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A Faster Method of Moments Solution to Double Layer Formulation of Acoustic Scattering

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Abstract: In this work, the acoustic scattering problem based on double layer formulation is solved with a novel numerical technique using method of moment's solution. A new set of basis functions, namely, Edge based Adaptive Basis Functions (EABF) are defined in the method of moment's solution procedure. The geometry of the body is divided into triangular patches and basis functions are defined on the edges. Since the double layer formulation involves the evaluation of the hyper-singular integral, the edge based adaptive basis functions are used to make the solution faster. The matrix equations are derived for the double layer formulation. The edge based adaptive basis functions used in this work generates a diagonal moment matrix and hence do not need any matrix inversion to be carried out. The scattering cross section of the canonical shapes is used to validate the numerical solution developed and the plots are presented.

Keyword: Acoustic scattering, Method of moments, Adaptive basis function, Boundary integral equations.

1 Introduction

Burton and Miller (BM) [Burton and Miller (1971)] has proposed a mathematical formulation to address the non-uniqueness of the solution that is inherent in the boundary integral equation formulations of exterior acoustic scattering/radiation problems. Non-uniqueness of the solutions may be defined as failure of these formulations when the frequency of incident acoustic wave matches with the characteristic / eigen-frequencies of the corresponding interior problem. BM approach [Burton and Miller (1971)] is based on linearly combining the Helmholtz integral equation and its normal derivative with a complex coupling parameter. It has been mathematically proved by them that, this linearly combined integral equa-

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tion insures unique solution at all frequencies. A major drawback of this formulation is the presence of the hyper-singular integral in the normal derivative of the Helmholtz integral equation. Many researchers [Amini and Wilton (1986), Meyer, Bell, Zinn, and Stallybras (1978), Chien, Raliyah and Alturi (1990), Yan, Cui and Hung (2005)] have attempted to evaluate the hyper-singular integral that may be used in the BM procedure to overcome the non-uniqueness problem. The usual procedure that has been proposed by these researchers is to regularize the hyper–singular integral. The regularization technique is computationally very expensive and it is difficult to incorporate in a general-purpose code. Also, there are other methods which reduce the hyper singular kernel to a strongly singular kernel and their solution is based on based on Petrov-Galerkin schemes [Qian, Han, and Atluri (2004)] and collocation-based boundary element method [Qian, Han, Ufimtsev, and Atluri (2004)]. A de-singularized boundary integral formulation is also one of the recently proposed method [Callsen, von Estorff, and Zaleski (2004)] to overcome the problems of singularity.

Chandrasekhar and Rao [Chandrasekhar and Rao (2004b)] have attempted to extend the concept of BM approach by linearly combining the integral equations based on layer potentials, namely, Single Layer and Double Layer Formulations, in contrast to combining the Helmholtz integral equation and the its normal derivative as suggested by Burton and Miller. The double layer formulation that they defined [Chandrasekhar and Rao (2004a)] also has the hyper-singular integral. They used a unique method of moments solution procedure along with a simple vector calculus operations to circumvent the hyper-singular nature of the integral. The method of moments solution procedure that they used, has basis functions defined on edges and it results in a moment matrix of the order equal to the number of edges generated in the triangular patch modeling of an arbitrarily shaped three dimensional body.

Chandrasekhar [Chandrasekhar (2005)] has tried to reduce the solution time of the acoustic scattering problem by defining the basis functions on the nodes in contrast to defining it on the edges. Since the number of nodes that are generated in the triangular patch modeling of a closed body is almost one third of the number of edges, the order of the moment matrix is equal to the number of nodes or approximately equal to one third of the number edges. In both the ways of defining the basis functions, i.e. on edges and on nodes, the moment matrix that is generated is a full matrix and matrix inversion is required to be carried out to solve the linear system of equations. Matrix inversion is the most expensive step computationally and is one of the major drawbacks of the method of moments procedure compared to differential equation formulation and finite element procedures.

In this work, the double layer formulation is solved with edge based basis functions, but without using the step of matrix inversion. For this purpose, a new set of basis functions are defined in such a way that the final matrices generated by the method of moment's solution procedure is diagonal and hence they do not need any matrix inversion algorithms to solve the linear system of equations. Since the matrix inversion is eliminated, it does not matter much from computational perspective whether the basis functions are defined on the edges or nodes as the matrix generated is a diagonal one. The new basis functions, popularly known as adaptive basis functions in the electromagnetic scattering/radiation areas [Waller and Rao (2002)], are used as basis functions while the pulse functions are used as testing functions. It is not the intent of this work to prove the non-uniqueness of the solution, but only to solve the double layer formulation (DLF) faster.

