 CMES, vol.23, no.3, pp.175-186, 2008

Modeling and Bending Vibration of the Blade of a Horizontal-Axis Wind Power Turbine

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Abstract: The blade of a horizontal-axis wind power turbine is modeled as a rotating beam with pre-cone angles and setting angles. Based on the Bernoulli-Euler beam theory, without considering the axial extension deformation and the Coriolis forces effect, the governing differential equations for the bending vibration of the beam are derived. It is pointed out that if the geometric and the material properties of the beam are in polynomial forms, then the exact solution for the system can be obtained. Based on the frequency relations as revealed, without tedious numerical analysis, one can reach many general qualitative conclusions between the natural frequencies and the physical parameters of the beams. The validity of the conclusions is not limited in specialized domains. Finally, the influences of the pre-cone angle, the angular speed and the setting angle on the natural frequencies of the beam are studied by the proposed numerical method. The phenomenon of divergence instability is also discussed.

Keyword: rotating beam, bending vibration, pre-cone angle

0.0.1 Introduction

Due to the increasing demand on the clean energy, wind power turbines are widely installed around the world. In the dynamic analysis of the horizontal-axis wind power turbines (HAWTs) [Egglestion and Stoddard (1987)], the blade can be modeled as a rotating non-uniform beam with pre-cone angles and setting angles.

Rotating beams are of importance in many practical applications such as turbine blades, helicopter rotor blades, airplane propellers, and robot manipulators. Such beams can also be presented as elements of multi-body dynamic systems (Huston and Liu (2005)). The problems have been studied for a long time. An interesting review of the subject can be found in the papers by Leissa (1981), Ramamurti and Balasubramanian (1984) and Rosen (1991).

Based on the Bernoulli-Euler beam theory, the governing characteristic differential equation for bending vibrations of rotating non-uniform beams is a fourth-order ordinary differential equation with variable coefficients expressed in terms of the flexural displacement [Lo et al. (1960)]. Carlson and Wang (1978) obtained an exact solution for the static bending of a rotating uniform Bernoulli-Euler beam. Rao and Carnegie (1970) and Hodges (1979) studied the steady response of a rotating cantilever non-uniform Bernoulli-Euler beam by using the Rayleigh-Ritz method. Ko (1989) studied the flexural behavior of rotating sandwich tapered beams with linearly distributed loads by using the finite difference technique. Hernried (1991) determined the in-plane (lag) and out-of-plane (flap) dynamic deflections of a flexible twisted non-uniform rotating blade through a mode superposition approach. Lee and Kuo (1992) and Lee and Lin (1994) provided the exact power series solution for the vibration of a rotating non-uniform beam. Recently, Vinod, etc. (2007) used the spectrally formulated finite element method to study the vibration and wave propagation of rotating beams. Singh, etc. (2007)

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used Genetic Programming to generate the empirical model of a finite element model for finding the natural frequencies of rotating beams.

The influence of tip mass, angular speed , hub radius, setting angle, taper ratio, pretwisted angle, inclined angle, and elastic root restraints on the natural frequencies of transverse vibrations of a rotating beam were investigated by many investigators [Pnueli (1972); Hodges and Rutkowshi (1981); Wright, etc. (1982); Liu and Yeh (1987); Storti and Aboelnaga (1987); Lee and Kuo [(1991), (1992)]; Lee, etc. (2004); Lee and Sheu (2007[1,2]). Wave propagation characteristics of rotating beams were studied by Vinod, etc (2006). Some other relevant researches about the rotating structures can be found in the works by Thakkar and Ganguli (2004, 2007) and Leu and Chen (2006).

From the existing literature, it can be found that no analytical solution for the vibration of a rotating beam with pre-cone angle had been presented. In addition, little attention has been focused on the investigation of the mechanism of rotating instability (divergence instability). In this paper, the blade of a horizontal-axis wind turbine is modeled as a rotating beam with pre-cone angles and setting angles. For simplicity, the beam theory employed is the Bernoulli-Euler beam theory. The extensional deformation and the Coriolis force effect are not considered. The beam considered is doubly symmetric such that the centroidal axis and the neutral axis are coincident. In addition, the width of the beam is considerably greater than the thickness of the beam. The analytical method given by Lee and Lin (1994) will be used to study the bending vibration of the beam system.

It is known that most of the numerical results can only provide partial qualitative conclusions. The conclusions are valid only in the specified domains those numerical analysis were performed. In addition, it requires tremendous computer calculation. In this paper, several frequency relations those provide general qualitative relations between the natural frequencies and the physical parameters are to be revealed without numerical analysis. Moreover, the influence of the coupling effect of the pre-cone angle and the setting angle and the angular speed on the natural frequencies will be investigated. The phenomenon of divergence instability will also be discussed.

1 Governing Equations and Boundary Conditions

Consider the pure bending vibration of a rotating Bernoulli-Euler beam, as shown in Figure 1. The beam is elastically restrained and mounted with a setting angle θ and a pre-cone angle ϕ on a hub with radius r_h. It rotates with constant angular velocity Ω.

