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On the solution method for problems related to the micro-mechanics of a periodically curved fiber near a convex cylindrical surface

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Abstract: Within the scope of the piecewise homogeneous body model through the use of the three-dimensional geometrically non-linear exact equations of the theory of elasticity, an approach for the investigation of problems with the micromechanics of a periodically curved fiber near the free convex cylindrical surface is proposed and employed. The main difficulties in finding the solution to these problems are caused by the impossibility of employing the summation theorem for cylindrical functions to satisfy the boundary conditions on the cylindrical surface. For this purpose the cosine and sine Fourier series presentation of the sought values is proposed to satisfy the boundary conditions. The coefficients of these series are calculated numerically through the integrals of the cylindrical functions whose argument depends on the integrating variable in the complicated form. This approach is employed successfully both for the solution to the corresponding boundary value problems in the determination of the self-balanced stress, which is caused primarily by the periodical curving of the fiber, and for the solution to the problems of micro-buckling near the surface through the use of the initial imperfection criterion.

Keywords: Micro/nano-fiber, micro-buckling, periodically curved fiber, self-balanced stress, convex cylindrical surface.

1 Introduction

One of the especially important particularities of fiber reinforced micro- or nano-polymer materials is a curving of the micro- or nano-fibers in the structure of these materials. It is known that this curving may be due to design features or to the

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technological processes resulting from the action of various factors (see: Akbarov and Guz (2000), Corten (1967), Feng, Allen and Moy (1998), Fisher, Bradshow and Brinson (2003a, 2003b), Guz, Tomashevski, Shulka and Yakovlev (1998), Qian, Dickey, Andrews and Rantell (2000), Tarnopolsky, Jigun and Polyakov (1987) and many others).

Studies of the influence of the fiber curvature on the mechanical behavior of the aforementioned reinforced materials have been made at the macro as well as micro/nano levels. In the investigations regarding the macro-levels such levels. In the investigations regarding the macro-levels such as in the papers by Bazant (1968), Hsaio and Daniel (1996), Manusfied and Purslow (1974), etc., the influence of the fiber curvature on the effective modulus of elasticity of the reinforced materials was studied. But in the investigations regarding the micro levels of the foregoing materials the corresponding studies were made within the scope of the piecewise homogeneous body model. These investigations can be divided into two groups: the first contains the investigations which have been made with the use of the approximate approach through writing the field equations of the corresponding mechanical problems (for example, Hsaio and Daniel (1996), Jochum and Grandider (2004), Guz, Tomashevsky, Shulka and Yakovlev (1988) and others); however, the second group contains those investigations which have been made employing the exact three-dimensional field equations (Akbarov (1986a, 1986b, 1990), Akbarov and Guz (1985a,1985b, 2000), Akbarov and Kosker (2003a, 2003b), Akbarov, Kosker and Ucan (2004, 2006) and others). The detailed consideration and analysis of the results attained in the studies carried out before the year 2000 were given in the monograph by Akbarov and Guz (2000). The review of the mentioned investigations was also given in the survey paper by Akbarov and Guz (2004). It follows from the foregoing references that before the end of the 20^{th} century the corresponding investigations were made for a small fibre volume fraction and the results obtained for a single periodically curving fiber which is in the infinite elastic and viscoelastic medium under action of uniformly distributed normal forces along the fiber at infinity. Consequently, in the mentioned investigations, the interaction between the curving fibers under determination of the self-balanced stresses was not taken into account. But in recent years, in papers by Akbarov and Kosker (2003a, 2003b), and Akbarov, Kosker and Ucan (2004, 2006) the method of solution and investigations proposed in the studies by Akbarov (1986a,1986b, 1990), Akbarov and Guz (1985a,1985b, 2000), and Kosker and Akbarov (2003) were developed for two neighboring periodically curved fibers as well as a row of them. Note that in these studies the self-balanced stresses caused by the curve of the fibers were investigated and the influence of the micro-structural parameters such as the relation of the fiber radius on the period of the curve, the distance between the fibers, the ratio of the modulus of elasticity of the constituents, etc., were determined. But the corresponding investigations regarding those cases where the periodically curved fibers are near the bounding surface as well as the corresponding working out of the solution method to these problems, have been completely absent up to now. At the same time, the following must first be investigated.

The fibers' curvature with a very small amplitude can be taken as a model for the investigation of the internal stability loss problems for the time-dependent micro- or nano-bi-materials within the scope of the initial imperfection criterion. Such an approach was developed in papers by Akbarov and Kosker (2001, 2004) in which it was shown that this approach can be applied not only for time dependent materials, but also for time independent materials. The review of these studies is given in the paper by Akbarov (2007). However, in the foregoing investigations it was assumed that the fibers are embedded in an infinitely elastic and viscoelastic medium. Consequently, in these investigations the effect of the bounding surface on the internal stability loss of the fibers has not been taken into account. Consequently, the results of the foregoing investigations can not be applied for the cases where the curved fibers are near the surface bounding the material containing these fibers. It should be noted that the solution method and the investigations of the corresponding micro-buckling problems of near-surface fibers were made in papers by Guz and Lapusta (1986, 1988) and Lapusta (1988) and others listed and detailed in the survey papers by Babich, Guz and Chekhov (2001) and Guz and Lapusta (1999). However in these papers, the corresponding investigations on the stability loss were made within the scope of the eigen-value (bifurcation) approach and it was assumed that the material suurounding the fibers occupies an infinite three dimensional space (for the case where the fibers are near the cylindrical cavity (Guz and Lapusta (1988); Lapusta (1988))) or a semi-infinite three dimensional space (for the case where the fibers are near the plane-surface (Guz and Lapusta (1986))). In other words, in the papers by Guz and Lapusta (1986, 1988), Lapusta (1988), and others, it was assumed that the size of the regions occupied by the material considered in perpendicular directions to that in which the fibers are lying is infinite or semi-infinite. Consequently, in these studies, investigations were also not carried out for the cases where the mentioned sizes are finite.

Taking the above discussion into account, in the present paper within the scope of the piecewise homogeneous body model and through the use of the three-dimensional geometrically non-linear exact equations of the theory of elasticity, the solution method proposed in Akbarov and Guz (2000), and Akbarov and Kosker (2001, 2003a, 2003b) is developed in order to determine the self-balanced stresses caused by a periodical curving of the near convex cylindrical surface micro- or nano-fibers and for the study of the near-surface micro-buckling of these fibers. It is assumed that the region occupied by the material considered is bounded by the cylindrical free surface. Consequently what is considered is the case where the size of the region occupied by the material in the directions perpendicular to that in which the fibers are lying is finite. The numerical results obtained by employment of the method developed are presented and discussed.

