Vibration Analysis of Membranes with Arbitrary Sapes Using Discrete Singular Convolution

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Abstract: In this paper, free vibration analysis of curvilinear or straight-sided quadrilateral membranes is presented. In the proposed approach, irregular physical domain is transformed into a rectangular domain by using geometric coordinate transformation. For demonstration of the accuracy and convergence of the method, some numerical examples are provided on membranes with different geometry such as skew, trapezoidal, sectorial, annular sectorial, and membranes with four curved edges. The results obtained by the DSC method are compared with those obtained by other numerical and analytical methods.

keyword: Discrete singular convolution, membrane, irregular domain, vibration.

1 Introduction

Numerical approaches are widely used in various engineering problems. Therefore, various methods have been used for numerical solution of mathematical physics and engineering problems (Cho et al., 2004; Liu 2007; Jin 2004; Smyrlis and Karageorghis, 2003; Lie et al., 2001; Parussuni 2007; Young et al., 2006; 2007).

Membranes are widely used in various engineering applications. Therefore, free vibration analysis of such structures is a most important task for engineer in the design stage of microphones, pumps, pressure regulators, and other acoustical applications. Free vibration analysis of membrane has been solved by several authors. An analysis of the free vibration of circular and annular membranes has been presented by Laura et al. (1997). Jabareen and Eisenberger (2001) proposed an exact method for free vibration analysis of non-homogeneous circular and annular membranes. Buchanan and Peddieson (1999, 2005) and Buchanan (2005) used Ritz and finite element method respectively, for vibration analysis of circular and elliptic membranes with variable density. An experimental study has been made for vibrations of circular membrane by Casperson and Nicolet (1968). Exact power series solutions for axisymmetric vibrations of circular and annular membranes with continuously varying density were presented by Willatzen (2002). Mei (1969) presented a finite element solution of free vibration problem of circular membranes under arbitrary tension. Oden and Sato (1967) applied the finite element method for static analysis of elastic membranes. Analytical solutions of the free vibration problems of arbitrarily shaped membranes have been investigated by Kang et al. (1999) and Kang and Lee (2000) using nondimensional dynamic influence function. Radial basis function-based differential quadrature method was used for free vibration analysis of arbitrary shaped membrane by Wu et al. (2007). Mashad (1996) proposed finite difference and perturbation method for vibration problem of membranes. The method differential quadrature was applied for frequency analysis of rectangular and circular membranes by Laura et al. (1997). Ho and Chen (2000) applied a hybrid method for vibration of non-homogeneous membranes. Some important studies concerning analysis of membranes have been carried out, namely by Leung et al. (2000), Houmat (2001, 2006), Pronsato et al. (1999), Gutierrez et al. (1998), Nagaya and Yamaguchi (1991), Irie et al. (1981), Iura and Atluri (1992), and Cazzani and Atluri (1993), Woo et al. (2004). Because of its relationship to the wave equation, the Helmholtz equation arises in problems in such areas of mathematical physics as the

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study of acoustics. As a result, it has been extensively treated both analytically and numerically (Shu and Chew, 1997; Shu and Xue, 1999; Bao et al., 2004).

