 CMES, vol.42, no.3, pp.177-186, 2009

Stability Problem of Composite Material Reinforced by Periodical Row of Short Fibers

A.N.Guz[1] and V.A.Dekret[1]

Abstract: Stability problem of composite material reinforced by periodical rows of short fibers is solved. The problem is formulated with application of equations of linearized three-dimensional theory of stability. The composite is modeled as piecewise-homogeneous medium. The influence of geometrical and mechanical parameters of the composite to the critical strain is investigated.

Keywords: Stability problem, composite material, nanocomposite, short fibers, nanotube, three-dimensional linearized theory of stability.

1 Introduction

The most strict and physically correct results on the theory of stability of the layered and fibred composite materials under compression are obtained with constitutive equations of the three-dimensional linearized theory of stability of deformable bodies (for example, monograph [Guz A.N. (1999)]) and the model of piecewise-homogeneous medium, such approach is offered first in [Guz A.N. (1969)]. The modern analysis of construction of the three-dimensional linearized theory of stability of deformable bodies (and in more wide sense – the three-dimensional linearized mechanics of deformable bodies) is presented in [Guz A.N. (2001), (2002), (2004)]; here in [Guz A.N. (2004)] basic attention is given to the construction of linearized constitutive equations for the elastic and elastoplastic bodies. With this approach [Guz A.N. (2001), (2002), (2004)] it is solved the series of problems of linearized mechanics of deformable bodies and the analysis of the obtained results with such approach is executed, as an example it is possible to specify modern analysis of [Guz A.N., Guz I.A. (2004)] exact solutions of the plane mixed problems of the linearized mechanics of deformable bodies that is presented in [Guz A.N., Menshykov O.V., Zozulya V.V. and Guz I.A. (2007)], [Guz A.N., Rushchitsky J.J.

[1] The Institute of Mechanics National Academy of Sciences of Ukraine, Nesterov str., 3, 03680, Kiev, Ukraine Tel.: (38044) 4569351, Fax: (38044) 4560319, E-mail: guz@carrier.kiev.ua, dekret@inmech.kiev.ua

and Guz I.A. (2008)], [Guz A.N., Zozulya V.V. (2007)] and in other publications.

At present the researches on the three-dimensional linearized theory of stability of the layered and fibred composite materials within the framework of approach [Guz A.N. (1999)] are conducted with applying the models of *infinite fibers* or *short fibers*. The comparative modern analysis of the results which are obtained with foregoing two models (*infinite fibers, short fibers*) is presented in [Guz A.N., Dekret V.A. (2008)]. It is necessary to mark that within the framework of approach [Guz A.N. (1999)] for the layered and fibred materials the overwhelming number of results on the three-dimensional theory of stability is obtained with applying the *infinite fibers* model. The modern analysis of these results are presented in [Babich I.Yu, Guz A.N. and Chekhov V.N. (2001)], [Guz A.N., Chekhov V.N. (2007)], [Guz A.N., Lapusta Yu.N. (1999)] and in a row of other summarizings publications. Only last years the results were obtained within the framework of *short fibers* model, the results are presented, for example, in [Dekret V.A. (2006), (2008)] and in a row of other publications in magazines. In connection with it the results within the framework of *short fibers* model, that obtained presently and partly indicated in [Guz A.N., Dekret V.A. (2008)], it is possible to consider only beginning of development of the investigated problem.

It is necessary to mark the results of research with models of *infinite fibers* and *short fibers* essential depend on the chosen values which are analysed. So in the work [Guz A.N., Dekret V.A. (2008)] the comparative analyze is conducted with *infinite fibers* and *short fibers* models for values $|\varepsilon_{11}^{cra}|$. In the case of *infinite fibers* model the value corresponds the critical value of strain along an ax Ox_1 both for reinforcing elements (fibres) and for a matrix. In the case of *short fibers* model the value is entered as

$$\varepsilon_{11}^{cra} = \varepsilon_{11}^{cra}(x_1, x_2), \quad x_1 = 0,\, x_2 = 0. \tag{1}$$

Thus in this case the value (1) corresponds to the critical value of strain along an ax Ox_1 in the middle point of reinforcing element (fibers). For the *short fibers* model a value (1) characterizes only the critical value of strain along an ax Ox_1 for a fiber and does not characterize the critical value of strain along an ax Ox_1 for a matrix, which can arrive at other values substantially. Obviously, for a matrix the value of strain "at infinity", as it applies to the *short fibers* model, it would get substantially larger values, because matrix is substantially less hard as compared to fibers.