2 Formulation of the problem

We consider a cylinder with infinite length and assume that the cross section of this cylinder is a circle the radius of which is R (Fig. 1). Assuming that this cylindrical body contains a fiber which has periodic curving along its length, we suppose that the fiber's cross section which is perpendicular to its middle line tangent vector, is a circle of constant radius R_0. In the natural state, we associate the Lagrangian cylindrical system of coordinates $Or\theta z\,(O_0r_0\theta_0z_0)$ and the Cartesian system of coordinates $Ox_1x_2x_3\,(O_0x_{10}x_{20}x_{30})$ with the cylinder (the fiber). Between these coordinates the following relations are satisfied.

$$x_2 = x_{20}, \quad x_3 = x_{30}, \quad z = z_0, \quad re^{i\theta} = R_{10} + r_0e^{i\theta_0}. \tag{1}$$

The middle line of the fiber is given by the equation

$$x_{30} = t, \quad x_{10} = A\sin\left(\frac{2\pi}{\ell}t\right)\cos\beta, \quad x_{20} = A\sin\left(\frac{2\pi}{\ell}t\right)\sin\beta \tag{2}$$

where t is a parameter and $t \in (-\infty, +\infty)$, A is the amplitude of the periodic curving form, ℓ is a period of the curving form and β is an angle between the plane $O_0x_{10}x_{30}$ and the plane on which the middle line of the fiber lies. Suppose that $A << \ell$, we introduce the small parameter

$$\varepsilon = \frac{A}{\ell}, \quad 0 \leq \varepsilon << 1.0. \tag{3}$$

Below, the values related to the cylinder and the fiber will be denoted by upper indices (1) and (0).

We assume that the fiber and the surrounding cylinder materials are isotropic and homogeneous. Within the scope of the piecewise homogeneous body model through the use of the three-dimensional geometrically non-linear exact equations of the theory of elasticity, we investigate the stress-strain state and the development of the infinitesimal initial waviness of the fiber in the case where the body is loaded (compressed or stretched) to infinity by uniformly distributed normal forces with

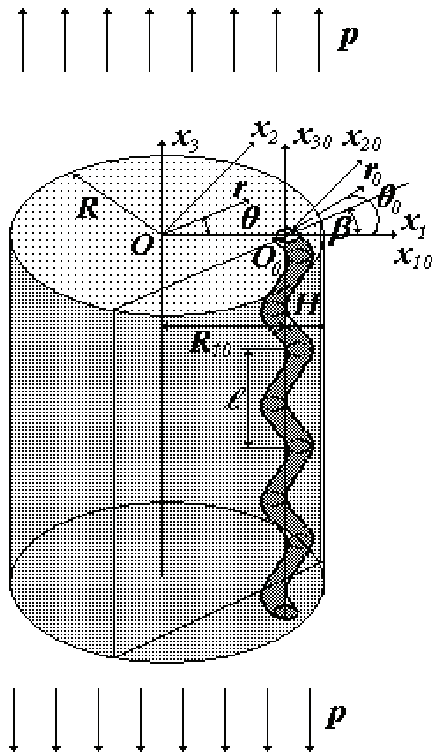

Figure 1: The geometry of the structure of the considered body

an intensity p acting in the direction of the Oz axis. For this purpose we write the field equations which are satisfied within the fiber and surrounding cylinder separately. Note that in writing the mentioned equations we use the conventional tensor notation below.

$$\nabla_i \left[\sigma^{(k)in}\left(\delta_n^j + \nabla_n u^{(k)j}\right)\right] = 0, \quad 2\varepsilon_{ij}^{(k)} = \nabla_j u_m^{(k)} + \nabla_m u_j^{(k)} + \nabla_j u^{(k)n} \nabla_m u_n^{(k)},$$

$$\sigma_{(in)}^{(k)} = \lambda^{(k)}\left(e^{(k)}\delta_i^n\right) + 2\mu^{(k)}\varepsilon_{(in)}^{(k)},$$

$$e^{(k)} = \varepsilon_{(11)}^{(k)} + \varepsilon_{(22)}^{(k)} + \varepsilon_{(33)}^{(k)}, \quad k = 0,1; \quad i;n;m;j = 1,2,3. \tag{4}$$

Here ∇_i shows the covariant derivatives with respect to the $i-th$ cylindrical coordinate, $\sigma^{(k)in}$ is a contravariant component of the stress tensor, $\varepsilon_{ij}^{(k)}$ is a covariant component of the Green's strain tensor, $u_m^{(k)}$ $\left(u^{(k)n}\right)$ is a covariant (contravariant) component of the displacement vector, $\sigma_{(ij)}^{(k)}\left(\varepsilon_{(ij)}^{(k)}\right)$ is a physical component of the stress (strain) tensor in the $k-th$ cylindrical system of coordinates; $\lambda^{(k)}$, $\mu^{(k)}$ are the Lamé's constants of the k-th material and δ_n^j is the Kronecker symbol.

Assume that on the interface between the fiber and surrounding medium (this surface is denoted by S_0) the complete contact conditions are satisfied.

$$\left[\sigma^{(1)in}\left(\delta_n^j+\nabla_n u^{(k)j}\right)\right]\Big|_{S_0} n_{oj}=\left[\sigma^{(0)in}\left(\delta_n^j+\nabla_n u^{(k)j}\right)\right]\Big|_{S_0} n_{oj}, \quad u_{(j)}^{(1)}\Big|_{S_0}=u_{(j)}^{(0)}\Big|_{S_0}. \tag{5}$$

In (5) $u_{(j)}^{(k)}$ is a physical component of the displacement vector, n_{0j} is a covariant component of the unit normal vector to the surface S_0.

Moreover on the cylindrical free surface$r=R$, so the following conditions are also satisfied.

$$\left[\sigma^{(1)in}\left(\delta_n^j+\nabla_n u^{(k)j}\right)\right]\Big|_{r=R} n_j=0, \tag{6}$$

where n_j is a covariant component of the unit normal vector to the cylindrical surface $r=R$.

It is known that

$$\sigma_{(ij)}^{(k)}=\sigma^{(k)^{ij}}H_i^{(k)}H_j^{(k)}, \quad \varepsilon_{(ij)}^{(k)}=\varepsilon_{(ij)}^{(k)}\left(H_i^{(k)}H_j^{(k)}\right)^{-1},$$

$$u_{(i)}=u^{(k)^i}H_i^{(k)}=u_i^{(k)}\left(H_i^{(k)}\right)^{-1}, \tag{7}$$

where $(ij)=rr,\theta\theta,zz,r\theta,rz,z\theta$, $(i)=r,\theta,z$. Here $H_i^{(k)}$ are Lamé's coefficients and $H_1^{(k)}=1.0$, $H_2^{(0)}=r_0$, $H_2^{(1)}=r$, $H_3^{(k)}=1.0$for the cylindrical system of coordinates. Thus, with this the formulation of the considered problem has been exhausted.