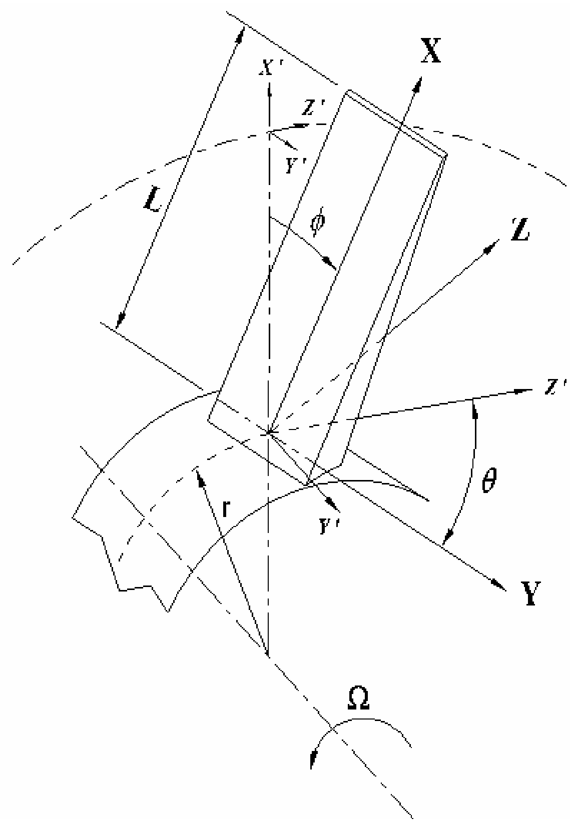

Figure 1: Geometry and coordinate system of a rotating non-uniform beam with an elastically restrained root.

The displacement fields of the beam in the x, y, and z-directions are

$$
\begin{aligned}
u &= z\frac{dw}{dx}, \\
v &= 0, \\
w &= w(x,t),
\end{aligned}
\tag{1}
$$

where z is the lateral distance of a point to the centroidal axis and t is the time variable. The velocity vector of a point (x, y, z) in a beam is given by

$$\begin{aligned}\vec{V} = & \\ & \left[\frac{du}{dt} + (z+w)\,\Omega \sin\theta \cos\phi + y\Omega \cos\theta \cos\phi\right]\vec{i} \\ & + \left[-(x+r_h+u)\,\Omega \cos\theta \cos\phi - (z+w)\,\Omega \sin\phi\right]\vec{j} \\ & + \left[\frac{dw}{dt} + y\Omega \sin\phi - (x+r_h+u)\,\Omega \sin\theta \cos\phi\right]\vec{k}\end{aligned} \tag{2}$$

and the kinetic energy T of the rotating beam can be expressed as

$$T = \frac{1}{2}\int_0^L \rho A(\vec{V}\cdot\vec{V})dx, \tag{3}$$

where ρ, A and L are the mass per unit length, the cross sectional area and the length of the beam, respectively.

Based on the Bernoulli-Euler beam theory, only the normal strain ε_x is considered. The nonlinear strain-displacement relation yields

$$\varepsilon_x = -z\frac{\partial^2 w}{\partial x^2} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2. \tag{4}$$

The potential energy U of the rotating beam is

$$U = \frac{1}{2}\iint E{\varepsilon_x}^2 dAdx \tag{5}$$

where E is the Young's modulus of the beam.

Application of Hamilton's principle, without considering the Coriolis force, yields the following governing differential equation:

$$\begin{aligned}&\frac{\partial^2}{\partial x^2}\left(EI\frac{\partial^2 w}{\partial x^2}\right) - \frac{\partial}{\partial x}\left(N\frac{\partial w}{\partial x}\right) \\ &-\rho A\left(w\Omega^2(\sin^2\theta\cos^2\phi + \sin^2\phi) - \frac{\partial^2 w}{\partial t^2}\right) = 0\end{aligned} \tag{6}$$

and the corresponding boundary conditions at $x = 0$:

$$-EI\frac{\partial^2 w}{\partial x^2} + k_\theta\frac{\partial w}{\partial x} = 0, \tag{7}$$

$$\frac{\partial}{\partial x}\left(EI\frac{\partial^2 w}{\partial x^2}\right) - N\frac{\partial w}{\partial x} + k_T w = 0. \tag{8}$$

and at $x = L$:

$$EI\frac{\partial^2 w}{dx^2} = 0, \tag{9}$$

$$\frac{\partial}{\partial x}\left(EI\frac{\partial^2 w}{\partial x^2}\right) = 0. \tag{10}$$

where I, k_θ, k_T are the area moment of inertia, the rotational spring constant and the translational spring constant of the beam, respectively. Here, $N(x)$ is the centrifugally stiffened force $N = EA(dw/dx)$. The second term in equation (7) is a nonlinear term induced from the nonlinear strain displacement relation (4). The centrifugally stiffened force $N(x)$ is used to be considered as the steady state normal force and is derived as

$$N(x) = \Omega^2\cos^2\phi\int_x^L \rho A\,(s+r_h)\,ds \tag{11}$$

Consequently, the governing differential equation (6) is reduced to a linear one. If the pre-cone angle is zero, the $N(x)$ will be the same as that of a conventional rotating beam [Lee and Lin (1994)]. It can be observed that if the pre-cone angle is increased, the axial centrifugal force decreases.