The method developed in this work can be used to improve or in combination with other methods like Mesh Less Petrov-Galerkin schemes [Vavourakis, Sellountos and Polyzos (2006); Sladek, Sladek, Wen and Aliabadi (2006); Gao, Liu and Liu (2006); Zhang and Chen (2008); Sladek, Sladek, Solek and Wen (2008); Dang and Bhavani Sankar (2008); and Arefmanesh, Najafi and Abdi, (2008)], boundary element formulation but not limited to acoustic scattering [Owatsiriwong, Phansri, and Park (2008); Criado, Ortiz, Manti c, Gray, and Paris (2007); Zai You Yan (2006); and Soares Jr, and Vinagre (2008):] or other techniques [Gergidis, Kourounis, Mavratzas and Charalambopoulos (2007); Chandrasekhar and Rao. (2007); Christov, Christov and Jordan (2007) and Fabian and Duddeck (2006)].

2 Organization of a paper

In this paper, next section briefly describes the method of moment's solution procedure [Harrington (1968)]. References to mathematical formulation and derivation of matrix equations are given in section 4 for the double layer formulation (DLF). In section 5, we describe the numerical solution procedure and develop edge based adaptive basis functions. Numerical results, based on the development of new basis functions are given in section 6. Lastly we present some important conclusions drawn from the present work.

3 Outline of Method of Moments (MoM)

Consider the deterministic equation

$$Lf = g \tag{1}$$

where L is a linear operator, g is a known function and f is an unknown function to be determined. Let f be represented by a set of known functions f_j, $j = 1,2,\ldots,N$

termed as basis functions in the domain of L as a linear combination, given by

$$f = \sum_{n=1}^{N} \beta_j f_j \tag{2}$$

where β_j are scalar coefficients to be determined. Substituting Eq. 2 into Eq. 1, and using the linearity of L, we have

$$\sum_{n=1}^{N} \beta_j L f_j = g \tag{3}$$

where the equality is usually approximate. Let $(w_1, w_2, w_3, ...)$ define a set of testing functions in the range of L. Now, taking the inner product of Eq. 3 with each w_i and using the linearity of inner product defined as $\langle f, g\rangle = \int_s f \bullet g ds$, we obtain a set of linear equations, given by

$$\sum_{n=1}^{N} \beta_j \left\langle w_i, L f_j \right\rangle = \langle w_i, g\rangle \quad i = 1, 2, \ldots, N \tag{4}$$

The set of equations in Eq. 4 may be written in the matrix form as

$$ZX = Y \tag{5}$$

which can be solved for X using any standard linear equation solution methodologies. The simplicity, accuracy and efficiency of the method of moments lies in choosing proper set of basis/testing functions and applying to the problem at hand. In this work, we propose a special set of basis functions and a novel testing scheme to obtain accurate results using DLF.

4 Matrix Equations

Consider an acoustic wave, with a pressure and velocity $\left(p^i, u^i\right)$, incident on a three-dimensional arbitrarily shaped rigid body placed in a source free homogeneous medium of density ρ and speed of sound c through the medium. When the incident wave interacts with the body, the acoustic wave gets scattered with a pressure and velocity (p^s, u^s). Here, we note that, incident fields are defined in the absence of the scattering body. Φ is the scalar velocity potential satisfying the Helmholtz differential equation $\nabla^2\Phi + k^2\Phi = 0$ for the time harmonic waves present in the region exterior to the surface of the body. One more condition on velocity potential is that it should satisfy the appropriate boundary conditions on the surface of the body along with the Sommerfeld radiation condition. The pressure and velocity fields of acoustic wave is related to the scalar velocity potential Φ as $u = -\nabla\Phi$ and $p = j\omega\rho\Phi$. The mathematic formulation and the derivation of the matrix equations for the double layer formulation is already reported in ref [Chandrasekhar and Rao (2004a)]. In ref [Chandrasekhar and Rao (2004a)], the matrix equations derived are based on defining the basis functions on each edge and testing is carried out using Galerkin's method. Where as in this work, the basis functions defined on each edge are grouped into different clusters to generate a diagonal moment matrix.

5 Numerical Solution Procedure

In a method of moments solution procedure, the basis functions can be defined on patches [Raju, Rao, Sun (1991), Rao and Sridhara (1991), Rao, Raju, and Sun (1992), Rao and Raju, (1989)], on edges [Chandrasekhar and Rao (2004a)], or on nodes [Chandrasekhar (2005)]. The effect of a basis function defined on any of the geometric entity like a patch, an edge or a node would be a non-zero at any point somewhere else on the structure. That means the effect of a unit source located at one point will be experienced by every other point on the structure. Imagine a case where there is cluster of basis functions, formed by grouping the basis functions and by assigning different weights to each of the basis functions in the cluster, net effect of this entire cluster at any desired point on the structure can be made to zero or less than a threshold value thus producing a null-field. This cluster of basis functions which produces a null-field at any desired point on the structure is called as Adaptive Basis Functions. Since the basis functions are defined on the edges in this work, and grouping them into a cluster, they are called as Edge based Adaptive Basis Functions (EABF). The procedure to develop the EABF is described in the following paragraphs.