3 Method of solution

Up to now, various types of numerical and semi-analytical methods have been developed for computer modeling to solve the various types of problems concerning deformable solid body mechanics. The present level of such methods is described, for example, in the papers by Yoda and Kodama (2006), Lu and Zhu (2007), Chen, Fu and Zhang (2007), Gato and Shie (2008), Liu, Chen, Li and Cen (2008), Lin, Lee, Tsai, Chen, Wang and Lee (2008), Wang and Wang (2008), Guz and Dekret (2008), Dekret (2008a, 2008b) and in many others. Note that all these methods are based on the discretization or semi- discretization of the domain occupied by the body considered and field equations which are satisfied in this domain. It is known that as a result of the mentioned discretization the solution to the problem considered is reduced to the solution of the system of algebraic equations. At the same time, there are also other methods, so called analytical + numerical methods, according to which the mentioned system of algebraic equations are attained without any discretization through the employment of the analytical solution method (see: Akbarov and Guliev (2009), Akbarov and Kosker (2003a, 2003b), Akbarov, Kosker and Ucan (2004, 2006), Guz and Lapusta (1986, 1988) and many others). But, the solution to this system of algebraic equations is realized by employing modern PC modeling. In the present paper, the latest version of computer modeling has been employed.

Thus, we consider the method of solution to the problem formulated in the previous section. First we derive the equation for the interface surface S_0. According to the condition of the fiber's cross section we can conclude that the coordinates of this surface must simultaneously satisfy the following equations.

$$\varepsilon f'(t)\,(x_{10}-\varepsilon f(t)\cos\beta)\cos\beta+\varepsilon f'(t)\,(x_{20}-\varepsilon f(t)\sin\beta)\sin\beta+x_{30}-t=0,$$
$$(-x_{10}\sin\beta+x_{20}\cos\beta)^2+(x_{30}-t)^2+(x_{10}\cos\beta+x_{20}\sin\beta-\varepsilon f(t))^2=R_0^2, \quad (8)$$

where $f(t)=\ell\sin(2\pi t/\ell)$, $f'(t)=2\pi\cos(2\pi t/\ell)$; x_{10}, x_{20}, x_{30} are coordinates of the surface S_0. Note that the first equation in (8) is an equation of the plane perpendicular to the vector which is the tangent vector to the middle line of the fiber at the point that corresponds to the fixed value of the parameter t; but the second equation in (8) is an equation of the circle which is counter to the cross section of the fiber which rises on the foregoing plane.

Using the relations $x_{10}=r_0\cos\theta_0$, $x_{20}=r_0\sin\theta_0$ we obtain the following equation for the surface S_0 in the cylindrical system of coordinates $O_0r_0\theta_0z_0$:

$$r_0=r_0(\theta_0,t,\varepsilon), \quad z_0=t+z_{01}(\theta_0,t,\varepsilon). \quad (9)$$

The explicit expressions of the functions $r_0(\theta_0,t,\varepsilon)$, $z_{01}(\theta_0,t,\varepsilon)$can also be attained from equation (8); in order not to take up too much space here, we will not present these expressions.

After some mathematical manipulations, we obtain the following equations.

$$n_{0r} = r_0(\theta_0, t, \varepsilon) \frac{\partial z_0(\theta_0, t, \varepsilon)}{\partial t} [B(\theta_0, t, \varepsilon)]^{-1},$$

$$n_{0\theta} = \left(\frac{\partial z_0(\theta_0, t, \varepsilon)}{\partial \theta_0} \frac{\partial r_0(\theta_0, t, \varepsilon)}{\partial t} - - \frac{\partial z_0(\theta_0, t, \varepsilon)}{\partial t} \frac{\partial r_0(\theta_0, t, \varepsilon)}{\partial \theta_0} \right) [B(\theta_0, t, \varepsilon)]^{-1},$$

$$n_{0r} = -r_0(\theta_0, t, \varepsilon) \frac{\partial r_0(\theta_0, t, \varepsilon)}{\partial t} [B(\theta_0, t, \varepsilon)]^{-1}, \tag{10}$$

where n_{0r}, $n_{0\theta}$, $n_{0z}$are physical components of the unit normal vector to the surface S_0 and

$$B(\theta_0, t, \varepsilon) = \left[\left(r_0(\theta_0, t, \varepsilon) \frac{\partial z_0(\theta_0, t, \varepsilon)}{\partial t} \right)^2 + \left(\frac{\partial z_0(\theta_0, t, \varepsilon)}{\partial \theta_0} \frac{\partial r_0(\theta_0, t, \varepsilon)}{\partial t} - \frac{\partial z_0(\theta_0, t, \varepsilon)}{\partial t} \frac{\partial r_0(\theta_0, t, \varepsilon)}{\partial \theta_0} \right)^2 + \left(r_0(\theta_0, t, \varepsilon) \frac{\partial r_0(\theta_0, t, \varepsilon)}{\partial t} \right)^2 \right]^{\frac{1}{2}}. \tag{11}$$

As in the monograph by Akbarov and Guz (2000), we attempt to solve the considered problem by employing the boundary form perturbation method, according to which the unknowns are presented in series form in ε (3).

$$\left\{ \sigma^{(m)ij}; \varepsilon_{ij}^{(m)}; u_i^{(m)}; u^{(m)i} \right\} = \sum_{q=0}^{\infty} \varepsilon^q \left\{ \sigma^{(m)ij,q}; \varepsilon_{ij}^{(m),q}; u_i^{(m),q}; u^{(m)i,q} \right\}. \tag{12}$$

Moreover, the expressions (9) and (10) are also presented in the series form in ε as follows.

$$r_0 = R_0 + \sum_{k=1}^{\infty} \varepsilon^k a_{0k}(\theta_0, t), \quad z_0 = t + \sum_{k=1}^{\infty} \varepsilon^k b_{0k}(\theta_0, t),$$

$$n_{0r} = 1 + \sum_{k=1}^{\infty} \varepsilon^k c_{0k}(\theta_0, t), \quad n_{0\theta} = \sum_{k=1}^{\infty} \varepsilon^k b_{0k}(\theta_0, t), \quad n_{0z} = \sum_{k=1}^{\infty} \varepsilon^k d_{0k}(\theta_0, t). \tag{13}$$

The expressions of functions $a_{ok}(\theta_0, t)$,..., $d_{0k}(\theta_0, t)$ in Eq. (13) can easily be obtained from Eqs. (9), (10) and (11); therefore these expressions are not given here.