The method of discrete singular convolution (Wei 1999) has been used recently for the vibration analysis of structures. Discrete singular convolution (DSC) method has emerged as a new approach for numerical solutions of differential equations. This new method has a potential approach for computer realization as a wavelet collocation scheme (Wei 2000; Wei 2001). The use of the discrete singular convolution method for vibration analysis of beams, plates and shells (Wei 2001a; Wei 2001b; Wei et al., 2002; Zhao et al., 2002; Xiang et al., 2002) have been proven to be quite satisfactory. Free vibration analysis of plates and shells has been investigated by the present author (Civalek, 2006, 2006a, 2007, 2007a, 2007b, 2007c, 2007d). In the past ten years, the meshless methods for numerical analysis of partial differential equations have become quite popular (Atluri and Shen, 2002; Atluri et al., 1999; 2004; 2006; 2006a; Soric et al. 2004; Han et al., 2005). A number of meshless methods have been proposed by this time. They can be subdivided in accordance with the definition of the shape functions. Important type of meshless methods are the Smoothed particle hydrodynamics, diffuse element method, element-free Galerkin method, reproducing kernel particle methods, meshless local Petrov-Galerkin (Zhu et al., 1998; Kim et al., 1999; Dai et al., 2004; Liu and Chen, 2001, 2002; Sladek et al., 2002; Han and Atluri, 2004; 2004a; Shen and Atluri, 2004; Long and Atluri, 2002; Gu and Liu, 2001; Sladek et al., 2006; Andreaus et al., 2004; 2005). A detailed discussion and comparison of different type meshless methods can be found in References (Atluri and Zhu, 1998; 2000; Zhu et al., 1999; Atluri and Shen, 2005). The meshless methods are widely used for beam and plate problems (Qian et al., 2003; Suateke, 2006; Wen and Hon, 2007; Wu et al., 2004; Jarak et al., 2007; Mai-Duy et al., 2007; Hon et al., 2005; Shu et al., 2005; Raju and Phillips, 2003). The aim of the present paper is to present the DSC method for free vibration analysis of membranes having different geometries. The physical domain is transformed into a rectangular domain by using geometric coordinate transformation. After this, the method of DSC is applied to discretization of the transformed set of governing equation and boundary conditions. Finally, a series of numerical examples are presented to validity and accuracy of the proposed method.

2 Discrete singular convolution (DSC)

Numerical methods are important tools in the analysis of science and engineering problems. Many numerical methods are available, but the finite element and finite difference methods are the most common. In general, these methods are either global or local approach. The local methods include finite difference, finite element, boundary element and many other methods. Discrete singular convolution has the accuracy of global methods and simplicity of local methods. Discrete singular convolution was proposed as potential and effective numerical approach for solving many engineering and mathematical physic problems. A detailed comparison of local and global methods was given by Wei (1999). The discrete singular convolution (DSC) method is an efficient and useful approach for the numerical solutions of differential equations. This method introduced by Wei (1999). Following the notations given by Wei (2000) consider a distribution, T and $\eta(t)$ as an element of the space of the test function. A singular convolution can be defined by (Wei 2001)

$$F(t) = (T * \eta)(t) = \int_{-\infty}^{\infty} T(t-x)\eta(x)dx, \qquad (1)$$

where $T(t-x)$ is a singular kernel. For example, singular kernels of delta type (Wei 2001b)

$$T(x) = \delta^{(n)}(x); \quad (n = 0, 1, 2, ...,). \qquad (2)$$

Kernel $T(x) = \delta(x)$ is important for interpolation of surfaces and curves, and $T(x) = \delta^{(n)}(x)$ for $n > 1$ are essential for numerically solving differential equations. With a sufficiently smooth approximation, it is more effective to consider a discrete singular convolution

$$F_\alpha(t) = \sum_k T_\alpha(t - x_k) f(x_k), \tag{3}$$

where $F_\alpha(t)$ is an approximation to $F(t)$ and $\{x_k\}$ is an appropriate set of discrete points on which the DSC (3) is well defined. Note that, the original test function $\eta(x)$ has been replaced by $f(x)$. This new discrete expression is suitable for computer realization. Recently, the use of some new kernels and regularizer such as delta regularizer was proposed to solve applied mechanics problem. The Shannon's kernel is regularized as (Wei 2000)

$$\delta_{\Delta,\sigma}(x - x_k) = \frac{\sin[(\pi/\Delta)(x - x_k)]}{(\pi/\Delta)(x - x_k)} \exp\left[-\frac{(x - x_k)^2}{2\sigma^2}\right]; \tag{4}$$

where Δ is the grid spacing. It is also known that the truncation error is very small due to the use of the Gaussian regularizer, the above formulation given by Eq. (4) is practically and has an essentially compact support for numerical interpolation. In the DSC method, the function $f(x)$ and its derivatives with respect to the x coordinate at a grid point x_i are approximated by a linear sum of discrete values $f(x_k)$ given by (Wei et al., 2002)

$$f^{(n)}(x) \approx \sum_{k=-M}^{M} \delta^{(n)}_{\Delta,\sigma}(x_i - x_k) f(x_k); \tag{5}$$

where $\delta_\Delta(x - x_k) = \Delta\delta_\alpha(x - x_k)$ and superscript (n) denotes the n*th*-order derivative, and $2M$+1 is the computational bandwidth which is centered around x and is usually smaller than the whole computational domain. For example the second order derivative at $x = x_i$ of the DSC kernels for directly given

$$f^{(2)}(x)\Big| \approx \sum_{k=-M}^{M} \delta^{(2)}_{\Delta,\sigma}(k\Delta x_N) f_{i+k,j}. \tag{6}$$

