\
\sigma_{rz}&=G^*\left(\frac{\partial u_r}{\partial z}+\frac{\partial u_z}{\partial r}\right)
\end{aligned} \tag{4}$$

Where λ^* and G^* are complex Lame coefficients, for simplicity, the hysteretic type dissipation (frequency-independent) in the media is assumed. It is further assumed that this dissipation is the same in bulk (volumetric) and shear straining. So λ^* and G^* are defined as:

$$\begin{aligned} \lambda^* &= \lambda(1+2\delta i) \\ G^* &= G(1+2\delta i) \end{aligned} \tag{5}$$

Where δ is the hysteretic damping coefficient. The motion is time harmonic. So

$$\begin{aligned} u_r(r,z,t) &= u_r(r,z)e^{i\omega t} \\ u_z(r,z,t) &= u_z(r,z)e^{i\omega t} \end{aligned} \tag{6}$$

Using the above relation and the stress-displacement relations i.e. Eq. 4, the equations of motions, Eq. 2 and Eq. 3, after some mathematical operation are converted as following governing differential equations :

$$\overline{v}G^*\left(\frac{\partial^2 u_r}{\partial r^2}+\frac{1}{r}\frac{\partial u_r}{\partial r}-\frac{1}{r^2}u_r\right)+\rho\omega^2 u_r+G^*\frac{\partial^2 u_r}{\partial z^2} +\frac{\partial G^*}{\partial z}\frac{\partial u_r}{\partial z}+\left(\frac{G^*}{1-2v}\right)\frac{\partial^2 u_z}{\partial r\partial z}+\frac{\partial G^*}{\partial z}\frac{\partial u_z}{\partial r}=0 \tag{7}$$

$$\left(\frac{G^*}{1-2v}\right)\left(\frac{\partial^2 u_r}{\partial r\partial z}+\frac{1}{r}\frac{\partial u_r}{\partial z}\right)+\overline{v}G^*\frac{\partial^2 u_z}{\partial z^2} +\frac{2v}{1-2v}\frac{\partial G^*}{\partial z}\left(\frac{\partial u_r}{\partial r}+\frac{u_r}{r}\right)+G^*\left(\frac{\partial^2 u_z}{\partial r^2}+\frac{1}{r}\frac{\partial u_z}{\partial r}\right) +\overline{v}\frac{\partial G^*}{\partial z}\frac{\partial u_z}{\partial z}+\rho\omega^2 u_z=0 \tag{8}$$

Where

$$\overline{v}=\frac{2(1-v)}{1-2v}=\frac{\lambda^*+2G^*}{G^*} \tag{9}$$

The boundary conditions of problem are:

$$@z=0\ \ \sigma_{zz}(r,z)=d_{zz}(r)=\begin{cases}\sum\limits_{j=1}^{4}\alpha_j f_j(r) & r\le R\\ 0 & r>R\end{cases} \tag{10}$$

$$@z=0\ \ \sigma_{rz}(r,z)=0 \tag{11}$$

Where R is the radius of circular foundation and d_{zz} is the stress distribution under the foundation, as mentioned before is a functional expansion of f_j ($j=1,2,3$ and 4). These functions are defined as follows and are depicted in figure 2 [Nourzad (1994)]:

$$f_j(r)=\begin{cases}\frac{p+1}{\pi R^2}\left[1-\left(\frac{r}{R}\right)^2\right]^p, \quad p=\frac{j-2}{2} & r\le R\\ 0 & r>R\end{cases} \tag{12}$$

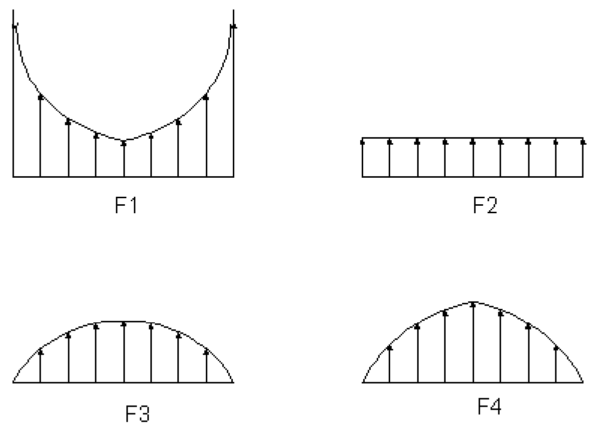


Figure 2: The base functions for the considered stress distribution

In addition, the solution must be such that the stresses and displacements are bounded at a remote distance and only outward waves propagating from the source appear (radiation condition).