Substituting Eq. (12) into Eq. (4), we obtain set equations for each approximation (12). Using Eq. (13) we expand the values of each approximation (12) in series form in the vicinity of the point $\{r_0 = R_0; z_0 = t\}$. Substituting these last expressions in the contact conditions in (5) and using the expressions of n_{0r}, $n_{0\theta}$ and n_{0z} given in (13), after some mathematical transformations we obtain contact conditions which are satisfied at $\{r_0 = R_0; z_0 = t\}$ for each approximation in Eq.(12). It is evident that for the zeroth approximation, Eq.(4) is valid and condition (5) is replaced by the same one satisfied at point $\{r_0 = R_0; z_0 = t\}$. We assume that $\nabla_n u^{(k),0} << 1$ and therefore we replace the terms $\delta_n^j + \nabla_n u^{(k),0}$ by δ_n^j where δ_n^j is the Kronecker symbols. According to this assumption, for the zeroth approximation, we obtain the following system of equations:

$$\nabla_i \sigma^{(k)ij,0} = 0, \quad 2\varepsilon_{ij}^{(k),0} = \nabla_j u_i^{(k),0} + \nabla_i u_j^{(k),0}, \tag{14}$$

contact conditions

$$\sigma_{(ij)}^{(0),0}\Big|_{r_0=R_0} = \sigma_{(ij)}^{(1),0}\Big|_{r_0=R_0}, \quad u_{(i)}^{(0),0}\Big|_{r_0=R_0} = u_{(i)}^{(1),0}\Big|_{r_0=R_0}, \tag{15}$$

and boundary conditions

$$\sigma_{(ij)}^{(1),0}\Big|_{r=R} = 0, \tag{16}$$

where $(ij) = rr, r\theta, rz, \ (i) = r, \theta, z$.

Taking the last assumption into account, for the subsequent approximations we obtain the following system of equations.

$$\nabla_i \left[\sigma^{(k)ij,q} + \sigma^{(k)in,0}\nabla_n u^{(k)j,q}\right] = -\underline{\sum_{m=1}^{q-1} \nabla_i \left(\sigma^{(k)in,q-m}\nabla_n u^{(k)j,m}\right)},$$

$$2\varepsilon_{ij}^{(k),q} = \nabla_j u_i^{(k),q} + \nabla_i u_j^{(k),q} + \underline{\sum_{m=1}^{q-1} \nabla_j u^{(k)n,q-m}\nabla_i u_k^{(k),m}}. \tag{17}$$

Moreover the boundary conditions in (6) for the first and subsequent approximations can be written as follows

$$\left[\sigma^{(k)ij,k}+\sigma^{(k)in,0}\nabla_n u^{(k)j,q}\right]\Big|_{r=R}=\underline{-\sum_{m=1}^{q-1}\sigma^{(k)in,q-m}\nabla_n u^{(k)j,m}}. \tag{18}$$

Note that the underlined terms in Eqs. (17) and (18) are equal to zero for the first approximation.

It is necessary to add to these equations the constitutive relations

$$\sigma_{(in)}^{(k),q}=\lambda^{(k)}e^{(k),q}\delta_i^n+2\mu^{(k)}\varepsilon_{(in)}^{(k),q} \tag{19}$$

which are satisfied for each approximation separately. Now we write the contact conditions for the first approximation by the physical components of the stress tensor and displacement vector.

$$\left[\sigma_{(i)r}\right]_{1,1}^{1,1}+f_1\left[\frac{\partial\sigma_{(i)r}}{\partial r}\right]_{1,0}^{1,0}+\phi_1\left[\frac{\partial\sigma_{(i)r}}{\partial z}\right]_{1,0}^{1,0}+\gamma_r\left[\sigma_{(i)r}\right]_{1,0}^{1,0}+\gamma_\theta\left[\sigma_{(i)\theta}\right]_{1,0}^{1,0}+\gamma_z\left[\sigma_{(i)z}\right]_{1,0}^{1,0}=0,$$

$$\left[u_{(i)}\right]_{1,1}^{1,1}+f_1\left[\frac{\partial u_{(i)}}{\partial r}\right]_{1,0}^{1,0}+\phi_1\left[\frac{\partial u_{(i)}}{\partial z}\right]_{1,0}^{1,0}=0, \tag{20}$$

where $(i)=r,\theta,z$. In Eq. (20) replacing $(i)$with r,θ and z we obtain the explicit form of the corresponding contact conditions in the considered approximation. Moreover, in Eq. (20) the following notation is used.

$$[\phi]_{1,s}^{1,s}=\phi^{(1),s}-\phi^{(0),s},\quad \gamma_z=-f'(t)\cos(\theta_0-\beta),\quad f_1=f(t)\cos(\theta_0-\beta),$$

$$\phi_1=-R_0 f'(t)\cos(\theta_0-\beta),$$

$$\gamma_r=\left(\frac{f(t)}{R_0}-f''(t)\right)\cos(\theta_0-\beta),\quad \gamma_\theta=-\frac{f(t)}{R_0}\sin(\theta_0-\beta),$$

$$f'(t)=\frac{df(t)}{dt},\quad f''(t)=\frac{d^2f(t)}{dt^2}. \tag{21}$$

The boundary conditions at $r=R$ for the first approximation, according to (16) and (17), can be written as follows.

$$\sigma_{rr}^{(1),1}\Big|_{r=R}=0,\quad \sigma_{r\theta}^{(1),1}\Big|_{r=R}=0,\quad \sigma_{zr}^{(1),1}\Big|_{r=R}=0. \tag{22}$$

In a similar way we can write the contact and boundary conditions for the second and subsequent approximations for the problem considered.

We determine the unknown values belonging to the considered approximations. Assume that the Poisson coefficient of the materials' fiber and surrounding medium are equal to each other, i.e. $\nu^{(1)} = \nu^{(0)}$, according to which, in the zeroth approximation, the stresses acting on those areas where the normal is perpendicular to the Oz axis direction vanish. In the cases where $\nu^{(1)} \neq \nu^{(0)}$ the mentioned stresses arise and have the order $O\left(\nu^{(1)} - \nu^{(0)}\right)$ and, according to the monograph by Akbarov and Guz (2000), do not have a considerable effect on the numerical results. Thus, taking this discussion into account, the determination of each approximation is considered separately.

The zeroth approximation. This approximation has the following exact solution.

$$\sigma_{zz}^{(1),0} = p\left[1 + \frac{R_0^2}{R^2}\left(\frac{E^{(0)}}{E^{(1)}} - 1\right)\right]^{-1}, \quad \sigma_{zz}^{(0),0} = \sigma_{zz}^{(1),0}\frac{E^{(0)}}{E^{(1)}},$$

$$\varepsilon_{zz}^{(1),0} = \varepsilon_{zz}^{(0),0} = \frac{\sigma_{zz}^{(0),0}}{E^{(0)}}, \quad u_z^{(1),0} = u_z^{(0),0} = \varepsilon_{zz}^{(0),0} z,$$

$$\sigma_{(ij)}^{(1),0} = \sigma_{(ij)}^{(0),0} = 0, \quad (ij) = rr, \theta\theta, r\theta, \theta z, rz. \tag{23}$$

The first approximation. According to Eqs. (23), Eq.(17) for this approximation can be written as follows.