2 Solution Method

For time-harmonic vibration of a rotating beam with angular frequency ω, one assumes

$$w(x,t) = \tilde{w}(x)e^{i\omega t} \tag{12}$$

In terms of the following dimensionless parameters:

$$\begin{aligned}&\xi = \frac{x}{L},\ b(\xi) = \frac{E(x)I(x)}{E(0)I(0)},\ m(\xi) = \frac{\rho(x)A(x)}{\rho(0)A(0)}, \\ &n(\xi) = \frac{N(x)}{\rho(x)A(x)\Omega^2L^2}, \quad \Lambda = \sqrt{\frac{\rho(0)A(0)}{E(0)I(0)}}\omega L^2, \\ &\mu = \frac{r_h}{L}, \quad \alpha = \sqrt{\frac{\rho(0)A(0)}{E(0)I(0)}}\Omega L^2, \quad W = \frac{\tilde{w}}{L},\end{aligned} \tag{13}$$

the governing characteristic differential equation can be rewritten in the following dimensionless form:

$$\frac{d^2}{d\xi^2}\left[b(\xi)\frac{d^2W}{d\xi^2}\right]-\frac{d}{d\xi}\left[n(\xi)\frac{dW}{d\xi}\right]$$
$$-m(\xi)\left[\alpha^2\left(\sin^2\theta\cos^2\phi+\sin^2\phi\right)+\Lambda^2\right]W=0. \tag{14}$$

Here, $n(\xi)=\alpha^2\cos^2\phi\int_\xi^1 m(\mu+\chi)d\chi$. The associated boundary conditions become at $\xi=0$:

$$\frac{d}{d\xi}\left(\frac{d^2W}{d\xi^2}\right)-n\frac{dW}{d\xi}+\beta_T W=0, \tag{15}$$

$$\beta_\theta\frac{dW}{d\xi}-\frac{d^2W}{d\xi^2}=0. \tag{16}$$

and at $\xi=1$:

$$\frac{d^2W}{d\xi^2}=0, \tag{17}$$

$$\frac{d}{d\xi}\left(b\frac{d^2W}{d\xi^2}\right)=0. \tag{18}$$

If the four linearly independent fundamental solutions $V_j(\xi)$, j = 1, 2, 3, 4, of the governing characteristic equations (14) are chosen such that they satisfy the following normalization conditions at the origin of the coordinate system:

$$\begin{bmatrix} V_1 & V_2 & V_3 & V_4 \\ V_1' & V_2' & V_3' & V_4' \\ V_1'' & V_2'' & V_3'' & V_4'' \\ V_1''' & V_2''' & V_3''' & V_4''' \end{bmatrix}_{\xi=0}=\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \tag{19}$$

where primes indicate differentiation with respect to the dimensionless spatial variable ξ, then after substituting the homogeneous solution which is a linear combination of the four fundamental solutions into the four associated boundary conditions, one obtains the frequency equation of the system

The frequency equation is tabulated in the Appendix and the natural frequencies of the system can be determined via the frequency equation.

The governing characteristic differential equation of the system is a fourth-order differential equation with variable coefficients. In general, the closed-form fundamental solutions of the differential equation are not available. However, if the coefficients of the differential equation can be expressed in polynomial forms, then the closed-form power series solutions can be obtained by following the algorithm developed by Lee and Lin (1992).

3 Frequency Relations

The natural frequencies of the system can be numerically determined by the method revealed in the previous section and many other approximated methods such as the finite element method, the finite difference method, the Galerkin method and the dynamic stiffness method, .. etc.. However, most of the numerical results can only provide partial qualitative conclusions. The conclusions are valid only in the specialized domains those numerical analysis are performed. In addition, it requires tremendous computer calculation. In this section, several qualitative relations are explored and many general qualitative conclusions are revealed without numerical analysis.

3.1 Frequency relations for the systems with different pre-cone angle rotational speed, setting angle and natural frequency

Consider two dynamic systems with the same physical parameters except the dimensionless rotational speed α, the setting angle θ, the pre-cone angle ϕ and the dimensionless natural frequency Λ_i. Here Λ_i denotes the $i-$th dimensionless natural frequency. To specify two different systems, subscripts "a" and "b" are added to the associated physical parameters.

It is observed that if the following relations exist

$$\alpha_a^2\cos^2\phi_a = \alpha_b^2\cos^2\phi_b, \tag{20}$$

$$\alpha_a^2\left(\sin^2\theta_a\cos^2\phi_a+\sin^2\phi_a\right)+\Lambda_{a,i}^2$$
$$=\alpha_b^2\left(\sin^2\theta_b\cos^2\phi_b+\sin^2\phi_b\right)+\Lambda_{b,i}^2, \tag{21}$$

then the governing characteristic differential equation (14) and the associated boundary conditions (15-18) will be the same. Therefore the fundamental solutions of the two systems will be the same. It implies that

a. If all the physical parameters of the system "a"; are known and the dimensionless natural frequencies $\Lambda_{a,i}$ of the system are determined, then the dimensionless natural frequencies $\Lambda_{b,i}$ of the system "b" with physical parameters, $\{\alpha_b,\ \theta_b,\ \phi_b\}$, satisfying the relations (20-21) can be easily determined via the relation (21).

The $(i+j)$-th dimensionless natural frequency Λ_{i+j} will satisfy the relation (21) as well

$$\alpha_a^2\left(\sin^2\theta_a\cos^2\phi_a+\sin^2\phi_a\right)+\Lambda_{a,i+j}^2 = \alpha_b^2\left(\sin^2\theta_b\cos^2\phi_b+\sin^2\phi_b\right)+\Lambda_{b,i+j}^2. \quad (22)$$

Subtracting equation (21) from equation (22), one has the following frequency relation

$$\Lambda_{a,i+j}^2-\Lambda_{a,i}^2 = \Lambda_{b,i+j}^2-\Lambda_{b,i}^2. \quad (23)$$

This relation shows that

b. The difference between the square of the two dimensionless natural frequencies of two systems those satisfy the relations (20-21) are the same.

