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Analysis and Prediction of Multi-Heating Lines Effect on Plate Forming by Line Heating

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Abstract: Experimental observations have shown that the inherent deformation produced by multi-heating lines is not a simple addition of the inherent deformation produced by single heating lines. Therefore, to accurately predict inherent deformation, the method of superposing inherent deformation of single heating lines is not appropriate. To overcome this difficulty, the authors investigate the influence of multi-heating lines on line heating inherent deformation. First, the influence of previous heating lines on inherent deformation of overlapping, parallel and crossing heating lines is clarified. The influence of the proximity to plate side edge on inherent deformation is also taken into account in the analysis. It is demonstrated that residual stresses produced by previous heating lines is the main factor influencing the inherent deformation of multi-heating lines, while the influence of the proximity to plate side edge on multi-heating lines is small and can be neglected.

Keyword: Line heating, inherent deformation, FEM, multi-heating, residual stresses, side edge effect.

1 Introduction

Forming by line heating has been an active research topic in manufacturing, especially in shipbuilding. The problem of forming by line heating can be divided into two sub-problems: the heat transmission problem and the elasto-plastic deformation problem. The first problem has been widely studied and efficient techniques have been presented (Moshaiov and Latorre (1985), Tsuji and Okumura (1989), Terasaki, Kitamura and Nakai (1999), Chang, Liu, and Chang (2005), Ling and Atluri (2006), Liu (2006), Liu, Liu, and Hong (2007), Osawa, Hashimoto, Sawamura, Kikuchi, Deguchi, and Yamaura (2007)). The elasto-plastic deformation problem has been also widely studied. Theoretical researches on the mechanism of line heating process aimed to predict the final shape of a metal plate when given the heating conditions and mechanical properties of the plate material exists (e.g. Moshaiov and Vorus (1987), Moshaiov and Shin (1991), Jang, Seo and Ko (1997), Kyrsanidi, Kermanidis and Pantelakis (1999)).

The finite element method or simplified analysis using the beam or plate theory is usually applied. The relationships between bending deformation, heating parameters and plate thickness have been developed in empirical models and inherent strain data bases (e.g. Ueda, Murakawa, Rashwan, Okumoto and Kamic, (1994), Jang and Moon (1998)).

Additional information, such as the influence of strain hardening, edge effect, and size effect, has also been reported in experimental and numerical investigations (Magee, Watkins and Steen (1997), Bao and Yao (2001), Cheng, Yao, Liu, Pratt and Fan (2005)). However, most of these investigations have focused on the deformation produced by single heating lines on small plates, no empirical method or inherent strain database has been developed for actual size plates.

The authors aim to propose a practical and accurate method to predict deformation of actual size plates such as those used in shipbuilding. As a fundamental component of this method, a line heating inherent deformation database is indeed necessary. This inherent deformation database besides being mainly dependent on primary fac-

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tors such as the plate thickness, the heat source speed and the heat input, it also takes into account secondary factors such as the geometry of the plate, the cooling condition, the location of the heating line, multi-heating lines, heat-induced curvature, residual stresses and inter-heating temperature. Here, it is to be noted that the influences of these secondary factors are not so simple that they can be linearly related to primary factors. Also, it is difficult to obtain these influences by experiments because of the large scatter in test results. Therefore, to clarify the influence of these secondary factors on inherent deformation a 3D thermal-elastic-plastic FEM using an iterative substructure method (ISM) is utilized (Nishikawa, Serizawa and Murakawa (2005)).

In this report, the effect of multi-heating lines on inherent deformation is investigated and clarified. At first, the inherent deformation produced by single heating lines applied to flat plates is predicted by integrating the inherent strain obtained through three dimensional thermal elastic-plastic finite element analyses. Then, a combination of multi-heating lines is applied and the resulting computed inherent deformation is compared with that obtained by superposing the inherent deformation of single heating lines. Three cases (overlapped, parallel and crossed heating lines) are considered. For each case the influence of previous heating lines on the inherent deformation of current heating line is investigated. The influence of the plate side edge on inherent deformation of multi-heating lines is also investigated. Finally, through the analysis of the results, conclusions of the influence of multi-heating lines on inherent deformation of plates undergoing line heating are presented.

2 Prediction of line heating inherent deformation using 3D thermal elastic-plastic FEM

2.1 *Inherent strain due to the line heating*

When a continuous body is subjected to a non-uniform heat cycle that produces permanent deformation, the strain in the deformed body has elastic and plastic components. The total strain is the summation of these components.

$$\varepsilon = \varepsilon^{e} + \varepsilon^{p} \tag{1}$$

The elastic components correspond to the residual stresses, and the plastic components are referred to as the inherent strain. It is to be noted that although the total strain is compatible in the body, the inherent strain (plastic strain caused by heating and subsequent cooling) is not compatible.

2.2 *Method of analysis*

Thermal-elastic-plastic FEM is a powerful tool for predicting line heating inherent deformation. However, three-dimensional thermal-elastic-plastic FE analysis requires very long computation time and large memory. To overcome this problem, an in-house three dimensional thermal elastic-plastic finite element code based on an iterative substructure method (Nishikawa, Serizawa and Murakawa (2005)) is employed in this research. The iterative substructure method (ISM) takes full advantage of the fact that the region which exhibits strong nonlinearity is limited to a very small area compared to the whole model to be analyzed, and the remaining part is mostly linear. In this way a large saving of computer time is achieved.

2.2.1 Analysis Procedure

The thermo-mechanical behavior of plate forming by line heating is analyzed using uncoupled formulation. However, the uncoupled formulation considers the contribution of the transient temperature field to stresses through thermal expansion, as well as temperature-dependent thermo-physical and mechanical properties. The solution procedure consists of two steps. First, the temperature distribution history is computed using transient heat transfer analysis. Second, the transient temperature distribution history obtained from the heat transfer analysis is employed as a thermal load in a subsequent mechanical analysis. Stresses, strains and displacements are then evaluated. The same finite element model used in the thermal analysis is employed in mechanical analysis.

2.2.2 Analyzed model

All analysis cases are carried out using rectangular flat plates as shown in Fig. 1. Finite Element models as shown in Fig. 2 are employed. Heating is applied over a moving zone that consists of two elements in the longitudinal (moving) direction, 4 elements in the transverse direction, and one element in the thickness direction (40x80x10 mm).