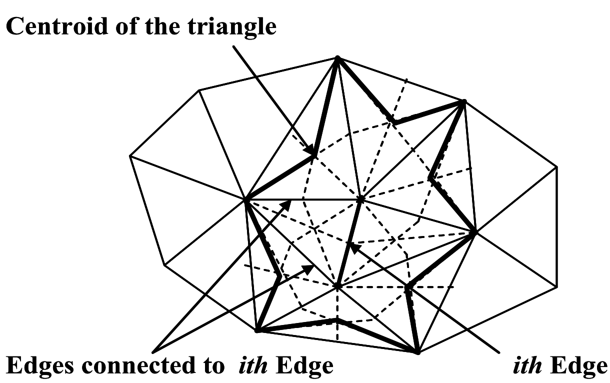

Figure 1: Cluster of edges grouped for defining adaptive basis function (EABF).

Initially, a cluster of basis functions is formed by grouping the neighborhood edges and weights are assigned to each of basis functions in the cluster. One may note that, while choosing the cluster of basis functions, the condition of neighborhood is not necessary. Let there are K basis functions in the cluster and the complex weights attached to these basis functions are α_k, $k = 1,2,......,K$. Now let us assume that there are N_e edges on the surface of the structure and the number of basis functions K in the cluster is very less compared to N_e i.e. $K \ll N_e$. By assigning complex weights α_k to each of the basis functions in the cluster, and when tested at any point outside the cluster, it results in a null field. Here, one may note that the points that are chosen for testing the basis functions need not completely lie outside the cluster, which is made clear in the following paragraphs.

Next task would be to evaluate the weights α_k, which requires the cluster of basis functions to be tested at number points equal to or grater than the number of basis functions in the cluster. This results in a linear system of equations equal to or greater than the number of weights to be determined depending on the case where the number of testing points chosen are equal to or greater than number of basis functions in the cluster respectively. In either case, the weights can be uniquely solved for. In the next paragraph, an example of a cluster of edge based basis functions and the procedure to evaluate the weights α_k are explained.

Consider an edge i and a set of edges in the neighborhood of edge i in the triangular patch modeling which has a total of N_e edges as shown in Fig. 1. The procedure to define an edge based basis function is already reported in ref [Chandrasekhar and Rao (2004a)]. Let there are K number of edges in the neighborhood of edge i. Let a cluster of edges be formed by grouping the edge i and its neighborhood edges. Hence there are totally $K+1$ number of edges in the cluster. By assigning a weight of unity or one to the edge i and weights α_k to rest of the edges in the cluster, α_k becomes the unknowns which need to be determined uniquely by testing the cluster at number of points equal to or greater than K. For the sake of clarity let us assume $K = 10$. Then for notational purposes the indices of the edges range from $i-5, i-4, \ldots, i, \ldots, i+4, i+5$ which is arbitrary. When the testing is carried out, it results in a system of linear equations given by

$$
\begin{aligned}
Z_{1,i-5}\alpha_{i,i-5} + Z_{1,i-4}\alpha_{i,i-4} + \ldots + Z_{1,i} + \ldots \\
+Z_{1,i+4}\alpha_{i,i+4} + Z_{1,i+5}\alpha_{i,i+5} = 0 \\
Z_{2,i-5}\alpha_{i,i-5} + Z_{2,i-4}\alpha_{i,i-4} + \ldots + Z_{2,i} + \ldots \\
+Z_{2,i+4}\alpha_{i,i+4} + Z_{2,i+5}\alpha_{i,i+5} = 0 \\
\ldots = 0 \\
Z_{N,i-5}\alpha_{i,i-5} + Z_{N,i-4}\alpha_{i,i-4} + \ldots + Z_{N,i} + \ldots \\
+Z_{N,i+4}\alpha_{i,i+4} + Z_{N,i+5}\alpha_{i,i+5} = 0
\end{aligned}
\tag{6}
$$

Eq. 6 can be represented in matrix form as

$$\begin{bmatrix} Z_{1,i-5} & Z_{1,i-4} & \dots & Z_{1,i+5} \\ Z_{2,i-5} & Z_{2,i-4} & \dots & Z_{2,i+5} \\ \dots & \dots & \dots & \dots \\ Z_{N_e,i-5} & Z_{N_e,i-5} & \dots & Z_{N_e,i-5} \end{bmatrix} \begin{Bmatrix} \alpha_{i,i-5} \\ \alpha_{i,i-4} \\ \dots \\ \alpha_{i,i+5} \end{Bmatrix} = \begin{Bmatrix} -Z_{1,i} \\ -Z_{2,i} \\ \dots \\ -Z_{N_e,i} \end{Bmatrix} \tag{7}$$