3 DSC method for irregular domains

Consider an eight-node curvilinear quadrilateral domain as shown in Fig. 1. The geometry of this domain can be mapped into a rectangular domain in the natural $\xi - \eta$ plane, as shown in Fig. 1. By employing the following transformation equations the physical domain is mapped into the computational domain

$$x = \sum_{i=1}^{N} x_i \Phi_i(\xi, \eta) \tag{7}$$

and

$$y = \sum_{i=1}^{N} y_i \Phi_i(\xi, \eta) \tag{8}$$

where x_i and y_i are the coordinates of node i in the physical domain, N is the number of grid points, and $\Phi_i(\xi, \eta)$; $i = 1, 2, 3, \ldots, N$ are the interpolation or shape functions.

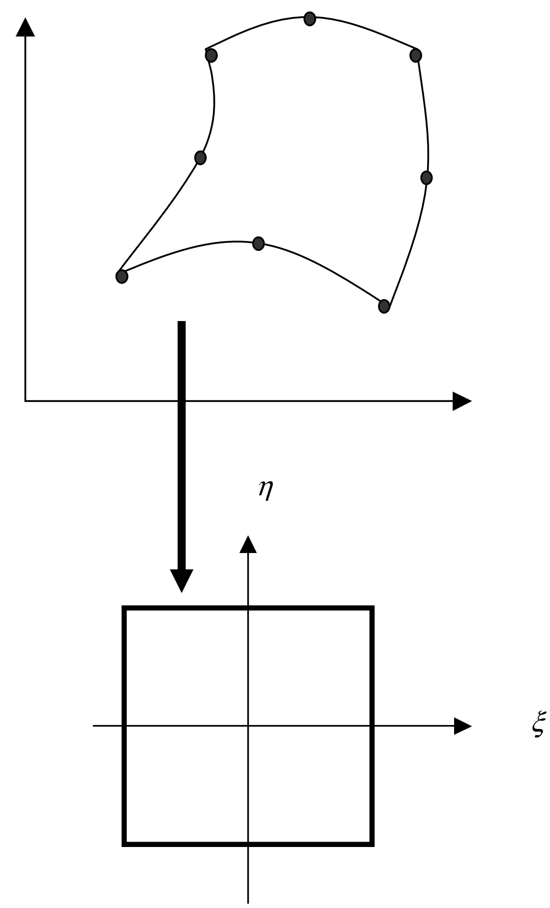

Figure 1: Mapping of arbitrary plates into natural coordinates

These are given for node i;

$$\Psi_i(\xi,\eta)=\frac{1}{4}(1+\xi\xi_i)(1+\eta\eta_i)(\xi\xi_i+\eta\eta_i-1) \quad \text{for} \quad i=1,3,5,7 \tag{9}$$

$$\Psi_i(\xi,\eta)=\frac{1}{4}(1-\xi^2)(1+\eta\eta_i) \quad \text{for} \quad i=2,6 \tag{10}$$

$$\Psi_i(\xi,\eta)=\frac{1}{4}(1+\xi\xi_i)(1-\eta^2) \quad \text{for} \quad i=4,8 \tag{11}$$