$$\frac{\partial \sigma_{rr}^{(k),1}}{\partial r} + \frac{1}{r}\frac{\partial \sigma_{r\theta}^{(k),1}}{\partial \theta} + \frac{\partial \sigma_{rz}^{(k),1}}{\partial z} + \frac{1}{r}\left(\sigma_{rr}^{(k),1} - \sigma_{\theta\theta}^{(k),1}\right) + \sigma_{zz}^{(k),0}\frac{\partial^2 u_z^{(k),1}}{\partial z^2} = 0,$$

$$\frac{\partial \sigma_{r\theta}^{(k),1}}{\partial r} + \frac{1}{r}\frac{\partial \sigma_{\theta\theta}^{(k),1}}{\partial \theta} + \frac{\partial \sigma_{\theta z}^{(k),1}}{\partial z} + \frac{2}{r}\sigma_{r\theta}^{(k),1} + \sigma_{zz}^{(k),0}\frac{\partial^2 u_\theta^{(k),1}}{\partial z^2} = 0,$$

$$\frac{\partial \sigma_{rz}^{(k),1}}{\partial r} + \frac{1}{r}\frac{\partial \sigma_{\theta z}^{(k),1}}{\partial \theta} + \frac{\partial \sigma_{zz}^{(k),1}}{\partial z} + \frac{1}{r}\sigma_{rz}^{(k),1} + \sigma_{zz}^{(k),0}\frac{\partial^2 u_z^{(k),1}}{\partial z^2} = 0. \tag{24}$$

The constitutive relations remain the same as in Eq. (19). Moreover, the geometrical relations for both the fiber and matrix have the following form.

$$\varepsilon_{rr}^{(k),1} = \frac{\partial u_r^{(k),1}}{\partial r}, \quad \varepsilon_{\theta\theta}^{(k),1} = \frac{\partial u_\theta^{(k),1}}{r\partial \theta} + \frac{u_r^{(k),1}}{r}, \quad \varepsilon_{zz}^{(k),1} = \frac{\partial u_z^{(k),1}}{\partial z},$$

$$\varepsilon_{r\theta}^{(k),1}=\frac{1}{2}\left(\frac{\partial u_r^{(k),1}}{r\partial\theta}+\frac{\partial u_\theta^{(k),1}}{\partial r}-\frac{u_\theta^{(k),1}}{r}\right),\quad \varepsilon_{\theta z}^{(k),1}=\frac{1}{2}\left(\frac{\partial u_\theta^{(k),1}}{\partial z}+\frac{\partial u_z^{(k),1}}{r\partial\theta}\right),$$

$$\varepsilon_{zr}^{(k),1}=\frac{1}{2}\left(\frac{\partial u_z^{(k),1}}{\partial r}+\frac{\partial u_r^{(k),1}}{\partial z}\right). \tag{25}$$

According to Eqs. (20) and (23), we have the following contact conditions for the first approximation:

$$[\sigma_{rr}]\,_{1,1}^{1,1}=0,\quad [\sigma_{r\theta}]\,_{1,1}^{1,1}=0,$$

$$[\sigma_{rz}]\,_{1,1}^{1,1}=2\pi\sin(\alpha z)\left(\sigma_{zz}^{(0),0}-\sigma_{zz}^{(1),0}\right)\times(\cos\theta_0\cos\beta+\sin\theta_0\sin\beta),$$

$$[u_r]\,_{1,1}^{1,1}=0,\quad [u_\theta]\,_{1,1}^{1,1}=0,\quad [u_z]\,_{1,1}^{1,1}=0. \tag{26}$$

As has been noted above, Eqs. (24) and (25) coincide with the corresponding equations of the Three-dimensional Linearized Theory of Elasticity; therefore to solve the obtained system of equations in (24) and (26), we can use the following representation in the cylindrical system of coordinates (Guz (1999)).

$$u_r^{(k),1}=\frac{1}{r}\frac{\partial}{\partial\theta}\psi^{(k),1}-\frac{\partial^2}{\partial r\partial z}\chi^{(k),1},\quad u_\theta^{(k),1}=-\frac{\partial}{\partial r}\psi^{(k),1}-\frac{1}{r}\frac{\partial^2}{\partial\theta\partial z}\chi^{(k),1},$$

$$u_3^{(k),1}=\left(\lambda^{(k)}+\mu^{(k)}\right)^{-1}\times\left(\left(\lambda^{(k)}+2\mu^{(k)}\right)\Delta_1+\left(\mu^{(k)}+\sigma_{zz}^{(k),0}\right)\frac{\partial^2}{\partial z^2}\right)\chi^{(k),1},$$

$$\Delta_1=\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}+\frac{1}{r^2}\frac{\partial^2}{\partial\theta^2}. \tag{27}$$

The functions $\psi^{(k),1}$ and $\chi^{(k),1}$ are determined from the equations.

$$\left(\Delta_1+\left(\xi_1^{(k)}\right)^2\frac{\partial^2}{\partial z^2}\right)\psi^{(k),1}=0,$$

$$\left(\Delta_1+\left(\xi_2^{(k)}\right)^2\frac{\partial^2}{\partial z^2}\right)\left(\Delta_1+\left(\xi_3^{(k)}\right)^2\frac{\partial^2}{\partial z^2}\right)\chi^{(k),1}=0, \tag{28}$$

where

$$\xi_1^{(k)} = \sqrt{1+\frac{\sigma_{zz}^{(k),0}}{\mu^{(k)}}}, \quad \xi_2^{(k)} = \sqrt{1+\frac{\sigma_{zz}^{(k),0}}{\mu^{(k)}}}, \quad \xi_3^{(k)} = \sqrt{1+\frac{\sigma_{zz}^{(k),0}}{\lambda^{(k)}+2\mu^{(k)}}} \tag{29}$$

The solution of the equations in (27) for the considered problem can be presented as follows.

$$\begin{aligned}
\psi^{(0),1} &= \alpha \sin\alpha z \sum_{n=-\infty}^{+\infty} C_{n1}^{(0)} I_n\left(\xi_1^{(0)}\alpha r_0\right) e^{in\theta_0}, \\
\chi^{(0),1} &= \cos\alpha z \sum_{n=-\infty}^{+\infty} \left[C_{n2}^{(0)} I_n\left(\xi_2^{(0)}\alpha r_0\right) + C_{n3}^{(0)} I_n\left(\xi_3^{(0)}\alpha r_0\right)\right] e^{in\theta_0}, \\
\psi^{(1),1} &= \alpha \sin\alpha z \sum_{n=-\infty}^{+\infty} \left[B_{n1}^{(1)} I_n\left(\xi_1^{(1)}\alpha r\right) e^{in\theta} + C_{n1}^{(1)} K_n\left(\xi_1^{(1)}\alpha r_0\right) e^{in\theta_0}\right],
\end{aligned} \tag{30}$$

$$\begin{aligned}
\chi^{(1),1} = \cos\alpha z \sum_{n=-\infty}^{+\infty} \Big\{ & \left[B_{n2}^{(1)} I_n\left(\xi_2^{(1)}\alpha r\right) + B_{n3}^{(1)} I_n\left(\xi_3^{(1)}\alpha r\right)\right] e^{in\theta} \\
& + \left[C_{n2}^{(1)} K_n\left(\xi_2^{(1)}\alpha r_0\right) + C_{n3}^{(1)} K_n\left(\xi_3^{(1)}\alpha r_0\right)\right] e^{in\theta_0} \Big\},
\end{aligned} \tag{31}$$

where $\alpha = 2\pi/\ell$, $I_n(x)$ and $K_n(x)$ are a Bessel function of a purely imaginary argument and Macdonald functions, in turn. Moreover, the unknowns $C_{ni}^{(k)}$, $B_{ni}^{(1)}$ $(i = 1,2,3;\ k = 0,1)$ are complex numbers and satisfy the relations:

$$C_{ni}^{(k)} = \overline{C_{-ni}^{(k)}}, \quad B_{ni}^{(1)} = \overline{B_{-ni}^{(1)}}, \quad ImC_{01}^{(k)} = ImB_{01}^{(1)} = 0. \tag{32}$$