In the above matrix, the first suffix of Z ranging from 1 to N_e represents the index of an edge on which the testing is carried out while the second index represents the source edge on which the basis function is defined. The $Z_{j,i}$ are computed in the same procedure as described in ref [Chandrasekhar and Rao (2004a)]. By solving the Eq. 6 in the least square sense, the $\alpha_{i,k}$'s can be evaluated. Once the $\alpha_{i,k}$'s are obtained, one can construct the i^{th} adaptive basis function, given by

$$h_i = f_i + \sum_{k=1}^{K} \alpha_{i,k} f_k \tag{8}$$

By using h_i as the basis function, it produces null field at every point other than at i^{th} edge. Thus, it results whole i^{th} column of [Z]-matrix as zero, or below a certain threshold value, barring the diagonal term.

The above mentioned procedure for constructing the adaptive basis function for i^{th} edge may be repeated for all the N_e edges and thus N_e adaptive basis functions $h_i, i = 1, \dots, N_e$ can be constructed. The MoM procedure with EABF produces a diagonal $[Z]$-matrix which can be easily solved for the source distribution without using any matrix inversion algorithms or linear equations solvers. The off-diagonal elements of the $[Z]$-matrix is not zero, but below certain threshold value which can be set to zero.

Two important questions that need to be answered in the usage of EABF is 1). How many basis functions one should use to obtain the satisfactory results and 2). The criterion in choosing the K basis functions. Answer to the first question is explained with an example in the next section which proves that when number of basis functions in the cluster is about 10% of total number of edges generated in the triangulated model, the solution gives satisfactory results for the simple and acoustically small problems. Higher the number of basis functions in the cluster, higher the accuracy of the solution with respect to the traditional MoM solution based on the independent basis functions. The criterion in choosing the basis functions is, it is always better to choose the basis functions into the cluster which are in the neighborhood of the i^{th} edge since the neighborhood basis functions have almost same amount of effect at any given point as the i^{th} edge, if they are very closely spaced. However, it is not a necessary condition and one can choose the basis functions into the cluster which are not adjacent to each other.

In case, when number of basis functions chosen is less than the required minimum number, the results may not be satisfactory. This is due to the fact that off-diagonal elements will be having values greater than the threshold limit and may not be close to zero as defined by the threshold value. This is because, one may have to gather certain minimum number of basis functions into the cluster to get the given threshold limit. In case this is not possible, then one may choose the solution obtained as an initial guess to solve the Eq. 5 iteratively. This is an effective method of solving the Eq. 5 and it will take only a fewer number of iterations as proved in the next section. In this work, conjugate gradient method is chosen as the iterative scheme and one may choose more efficient algorithms for the same purpose.

It is also possible to use the adaptive basis functions as testing functions as well. Further for Galerkin procedure, one can use these functions both as basis as well as testing functions. But it is more computationally expensive to use these functions in Galerkin procedure.

The salient features of the proposed method are:

Less storage memory: The computer storage requirement for the method of moments solution with EABF is considerably less than the MoM solution based on using ungrouped edge based basis functions [Chandrasekhar and rao (2004a)].The storage requirements for the new method has two parts. The first one being the space required to store the weights of the basis functions which is KXN_e, where$K \ll N_e$. The second one being the storage of the diagonal matrix, which can be a vector of size N_e as the off-diagonal elements of the $[Z]$-matrix are assumed to be below the threshold value or zero. To refine the solution, the solution of the adaptive basis functions can be used as an initial guess for an iterative solution of Eq. 5, then an additional storage space of N_eXN_e for $[Z]$-matrix may be needed, which is worth as the computational time required for the solution of the linear system of equations is greatly reduced.

Less computational time: Computationally most expensive step in the solution of the acoustic scattering problem based on the integral equation formulation and method of moment's solution is the inversion of the moment matrix. Especially when the number of edges in the triangulated model is large, hardware of the computer in use may have a limitation in solving the linear system of equations. Since the matrix inversion is eliminated in this new method, it can handle much larger problems. For a case of number of unknowns of 100,000 time required for the generation of $[Z]$-matrix is very less compared to the time required to invert it. Since this step of inversion is completely eliminated in the proposed method, it is more efficient than the traditional MoM. However, one may have to solve the linear system of equations given by Eq. 6, number of times equal to N_e. This step is not computationally expensive as $K \ll N_e$ when N_e is very large. The operation count to solve the linear system of equations given Eq. 6, is estimated to be of the order $\left(K^2N_e + KN_e + cK^3\right)$, where c is the constant. As long as $K \ll N_e^{1/2}$, the operation count is similar to the MoM solution. However, for acoustically large problems, with $N \approx$100,000, we envisage $K \approx N_e^{1/3}$ and the proposed method gives a more efficient solution than the traditional MoM. Further, the number of equations in Eq. can be reduced to K[Rius, Parron, Ubeda and Mosig (1997)] which implies a very large reduction in the computational operations. Thus when the computer storage and solution time is a factor in solving the large problems, the capability of the proposed method can be experienced.