Using the chain rule, the first-order, and second order derivatives of a function are given

$$\begin{Bmatrix} u_x \\ u_y \end{Bmatrix} = [J_{11}]^{-1}\begin{Bmatrix} u_\xi \\ u_\eta \end{Bmatrix} \tag{12}$$

$$\begin{Bmatrix} u_{xx} \\ u_{yy} \\ 2u_{yx} \end{Bmatrix} = [J_{22}]^{-1}\begin{Bmatrix} u_{\xi\xi} \\ u_{\eta\eta} \\ 2u_{\xi\eta} \end{Bmatrix} - [J_{22}]^{-1}[J_{21}][J_{11}]^{-1}\begin{Bmatrix} u_\xi \\ u_\eta \end{Bmatrix} \tag{13}$$

where ξ_i and η_i are the coordinates of Node i in the $\xi-\eta$ plane, and J_{ij} are the elements of the Jacobian matrix. These are expressed as follows:

$$[J_{11}]=\begin{bmatrix} x_\xi & y_\xi \\ x_\eta & y_\eta \end{bmatrix}, \tag{14}$$

$$[J_{21}]=\begin{bmatrix} x_{\xi\xi} & y_{\xi\xi} \\ x_{\eta\eta} & y_{\eta\eta} \\ x_{\xi\eta} & y_{\xi\eta} \end{bmatrix} \tag{15}$$

$$[J_{22}]=\begin{bmatrix} {x_\xi}^2 & {y_\xi}^2 & x_\xi y_\xi \\ {x_\eta}^2 & {y_\eta}^2 & x_\eta y_\eta \\ x_\xi x_\eta & y_\xi y_\eta & \frac{1}{2}(x_\xi y_\eta + x_\eta y_\xi) \end{bmatrix}. \tag{16}$$

The above transformations will be used later to transform the governing differential equations and related boundary conditions from the physical domain x-y into the computational domain $\xi-\eta$. Thus an arbitrary-shaped quadrilateral plate may be represented by the mapping of a square plate defined in terms of its natural coordinates. For example, the second-order derivatives with respect to the -x coordinate can be written, as

$$\frac{\partial^2 w}{\partial x^2}=[J_{22}]^{-1}\sum_{i=-M}^{M}\delta_{\Delta,\sigma}^{(2)}(k\Delta\xi)w_{ik} - [J_{22}]^{-1}[J_{21}][J_{11}]^{-1}\sum_{i=-M}^{M}\delta_{\Delta,\sigma}^{(1)}(k\Delta\xi)w_{ik} \tag{17}$$

4 Problem formulations and solution

The governing differential equation for free vibration of membranes is (Chen and Wu, 1996)

$$\frac{\partial^2 W}{\partial x^2}+\frac{\partial^2 W}{\partial y^2}+\frac{\rho}{T}\omega^2 W=0, \tag{18}$$

where W is the transverse deflection, ρ is the mass per unit area, ω is the circular frequency, and T is the tension per unit length. The density of the membrane is the linear function of the x. Eq. (18) can be given in non-dimensional form written as follows:

$$\frac{\partial^2 W}{\partial X^2}+\lambda^2\frac{\partial^2 W}{\partial Y^2}+\Omega^2 W=0, \tag{19}$$

In Eq. (19) the non-dimensional variables have been used given below

$$X=x/a, \qquad Y=y/b, \tag{20}$$

$$\Omega^2=\rho\omega^2 a^2/T, \quad \lambda=a/b, \tag{21}$$

Applying the discrete singular convolution to the governing equation yields

$$\sum_{k=-M}^{M}\delta_{\Delta,\sigma}^{(2)}(k\Delta x)W_{i+k,j}+\lambda^2\sum_{k=-M}^{M}\delta_{\Delta,\sigma}^{(2)}(k\Delta y)W_{i,j+k} + \Omega^2 W_{ij}=0, \tag{22}$$

The boundary conditions are as follows:

$$W=0 \quad \text{at edges} \tag{23}$$

Employing the transformation rule, the governing Eq. (21) becomes,

$$[J_{22}]^{-1}\sum_{i=-M}^{M}\delta_{\Delta,\sigma}^{(2)}(k\Delta\xi)W_{ik}$$
$$-[J_{22}]^{-1}[J_{21}][J_{11}]^{-1}\sum_{i=-M}^{M}\delta_{\Delta,\sigma}^{(1)}(k\Delta\xi)W_{ik}$$
$$+\lambda^2\left[[J_{22}]^{-1}\sum_{i=-M}^{M}\delta_{\Delta,\sigma}^{(2)}(k\Delta\eta)W_{jk}\right]$$
$$-\lambda^2\left[[[J_{22}]^{-1}[J_{21}][J_{11}]^{-1}\sum_{i=-M}^{M}\delta_{\Delta,\sigma}^{(1)}(k\Delta\eta)W_{jk}\right]$$
$$+\Omega^2 W_{ij}=0 \quad (24)$$