Heat input per unit length of the heating line is adjusted such that the highest temperature on the surface in the heating zone is kept at 800°C. Cooling is defined corresponding to natural cooling in air. Mild steel thermal, physical and mechanical properties with temperature dependency are used. Necessary constraints are added to eliminate rigid body motion. The analysis conditions shown in Tab. 1 are used. In all cases, heating and cooling condition are kept constant.

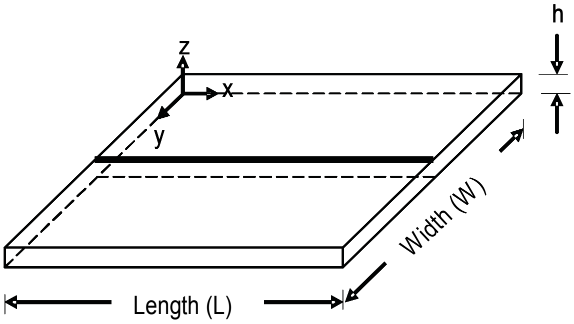


Figure 1: Schematic of the plate model

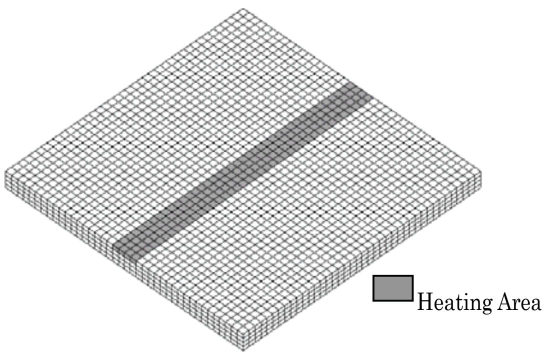


Figure 2: Example of the finite element model

Table 1: Heating Conditions and Plate Models

Heating conditions			
Heat input (J/mm)	**Traveling speed (mm/s)**	**Max. surface Temperature (°C)**	**Max. back surface temperature (°C)**
5,000	3.0	800	260
Plate models used to study the effect of multiple-heating lines			
Model	**L (mm)**	**W (mm)**	**h (mm)**
1	2000	2000	40
2	1400	1400	40

2.3 Line heating Inherent deformation

The Deformation of the plate is expressed in terms of the inherent deformation which is defined as the integration of the plastic strain over the cross section of the plate. The inherent deformation can be divided into four components; longitudinal shrinkage (δ_x^i), transverse shrinkage (δ_y^i), longitudinal bending (θ_x^i), and transverse bending (θ_y^i). These four components are defined by the following equations:

$$\delta_x^i = \int \varepsilon_x^i dydz/h \tag{2}$$

$$\delta_y^i = \int \varepsilon_y^i dydz/h \tag{3}$$

$$\theta_x^i = \int \varepsilon_x^i (z-h/2)/(h^3/12)dydz \tag{4}$$

$$\theta_y^i = \int \varepsilon_y^i (z-h/2)/(h^3/12)dydz \tag{5}$$

Where h is the plate thickness. It is to be noted that the inherent deformation at a cross section is not a characteristic value of the material and heat input. It depends on many other factors such as heat transfer conditions, displacement constrains, geometry of the structure and location of heating. Figure 3 shows a comparison between the distributions of analytically obtained apparent deformation produced by a straight heating line (Model 1 in Tab. 1) and that obtained through line heating experiments. We used apparent deformation instead of inherent deformation due to the fact that inherent deformation is difficult to obtain experimentally.

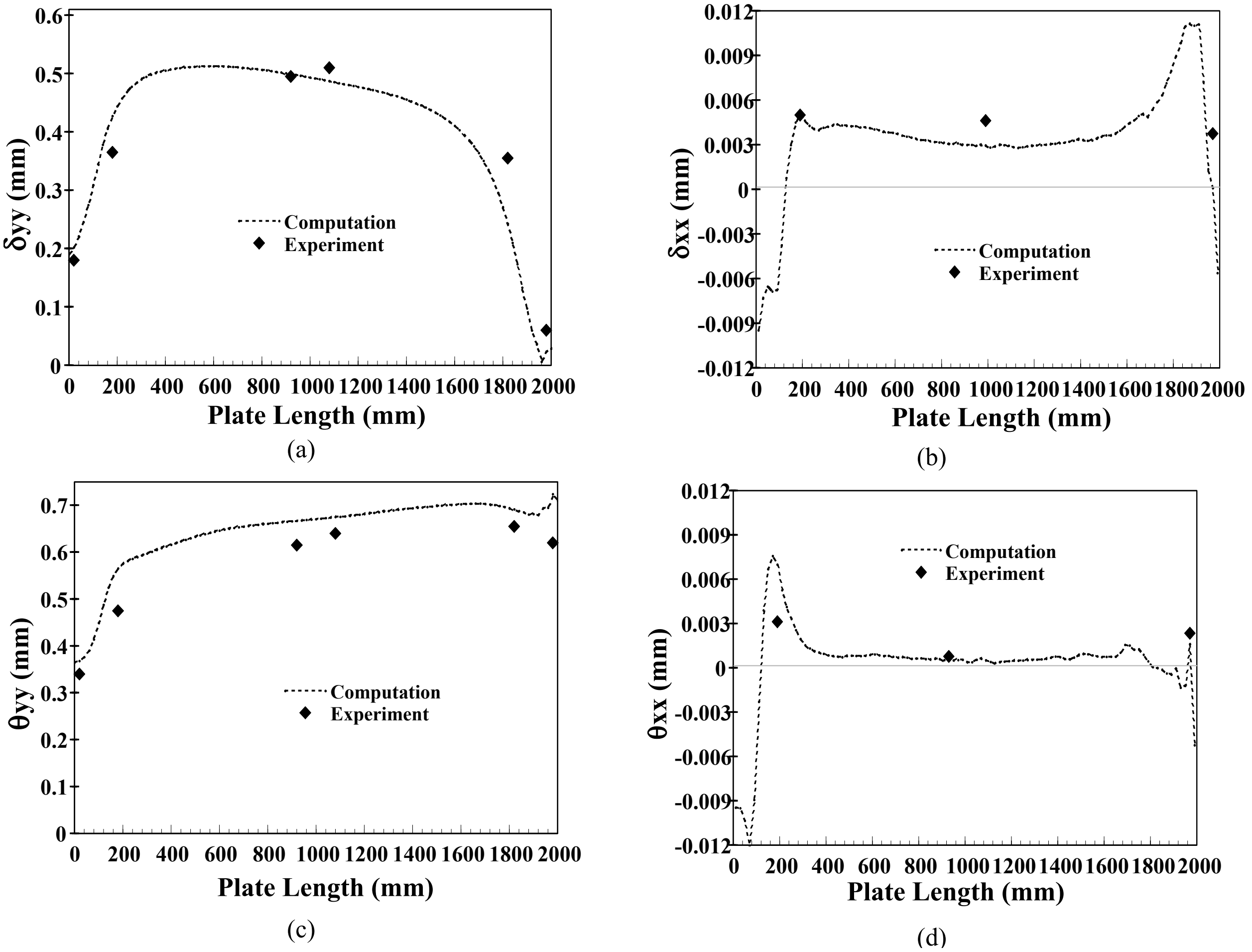


Figure 3: Distribution of the apparent deformation produced by a single heating line (a) Transverse shrinkage, (b) Longitudinal shrinkage, (c) Transverse bending, and (d) Longitudinal bending