At the present time, there are considerable interest in short fiber-reinforced composites in various engineering application. In general, fibers (e.g., glass, carbon, SiC, TiB, carbon nanotube, etc.) are embedded in the form of whiskers or short fibers in the matrix. In [Chen W.H., Cheng H.C. and Hsu Y.C. (2007)], [Pyo S.H., Lee H.K. (2009)], [Wu C. J., Chou C. Y., Han C. N. and Chiang K. N. (2009)]

several approaches described in order to predict mechanical behavior of composite materials and its components. Investigations of stability loss in structure of composite materials have significant interest as one of the basic destruction mechanisms of composite materials at compression.

In the works [Dekret V.A. (2006), (2008)] the results of investigation of stability loss of the composite materials, reinforced by rows of fibers within the framework of *short fibers* model are presented, where the value of the critical strain for a matrix "at infinity" was analysed.

$$|\varepsilon_{11}^{crm}|^{\infty} = |\varepsilon_{11}^{crm}|, \quad x_1 \to \pm\infty \tag{2}$$

The primary objective of the present study is to research of the critical strain value ε_{11}^{cra} of the composite materials reinforced by periodic row of short fibers in dependence on the mechanical and geometrical parameters of components.

2 Statement and technique of the solution of problem

Let us consider the stability problem of composite material reinforced by periodic row of serial placed short fibers under compression along fibers. Such statement is appeared at research of composite materials of regular structure, which consist of separate rows where short fibers are placed serially that within the one row the fibers are placed densely enough, and in the separate rows the fibers are placed at a sufficient distance one from other. At that we will conduct the research of the phenomenon of internal instability at plane deformation, which does not connect with influence of surfaces, therefore a matrix and row of fibers can be considered endless. In addition, we will examine only the periodic forms of stability loss along the loadings direction. Thus, in the Cartesian coordinates x_1Ox_2, the composite material is modelled by the infinite matrix, filled with infinite periodic row of short fibers which are directed along the Ox_1 axis. At infinity in a direction Ox_1 the composite is loaded by compression of constant intensity P (Fig.1).

For a statement of problem we will select the rectangular area of composite material that the conditions of periodicity of composite structure were satisfied as shown on Fig.1. The width of calculation area is determined from the condition of periodicity of structure $l_1 = L + r$, where L is length of fiber, r is distance between nearby fibers (Fig.1); the height of calculation area is determined from the condition of attenuation of perturbations that the condition $l_2 >> D$ was satisfied and it is determined as a result of calculative experiment.

The composite material will model as piecewise-homogeneous medium, when material within the limits of component is considered as homogeneous and contact

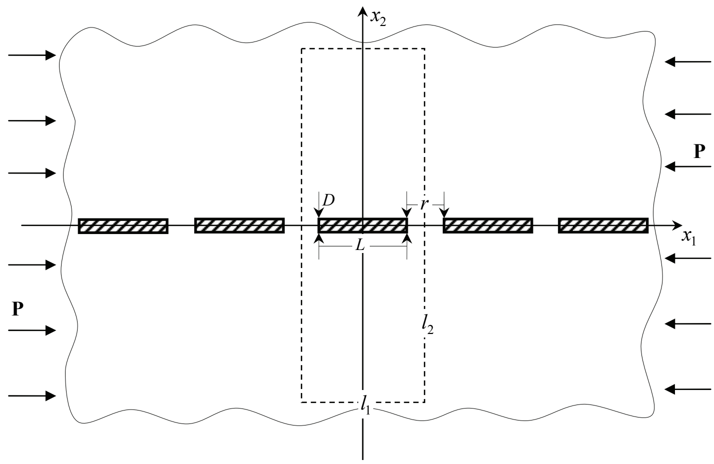

Figure 1: Plane model of composite

conditions are executed on contact of components. At that we will consider the components of composite as the linearly elastic isotropic homogeneous body.

Let us execute the research of stability of composite material with application of static method of the three-dimensional linearized theory of stability of the deformable bodies [Guz A.N. (1999)]. It is necessary to mark that within the framework of *short fiber* model we get the stability problem of the fibred materials with non-homogeneous subcritical state. To determine the components of subcritical state we need to solve the problem about the concentration of tensions near the *short fiber*.