5 Numerical results

In this section, free vibration of membranes having different geometries is analyzed. Numerical results are presented for the frequencies analysis of trapezoidal, rhombic, skew, sectorial, annular sectorial, elliptic, convex and concave membranes (Figs. 2-9). A few investigators have attempted to analyze the vibration of membranes by using the different numerical and analytical methods. These methods are the finite element, differential quadrature, Ritz and radial basis function based differential quadrature.

To check the validity of the fundamental solution and the proposed formulation, the results for frequencies of trapezoidal and square membrane are obtained with different grid numbers. The results are compared with the available results and are listed here. Free vibration analysis of trapezoidal membrane (Fig.3) is considered. The results obtained by the present method are compared with the finite element solution (Kang and Lee, 2004). The frequency values are given in Table 1. The results are matching very well with the results given by Kang and Lee (2004).

Another convergence study is presented in Table 2 for the first six modes of vibration for the rectangular (*b*/*a*=0.75) plate. For the using 15 grids, the rate of convergence remains reasonable and the agreement with the results from references (Kang et al., 1999; Kang and Lee, 2000; Wu et al.,2007) is excellent. First five frequency values of the skew membrane are listed in Table 3 for four different skew angles.

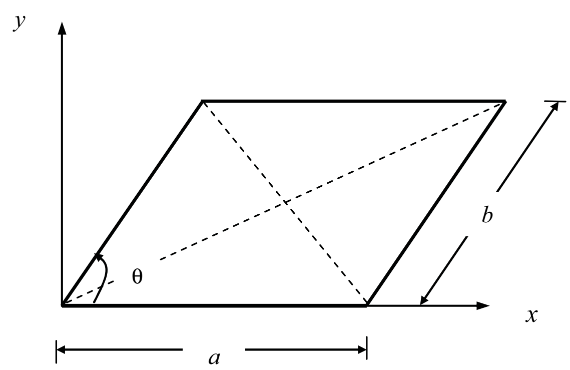

Figure 2: Skew membrane

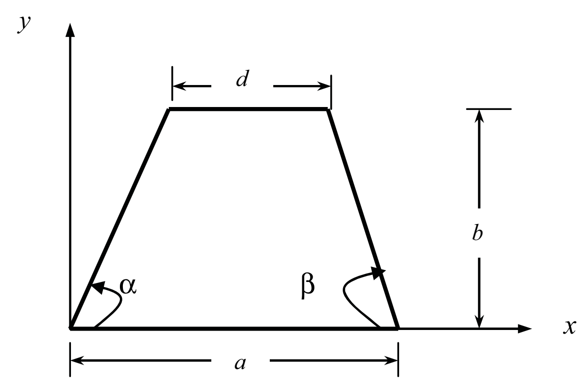

Figure 3: Trapezoidal plate

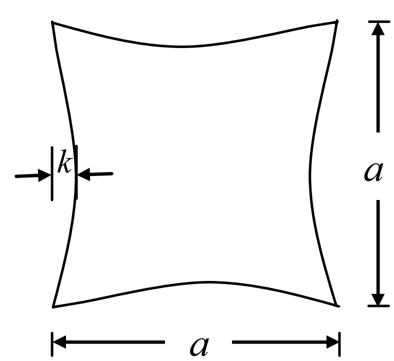

Figure 4: Membrane with four concave edges

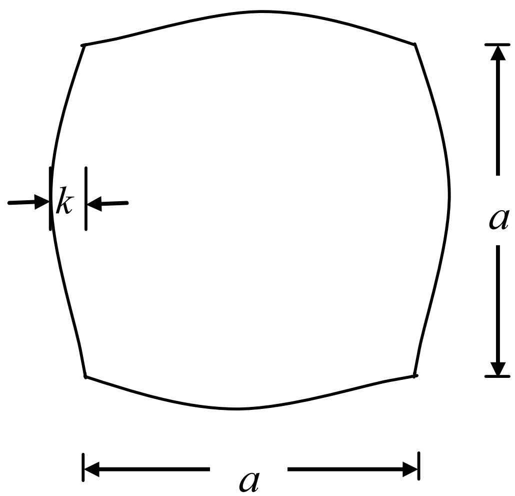

Figure 5: Membrane with four convex edges

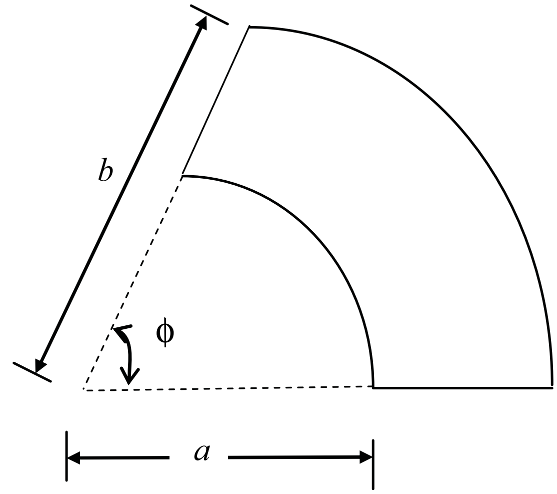

Figure 8: Annular sectorial membrane

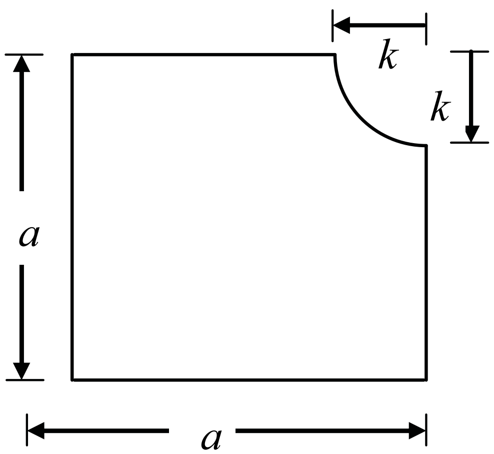

Figure 6: Membrane with a quarter-circular edge cuts