6 Numerical Results

In this section, the numerical solution developed using EABF is validated for the cases of a sphere, cylinder and a cube which are three dimensional bodies. For all these cases, the body is placed at the center of the co-ordinate system and a plane wave, traveling along –Z axis is incident on the body. Here, one may note that, no convergence study is carried out to ascertain the optimum number of basis functions required in the cluster to obtain certain degree of accuracy. The scattering cross section is defined by

$$\begin{aligned} S &= 4\pi \left| \frac{\Phi^s}{\Phi^i} \right|^2 \\ &\approx \frac{1}{4\pi} \left| \sum_{n=1}^{N_e} \alpha_n A_n n_n \bullet r_n e^{jkn_n \bullet r_n} \right|^2 \end{aligned} \tag{9}$$

As a first case, a sphere of radius $1m$ is considered and it is approximated with triangular patch modeling. The modeling is done by dividing the θ and ϕ direction equal segments each and the complete modeling procedure is described in [Chandrasekhar and Rao (2004a)]. Also the geometries with sharp corners and sharp edges such as a cube and a cylinder are considered in order to demonstrate the capability of method of moments solution based on EABF.

The sphere considered for the validation of the EABF solution is modeled with 178 nodes, 352 patches and 528 edges. Fig.2 shows the scattering cross section versus polar angle for an acoustically rigid sphere of radius 1m, subjected to an axially incident plane wave of $k = 1 rad/m$, based on double layer formulation (DLF). Solutions based on the edge based adaptive basis functions (EABF) are compared with the exact and traditional MoM solution with out any refinement to the ABF solutions. The ABF solution with K =25, 37 and 53 are validated and it can be concluded that the results with K=53 is very good agreement with the exact solution which is 10% of the N_e and the traditional MoM solution. In case of the traditional MoM solution, the order of the matrix is 528 which is inverted/solved iteratively. Where as in the cases of solution based on EABF, the number of basis functions used is 53 and it generates an almost diagonal matrix.

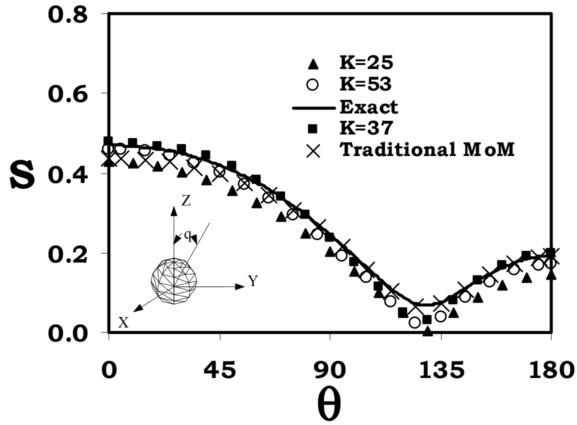

Figure 2: Scattering cross section versus polar angle for an acoustically rigid sphere of radius 1m, subjected to an axially incident plane wave of $k = 1rad/m$, based on double layer formulation (DLF) with no refinement of ABF sloution.

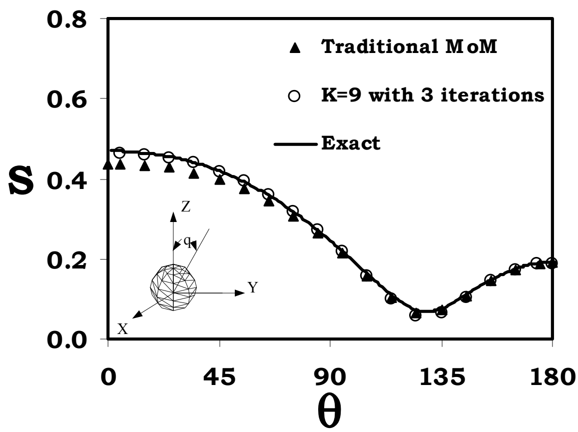

Figure 3: Scattering cross section versus polar angle for an acoustically rigid sphere of radius 1m, subjected to an axially incident plane wave of $k = 1rad/m$, based on double layer formulation (DLF) with refinement of ABF solution.