3.2 Frequency relations for the systems with the same angular speed and pre-cone angle

If two systems have the same angular speed and pre-cone angle, then $\alpha_a=\alpha_b=\alpha$, $\phi_a=\phi_b=\phi$ and the relation (20) is satisfied. Relation (21) can be rewritten as

$$\Lambda_{b,i}^2=\alpha^2\cos^2\phi\left(\sin^2\theta_a-\sin^2\theta_b\right)+\Lambda_{a,i}^2. \quad (24)$$

This relation reveals the following conclusions:

a. When the setting angle is less than 90°, the natural frequencies of a beam with constant angular speed and pre-cone angle will decrease as the setting angle is increased.

b. The influence of the setting angle on the natural frequencies of a beam rotating at high speed is greater than that of a beam rotating at low speed.

c. The influence of the setting angle on the natural frequencies of a beam with small pre-cone angle is greater than that of a beam with large pre-cone angle.

d. The smaller the axial centrifugal factor $\alpha^2\cos^2\phi$ is, the less influence of the setting angle on the natural frequencies is.

e. For a non-rotating beam, the setting angle and the pre-cone angle will have no influence on the natural frequencies of the beam.

f. When the pre-cone angle ϕ = 90°, the setting angle will have no influence on the natural frequencies of the beam.

g. For a beam with constant axial centrifugal factor $\alpha^2\cos^2\phi$, the influence of the setting angle on the natural frequency of higher mode is less significant than that of lower mode.

For a beam with constant angular speed and pre-cone angle $\phi_a=\phi b=0$, the frequency relation (21) is reduced to

$$\alpha^2\sin^2\theta_a+\Lambda_{a,i}^2=\alpha^2\sin^2\theta_b+\Lambda_{b,i}^2 \quad (25)$$

It is exactly the same as that revealed by Lee and Sheu (2007).

4 Numerical Results

To illustrate the previous analysis and investigate the influence of the parameters on the natural frequencies of the rotating non-uniform beam, several numerical results are presented and discussed. In the following numerical analysis, the material properties and the width of the beam are assumed to be constants and the depth of the beam varied linearly with the taper ratio λ. Therefore, the dimensionless mass per unit length and the dimensionless bending rigidity of the beam are $m=(1+\lambda\xi)$ and $b=(1+\lambda\xi)^3$, respectively. When $\lambda=0$, it represents a uniform beam.

In Table 1, the first four natural frequencies of a cantilevered non-uniform beam determined by the method proposed in this paper are compared with those in the existing literatures. It shows that the results are very consistent.

Table 1: Natural Frequencies of a cantilevered non-uniform beam [$m=(1-0.1\xi)$, $b=(1-0.1\xi)^3$, $\mu=0$, $\theta=0°$, $\phi=0°$]

α	Λ	*	#	##
0	Λ_1	3.8238	3.8238	3.8238
	Λ_2	18.317	18.317	18.317
	Λ_3	47.265	47.265	47.265
	Λ_4	90.450	90.450	-
3	Λ_1	5.0927	5.0927	5.0927
	Λ_2	19.684	19.684	19.684
	Λ_3	48.619	48.619	48.619
	Λ_4	91.822	91.822	-
5	Λ_1	6.7434	6.7434	6.7345
	Λ_2	21.905	21.905	21.905
	Λ_3	50.934	50.934	50.934
	Λ_4	94.206	94.206	-
10	Λ_1	11.502	11.502	11.502
	Λ_2	30.183	30.183	30.183
	Λ_3	60.564	60.564	60.564
	Λ_4	104.61	104.61	-

*: given by the proposed method
#: given by Lee and Lin (1994)
##: given by Hodges and Rutkowski (1981)

Table 2: Prediction of the first two natural frequencies Λ_b of uniform cantilevered Bernoulli-Euler beams. [$m=b=1$, $\mu=0$, $\alpha_a=5$, $\phi_a=0°$, $\theta_a=45°$, $\Lambda_{a,1}=5.3941$, $\Lambda_{a,2}=25.1993$]

$\alpha^2\cos^2\phi$	α_b	ϕ_b	θ_b	$\Lambda_{b,1}$	$\overline{\Lambda}_{b,1}$	$\Lambda_{b,2}$	$\overline{\Lambda}_{b,2}$
	5.0771	10	0	6.3890	6.3890	25.4308	25.4308
			30	5.8796	5.8796	25.3076	25.3076
			60	4.6978	4.6978	25.0594	25.0594
			90	3.9774	3.9774	24.9344	24.9344
25	5.3209	20	0	6.1875	6.1875	25.3809	25.3809
			30	5.6599	5.6599	25.2575	25.2575
			60	4.4198	4.4198	25.0088	25.0088
			90	3.6448	3.6448	24.8836	24.8836
	5.7735	30	0	5.7674	5.7674	25.2818	25.2818
			30	5.1974	5.1974	25.1579	25.1579
			60	3.8096	3.8096	24.9082	24.9082
			90	2.8746	2.8746	24.7824	24.7824

$\Lambda_{b,i}$: ¡Gdimensionless natural frequencies determined by the proposed numerical method
$\overline{\Lambda}_{b,i}$: ¡Gdimensionless natural frequencies evaluated via the relations (21, 22)

Table 3: Frequency relations between rotating cantilevered Bernoulli - Euler beams [$m=(1+\lambda\xi)$, $b=(1+\lambda\xi)^3$, $\mu=0$]