The two coupled second-order partial differential equations i.e. Eqs. (8) and (9) subjected to above boundary conditions defines the boundary value problem for the vertical response of a rigid circular foundations resting on a continuously non-homogenous half space.

3 General Solution

The general solution of the system of governing differential equations can be obtained by employing a Hankel transforms for the radial coordinate r. So we have:

$$\begin{aligned} H_1(u_r(r,z)) &= \overline{u}_r(k,z)=\int_{\circ}^{+\infty} r u_r(r,z)J_1(kr)dr \\ H_{\circ}(u_z(r,z)) &= \overline{u}_z(k,z)=\int_{\circ}^{+\infty} r u_z(r,z)J_{\circ}(kr)dr \end{aligned} \tag{13}$$

Where k is the Hankel transforms parameter and J_n is the Bessel function of the first kind of order n. Substituting of above equations in Eqs. (8) and (9) and making use of the expressions for Hankel transform of the derivatives of a function yields:

$$G^*\frac{\partial^2\overline{u}_r}{\partial z^2}+\frac{\partial G^*}{\partial z}\frac{\partial\overline{u}_r}{\partial z}+\left(\rho\omega^2-k^2\overline{v}G^*\right)\overline{u}_r -k\left(G^*(\overline{v}-1)\right)\frac{\partial\overline{u}_z}{\partial z}-k\frac{\partial G^*}{\partial z}u_z=0 \quad (14)$$

$$((\overline{v}-1)G^*)\,k\frac{\partial\overline{u}_r}{\partial z}+(\overline{v}-2)\frac{\partial G^*}{\partial z}k\overline{u}_r+\overline{v}G^*\frac{\partial^2\overline{u}_z}{\partial z^2} +\overline{v}\frac{\partial G^*}{\partial z}\frac{\partial u_z}{\partial z}+\left(\rho\omega^2-k^2G^*\right)\overline{u}_z=0 \quad (15)$$

As it is mentioned before, employing the Hankel transforms made the system of governing partial differential equations to the above ordinary equations. To solve the latter system a subsidiary depth variable is introduced as follows

$$\xi=E_0e^{-\alpha z} \quad (16)$$

Where

$$E_0=1-\frac{G_0^*}{G_\infty^*} \quad (17)$$

Which transforms the interval $0\leq z\leq H$ onto $E\geq\xi\geq 0$, then shear modulus variation, Eq. 1, reduces to:

$$G^*=G_\infty^*(1-\xi) \quad (18)$$

E_0 can be regarded as a measure of the non-homogeneity of the half space medium. $E_0\to 0$ corresponds to the homogeneous half space ($G_0\to G_\infty$). Inserting the above transformations into the differential equations i.e. Eqs. (14) and (15) results in:

$$\alpha^2\xi^3\overline{u}_r''-\alpha^2\xi^2\overline{u}_r''+2\alpha^2\xi^2\overline{u}_r'-\alpha^2\xi\overline{u}_r'-k^2\overline{v}\xi\overline{u}_r -\left(\frac{\rho\omega^2}{G_\infty^*}-k^2\overline{v}\right)\overline{u}_r+k\alpha(\overline{v}-1)\xi^2\overline{u'}_z -(k\alpha(\overline{v}-1))\,\xi\overline{u'}_z+k\alpha\xi\overline{u}_z=0 \quad (19)$$

$$-\alpha k\left(\overline{v}-1\right)\xi^2\overline{u'}_r+\left(k\alpha\left(\overline{v}-1\right)\right)\xi\overline{u'}_r -\alpha k\left(\overline{v}-2\right)\xi\overline{u}_r+\overline{v}\alpha^2\xi^3\overline{u''}_z+\left(\alpha^2\overline{v}\right)\xi^2\overline{u''}_z +2\overline{v}\left(\alpha^2\xi^2\right)\overline{u'}_z-\left(\alpha^2\overline{v}\right)\xi\overline{u'}_z-k^2\xi u_z -\left(\frac{\rho\omega^2}{G_\infty^*}-k^2\right)\overline{u}_z=0 \quad (20)$$