We will determine the subcritical state within the framework of classic linear elasticity theory of isotropic body, thus we will write down equations of equilibrium and elasticity in a kind

$$\frac{\partial}{\partial x_i}\sigma_{ij}^0 = 0; \quad \sigma_{ij}^0 = \delta_{ij}\lambda\varepsilon_{nn}^0 + 2\mu\varepsilon_{ij}^0, \quad 2\varepsilon_{ij}^0 = \frac{\partial u_i^0}{\partial x_j} + \frac{\partial u_j^0}{\partial x_i} \tag{3}$$

For a transition in expressions (3) to the matrix and the reinforcing elements (fibers) we needs all values in (3) to provide with the proper indexes "*m*" and "*a*". As for a matrix in accordance with a calculation scheme on Fig.1 the research is conducted for an endless area, strengths and displacements in a matrix comfortably to present as a sum

$$\sigma_{ij}^{0^m} = \sigma_{ij}^{\infty} + \sigma_{ij}^{10^m}; \quad u_j^{0^m} = u_j^{\infty} + u_j^{10^m}; \tag{4}$$

where: σ_{ij}^{∞} and u_j^{∞} correspond the external loading **P** (Fig.1), to set for a matrix “at infinity”; $\sigma_{ij}^{10^m}$ and $u_j^{10^m}$ correspond the perturbations of the strength-strain state caused by short fibres presence. We will notice the values with indexes “∞” and “1” also determined by the ratios (3). Valuesσ_{ij}^{∞} and u_j^{∞} in accordance to the calculation scheme on Fig.1 determined by the next expressions

$$\sigma_{11}^{\infty} = -P; \quad \sigma_{22}^{\infty} = 0; \quad \sigma_{12}^{\infty} = 0; \quad u_1^{\infty} = A_1 x_1; \quad u_2^{\infty} = A_2 x_2; \tag{5}$$

where values A_1 and A_2 determined from the second expression (3) taking into account the first three expressions (5). Thus for a matrix get expressions (3) (values $\sigma_{ij}^{10^m}$, $\varepsilon_{ij}^{10^m}$ and $u_j^{10^m}$, constants λ_m and μ_m) and for a fiber (reinforcing element) get expressions (3) (values $\sigma_{ij}^{0^a}$, $\varepsilon_{ij}^{0^a}$ and $u_j^{0^a}$, constants λ_a and μ_a). Consequently, the research of the subcritical state is conducted with foregoing values and basic ratios (3).

The complete formulation of problem also includes the continuity conditions of strengths and displacements at the components contact, which for a calculative scheme on Fig.1 are presented in a next form

$$\begin{aligned}
&\sigma_{11}^{\infty} + \sigma_{11}^{10^m} = \sigma_{11}^{0^a}, \quad \sigma_{12}^{10^m} = \sigma_{12}^{0^a}, \quad u_1^{\infty} + u_1^{10^m} = u_1^{0^a}, \quad u_2^{\infty} + u_2^{10^m} = u_2^{0^a},\\
&x_1 = \pm L/2, \quad |x_2| \le \pm D/2,\\
&\sigma_{22}^{10^m} = \sigma_{22}^{0^a}, \quad \sigma_{12}^{10^m} = \sigma_{12}^{0^a}, \quad u_1^{\infty} + u_1^{10^m} = u_1^{0^a}, \quad u_2^{\infty} + u_2^{10^m} = u_2^{0^a},\\
&|x_1| \le \pm L/2, \quad x_1 = \pm D/2.
\end{aligned} \tag{6}$$

The limit conditions and conditions of perturbations attenuation are presented in a next form

$$u_1^{10^m} = p x_1, \quad \sigma_{12}^{10^m} = 0, \quad x_1 = \pm l_1/2, \quad u_i^{\infty} \to 0, \quad \sigma_{ij}^{\infty} \to 0, \quad x_2 = \pm l_2/2. \tag{7}$$

The foregoing statement of problem of determination of the subcritical state corresponds to the generally accepted approach about the research of problems of concentration of tensions near including and holes.

After determination the subcritical state, let us execute the solution of the stability problem within the framework of the second variant of the theory of small subcritical deformations [Guz A.N. (1999)], when the model of matrix and short fibers as a linear elastic isotropic body.