				$\lambda=-0.5$					$\lambda=0$				
$\alpha^2\cos^2\phi$	ϕ	α	θ	Λ_1^2	Λ_2^2	Λ_3^2	$\Lambda_2^2-\Lambda_1^2$	$\Lambda_3^2-\Lambda_2^2$	Λ_1^2	Λ_2^2	Λ_3^2	$\Lambda_2^2-\Lambda_1^2$	$\Lambda_3^2-\Lambda_2^2$
4	0	2	0	19.685	358.596	2291.692	338.911	1933.096	17.117	511.435	3877.950	494.318	3366.515
			45	17.685	356.596	2289.692	338.911	1933.096	15.117	509.435	3875.950	494.318	3366.515
			90	15.685	354.596	2287.692	338.911	1933.096	13.117	507.435	3873.950	494.318	3366.515
	30	2.309	0	18.352	357.263	2290.359	338.911	1933.096	15.784	510.102	3876.617	494.318	3366.515
			45	16.352	355.263	2288.359	338.911	1933.096	13.784	508.102	3874.617	494.318	3366.515
			90	14.352	353.263	2286.359	338.911	1933.096	11.784	506.102	3872.617	494.318	3366.515
	60	4	0	7.685	346.596	2279.692	338.911	1933.096	5.117	499.435	3865.950	494.318	3366.515
			45	5.685	344.596	2277.692	338.911	1933.096	3.117	497.435	3863.950	494.318	3366.515
			90	3.685	342.596	2275.692	338.911	1933.096	1.117	495.435	3861.950	494.318	3366.515
36	0	6	0	58.601	543.322	2752.394	484.721	2209.072	54.175	718.727	4446.744	664.552	3728.017
			45	40.601	525.322	2734.394	484.721	2209.072	36.175	700.727	4428.744	664.552	3728.017
			90	22.601	507.322	2716.394	484.721	2209.072	18.175	682.727	4410.744	664.552	3728.017
	15	6.212	0	56.017	540.738	2749.810	484.721	2209.072	51.590	716.142	4444.159	664.552	3728.017
			45	38.017	522.738	2731.810	484.721	2209.072	33.590	698.142	4426.159	664.552	3728.017
			90	20.017	504.738	2713.810	484.721	2209.072	15.590	680.142	4408.159	664.552	3728.017
	30	6.928	0	46.601	531.322	2740.394	484.721	2209.072	42.175	706.727	4434.744	664.552	3728.017
			45	28.601	513.322	2722.394	484.721	2209.072	24.175	688.727	4416.744	664.552	3728.017
			90	10.601	495.322	2704.394	484.721	2209.072	6.175	670.727	4398.744	664.552	3728.017

In Table 2, the first two natural frequencies of the system "*b*" are determined by employing the proposed numerical method in section 3 and the relations (20-21), respectively. It can be found that the results are consistent.

In Table 3, the frequency relation (23) is illustrated.

Figure 2 shows the influence of the pre-cone angle on the first natural frequency of a rotating cantilevered beam with setting angle being zero and different angular speeds. One can observe that:

a. The pre-cone angle will have no influence on the natural frequencies of a non-rotating beam. This conclusion is obvious. Since when the angular speed is zero, the pre-cone angle will also disappeared from the coefficients of the governing characteristic differential equation (14).

b. The natural frequencies of a clamped rotating beam will decrease when the pre-cone angle is increased.

c. When the pre-cone angle is small, the natural frequencies of a beam with high angular speed are greater than those with low angular speed. However, when the pre-cone angle is greater than the critical value, the natural frequency of the beam with high angular speed will be less than those with low angular speed.

d. The influence of the pre-cone angle on the natural frequencies of a beam with high angular speed is greater than that of the beam with low angular speed.

e. The phenomenon of divergence instability that revealed by Lee and Kuo (1991) will happen as the angular speed and the pre-cone angle are greater than certain values.

It can be observed that the last 2^{nd} term, $-\rho A\Omega^2(\sin^2\theta\cos^2\phi+\sin^2\phi)w$ in the governing differential equation (6) acts as a negative spring. As the value of $\rho A\Omega^2(\sin^2\theta\cos^2\phi+\sin^2\phi)w$ is increased, the natural frequencies of the system will decrease. The decreasing rate of the natural frequencies for the beam with high angular speed will be greater than that with low angular speed as the pre-cone angle is increased. This explains the last three phenomena revealed in Figure 2.

In Figure 3, the influence of the pre-cone angle on the first three natural frequencies of a cantilevered rotating beam with setting angle being zero and different angular speeds is shown. It can be found that the critical pre-cone angle, as mentioned in the conclusion "c" in Figure 2, associated with higher vibration mode will be greater than that associated with lower vibration mode.

In Figure 4, the influence of the pre-cone angle, the setting angle and the angular speed on the first natural frequency of a cantilevered beam is shown. It can be found that:

a. When the pre-cone angle or the setting angle is increased, the associated natural frequencies decrease.

b. When the setting angle is the same, the influence of the pre-cone angle on the natural frequencies of a beam with high angular speed is greater than that of the beam with low angular speed. This conclusion is an extension of the conclusion "d" revealed in Figure 2, in which the setting angle of the beam is zero.

c. When the pre-cone angle ϕ = 90°, there is no axial centrifugal force. In this case, the setting angle θ will have no influence on the natural frequencies of the beam. This conclusion is consistent with our common physical sense.

d. When the setting angle is increased, the associated critical pre-cone angle for the happening of the divergence instability phenomenon will decrease.