Analytical solutions for the system of differential equations (33)-(36) can be found by using the Frobenius method (extended power series method). According to the method, the general solutions are given by a linear combination of power series as follows [Boyce and Diprima (1992)]:

$$\begin{aligned}\overline{u}_r&=\sum_{i=1}^{4}A_i(k)\sum_{n=0}^{\infty}a_n^i\xi^{n+m_i}\\ \overline{u}_z&=\sum_{i=1}^{4}A_i(k)\sum_{n=0}^{\infty}b_n^i\xi^{n+m_i}\end{aligned} \quad (21)$$

Where $A_i(k)$ are arbitrary functions to be determined from appropriate boundary conditions. If the above power series are used to Eqs. (19) and (20), we have:

$$\sum_{n=1}^{\infty}\left(\alpha^2(n+m-1)(n+m)-k^2\overline{v}\right)a_{n-1}\xi^{n+m} -\sum_{n=0}^{\infty}\left(\alpha^2(n+m)^2+\left(\frac{\rho\omega^2}{G_\infty^*}-k^2\overline{v}\right)\right)a_n\xi^{n+m} +\sum_{n=1}^{\infty}k\alpha\left((\overline{v}-1)(n+m-1)\right)b_{n-1}\xi^{n+m} -(k\alpha(\overline{v}-1))\sum_{n=1}^{\infty}b_n(n+m)\xi^{n+m}=0 \quad (22)$$

$$\sum_{n=1}^{\infty}-\alpha k\left((\overline{v}-1)(n+m-1)+(\overline{v}-2)\right)a_{n-1}\xi^{n+m} +\sum_{n=0}^{\infty}(k\alpha(\overline{v}-1))\,a_n(n+m)\xi^{n+m} +\sum_{n=1}^{\infty}\left((\overline{v}\alpha^2(n+m)(n+m-1)-k^2)\right)b_{n-1}\xi^{n+m} -\sum_{n=0}^{\infty}\left((\overline{v}\alpha^2)(n+m)^2+(\frac{\rho\omega^2}{G_\infty^*}-k^2)\right)b_n\xi^{n+m} =0 \quad (23)$$

So m_i are the complex roots of the following equations:

$$\det\begin{bmatrix} -\left(\alpha^2 m^2+(\frac{\rho\omega^2}{G^*_\infty}-k^2\overline{v})\right) & -(k\alpha(\overline{v}-1))m \\ (k\alpha(\overline{v}-1))m & -\left((\overline{v}\alpha^2)m^2+(\frac{\rho_f\omega^2}{G^*_\infty}-k^2)\right) \end{bmatrix} = 0 \quad (24)$$

and can be described as:

$$\begin{aligned} &m_1=+R+Ii \quad m_3=-R-Ii \\ &m_2=R'+I'i \quad m_4=-R'-I'i \quad R,I>0 \end{aligned} \quad (25)$$

The coefficients of the power series are determined for:

$$\begin{aligned} &a_0^i=1 \\ &b_0=-\frac{\left(\alpha^2 m^2+\left(\frac{\rho\omega^2}{G^*_\infty}-k^2\overline{v}\right)\right)}{(k\alpha(\overline{v}-1))m}a_0 \end{aligned} \quad (26)$$

By the two coupled recurrence relations:

$$\begin{aligned} &\left(\alpha^2(n+m-1)(n+m)-k^2\overline{v}\right)a_{n-1} \\ &-\left(\alpha^2(n+m)^2+\left(\frac{\rho\omega^2}{G^*_\infty}-k^2\overline{v}\right)\right)a_n \\ &+k\alpha\left((\overline{v}-1)(n+m-1)\right)b_{n-1} \\ &\qquad -\left(k\alpha(\overline{v}-1)\right)(n+m)b_n=0 \end{aligned} \quad (27)$$

$$\begin{aligned} &-\alpha k\left((\overline{v}-1)(n+m-1)\right)+(\overline{v}-2)a_{n-1} \\ &+\left(k\alpha(\overline{v}-1)\right)(n+m)a_n \\ &+\left(\overline{v}\alpha^2(n+m)(n+m-1)-k^2\right)b_{n-1} \\ &\qquad -\left((\overline{v}\alpha^2)(n+m)^2+\left(\frac{\rho\omega^2}{G^*_\infty}-k^2\right)\right)b_n=0 \end{aligned} \quad (28)$$