Thus the stability problem is described by the following equations

$$\frac{\partial}{\partial x_i}\left(\omega_{ij\alpha\beta}\frac{\partial}{\partial x_\beta}u_\alpha\right) = 0; \quad i, j, \alpha, \beta = 1, 2. \tag{8}$$

In this case the expressions of asymmetrical tensor of tensions have a place

$$t_{ij} = \omega_{ij\alpha\beta}\frac{\partial}{\partial x_\beta}u_\alpha. \tag{9}$$

Taking into account the expressions (8) and (9) are written down as a general view for a matrix and fibers. In order to solve the stability problem it is necessary to submit the expressions (8) and (9) separately for a matrix as it applies to values$\sigma_{ij}^{1\ m}$, $\varepsilon_{ij}^{1\ m}$, u_j^{1m} and to $\omega_{ij\alpha\beta}^{1\ \ \ m}$, λ_m and μ_m. Also it is necessary to submit the expressions (8) and (9) separately for a fiber as it applies to values σ_{ij}^a, ε_{ij}^a, u_j^a and $\omega_{ij\alpha\beta}^a$, λ_a and μ_a. Thus the next expressions take place for a matrix

$$\omega_{ij\alpha\beta}^{1\ \ \ m} = \delta_{ij}\delta_{\alpha\beta}\lambda_m + \left(\delta_{i\beta}\delta_{\alpha j} + \delta_{i\alpha}\delta_{\beta j}\right)\mu_m + \delta_{\alpha j}\sigma_{i\beta}^{0\ m}; \quad \sigma_{i\beta}^{0\ m} = -\delta_{i\beta}\sigma_{\beta\beta}^{0}P + \sigma_{i\beta}^{1\ m} \tag{10}$$

and for a short fibers

$$\omega_{ij\alpha\beta}^{a} = \delta_{ij}\delta_{\alpha\beta}\lambda_a + \left(\delta_{i\beta}\delta_{\alpha j} + \delta_{i\alpha}\delta_{\beta j}\right)\mu_a + \delta_{\alpha j}\sigma_{i\beta}^{0\ a}. \tag{11}$$

The complete definition of stability problem with basic ratios in a kind (8) and (9) with denotations (10) for a matrix and (11) for a short fiber also include the continuity conditions of strengths and displacements at the component contact, which for a calculative scheme on Fig.1 are presented in a next form

$$\begin{gathered} t_{11}^{1\ m} = t_{11}^a, \quad t_{12}^{1\ m} = t_{12}^a, \quad u_1^{1m} = u_1^a, \quad u_2^{1m} = u_2^a, \quad x_1 = \pm L/2, \quad |x_2| \le \pm D/2, \\ t_{22}^{1\ m} = t_{22}^a, \quad t_{21}^{1\ m} = t_{21}^a, \quad u_1^{1m} = u_1^a, \quad u_2^{1m} = u_2^a, \quad |x_1| \le \pm L/2, \quad x_1 = \pm D/2. \end{gathered} \tag{12}$$

The conditions of perturbations attenuation for a matrix are presented in a next form

$$u_j^{1m} \to 0, \quad x_1 = \pm l_1/2, \quad x_2 = \pm l_2/2. \tag{13}$$

The formulated problem of stability as a kind (10)-(13) includes equations (8) the coefficients of which depend on two variables x_1 and x_2, as by virtue of denotations (10) and (11) in these coefficients the values $\sigma_{i\beta}^{10m}$ and $\sigma_{i\beta}^{0\ a}$ are included, which are determined as a result of solution of the proper problem about the concentration of tensions at determination of the subcritical state.

Obviously, the solution of the problems by analytical methods is impossible; in this connection for the solution of problem of determination of the subcritical state and stability problem, as well as in [Dekret V.A. (2006), (2008)], [Guz A.N., Dekret V.A. (2008)] numeral methods are used.

3 The results of calculations

The obtained data of the research are presented as dependence of the value $|\varepsilon_{11}^{a}|$ on geometrical parameter LD^{-1}. In the case of *short fibers* model the value (1) corresponds to the critical value of strain along an ax Ox_1 in the middle point of reinforcing elements (fibers). Consequently, for the *short fibers* model the value (1) characterizes only the critical value of strain for a fiber and does not characterize the critical value of strain for a matrix.