In Figure 5, the influence of the translational spring constant and the pre-cone angle on the first natural frequency of a beam is revealed. One can observed that when the translational spring constant is decreased, the associated critical pre-cone angle for the happening of the divergence instability phenomenon will decrease.

Finally, it should be mentioned that in this paper, for simplicity, the linear theory is used to study dynamic behaviors of the beam system. All the

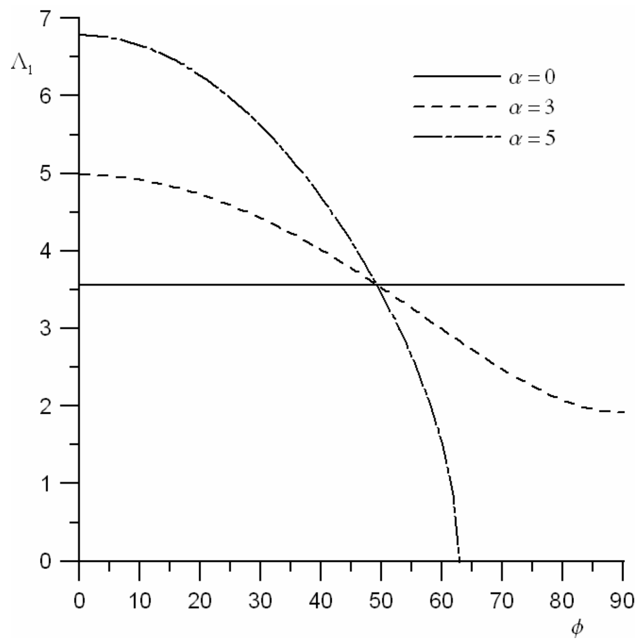

Figure 2: Influence of the pre-cone angle on the first natural frequency of a cantilevered rotating beam with different angular speeds. [$m = (1-0.1\xi), b = (1-0.1\xi)^3$, $\mu = 0.1$, $\theta = 0°$]

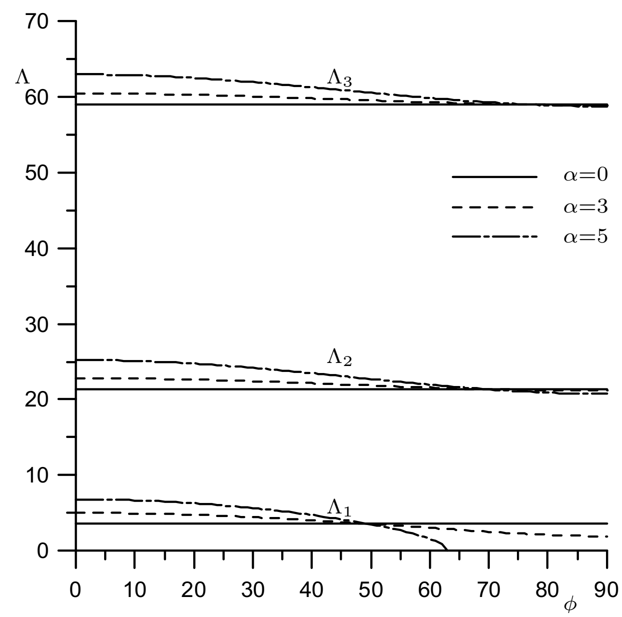

Figure 3: Influence of the pre-cone angle on the first three natural frequencies of a cantilevered rotating beam with different angular speeds. [$m = (1-0.1\xi), b = (1-0.1\xi)^3$, $\mu = 0.1$, $\theta = 0°$]

values of physical parameters in the present numerical analysis are mainly used to illustrate the qualitative information of this linear beam system. In practice, when the dimensionless angular speed α is greater than certain value, the axial extension deformation, the Coriolis force, the shear deformation, the rotatory inertia and the nonlinear effects will turn to be significant. If solutions of high accuracy are required, then an advanced theory should be adapted [Kaza and Kvaternik (1977)].

It is well known that most of the qualitative behav-

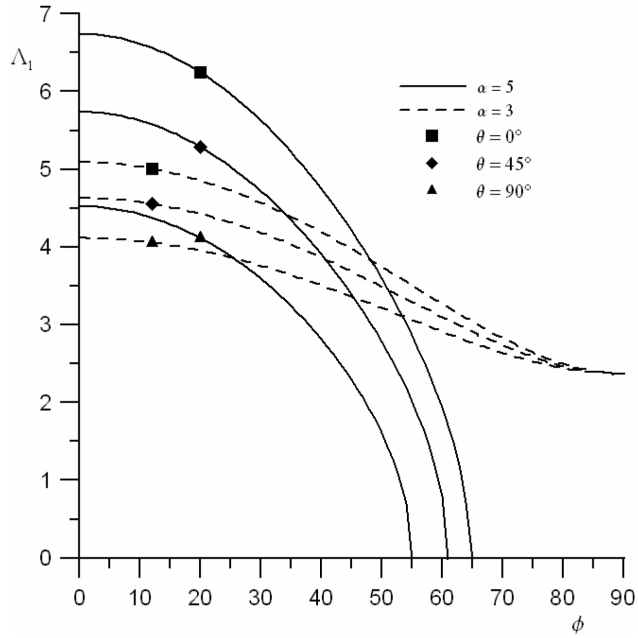

Figure 4: Influence of the angular speed and the pre-cone angle on the first natural frequency of a cantilevered beam. [$m = (1-0.1\xi)$, $b = (1-0.1\xi)^3$, $\mu = 0$]