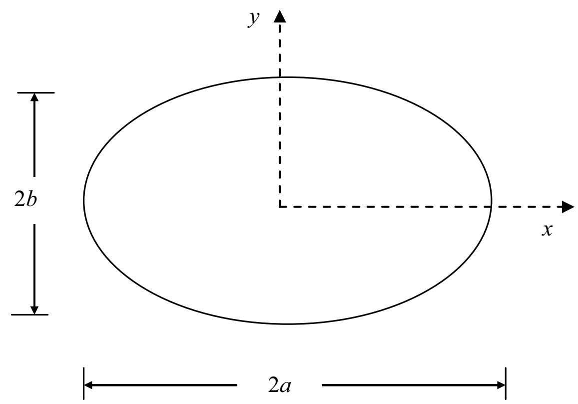

Figure 9: Elliptic membrane

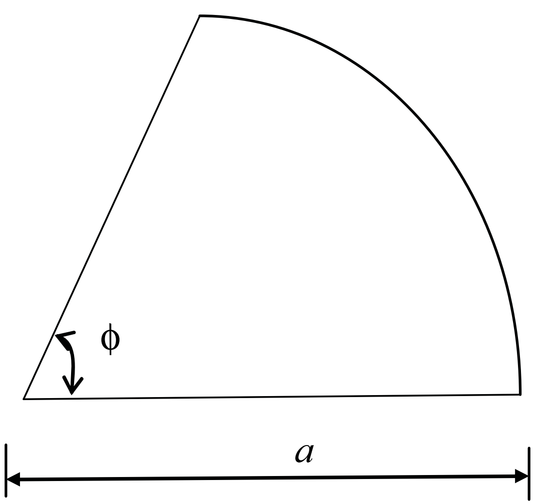

Figure 7: A typical sectorial membrane

Table 4 and Table 5 summarize the five smallest frequency values of the sectorial and annular sectorial membrane (Figs.7-8). The calculated results are compared with results given by Houmat (2006). It is shown that the good agreement is observed in all cases. The free vibration of elliptic membrane as shown in Fig. 9 was studied. The results obtained are presented in Table 6. The first three frequency values are presented in this table for four different a/b ratio of elliptic plate. Finally, the fundamental frequency values are listed in Table 7 for different curvilinear shaped membrane (Figs. 4-6). These are labeled as the membrane four concave edges, membrane four convex edges and square membrane with a quarter-circular edge cuts. It is found that the frequency parameters of membranes with concave shaped show larger than those of convex membrane. It is also concluded that, as the parameter k increase and the shape of membranes become more irregular, the frequency values monotonically increase due to the decrease of the area.

Table 1: Comparison of frequency values of the trapezoidal membrane (a/b=2.0; $\beta = 70°$; $\alpha = 60°$)

Mode	Methods			
	Kang and Lee(2004)	Present DSC N=11	Present DSC N=13	Present DSC N=15
1	3.81	3.83	3.82	3.82
2	5.29	5.30	5.27	5.27
3	6.58	6.58	6.56	6.56
4	7.07	7.07	7.05	7.05
5	7.62	7.64	7.62	7.60
6	8.75	8.76	8.74	8.73

Table 2: Comparison of frequencies of the rectangular membrane (b/a=0.75)

Mode	Methods			
	Kang et al. (1999)	Wu et al. (2007)	Kang et al. (1999)	Present DSC (N=15)
1	4.3633	4.3633	4.3651	4.3633
2	6.2929	6.2929	6.3006	6.2930
3	7.4560	7.4561	7.4669	7.4561
4	8.5947	8.5948	8.6213	8.5948
5	8.7266	8.7267	8.7407	8.7267
6	10.508	10.508	10.537	10.508

Table 3: First five frequency values of the skew membrane (b/a=1)

Mode	Skew angles			