Thus, using the solutions in (30) and (31) we obtain the expressions for the sought values from (27), (25) and (19). Note that these expressions contain the unknown constants in (32). For determination of these unknowns we must obtain the corresponding algebraic system of equations from the contact conditions (26) and boundary conditions (22). In this case under satisfaction of the contact conditions (26) all the expressions mentioned must be written in the coordinates (r_0, θ_0). In other words, the terms in these expressions which have been attained from the terms $B_{nk}^{(1)} I_n\left(\xi_k^{(1)}\alpha r\right) e^{in\theta}$ must be written in the coordinates (r_0, θ_0). For this purpose we use the summation theorem (Watson (1958)) for the $I_n(x)$ function, which can be written for the case at hand as follows.

$$I_{\nu}(cr)e^{i\nu\theta} = \sum_{m=-\infty}^{+\infty} I_{\nu-m}\left(cR_{10}\right) I_m\left(cr_0\right) e^{im\theta_0}, \quad c = const. \tag{33}$$

Consequently, in order to employ expression (33) to satisfy contact conditions (26) at $r_0 = R_0$, the inequality

$$R_0 < R_{10} \tag{34}$$

must occur.

Thus, as inequality (34) satisfies for the problem considered (Fig. 1) using the summation theorem (33), after some mathematical manipulations the functions in (31) can be rewritten as follows.

$$\begin{aligned}
\psi^{(1),1} =& \alpha \sin\alpha z \left[\sum_{n=0}^{+\infty} \varepsilon_n Y_{n1}^{(1)} K_n\left(\xi_1^{(1)}\alpha r_0\right) \cos n\theta_0 \right. \\
&+ \sum_{n=1}^{+\infty} X_{n1}^{(1)} K_n\left(\xi_1^{(1)}\alpha r_0\right) \sin n\theta_0 \\
&+ \sum_{n=0}^{+\infty} \left[\sum_{m=0}^{+\infty} \varepsilon_n Z_{m1}^{(1)} \lambda_{nm}^{+}\left(\xi_1^{(1)}\alpha R_{10}\right) I_m\left(\xi_1^{(1)}\alpha r_0\right) \right] \cos n\theta_0 \\
&\left. + \sum_{n=1}^{+\infty} \left[\sum_{m=1}^{+\infty} T_{m1}^{(1)} \lambda_{nm}^{-}\left(\xi_1^{(1)}\alpha R_{10}\right) I_m\left(\xi_1^{(1)}\alpha r_0\right) \right] \sin n\theta_0 \right\},
\end{aligned}$$

$$
\begin{aligned}
\chi^{(1),1} = \cos\alpha z \Bigg\{ & \sum_{n=0}^{+\infty} \varepsilon_n Y_{n2}^{(1)} K_n\left(\xi_2^{(1)}\alpha r_0\right)\cos n\theta_0 + \sum_{n=1}^{+\infty} X_{n2}^{(1)} K_n\left(\xi_2^{(1)}\alpha r_0\right)\sin n\theta_0 \\
& + \sum_{n=0}^{+\infty} \varepsilon_n Y_{n3}^{(1)} K_n\left(\xi_3^{(1)}\alpha r_0\right)\cos n\theta_0 + \sum_{n=1}^{+\infty} X_{n3}^{(1)} K_n\left(\xi_3^{(1)}\alpha r_0\right)\sin n\theta_0 \\
& + \sum_{n=0}^{+\infty}\left[\sum_{m=0}^{+\infty} \varepsilon_n Z_{m2}^{(1)} \lambda_{nm}^{+}\left(\xi_2^{(1)}\alpha R_{10}\right) I_m\left(\xi_2^{(1)}\alpha r_0\right)\right]\cos n\theta_0 \\
& + \sum_{n=0}^{+\infty}\left[\sum_{m=0}^{+\infty} \varepsilon_n Z_{m3}^{(1)} \lambda_{nm}^{+}\left(\xi_3^{(1)}\alpha R_{10}\right) I_m\left(\xi_3^{(1)}\alpha r_0\right)\right]\cos n\theta_0 \\
& + \sum_{n=1}^{+\infty}\left[\sum_{m=1}^{+\infty} T_{m2}^{(1)} \lambda_{nm}^{-}\left(\xi_2^{(1)}\alpha R_{10}\right) I_m\left(\xi_2^{(1)}\alpha r_0\right)\right]\sin n\theta_0 \\
& + \sum_{n=1}^{+\infty}\left[\sum_{m=1}^{+\infty} T_{m3}^{(1)} \lambda_{nm}^{-}\left(\xi_3^{(1)}\alpha R_{10}\right) I_m\left(\xi_3^{(1)}\alpha r_0\right)\right]\sin n\theta_0 \Bigg\},
\end{aligned}
\tag{35}
$$

where

$$
\varepsilon_0 = \frac{1}{2}, \quad \varepsilon_n = 1.0 \text{ for } n \geq 1, \quad \lambda_{nm}^{\pm}(x) = I_{n+m}(x) \pm I_{n-m}(x),
$$

$$
Y_{nj}^{(1)} = 2ReC_{nj}^{(1)}, \quad Z_{nj}^{(1)} = 2ReB_{nj}^{(1)}, \quad X_{nj}^{(1)} = -2ImC_{nj}^{(1)}, \quad T_{nj}^{(1)} = -2ImB_{nj}^{(1)}. \tag{36}
$$