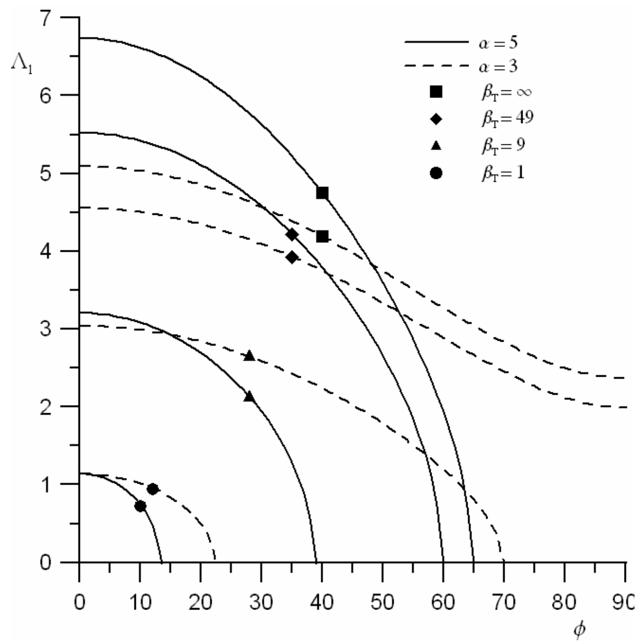

Figure 5: Influence of the translational spring constants β_T and the pre-cone angle on the first natural frequency of a beam. [$m = (1-0.1\xi)$, $b = (1-0.1\xi)^3$, $\mu = 0$, $\theta = 0°$, $\beta_\theta = 0$]

iors of a simple system will either exist or have similar behaviors while they are re-evaluated by advanced theories. Even though some of the numerical data presented in this paper may be not accurate enough in practice, it still provides valuable physical observations and information to the literature.

5 Conclusions

In this paper, a wind power turbine blade is modeled as a rotating beam with pre-cone angles and setting angles. Based on the Bernoulli-Euler beam theory, without considering the axial extension deformation and the Coriolis force effect, the governing differential equations for the pure bending vibrations of the rotating non-uniform beam are derived. It is pointed out that if the geometric and the material properties of the beam are in polynomial forms, then the exact solution for the system can be obtained.

In the previous analysis, most of the qualitative conclusions about the dynamic behavior of the beam are not general and valid only in the specialized domains that numerical analyses are performed. In the present analysis, based on the frequency relations as revealed, many general qualitative conclusions between the natural frequencies and the physical parameters of the beams are explored without numerical analysis. The conclusions are valid in the entire domains. In addition, the influences of the pre-cone angle, the angular speed and the setting angle on the natural frequencies of the beam are also investigated numerically. The phenomenon of divergence instability is discussed.

In this paper, for simplicity, the Bernoulli-Euler beam theory is employed to study dynamic behaviors of the beam system. To improve the accuracy of the analysis, one should extend the work by using the Timoshenko beam theory. In addition, the axial extension deformation, the Coriolis force and the nonlinear effects can also be considered.

Acknowledgement: This research work was supported by the National Science Council of Taiwan, Republic of China, under grant: NSC95-2221 -E-006-033 and the Ministry of Economic Affairs of Taiwan, Republic of China; under grant: TDPA: 95-EC-17-A-05-S1-0014.

References

Carlson, R.M.; Wong, J.T. (1978): An Exact Solution for the Static Bending of Uniform Rotating Beams. *Journal of the American Helicopter Society*, vol. 23 , no. 4, pp. 36-38.

Egglestion, D.M.; Stoddard, F.S. (1987): *Wind Turbine Engineering Design*. Van Nostrand Reinhold. New York.

Hernried, A.G. (1991): Forced Vibration Response of a Twisted Non-uniform Rotating Blade. *Computers & Structures*, vol. 41, no. 2, pp. 207-212.

Hodges, D.H. (1979): Vibration and Response of Non-uniform Rotating Beams with Discontinuities. *Journal of the American Helicopter Society*, vol. 24, pp. 43-50.

Hodges, D.H.; Rutkowski, M.J. (1981): Free-Vibration Analysis of Rotating Beams by a Variable-Order Finite-Element Method. *AIAA Journal*, vol. 19, no. 11, pp. 1459-1466.

Huston, R.L.; Liu, C.Q. (2005): Advances in Computational Methods for Multibody System Dynamics. *CMES: Computer Modeling in Engineering and Sciences*, vol. 10, no. 2, pp. 143-152.

Kaza, K.R.V.; Kvaternik, R.G. (1977): Nonlinear Flap-Lag-Axial Equations of a Rotating Beam. *AIAA Journal*, vol. 15, no. 6, pp. 871-874.

Ko, C.L. (1989): Flexural Behavior of a Rotating Sandwich Tapered Beam. *AIAA Journal*, vol. 27, no. 3, pp. 359-369.

Lee, S.Y.; Kuo, Y.H. (1992): Bending Vibrations of a Rotating Un-uniform Beam with an Elastically Restrained Root. *Journal of Sound and Vibration*, vol. 154, no. 3, pp. 441-451.

Lee, S.Y.; Lin, S.M. (1994): Bending Vibrations of Rotating Non-uniform Timoshenko Beam with an Elastically Restrained Root. *ASME Journal of Applied Mechanics*, vol. 61, no. 4, pp. 949-955.

Lee, S.Y.; Lin, S.M.; Wu, C.T. (2004): Free Vibration of a Rotating Non-uniform Beam with Arbitrary Pretwist, an Elastically Restrained Root and a Tip Mass. *Journal of Sound and Vibration*, vol. 273, pp. 477-492.