The calculations were executed at the followings values of parameters of components of composite: ratios of the Young modules are $E_aE_m^{-1} = 1000$; Poison's coefficients are $\nu_1 = \nu_2$; geometrical parameters of fibers are $LD^{-1} = 100, 200, 300, 500$. Dimensionless distance between fibers $r^* = rL^{-1}$ is consistently changed in an interval $0,2 \leq r^* \leq 4,5$.

In Fig.2 dependence of the value of critical strain $|\varepsilon_{11}^{a}|$ is shown on the geometrical parameters of fibers LD^{-1} for various values $r^* = 0,2; 1; 4$ (curve 1,2,3 accordingly).

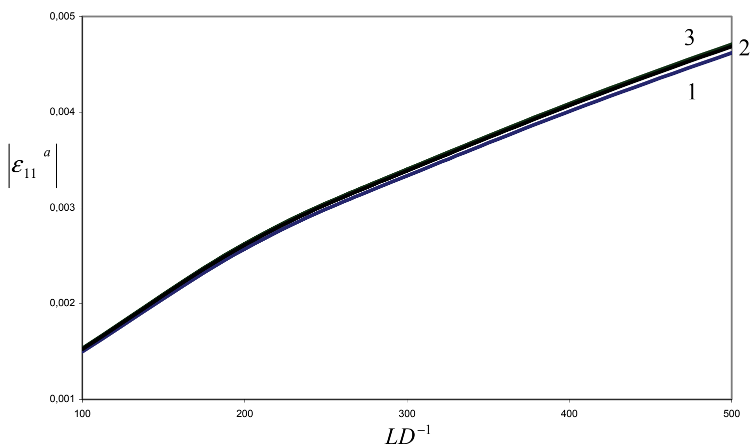

Figure 2: Dependence of the value of critical strain for a fiber on the geometrical parameters of fibers

For comparison, in Fig.3 dependence of the value of critical strain $|\varepsilon_{11}^{m}|$ (for a matrix "at infinity") is shown on the geometrical parameters of fibers LD^{-1} for various values $r^* = 0,2; 1; 4$(curve 1,2,3 accordingly).

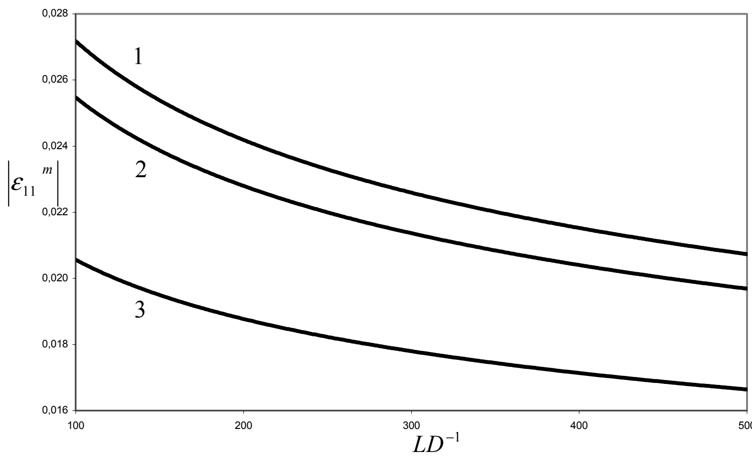

Figure 3: Dependence of the value of critical strain for a fiber on the geometrical parameters of fibers

4 Conclusion

The research of stability of the composite materials reinforced by periodic row of serially placed short fibers has been presented. The stability problem is formulated with application of the three-dimensional linearized theory of stability of deformable bodies and the model of piecewise-homogeneous medium, such approach is most strict and physically correct.

The results of research allow us to conclude that under compression "at infinity" directed along the fibers may result in fracture of the composite reinforced by periodic row of serially placed short fibers due to stability loss of its structure.

At this the critical strain of the composite substantially depends on the chosen values for the *short fibers* model. So the critical strain (1) in the middle point of the reinforcing element (fiber) were obtained substantially less values then for a matrix, because matrix substantially less hard as compared to fibers. It is necessary to note the values of critical strain in a fiber grow with increasing of fiber length whereas the values of critical strain in a matrix reduce with increasing of fiber length.

In addition the values of the critical strain in the middle point of a fiber (1) considerably less depends on the size of distance between fibers then in a matrix (2).

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