4 Solution of Boundary Value Problem

To satisfy the radiation condition at infinity we set:

$$A_3=A_4=0 \quad (29)$$

The other boundary conditions should be presented in the transformed domain, applying the integral transforms and also the subsidiary depth variable to relation (4), we have:

$$\begin{aligned} &\overline{\sigma}_{zz}(k,\xi) \\ &=\frac{2G^*v}{1-2v}k\overline{u}_r+\frac{2(1-v)}{1-2v}G^*(-\alpha\xi)\frac{d\overline{u}_z}{d\xi} \\ &\overline{\sigma}_{rz}(k,\xi)=G^*(-\alpha\xi)\left(\frac{d\overline{u}_r}{d\xi}\right)-kG^*\overline{u}_z \end{aligned} \quad (30)$$

Also by applying the Hankel transform to the base functions, we have:

$$H_0(f_j(r))=\frac{(p+1)2^p\Gamma(p+1)}{\pi(kR)^{p+1}}J_{p+1}(kR) \quad (31)$$

So the boundary conditions (10) and (11) in the transformed domain can be written as:

$$\begin{aligned} @\xi=E_0 \quad &\frac{2G^*v}{1-2v}k\overline{u}_r+\frac{2(1-v)}{1-2v}G^*(-\alpha\xi)\frac{d\overline{u}_z}{d\xi} \\ &=\sum_{j=1}^{4}a_j\frac{((p+1)2^p\Gamma(p+1))}{\pi(kR)^{p+1}}J_{p+1}(kR) \end{aligned} \quad (32)$$

$$@\xi=E_0 \quad G^*(-\alpha\xi)\left(\frac{d\overline{u}_r}{d\xi}\right)-kG^*\overline{u}_z=0 \quad (33)$$

Inserting the general solution, based relations (21) and (22) to the above boundary conditions we have:

$$\begin{aligned} &(\overline{v}-2)k\sum_{i=1}^{2}A_i\sum_{n=0}^{\infty}a_n^iE_0^{n+m_i} \\ &-\alpha\overline{v}\sum_{i=1}^{2}A_i\sum_{n=0}^{\infty}(n+m_i)b_n^iE_0^{n+m_i} \\ &\qquad=\sum_{j=1}^{4}a_j\frac{((p+1)2^p\Gamma(p+1))}{\pi(kR)^{p+1}G_0^*}J_{p+1}(kR) \end{aligned} \quad (34)$$

$$\begin{aligned} G_0^*\Bigg(&-\alpha\sum_{i=1}^{2}A_i\sum_{n=0}^{\infty}(n+m_i)a_n^iE_0^{n+m_i} \\ &-k\sum_{i=1}^{2}A_i\sum_{n=0}^{\infty}b_n^iE_0^{n+m_i}\Bigg)=0 \end{aligned} \quad (35)$$

Solving the above system of simultaneous equations, leads to determination of $A_1(k), A_2(k)$. Substituting these coefficients to Eqs. (21) and (22) and performing the inverse Hankel transforms, results in the following explicit solution for the displacement field at any point within domain of the non-homogeneous elastic half-space:

$$u_r(r,\xi)=\int_0^{+\infty}k\left(\sum_{i=1}^{2}A_i\sum_{n=0}^{\infty}a_n^i\xi^{n+m_i}\right)J_1(kr)dk \quad (36)$$

$$u_z(r,\xi)=\int_0^{+\infty}k\left(\sum_{i=1}^{2}A_i\sum_{n=0}^{\infty}b_n^i\xi^{n+m_i}\right)J_0(kr)dk \quad (37)$$