Fig. 3 shows the scattering cross section versus polar angle for an acoustically rigid sphere of radius 1m, subjected to an axially incident plane wave of $k = 1rad/m$, based on double layer formulation (DLF) where the refinement to the ABF solutions are considered. Again, the same problem is solved with a cluster of 9 basis functions and 3 iterations are used to refine the solution which is in well agreement with the exact solution as well as the traditional MoM solution. In this case the number of basis functions in the cluster are chosen as $N^{1/3}$, the solution obtained is used as an initial guess to solve Eq. 5 using conjugate gradient iterative scheme, which just needed 3 iterations to obtain a solution of good accuracy. The cases considered next do not have the closed form solution, hence the traditional MoM solution is compared with the exact solution for the case of sphere [Bowman, Senior and Uslenghi (1969)] to validate the traditional MoM solution.

Fig. 4 shows the scattering cross section versus polar angle for an acoustically rigid cylinder of radius 1m and height 1m, subjected to an axially incident plane wave of $k = 1rad/m$, based on double layer formulation (DLF). Here the cylinder is modeled with 202 nodes, 400 patches and 600 edges. The number of basis functions chosen in the cluster are 9 for calculating the initial guess and 3 iterations were used to solve Eq. 5 iteratively. The solution based on the EABF is in well agreement with the solution based on traditional MoM solution in which the independent pulse functions are used basis functions on each edge.

As a next example, to show the capability of the edge based adaptive basis func-

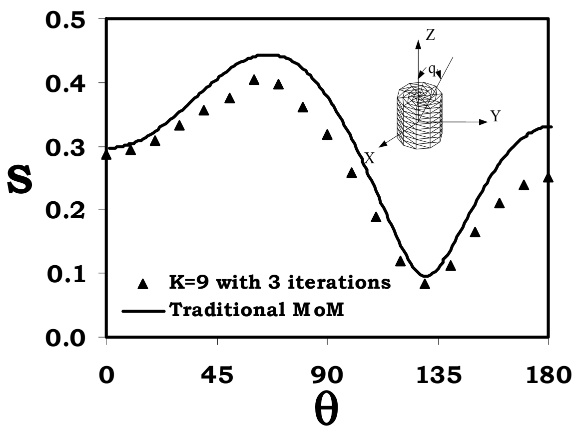

Figure 4: Scattering cross section versus polar angle for an acoustically rigid cylinder of radius 1m and height 1m, subjected to an axially incident plane wave of $k = 1$ rad/m, based on Double layer formulation (DLF) with refinement of ABF solution.

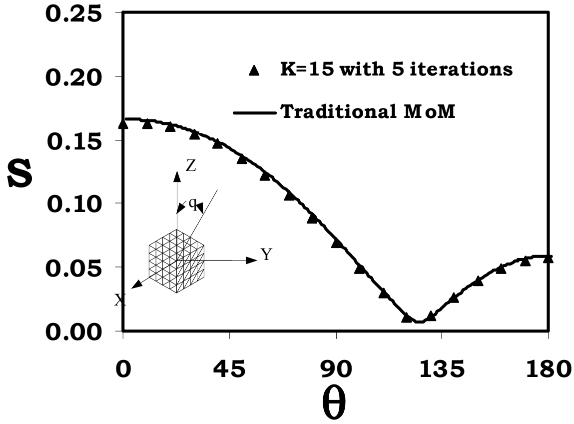

Figure 5: Scattering cross section versus polar angle for an acoustically rigid cube of length 1m, subjected to an axially incident plane wave of k = 1 rad/m, based on Double layer formulation (DLF) with refinement of ABF solution.

tions which can handle higher number of edges in the triangulated model, a cube is modeled with 866 nodes, 1728 patches and 2592 edges. Here, in case of the traditional MoM, one has to invert/solve a matrix of size 2592x2592 which is completely eliminated by using the edge based adaptive basis functions, where only 15 basis functions are considered in the cluster. The solution obtained by the cluster of basis functions is further refined 5 times to get more accurate solution. Fig. 5 shows the scattering cross section versus polar angle for an acoustically rigid cube of length 1m, subjected to an axially incident plane wave of k = 1 rad/m, based on double layer formulation (DLF). The solution obtained by the adaptive basis functions is in well agreement with the traditional MoM solution.

7 Conclusions

In this work, a new method to solve the acoustic scattering problem from arbitrarily-shaped 3-D rigid bodies, based on method of moments solution is presented. The new method uses new basis functions, namely, Edge based Adaptive Basis Functions (EABF), to generate a diagonal matrix considering non-zero elements of the off-diagonal elements less than a threshold value. The diagonal matrix generated by the EABF solution captures the essential features of the scattering phenomenon. To improve the solution further, a simple iterative scheme is proposed which gives accurate solution after relatively a few iterations when compared to the regular MoM solution. The double layer formulation which involves evaluation of hyper singular integral, needed edge based basis functions for the numerical solution based on method of moments procedure. Since the number of edges in a triangular patch modeling of a closed body is higher than that of patches and nodes, it generates largest size of the matrix compared to patch based or node based solutions. With the development of the edge based adaptive basis functions, the final size of the moment matrix is relatively irrelevant as the matrix inversion is eliminated. Presently work is in progress to extend the concept of cluster of basis functions to solve the combined layer formulation using node based basis functions.