Table 2: Influential factors on inherent deformation

Plate Geometry	Heating Condition	Cooling Condition	Location of the Heating	Multi-heating Lines	Others
- Plate Length - Plate Width - Plate Thickness	- Heat Input - Torch Speed - Torch Size	- Air Cooling - Water Cooling - Inter-heat Temperature	- Entrance Plate Edge - Exit Plate Edge - Side Plate Edge	- Overlapped - Parallel - Crossed	- Residual Stress - Initial Curvature - Material properties

3 Problem description and overall strategy

The plate forming process by line heating can be viewed as a process to form a plate into a desired shape using the shrinkage and angular distortion produced through the plastic deformation produced during line heating. To form plates with general three-dimensional geometry which make most of the curved parts of ship structures, it may be necessary to properly combine in-plane shrinkage and angular deformation according to each particular situation.

Values of in-plane shrinkage and angular distortion produced by line heating can be accurately obtained based on the geometry and the heating condition if the secondary factors affecting the deformation are taken into account.

Ueda, Murakawa, Rashwan, Okumoto and Kamichika (1993), proposed an automatic plate bending process which has been successfully used in ship plates forming. This process consists of four steps which can be summarized as follow. The first step is to determine, using FEM, the necessary inherent strain to form the plate from its initial shape to the desired shape. The second step is to determine the positions and directions of the heating lines, the kind of inherent deformation (inplane shrinkage/or angular distortion) and its magnitude. The third step is to determine the heating and cooling condition to produce the necessary inherent deformation from inherent deformation databases which is to be established by line heating analysis using FEM. The last step is to determine the condition to correct the geometrical error left after forming based on the first, second and third steps. However, to fully automate this process and minimize the work in the fourth step, accuracy improvements are necessary.

As an overall strategy to improve the accuracy of the forming process, the authors pay attention to those factors which may influence line heating inherent deformation. In order to identify important factors influencing inherent deformation, first we analyze the inherent deformation produced by single heating lines and compare it with that obtained by different combination of multi- heating lines. It is found that many factors influence the inherent deformation. A summary of these factors is presented in Tab. 2. In previous reports, the influence of the plate geometry and the edge effect on inherent deformation are studied and clarified (Vega, Rashed, Serizawa and Murakawa (Report 1, 2007), Vega, Tajima, Rashed and Murakawa (Report 2, 2007)). In this report we aim to clarify the influence of multi-heating lines on inherent deformation. Further reports will be presented aiming to propose a practical and accurate method to predict inherent deformation, and hence the formed shape of a plate.

4 Influence of multi-heating lines on inherent deformation

In forming 3-D shapes, ship curved plates for example, complicated heating patterns are needed. Combinations of heating lines in different direction are applied to finally get the plastic strain necessary to form the plate. On the other hand, when two heating lines are applied close to each other, it is observed that the resulting inherent deformation is not a simple addition of that produced by each heating line applied alone to a stress-free plate.

With the aim to clarify this difference, we base our discussion on the influence of previous heating lines on inherent deformation produced by a current heating line. This influence is mainly due to the fact that when the previous heating line cools down to room temperature, inherent deformation and residual stresses are produced. Both, the inherent deformation and the residual stresses influence each other. If the current heating line is applied over an area in which compressive residual stresses exist, additional compressive plastic strain will be produced in the heating process, therefore, at the end, the inherent deformation is larger than that produced by the previous heating line. In the same way, if the current heating line is applied over an area in which tensile residual stresses exist, additional tensile plastic strain is produced in the heating process. Then, after the plate cools down to room temperature, the inherent deformation is smaller than that of the previous heating line.

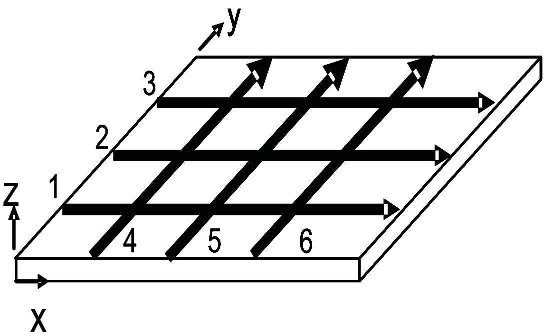


Figure 4: Schematic of the multi-heating lines

Figure 4 shows a square plate (Model 2 in Tab. 1) which is to be formed by a combination of six heating lines. The first three are applied in the X-direction, the remaining three lines are transverse to the first three (Y-direction). The heating and cooling conditions are the same for all the heating lines. Before applying each heating line the plate is at room temperature.

Figure 5 shows a comparison of the total inherent deformation obtained by computing the inherent deformation produced by all (6) heating lines and that obtained by simply superposing inherent deformation of each single heating line when applied alone to the stress-free plate. By simple inspection it may be seen that the difference between results, especially in the longitudinal components of inherent deformation is large.