As the definition of Impedance function, the normal displacement under the foundation should be considered as unit. So the normal displacement is assumed to unit at 4 optional points under the foundation. So we have:

$$u_z(r_j,\xi) = 1.0 \quad 0 < r_j < R \quad j = 1,2,3 \text{ and } 4 \tag{38}$$

If the above system of 4 linear equations is solved, the coefficients of α_j $(j = \overline{1,4})$ will be determined. The introduced base functions for the stress distribution under the foundations have an interesting characteristic as follows:

$$\int_0^R (2\pi r) f_j(r)dr = 1 \tag{39}$$

So the force under the foundation is derived as:

$$\begin{aligned} F &= \int_0^R \sigma_{zz} \times (2\pi r)dr \\ &= \int_0^R \sum_{j=1}^{4} \alpha_j f_j(r) \times (2\pi r)dr = \sum_{j=1}^{4} \alpha_j \end{aligned} \tag{40}$$

on the other hand as the definition of Impedance function, we have:

$$K(\omega) = \frac{F}{u_z} = \frac{F}{1} = \sum_{j=1}^{4} \alpha_j \tag{41}$$

5 Results

The presented analytical solution in the previous sections has been applied to investigate the depth non-homogeneity effect on dynamic response of the circular foundation or in the other words the Impedance function of the foundation utilizing the dimensionless variables as follows:

$$\begin{aligned} Re(\overline{U}_z) &= \frac{Re(U_z)}{R} \\ Im(\overline{U}_z) &= \frac{Im(U_z)}{R} \\ Re(\overline{K}) &= \frac{Re(K)}{G_0R} \\ Im(\overline{K}) &= \frac{Im(K)}{G_0R} \\ \overline{\omega} &= \frac{\omega R}{v_s} \\ \overline{r} &= \frac{r}{R} \end{aligned} \tag{42}$$

Where $Re(\overline{U}_z)$ and $Im(\overline{U}_z)$ are the real and imaginary parts of dimensionless displacement and also $Re(\overline{K})$ and $Im(\overline{K})$ are the real and imaginary parts of dynamic stiffness or impedance function and $\overline{\omega}$ and $\overline{r}$ are the dimensionless frequency and polar radius or distance from the center of the circular foundation.

In order to study the effect of depth non-homogeneity on the dynamic response of the media , the variation of dimensionless real and imaginary parts of vertical displacements versus dimensionless distance are illustrated in figures 3 and 4 for different values of depth non-homogeneity parameter in each figure and for the different frequencies. As it is clearly visible the displacement under the foundation $(0 < \overline{r} < 1.0)$ equals to unit. The model's properties are given in Tab. 1. Also in Fig. 5 and Fig. 6, The diagrams of real and imaginary parts of impedance function versus frequency are drawn to show clearly how the dynamic behavior of foundation is dependent to non-homogeneity parameters i.e. $\frac{G_0}{G_\infty}$ and α.