	$\theta = 75°$	$\theta = 70°$	$\theta = 60°$	$\theta = 45°$
1	4.5671	4.6708	4.9912	5.9019
2	6.9514	7.0061	7.2588	8.1477
3	7.4940	7.7520	8.4751	10.0261
4	8.8954	8.9403	9.1623	10.3592
5	10.2019	10.4162	11.0856	11.8780

Table 4: Comparison of five smallest frequency of the sectorial membrane (a=1; $\phi = 90°$)

Mode	Methods		
	Houmat (2006)	Houmat (2006)	Present DSC N=15
1	5.136	5.136	5.136
2	7.589	7.588	7.590
3	8.417	8.147	8.148
4	9.937	9.936	9.939
5	11.065	11.065	11.067

Table 5: Comparison of five smallest frequencies of the annular sectorial membrane (a/b=1/2; $\phi = 90°$)

Mode	Methods			
	Houmat (2006)	Houmat (2006)	Present DSC N=13	Present DSC N=15
1	6.813	6.813	6.814	6.8141
2	8.266	8.266	8.267	8.2670
3	10.189	10.188	10.189	10.189
4	12.311	12.311	12.311	12.311
5	12.855	12.855	12.870	12.862

Table 6: First three frequency values of the elliptic membrane

Mode	a/b			
	1	1.5	2	2.5
1	2.406	2.041	1.892	1.814
2	5.528	4.830	4.006	3.817
3	8.663	7.026	6.517	5.820

6 Conclusions

In the presented study, the method of discrete singular convolution has been proposed as a new numerical algorithm for the eigenvalue analysis of membranes having different geometries. The frequency values of skew, trapezoidal, sector, annular sector, elliptic, convex and concave membranes have been obtained. Where available, the obtained results have been compared with the results given by literature. Furthermore, some results provided are benchmark solutions for which future comparisons can be made. Finally, it is

Table 7: Fundamental frequency values of the membrane with different geometries

MembraneShape	k/a			
	0.1	0.15	0.20	0.25
Concave membrane	4.68	4.76	4.89	5.01
Convex membrane	4.34	4.29	4.27	4.24
Membrane with quarter-circular edge cuts	4.49	4.53	4.62	4.75

worth nothing that the present DSC method could be extended to free vibration analysis of triangular membranes using a different geometric mapping technique. This study will be reported in another paper.

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