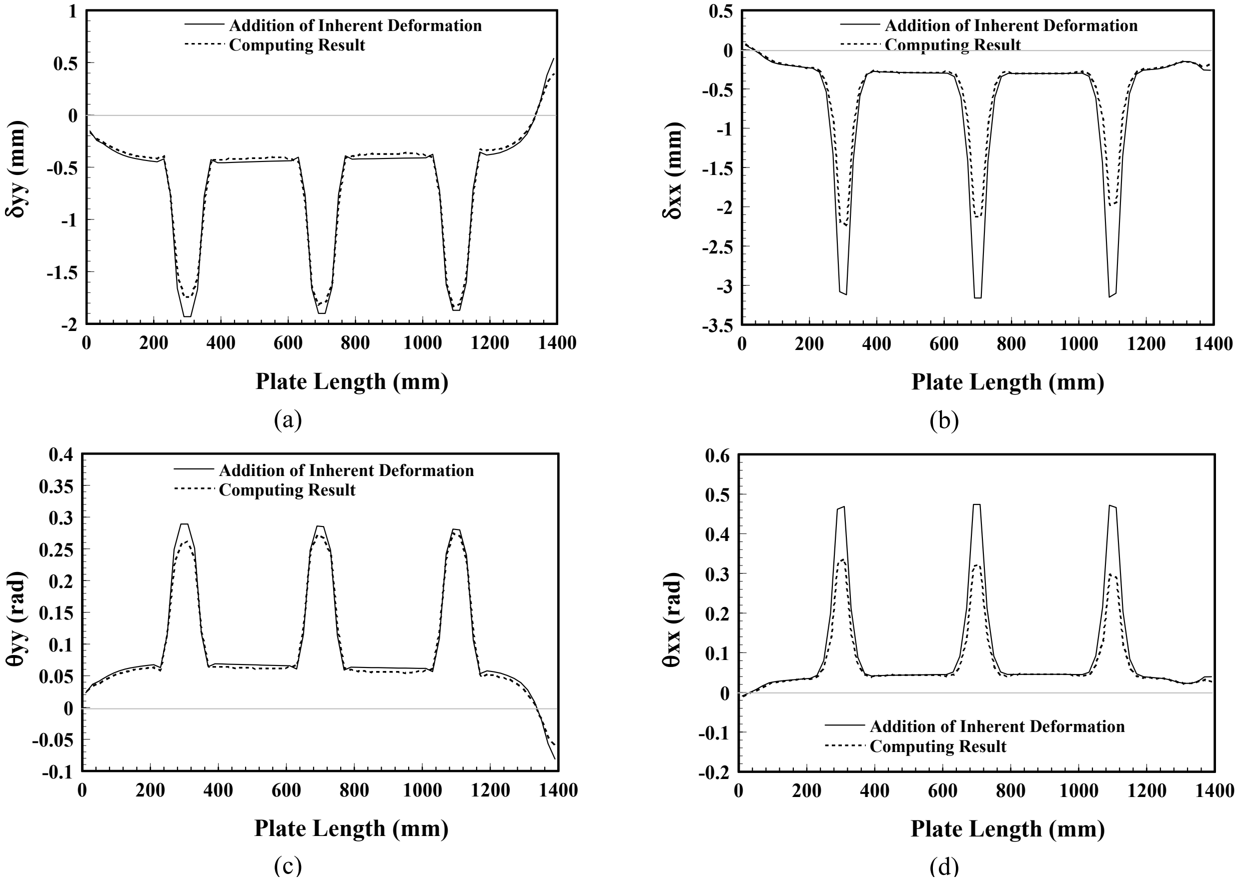
To better understand this influence, we consider three combinations of heating lines actually used in plate forming by line heating. At first, we examine the influence of a previous heating line on the inherent deformation of an overlapping heating line (for example heating 1 + 1 in Fig. 4). Then, we examine the case of parallel heating lines (for example heating 1 + 2 in Fig. 4) and the case of heating lines which cross each other (for example heating 1 + 4 in Fig. 4). The influence of side plate edges is investigated in the case of parallel heating lines. The influence of multi-crossed heating lines on inherent deformation is also studied and clarified.

4.1 Overlapped heating lines

The process of forming inherent deformation by overlapped heating lines has been explained using a simplified three bars mechanical model (see Vega, Tajima, Rashed and Murakawa (2007)). To illustrate that, let us suppose that a straight heating line is applied along the plate length (Model 1 in Tab. 1). After the heated area cools down to room temperature, a second overlapping heating line with the same heating and cooling condition as the first, is applied.

Figure 6 shows the average residual stresses (average over the thickness) σ_{xx} and σ_{yy} produced by the first heating line plotted along (a) and transverse to (b) the heating line. As it may be seen in Fig. 6 (a), large compressive residual stress σ_{yy} in Y-direction appear at the entrance and exit edges of the plate. While at both, entrance and exit edges of the plate, the residual stress σ_{xx} in X-direction is small. At the middle region of the plate, small amount of tensile residual stress σ_{yy} in Y-direction exists while the tensile residual stress σ_{xx} in X-direction is large. Figure 7 compares the computed results of inherent deformation produced by the first heating line and that produced by the two overlapped heating lines. From this figure it may be seen that the total inherent deformation produced by two overlapped heating lines is not a simple addition of inherent deformation produced by single heating lines. To clarify the cause of this variation we study the influence of the residual stresses produced by the previous heating line (Fig. 6) on the inherent deformation of the second (overlapping) heating line (Fig. 7).

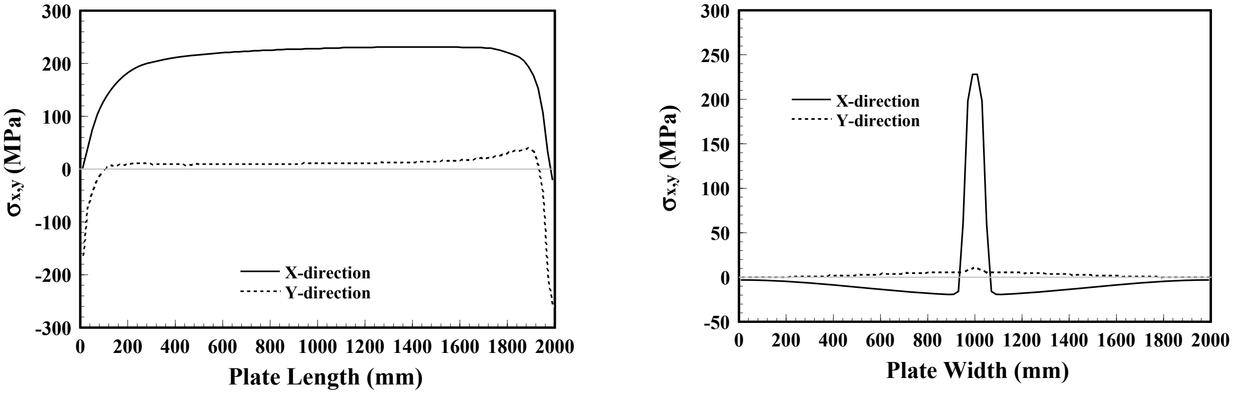
As mentioned above, when a heating line is applied over an area in which compressive residual stresses exist, at high temperature, additional compressive strain is produced, consequently, additional inherent deformation is created. This increment on inherent deformation is clearly observed for example, in the transverse components of inherent deformation at both edges of the plate (Fig. 7 (a) and (c)). At the central region of the plate, small amount of tensile residual stress σ_{yy} in Y-direction exists. This tensile residual stress slightly influences the transverse components of inherent deformation produced by the second heating line as shown in Fig. 7 (a) and (c). On the other hand, at the central region of the plate, large tensile residual stress σ_{xx} in X-direction is observed (Fig. 6). When a second (overlapping) heating line is applied, at high temperature, this tensile residual stress is transformed in tensile strain. Due to the fact that the tensile stress is large (close to the yield stress), the tensile strain is also large, therefore, the longitudinal plastic strain produced by the previous heating line, almost completely disappears. Then, when the plate cools down to room temperature, new compressive strain is created. This new compressive strain produces almost the same inherent deformation and residual stresses σ_{xx} in X-direction as the first heating line. If additional overlapped heating lines are applied, the same phenomenon occurs, therefore, the longitudinal components of inherent deformation at the central region of the plate, is almost the same as that obtained by the previous overlapped heating lines. At both, entrance and exit edges of the plate, the residual stress σ_{xx} in X-direction is small, therefore, no significant variation on the longitudinal components of inherent deformation is observed.