References

Amini, S.; Wilton, D.T. (1986): An investigation of Boundary element methods for the exterior acoustic problem. *Computational Methods in Applied Mechanical Engineering*, vol 54, pp. 49-65.

Arefmanesh, A.; Najafi, M.; Abdi, H. (2008): Meshless Local Petrov-Galerkin Method with Unity Test Function for Non-Isothermal Fluid Flow. *CMES: Computer Modeling in Engineering and Sciences*, vol. 25, No. 1, pp. 9-22.

Bowman, J.J.; Senior, T.B.A.; Uslenghi, P.L.E. (1969): Electromagnetic and Acustic Scattering by Simple Shapes North-holland, Amsterdam.

Burton, A.J.; Miller, G.F. (1971): The application of integral equation methods to the numerical solution of some exterior boundary value problems. *Proceedings of Royal Society, London*, vol A323, pp. 601-618 .

Callsen, S.; von Estorff, O.; Zaleski, O. (2004): Direct and Indirect Approach of a Desingularized Boundary Element Formulation for Acoustical Problems. *CMES: Computer Modeling in Engineering & Sciences*, Vol. 6, No. 5, pp. 421-430

Chandrasekhar, B.; Rao, S.M. (2004a): Acoustic scattering from rigid bodies of arbitrary shape –double layer formulation. *Journal of Acoustical Society of America*, vol 115, pp. 1926-1933.

Chandrasekhar, B.; Rao, S.M. (2004b): Elimination of internal resonance problem associated with acoustic scattering by three-dimensional rigid body, *Journal of Acoustical Society of America*, vol 115, pp. 2731-2737.

Chandrasekhar, B.; Rao, S.M. (2005): Acoustic Scattering from Complex Shaped Three Dimensional Structures, *CMES: Computer Modeling in Engineering & Sciences*, Vol. 8, No. 2, pp. 105-118.

Chandrasekhar, B. (2005): Acoustic scattering from arbitrarily shaped three dimensional rigid bodies using method of moments solution with node based basis functions, *CMES: Computer Modeling in Engineering & Sciences*, Vol. 9, No. 3, pp. 243-254.

Chandrasekhar, B.; Rao, S.M.. (2007): Acoustic Scattering from Fluid Bodies of Arbitrary Shape. *CMES: Computer Modeling in Engineering & Sciences*, vol. 21, No. 1, pp. 67-80.

Chien, C.C.; Raliyah, H.; Alturi, S.N. (1990): An effective method for solving the hyper singular integral equations in 3-D acoustics. *Journal of Acoustical Society of America*, vol 88, pp. 918-937.

Christov, I.; Christov, C.I.; Jordan, P.M. (2007): Cumulative Nonlinear Effects in Acoustic Wave Propagation, *CMES: Computer Modeling in Engineering & Sciences*, vol. 17, No. 1, pp. 47-54.

Criado, R; Ortiz, J.E.; Manti c, V.; Gray, L.J.; Paris, F. (2007): Boundary Element Analysis of Three-Dimensional Exponentially Graded Isotropic Elastic Solids. *CMES: Computer Modeling in Engineering & Sciences*, vol. 22, No. 2, pp. 151.

Dang, T.D.; Bhavani V. Sankar (2008): Meshless Local Petrov-Galerkin Micromechanical Analysis of Periodic Composites Including Shear Loadings. *CMES: Computer Modeling in Engineering & Sciences*, vol. 26, No. 3, pp. 169-188.

Fabian M.; Duddeck, E. (2006): An Alternative Approach to Boundary Element Methods via the Fourier Transform. *CMES: Computer Modeling in Engineering & Sciences*, vol. 16, No. 1, pp. 1-14.

Gao, L.; Liu, K.; Liu, Y. (2006): Applications of MLPG Method in Dynamic Fracture Problems. *CMES: Computer Modeling in Engineering & Sciences*, vol. 12, No. 3, pp. 181-196.

Gergidis, L.N.; Kourounis, D.; Mavratzas, S.; Charalambopoulos, A. (2007): Acoustic Scattering in Prolate Spheroidal Geometry via Vekua Tranformation – Theory and Numerical Results. *CMES: Computer Modeling in Engineering & Sciences*, vol. 21, No. 2, pp. 157-176.

Harrington, R.F. (1968): *Field computation by Method of Moments*. MacMillan, New York.