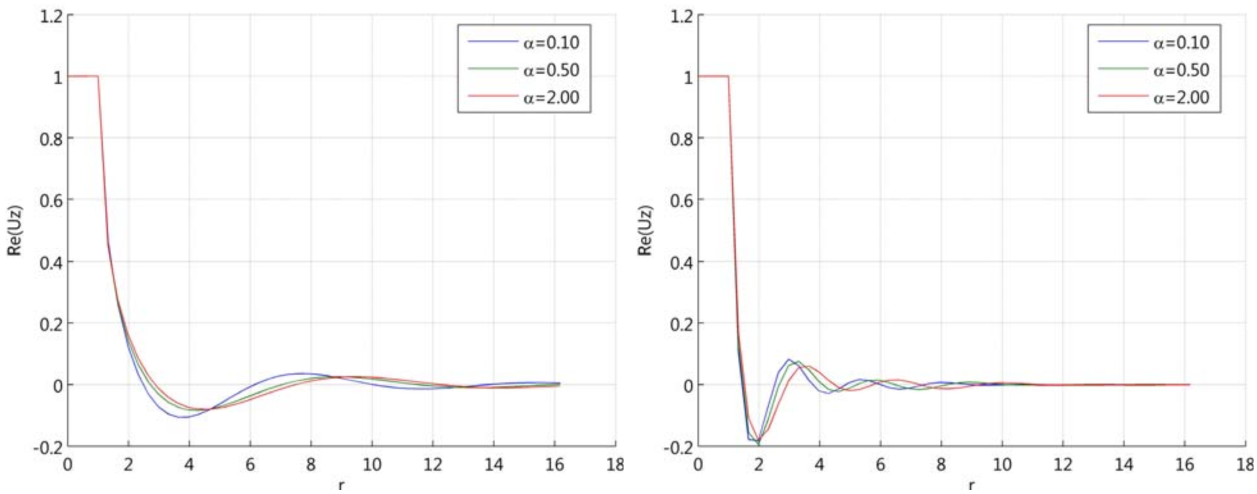
Table 1: Selected models' properties

No	Model's Parameter	Value	Unit
1	Shear modulus at surface (G_0)	18	Mpa
2	Mass density of soil (ρ)	1855	kg/m^3
3	Poisson ratio (v)	0.3	—
4	Hysteretic damping (δ)	0.04	—

To validate the formula derived above, the presented solution is computed for the state when $G_0/G_\infty \to 1$ and compared with the classic solutions available in literature for the homogeneous state. The agreement of the results is found to be excellent [Noorzad (1994)].

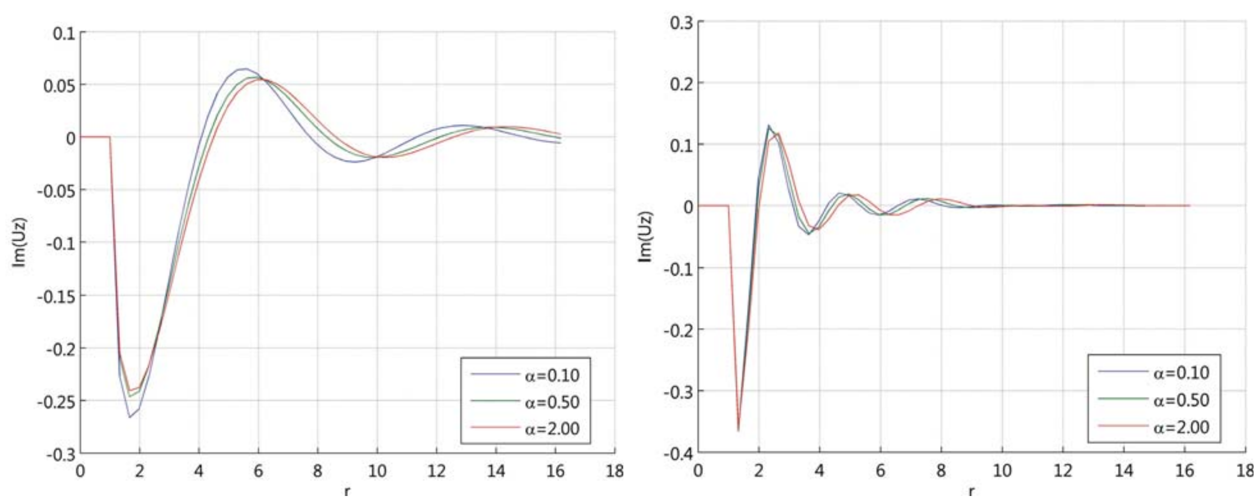
6 Conclusion

The effect of depth non-homogeneity on the stiffness of a circular shallow foundation is studied analytically. The mixed boundary problem has been solved by using a functional expansion for the stress distribution under the foundation. By choosing exponential function for the shear modulus depth-variation, the boundary value problem is solved by applying Hankel integral transform



$\overline{\omega} = 1$ $\overline{\omega} = 3$

Figure 3: Variation of real part of vertical displacements versus displacement for different depth non-homogeneity parameters ($\frac{G_0}{G_\infty} = 0.5$)



$\overline{\omega} = 1$ $\overline{\omega} = 3$

Figure 4: Variation of imaginary part of vertical displacements versus displacement for different depth non-homogeneity parameters ($\frac{G_0}{G_\infty} = 0.5$)

and using the extended power series method. Selected numerical results including the variation of real and imaginary parts of dynamic stiffness of the foundation versus frequency for different values of depth non-homogeneity parameter show the Impedance function of the media is dependant to shear modulus distribution and depth non-homogeneity.