(a) (b)

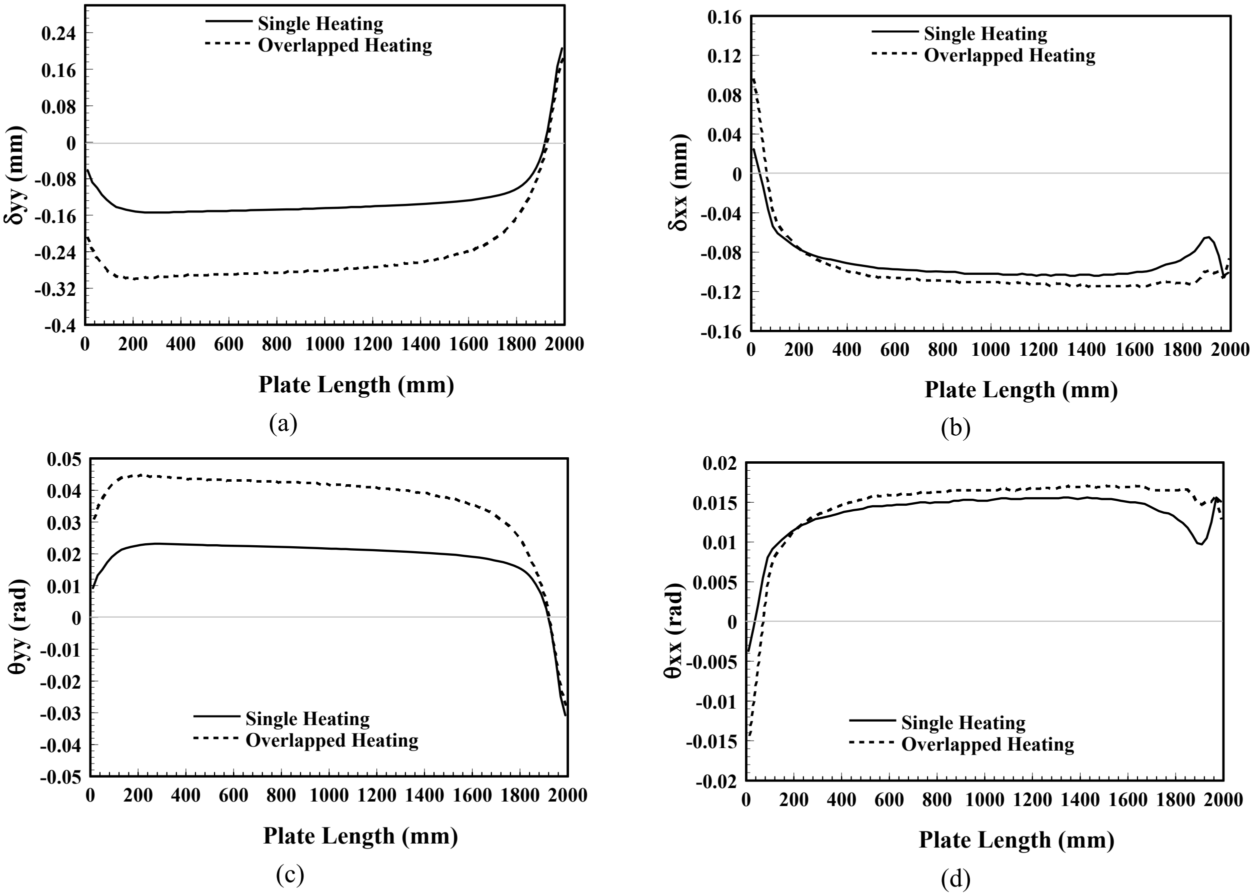
(c) (d)

Figure 5: Comparison of the inherent deformation obtained by computing the line heating pattern shown in Fig. 4 and that obtained by superposing inherent deformation of single heating lines. (a) Transverse inherent Shrinkage, (b) Longitudinal inherent Shrinkage, (c) Transverse inherent bending, (d) Longitudinal inherent bending



(a) (b)

Figure 6: Residual Stresses Distribution, (a) Along the Heating Line, (b) Transverse to the Heating Line



(a) (b) (c) (d)

Figure 7: Inherent deformation produced by overlapped heating lines, (a) Inherent transverse shrinkage, (b) Inherent longitudinal shrinkage, (c) Inherent transverse bending and (d) Inherent longitudinal bending

4.2 Parallel heating lines

In this section, a heating pattern consisting of parallel heating lines as shown in Fig. 8 (Model 1 in Tab. 1) is examined. Heating lines are applied at 1000, 1240, 1480 mm from plate side, which is 240 mm spacing between each two successive heating lines. It may be noted that if the spacing between heating lines is large enough, the inherent deformation can be predicted by simply superposing the inherent deformation produced by individual parallel heating lines (Vega, Tajima, Rashed and Murakawa, (Report 2 (2007)).

In Fig. 9 a comparison of the residual stresses produced by the first heating line, that produced after two parallel heating line and that produced after three parallel heating lines is shown (note that the scales are different). Comparing the residual stresses at a specific point (Y=1500mm for example), it may be seen that residual stresses in both, X – and Y – directions increase with additional parallel heating lines. Thus, the influence of the residual stresses on the inherent deformation of each additional parallel heating line is larger than that of the previous heating lines.