Meyer, W.L.; Bell, W.A.; Zinn, B.T.; Stallybras, M.P. (1978): Boundary integral solutions of three dimensional acoustic radiation problems. *Journal of Sound and Vibration*, vol 59, pp. 245-262.

Owatsiriwong, A.; Phansri, B.; Park, K.H. (2008): A Cell-less BEM Formulation for 2D and 3D Elastoplastic Problems Using Particular Integrals. *CMES: Computer Modeling in Engineering & Sciences*, vol. 31, No. 1, pp. 37-60.

Qian, Z.Y.; Han, Z.D.; Atluri, S.N. (2004): Directly Derived Non-Hyper-Singular Boundary Integral Equations for Acoustic Problems, and Their Solution through Petrov-Galerkin Schemes, *CMES: Computer Modeling in Engineering & Sciences*, Vol. 5, No. 6, pp. 541-562

Qian, Z.Y.; Han, Z.D.; Ufimtsev, P.; Atluri, S.N. (2004): Non-Hyper-Singular Boundary Integral Equations for Acoustic Problems, Implemented by the Collocation-Based Boundary Element Method, *CMES: Computer Modeling in Engineering & Sciences*, Vol. 6, No. 2, pp. 133-144.

Raju, P.K.; Rao, S. M.; Sun, S.P. (1991): Application of the method of moments to acoustic scattering from multiple infinitely long fluid filled cylinders. *Computers and Structures*, vol 39, pp. 129-134.

Rao, S.M.; Raju, P.K. (1989): Application of Method of moments to acoustic scattering from multiple bodies of arbitrary shape. *Journal of Acoustical Society of America*, vol 86, pp. 1143-1148.

Rao, S.M.; Sridhara, B.S. (1991): Application of the method of moments to acoustic scattering from arbitrary shaped rigid bodies coated with lossless, shearless materials of arbitrary thickness. *Journal of Acoustical Society of America*, vol 90, pp. 1601-1607.

Rao, S. M.; Raju, P.K.; Sun, S.P. (1992): Application of the method of moments to acoustic scattering from fluid-filled bodies of arbitrary shape. *Communications in Applied Numerical Methods*, vol 8, pp. 117-128.

Rius, J.M.; Parron, J; Ubeda, E.; Mosig, J.R. (1997): Integral equation MEI applied to three-dimensional arbitrary surfaces, *Electron. Lett.*, pp. 2029-2031.

Schenck, H.A (1968): Improved integral formulation for acoustic radiation problems. *Journal of Acoustical Society of America*, vol 44, pp. 41-58.

Sladek, J.; Sladek, V.; Wen, P. H.; Aliabadi, M.H. (2006): Meshless Local Petrov-Galerkin (MLPG) Method for Shear Deformable Shells Analysis, *CMES: Computer Modeling in Engineering & Sciences*, vol. 13, No. 2, pp. 103-118.

Sladek, J.; Sladek, V.; Solek, P.; Wen, P.H. (2008): Thermal Bending of Reissner-Mindlin Plates by the MLPG. *CMES: Computer Modeling in Engineering & Sciences*, vol. 28, No. 1, pp. 57.

Soares Jr, D.; Vinagre, M. P. (2008): Numerical Computation of Electromagnetic Fields by the Time-Domain Boundary Element Method and the Complex Variable Method. *CMES: Computer Modeling in Engineering & Sciences*, vol. 25, No. 1, pp. 1-8.

Vavourakis, V.; Sellountos, E. J.; Polyzos, D. (2006): A comparison study on different MLPG(LBIE) formulations. *CMES: Computer Modeling in Engineering & Sciences*, vol. 13, No. 3, pp. 171-184.

Waller, M.L.; Rao, S.M. (2002): Application of Adaptive Basis Functions for a Diagonal Moment Matrix Solution of Arbitrarily Shaped Three-Dimensional Conducting Body Problems, *IEEE transactions on antennas and propagation*, vol. 50, no. 10, pp 1445-1452.

Yan, Z.Y.; Cui, F. S.; Hung, K. C. (2005): Investigation on the Normal Derivative Equation of Helmholtz Integral Equation in Acoustics. *CMES: Computer Modeling in Engineering & Sciences*, Vol. 7, No. 1, pp. 97-106.

Zai You Yan (2006): Treatment of Sharp Edges {\&} Corners in the Acoustic Boundary Element Method under Neumann Boundary Condition. *CMES: Computer Modeling in Engineering & Sciences*, vol. 13, No. 2, pp. 81-90.

Zhang, Y.Y.; Chen, L. (2008): A Simplified Meshless Method for Dynamic Crack Growth, *CMES: Computer Modeling in Engineering & Sciences*, vol. 31, No. 3, pp. 189-200.