Thus, from equations (36), (26), (24), and (18) we obtain the expressions for the stresses and displacements of the surrounding cylinder and fiber. These expressions can be presented as follows

$$
\begin{aligned}
& \left\{\sigma_{rr}^{(k),1};\ \sigma_{r\theta}^{(k),1};\ \sigma_{rz}^{(k),1};\ u_r^{(k),1};\ u_\theta^{(k),1};\ u_z^{(k),1}\right\}^T = \\
& \{\sin\alpha z;\ \sin\alpha z;\ \cos\alpha z;\ \sin\alpha z;\ \sin\alpha z;\ \cos\alpha z\}^T \\
& \times \Bigg\{ \sum_{n=0}^{+\infty} \left\{\sigma_{rrc}^{(k),1}(n);\ \sigma_{r\theta c}^{(k),1}(n);\ \sigma_{rzc}^{(k),1}(n);\ \ u_{rc}^{(k),1}(n);\ u_{\theta c}^{(k),1}(n);\ u_{zc}^{(k),1}(n)\right\}^T \cos n\theta_0 \\
& + \sum_{n=1}^{+\infty} \left\{\sigma_{rrs}^{(k),1}(n);\ \sigma_{r\theta s}^{(k),1}(n);\ \sigma_{rzs}^{(k),1}(n);\ \ u_{rs}^{(k),1}(n);\ u_{\theta s}^{(k),1}(n);\ u_{zs}^{(k),1}(n)\right\}^T \sin n\theta_0 \Bigg\},
\end{aligned}
\tag{37}
$$

where

$$\sigma_{rrc}^{(0),1}(n) = X_{n1}^{(0),1} a_{11n}^{(0)}\left(\xi_1^{(0)}\alpha r_0\right) + Y_{n2}^{(0),1} a_{12n}^{(0)}\left(\xi_2^{(0)}\alpha r_0\right) + Y_{n3}^{(0),1} a_{13n}^{(0)}\left(\xi_3^{(0)}\alpha r_0\right),$$

$$\sigma_{rrs}^{(0),1}(n) = Y_{n1}^{(0),1} b_{11n}^{(0)}\left(\xi_1^{(0)}\alpha r_0\right) + X_{n2}^{(0),1} b_{12n}^{(0)}\left(\xi_2^{(0)}\alpha r_0\right) + X_{n3}^{(0),1} b_{13n}^{(0)}\left(\xi_3^{(0)}\alpha r_0\right),$$

$$\begin{aligned}\sigma_{rrc}^{(1),1}(n) =& X_{n1}^{(1)} a_{12n}^{(1)}\left(\xi_1^{(1)}\alpha r_0\right)\\ &+ \sum_{m=0}^{+\infty} T_{m1}^{(1)}\lambda_{nm}^{-}(\xi_1^{(1)}\alpha R_{10}) a_{11m}^{(1)}\left(\xi_1^{(1)}\alpha r_0\right)\\ &+ Y_{n2}^{(1)} a_{14n}^{(1)}\left(\xi_2^{(1)}\alpha r_0\right) + \sum_{m=0}^{+\infty} Z_{m2}^{(1)}\lambda_{nm}^{+}(\xi_2^{(1)}\alpha R_{10}) a_{13m}^{(1)}\left(\xi_2^{(1)}\alpha r_0\right)\\ &+ Y_{n3}^{(1)} a_{16n}^{(1)}\left(\xi_3^{(1)}\alpha r_0\right) + \sum_{m=0}^{+\infty} Z_{m3}^{(1)}\lambda_{nm}^{+}(\xi_3^{(1)}\alpha R_{10}) a_{15m}^{(1)}\left(\xi_3^{(1)}\alpha r_0\right),\\ \sigma_{rrs}^{(1),1}(n) =& Y_{n1}^{(1)} b_{12n}^{(1)}\left(\xi_1^{(1)}\alpha r_0\right)\\ &+ \sum_{m=0}^{+\infty} Z_{m1}^{(1)}\lambda_{nm}^{+}(\xi_1^{(1)}\alpha R_{10}) b_{11m}^{(1)}\left(\xi_1^{(1)}\alpha r_0\right)\\ &+ X_{n2}^{(1)} b_{14n}^{(1)}\left(\xi_2^{(1)}\alpha r_0\right) + \sum_{m=0}^{+\infty} T_{m2}^{(1)}\lambda_{nm}^{-}(\xi_2^{(1)}\alpha R_{10}) b_{13m}^{(1)}\left(\xi_2^{(1)}\alpha r_0\right)\\ &+ X_{n3}^{(1)} b_{16n}^{(1)}\left(\xi_3^{(1)}\alpha r_0\right) + \sum_{m=0}^{+\infty} T_{m3}^{(1)}\lambda_{nm}^{+}(\xi_3^{(1)}\alpha R_{10}) b_{15m}^{(1)}\left(\xi_3^{(1)}\alpha r_0\right).\end{aligned} \tag{38}$$

The expressions for $\sigma_{r\theta s}^{(k),1}(n)$, $\sigma_{rzc}^{(k),1}(n)$, $u_{rc}^{(k),1}(n)$, $u_{\theta s}^{(k),1}(n)$, $u_{zc}^{(k),1}(n)$ (for $\sigma_{r\theta s}^{(k),1}(n)$, $\sigma_{rzc}^{(k),1}(n)$, $u_{rc}^{(k),1}(n)$, $u_{\theta s}^{(k),1}(n)$, $u_{zc}^{(k),1}(n)$) are obtained from (38) by replacing $a_{1..}^{(k)}(\cdot)$ $\left(b_{1..}^{(k)}(\cdot)\right)$ with $a_{2..}^{(k)}(\cdot)$, $a_{3..}^{(k)}(\cdot)$, $a_{4..}^{(k)}(\cdot)$, $a_{5..}^{(k)}(\cdot)$, $a_{6..}^{(k)}(\cdot)$ ($b_{2..}^{(k)}(\cdot)$, $b_{3..}^{(k)}(\cdot)$, $b_{4..}^{(k)}(\cdot)$, $b_{5..}^{(k)}(\cdot)$, $b_{6..}^{(k)}(\cdot)$), respectively,
where

$$\begin{aligned}&a_{11n}^{(1)}\left(\xi_1^{(1)}x\right) = -b_{11n}^{(1)}\left(\xi_1^{(1)}x\right) = \left(\lambda^{(1)} + 2\mu^{(1)}\right)\\ &\times\left[\left(2n/x^2\right) I_n(\xi_1^{(1)}x) - \left(n\xi_1^{(1)}/2x\right)\left(I_{n+1}(\xi_1^{(1)}x) + I_{n-1}(\xi_1^{(1)}x)\right)\right]\\ &\qquad + \lambda^{(1)}\left[n\xi_1^{(1)}\left(I_{n+1}(\xi_1^{(1)}x) + I_{n-1}(\xi_1^{(1)}x)\right) - \left(2n/x^2\right) I_n(\xi_1^{(1)}x)\right],\end{aligned}$$

$$a_{12n}^{(1)}\left(\xi_1^{(1)}x\right)=-b_{12n}^{(1)}\left(\xi_1^{(1)}x\right)=\left(\lambda^{(1)}+2\mu^{(1)}\right)$$
$$\times\left[\left(2n/x^2\right)K_n(\xi_1^{(1)}x)+\left(n\xi_1^{(1)}/2x\right)\left(K_{n+1}(\xi_1^{(1)}x)+K_{n-1}(\xi_1^{(1)}x)\right)\right]$$
$$+\lambda^{(1)}\left[-n\xi_1^{(1)}\left(K_{n+1}(\xi_1^{(1)}x)+K_{n-1}(\xi_1^{(1)}x)\right)-\left(2n/x^2\right)K_n(\xi_1^{(1)}x)\right],$$