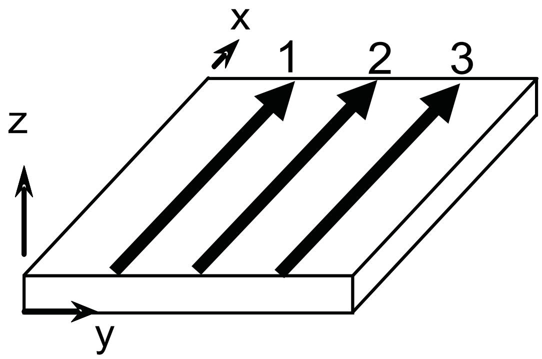


Figure 8: Schematic of the parallel heating lines

Figure 10 shows a comparison between the computed inherent deformation produced by a single heating line, that produced by two parallel heating lines, and that produced by three parallel heating lines, respectively. In Figures 10(a) and (c) it is observed that the transverse components of inherent deformation produced by two parallel heating lines are reduced by the tensile residual stress existing in the area in which the second heating line is applied (see Fig. 9 (b)). The same effect may be observed when three parallel heating lines are applied. However, in this case, the reduction of the transverse components of inherent deformation is larger due to the larger level of residual stresses (it increases with the number of parallel heating lines). In the same way the compressive residual stresses existing in the area in which the parallel heating lines are applied (Fig. 9(a)) cause the longitudinal components of inherent deformation to increase as shown Fig. 10 (b) and (d). This increase of the inherent deformation also depends on the level of residual stresses and increases with the number of parallel heating lines.

4.2.1 Influence of plate side edge on parallel heating lines

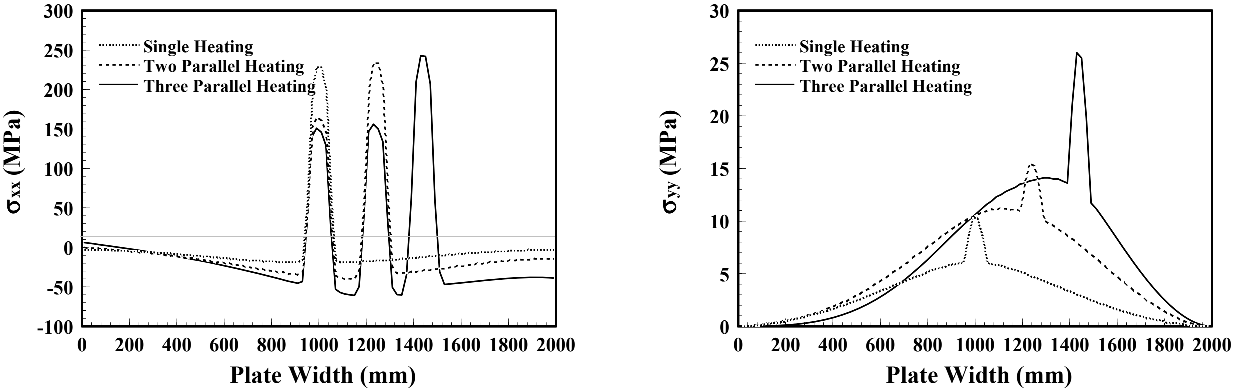
At first, inherent deformation of single heating lines applied at different distances from the free edge (100, 220, 340, 460, 580 and 700 mm) is computed. Figure 11 shows the results of the inherent deformation produced at the center of the plate by each single heating line. It may be clearly seen that both, transverse and longitudinal component of inherent deformation decrease as the heating line becomes closer to the free edge. However, for those cases which are spaced more than 400mm from the free edge (for a 40 mm thick plate), the inherent deformation does not significantly change. From these results, it may be concluded that despite the fact that the inherent deformation decreases when the heating line becomes closer to side edges, this is not an influential factor in the case of multi-heating lines examined in this report.

4.3 Crossed heating lines

In this section we examine the inherent deformation caused by heating lines which cross each other. At first, the case of two crossed heating lines is examined. Figure 12 (a) shows the heating pattern (Model 2 in Tab. 1) used in this analysis. The second heating line (crossing heating) is applied after the first heating area cools down to room temperature. After computing the inherent deformation produced by the two heating lines and comparing it with that obtained by superposing inherent deformation produced by two single heating lines, it is observed that the resulting inherent deformation is significantly influenced especially at the crossing area. The difference, which is also caused by the residual stresses produced by the previous heating line, is defined by the authors as the cross effect (see Vega, Rashed, Serizawa and Murakawa (Report 3, 2008)).