Lee, S.Y.; Sheu, J.J. (2007[1]): Free Vibrations of an Inclinded Rotating Beam. *ASME Journal of Applied Mechanics*, vol. 74, pp.406-414.

Lee, S.Y.; Sheu, J.J. (2007[2]): Free Vibrations of a Rotating Inclinded Timoshenko Beam. *Journal of Sound and Vibration*, vol. 304, pp. 606-624.

Leissa, A. (1981): Vibrational Aspects of Rotating Turbomachnery Blades. *ASME Applied Mechanics Reviews*, vol. 34, no. 5, pp. 629-635.

Leu, S.Y.; Chen, J.T. (2006): Sequential Limit Analysis of Rotating Hollow Cylinders of Nonlinear Isotropic Hardening. *CMES: Computer Modeling in Engineering and Sciences*, vol. 14, no. 2, pp. 129-140.

Liu, W.H.; Yeh, F.H. (1987): Vibrations of Non-Uniform Rotating Beams. *Journal of Sound and Vibration*, vol. 119, no. 2, pp. 379-384.

Lo, H.; Goldberg, J.E.; Bogdanoff, J.L. (1960): Effect of Small Hub-Radius Change on Bending Frequencies of a Rotating Beam. *ASME Journal of Applied Mechanics*, vol. 27, no. 3, pp. 548-550.

Pnueli, D. (1972): Natural Bending Frequency Comparable to Rotating Frequency in Rotating Cantilever Beam. *ASME Journal of Applied Mechanics*, vol. 39, pp. 602-604.

Ramamurti, V.; Balasubramanian, P. (1984): Analysis of Turbomachine Blades: A Review. *The Shock and Vibration Digest*, vol. 16, pp. 13-28.

Rao, J.S.; Carnegie, W. (1970): Non-Linear Vibration of Rotating Cantilever Beams. *The Aeronautical Journal of the Royal Aeronautical Society*, vol. 74, pp. 161-165.

Rosen, A. (1991): Structural and Dynamic Behavior of Pretwisted Rods and Beams. *Applied Mechanics Review*, vol. 44, no. 12, pp. 483-515.

Singh, A.P.; Mani, V.; Ganguli, R. (2008): Genetic Programming Metamodel for Rotating Beams. *CMES: Computer Modeling in Engineering and Sciences*, vol. 659, no. 1, pp. 1-17.

Storti, D.; Aboelnaga, Y. (1987): Bending Vibrations of a Class of Rotating Beams with Hypergeometric Solutions. *ASME Journal of Applied Mechanics*, vol. 54, no. 2, pp. 311-314.

Thakkar, D.; and Ganguli, R. (2004): Dynamic Response of Rotating Beams with Piezoceramic Actuation. *Journal of Sound and Vibration*, vol. 270, no. 4-5, pp. 729-753.

Thakkar, D.; and Ganguli, R. (2007): Induced Shear Actuation of Helicopter Rotor Blade for Active Twist Control. *Thin Walled Structures*, vol. 45, no. 1, pp. 111-121.

Vinod K.G.; Gopalakrishnan, S.; and Ganguli, R. (2006): Wave Propagation Characteristics of Rotating Euler-Bernoulli Beams. *CMES: Computer Modeling in Engineering and Sciences*, vol. 16, no. 3, 2006, pp 197-208.

Vinod K.G.; Gopalakrishnan, S.; and Ganguli, R. (2007): Free Vibration and Wave Propagation Analysis of Uniform and Tapered Rotating Beams using Spectrally Formulated Finite Elements. *International Journal of Solids and Structures*, vol. 44, no. 18-19, pp. 5875-5893.

Wright, A.D.; Smith, C.E.; Thresher, R.W.; Wang, J.C. (1982): Vibration Modes of Centrifugally Stiffened Beams. *ASME Journal of Applied Mechanics*, vol. 49, no. 1, pp. 197-202.

Appendix: Frequency Equation

$$\pi = \begin{vmatrix} D_{14} & D_{13} & D_{12} & D_{11} \\ D_{24} & D_{23} & D_{22} & D_{21} \\ G_{34} & G_{33} & G_{32} & G_{31} \\ G_{44} & G_{43} & G_{42} & G_{41} \end{vmatrix} = 0$$

where

$$D_{14} = \beta_T a_2|_{\xi=0},$$

$$D_{13} = \beta_T \left(2a_2 b' + a_2'\right) + a_1 a_2\big|_{\xi=0},$$

$$D_{12} = \beta_T \left(a_2 b'' + a_2' b' + a_3 a_2 - n\right) + a_1 a_2 b'\big|_{\xi=0},$$

$$D_{11} = \beta_T \left(a_3 a_2 - n\right)' + a_1 \left(a_3 a_2 - n\right)\big|_{\xi=0},$$

$$D_{21} = \beta_\theta,\ D_{22} = -1,\ D_{23} = D_{24} = 0,$$

$$G_{3j} = V_j'(1)$$

$$G_{4j} = b(1)V_j''(1) + b'(1)V_j'(1) + g(1)a_3 V_j(1), \qquad j = 1,2,3,4$$

in which

$$a_1 = \alpha^2 \left(\sin^2\theta \cos^2\phi + \sin^2\phi\right) + \Lambda^2,$$

$$a_2 = (1 + \mu n/q),$$

$$a_3 = \eta \left(\alpha^2 \cos^2\phi + \Lambda^2\right).$$