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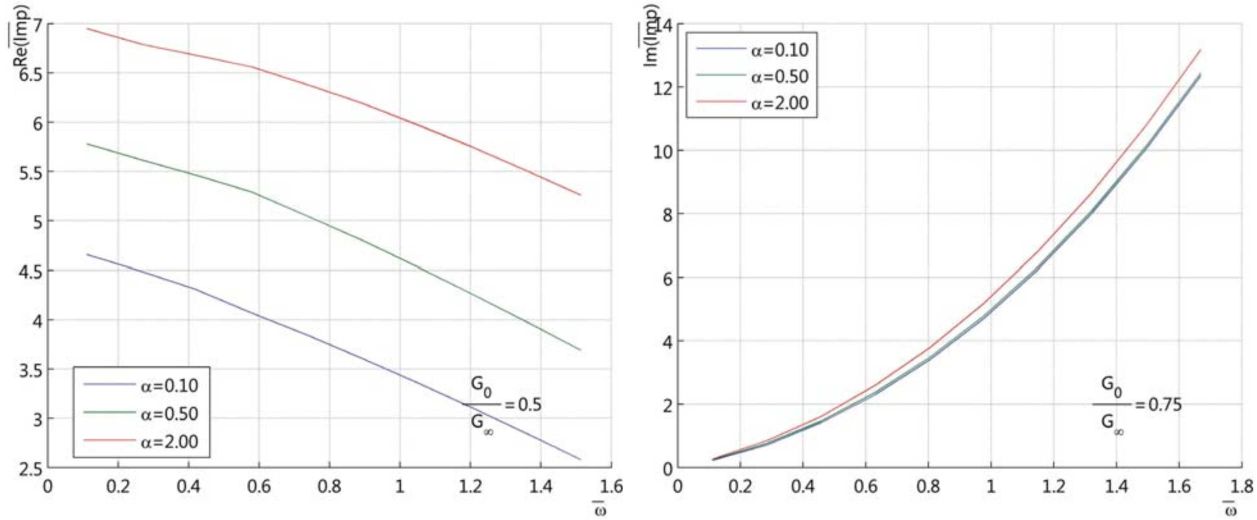

Figure 5: Variation of real part of Impedance function versus frequency for different depth non-homogeneity parameters

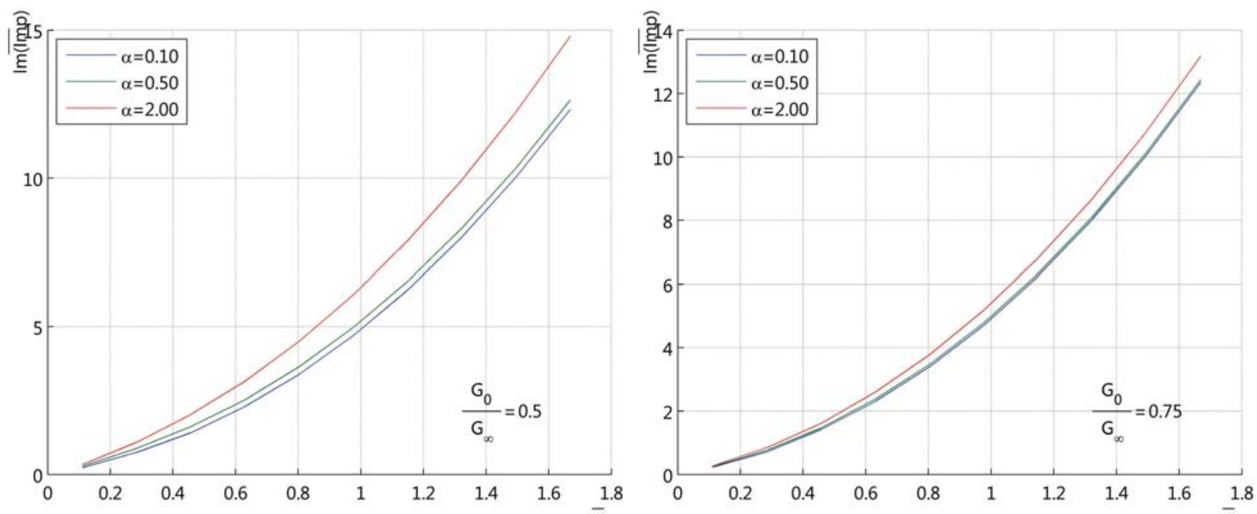

Figure 6: Variation of imaginary part of Impedance function versus frequency for different depth non-homogeneity parameters

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