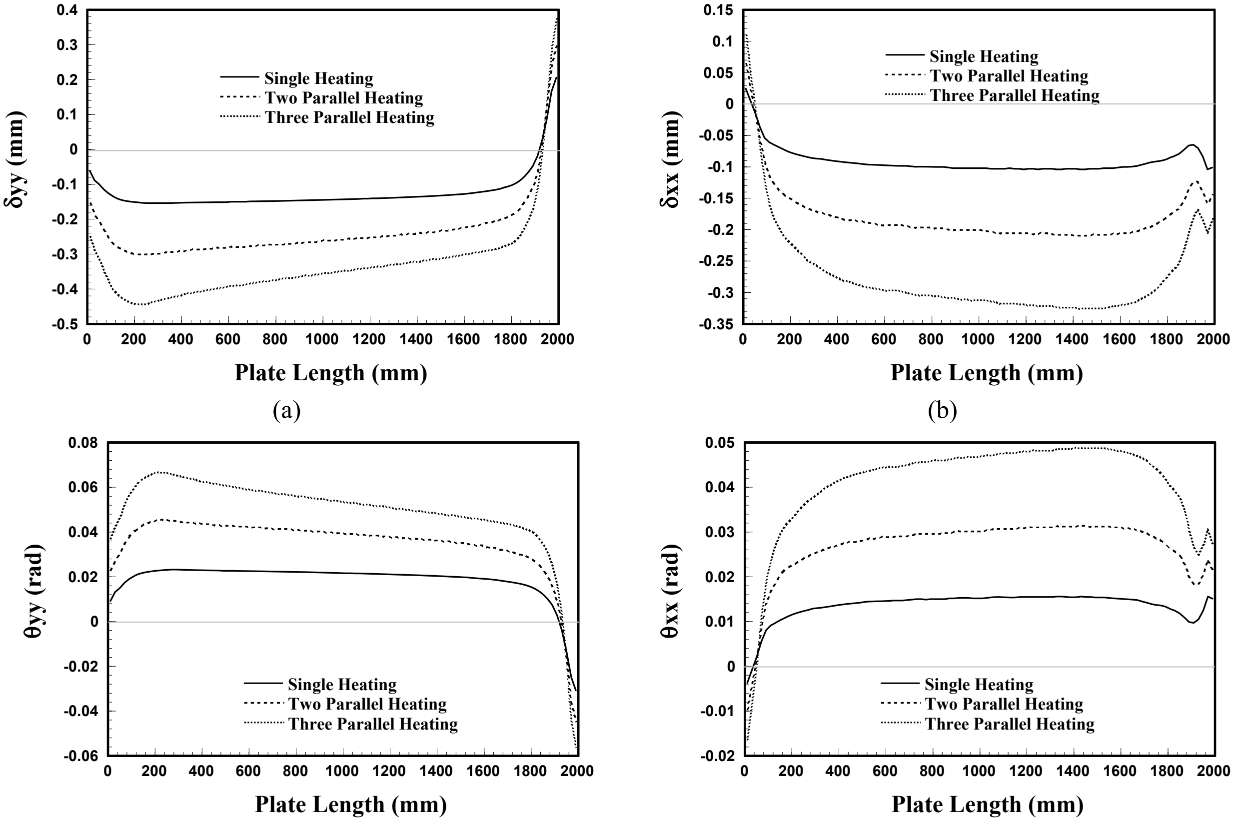
To better understand the cross effect, let us analyze the residual stress distribution produced by a straight heating line shown in Fig. 6. Now, if a transverse heating line (heating 2 in Fig. 12 (a)) is applied over the residual stress pattern shown in Fig. 6, and when the area in which both heating lines intersect is at high temperature, the residual tensile stress is transformed into additional tensile strain, as explain before. Due to the fact that this tensile strain is large, the longitudinal component of inherent deformation produced by the previous heating line at the crossed area is almost completely released. Then, when the plate cools down, new compressive strain is produced along the second heating line. This new compressive strain is also affected by the residual stresses far from the crossing area. Figure 13 shows the distribution of the cross effect along the second heating line (heating 2 in Fig. 12 (a)). Here, the cross effect ($\Delta\delta_x$, $\Delta\delta_y$, $\Delta\theta_x$ and $\Delta\theta_y$) is defined as the difference between the inherent deformation obtained by superposing inherent deformation of single heating lines and that obtained from computing the two crossed heating lines.

Despite the fact that at the crossing area, tensile residual stresses in X and Y direction exist, the cross effect on inherent transverse shrinkage, for example, is positive (decreases the inherent



(a) (b)

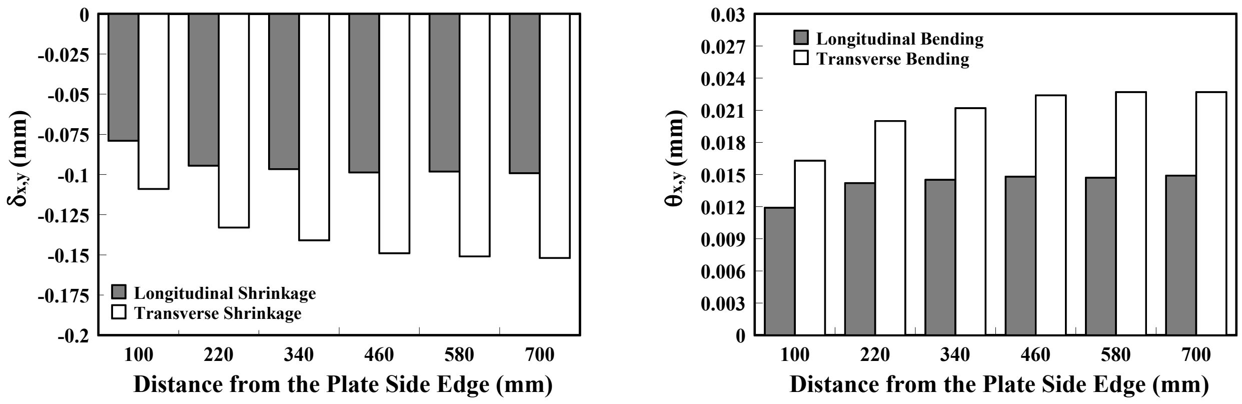
Figure 9: Residual Stresses Distribution (transverse to the heating line, at L/2), a) stress in X-direction, b) Stress in Y-direction



(a) (b)

(c) (d)

Figure 10: Inherent deformation produced by parallel heating lines; (a) Inherent transverse shrinkage, (b) Inherent longitudinal shrinkage, (c) Inherent transverse bending, (d) Inherent longitudinal bending



(a) (b)

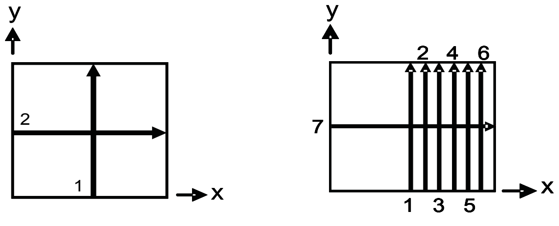
Figure 11: Influence of the plate side edge on inherent deformation of parallel heating lines (a) Inherent shrinkage, (b) Inherent bending

transverse shrinkage) while on inherent longitudinal shrinkage is negative (increases the inherent shrinkage). It is due to the fact that the component of residual stress in X-direction is much larger than that in Y-direction (see Fig. 6). The tensile X-direction residual stresses have a large effect while the Y-direction residual stress does not have a significant effect. The increment of inherent deformation in the longitudinal direction is due to the incompressibility of the plastic deformation of the material.

4.3.1 *Influence of multi-crossed heating lines on inherent deformation*

Here we examine the case in which after applying the heating lines 1 to 6 shown in Fig. 12 (b), a crossing heating line (heating line 7 in the same figure) is applied. Figure 14 shows a comparison of the resulting cross effect and that of the influence of parallel heating lines. To construct this figure, at first we compute the influence of parallel heating lines (heating 1 to 6 in Fig. 12 (b)) then we compute the influence of multi-crossed heating lines (heating 1 to 7 in the same figure).

The difference between both results is the cross effect. It is to be noted that the cross effect in this case is smaller than that produced in the case of single crossed heating lines. This is due to the fact that after applying additional parallel heating lines, the tensile residual stresses produced by previous heating lines is reduced as shown Fig. 9 (a). In Fig. 14 it is clearly seen that the influence of crossed heating lines on inherent deformation is small and it is concentrated at each crossed area, except at the last crossed area (where the residual stress is not reduced) while the influence of parallel heating lines is distributed along the heating line and it increases with the number of heating lines.



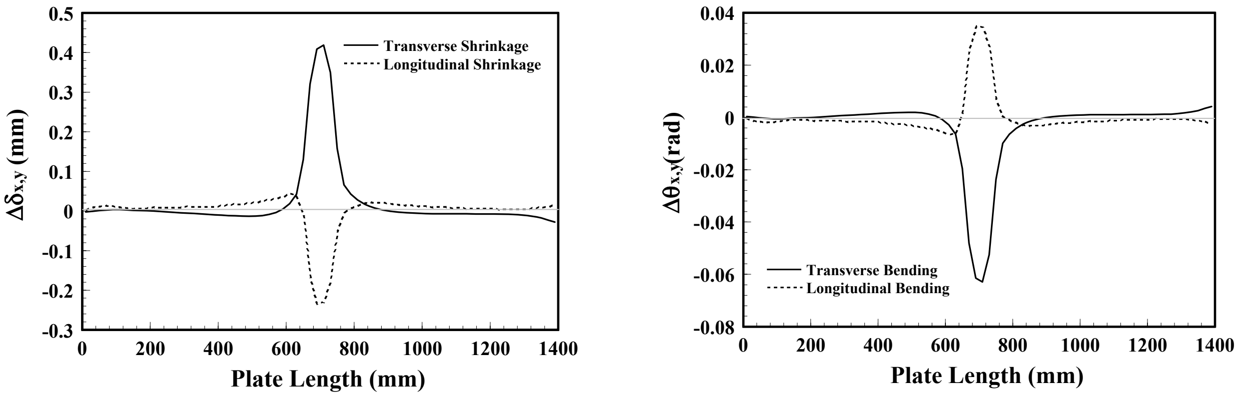
(a) (b)

Figure 12: Schematic of crossed heating lines (a) Single crossed heating lines, (b) Multi-crossed heating lines

5 Conclusions

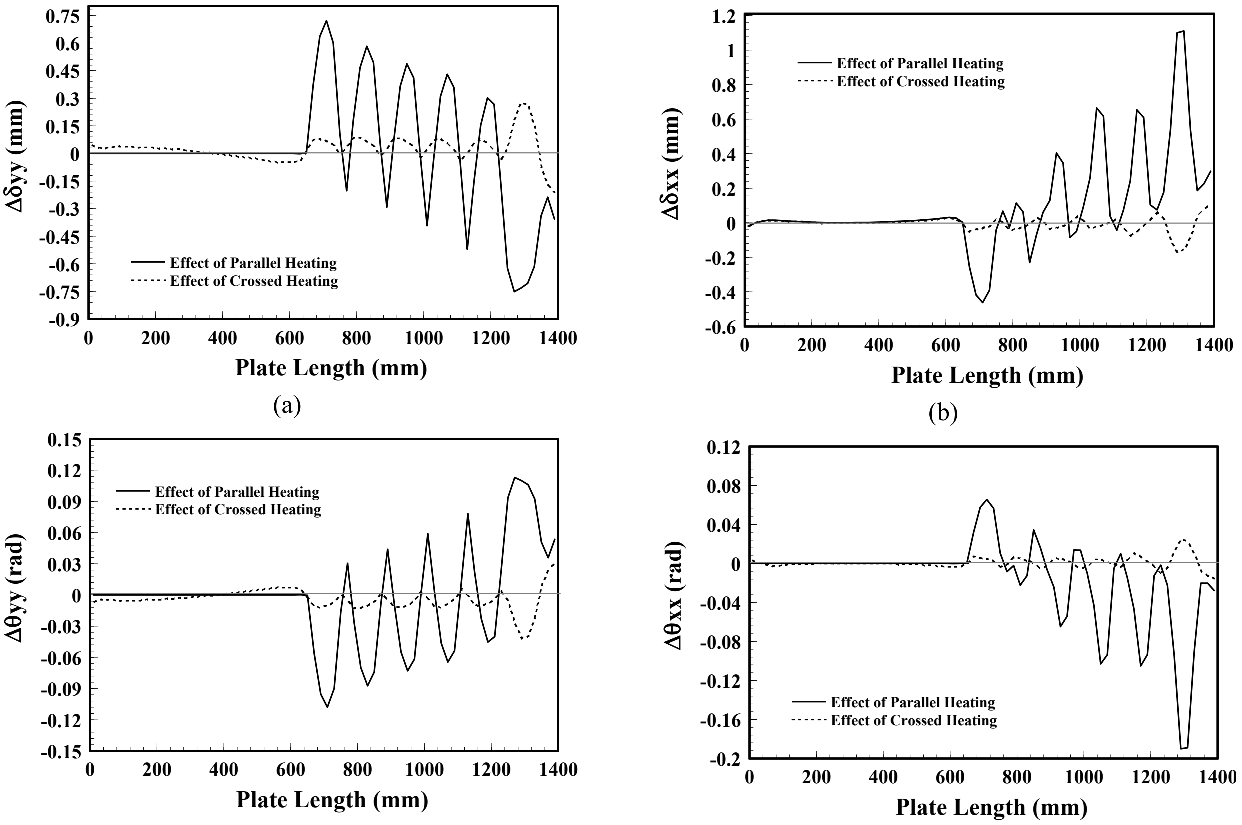
Through a 3D thermal elastic-plastic finite element analysis the inherent deformation of multi-heating lines is examined and clarified. From the results of this investigation, the following conclusions are drawn.

The inherent deformation of multi-heating lines is not a simple addition of inherent deformation of individual heating lines. Consequently, the



(a) (b)

Figure 13: The cross effect (a) Effect on inherent shrinkage, (b) Effect on inherent bending



(a) (b)

(c) (d)

Figure 14: Influence of multi-crossed heating lines on inherent deformation (a) Inherent transverse shrinkage, (b) Inherent longitudinal shrinkage, (c) Inherent transverse bending, and (d) Inherent longitudinal bending

method of superposing inherent deformation of single heating lines does not accurately predict inherent deformation.

The influences of multi-heating lines on inherent deformation is mainly due to the effect of residual stresses produced by previous heating lines on inherent deformation caused by following heating lines.

The effect of a previous heating line on the inherent deformation of a following heating line varies with the separation between the two heating lines. Three combinations of multi-heating lines need to be considered (overlapped, parallel and crossed heating lines).

The influence of the plate side edge on inherent deformation is appreciable only close to the side and its effect on inherent deformation of multi-heating lines is expected to be small.

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