$$a_{13n}^{(1)}\left(\xi_2^{(1)}x\right)=b_{13n}^{(1)}\left(\xi_2^{(1)}x\right)=\left(\lambda^{(1)}+2\mu^{(1)}\right)$$
$$\times\left[\varepsilon_n\left(\left(\xi_2^{(1)}\right)^2/2\right)\left(I_{n+2}(\xi_2^{(1)}x)+2I_n(\xi_2^{(1)}x)+I_{n-2}(\xi_2^{(1)}x)\right)\right]$$
$$+\lambda^{(1)}\left(2n^2/x\right)I_n(\xi_2^{(1)}x)+\lambda^{(1)}\left(\xi_2^{(1)}/x\right)\left(I_{n+1}(\xi_2^{(1)}x)+I_{n-1}(\xi_2^{(1)}x)\right)$$
$$-\lambda^{(1)}2c_2^{(1)}I_n(\xi_2^{(1)}x),$$

$$a_{14n}^{(1)}\left(\xi_2^{(1)}x\right)=b_{14n}^{(1)}\left(\xi_2^{(1)}x\right)=\left(\lambda^{(1)}+2\mu^{(1)}\right)$$
$$\times\left[\varepsilon_n\left(\xi_2^{(1)}\right)^2\left(K_{n+2}(\xi_2^{(1)}x)+2K_n(\xi_2^{(1)}x)+K_{n-2}(\xi_2^{(1)}x)\right)/2\right]$$
$$+\lambda^{(1)}\left(2n^2/x\right)K_n(\xi_2^{(1)}x)-\lambda^{(1)}\left(\xi_2^{(1)}/x\right)\left(K_{n+1}(\xi_2^{(1)}x)+K_{n-1}(\xi_2^{(1)}x)\right)$$
$$-\lambda^{(1)}2c_2^{(1)}K_n(\xi_2^{(1)}x),$$

$$a_{15n}^{(1)}(\xi_3^{(1)}x)=b_{15n}^{(1)}(\xi_3^{(1)}x)=a_{13n}^{(1)}(\xi_3^{(1)}x),\quad a_{16n}^{(1)}(\xi_3^{(1)}x)=b_{16n}^{(1)}(\xi_3^{(1)}x)=a_{14n}^{(1)}(\xi_3^{(1)}x),$$

$$a_{11n}^{(0)}\left(\xi_1^{(0)}x\right)=-b_{11n}^{(0)}\left(\xi_1^{(0)}x\right)=\left(\lambda^{(0)}+2\mu^{(0)}\right)$$
$$\times\left[\left(2n/x^2\right)I_n(\xi_1^{(0)}x)-\left(n\xi_1^{(0)}/2x\right)\left(I_{n+1}(\xi_1^{(0)}x)+I_{n-1}(\xi_1^{(0)}x)\right)\right]$$
$$+\lambda^{(0)}\left[n\xi_1^{(0)}\left(I_{n+1}(\xi_1^{(0)}x)+I_{n-1}(\xi_1^{(0)}x)\right)-\left(2n/x^2\right)I_n(\xi_1^{(0)}x)\right],$$

$$a_{12n}^{(0)}\left(\xi_2^{(0)}x\right)=b_{12n}^{(0)}\left(\xi_2^{(0)}x\right)=\left(\lambda^{(0)}+2\mu^{(0)}\right)$$
$$\times\left[\varepsilon_n\left(\xi_2^{(0)}\right)^2\left(I_{n+2}(\xi_2^{(0)}x)+2I_n(\xi_2^{(0)}x)+I_{n-2}(\xi_2^{(0)}x)\right)/2\right]$$
$$+\lambda^{(0)}\left(2n^2/x\right)I_n(\xi_2^{(0)}x)+\lambda^{(0)}\left(\xi_2^{(0)}/x\right)\left(I_{n+1}(\xi_2^{(0)}x)+I_{n-1}(\xi_2^{(0)}x)\right)$$
$$-\lambda^{(0)}2c_2^{(0)}\varepsilon_nI_n(\xi_2^{(0)}x);$$

$$a_{13n}^{(0)}(\xi_3^{(0)}x) = b_{13n}^{(0)}(\xi_3^{(0)}x) = a_{12n}^{(0)}(\xi_3^{(0)}x),$$

$$\begin{aligned} a_{21n}^{(1)}\left(\xi_1^{(1)}x\right) &= b_{21n}^{(1)}\left(\xi_1^{(1)}x\right) = \\ 2\mu^{(1)}&\left[\left(\xi_1^{(1)}\right)^2\left(I_{n+2}(\xi_1^{(1)}x)+2I_n(\xi_1^{(1)}x)+I_{n-2}(\xi_1^{(1)}x)\right)/2\right. \\ &\left.+\,2\left(n^2/x^2\right)I_n(\xi_1^{(1)}x)-\left(\left(\xi_1^{(1)}\right)^2/x\right)\left(I_{n+1}(\xi_1^{(1)}x)+I_{n-1}(\xi_1^{(1)}x)\right)\right], \end{aligned}$$

$$\begin{aligned} a_{22n}^{(1)}\left(\xi_1^{(1)}x\right) &= b_{22n}^{(1)}\left(\xi_1^{(1)}x\right) = \\ 2\mu^{(1)}&\left[\left(\xi_1^{(1)}\right)^2\left(K_{n+2}(\xi_1^{(1)}x)+2K_n(\xi_1^{(1)}x)+K_{n-2}(\xi_1^{(1)}x)\right)/2\right. \\ &\left.+\,2\left(n^2/x^2\right)K_n(\xi_1^{(1)}x)+\left(\left(\xi_1^{(1)}\right)^2/x\right)\left(K_{n+1}(\xi_1^{(1)}x)+K_{n-1}(\xi_1^{(1)}x)\right)\right], \end{aligned}$$

$$\begin{aligned} a_{23n}^{(1)}\left(\xi_2^{(1)}x\right) &= -b_{23n}^{(1)}\left(\xi_2^{(1)}x\right) = \\ 2\mu^{(1)}&\left[-\left(n/x\right)\xi_2^{(1)}\left(I_{n+1}(\xi_2^{(1)}x)+I_{n-1}(\xi_2^{(1)}x)\right)\right. \\ &\left.-\left(n/x\right)\xi_2^{(1)}\left(I_{n+1}(\xi_2^{(1)}x)+I_{n-1}(\xi_2^{(1)}x)\right)-\left(2n/x^2\right)I_n(\xi_2^{(1)}x)\right], \end{aligned}$$

$$\begin{aligned} a_{24n}^{(1)}\left(\xi_2^{(1)}x\right) &= -b_{24n}^{(1)}\left(\xi_2^{(1)}x\right) = \\ 2\mu^{(1)}&\left[\left(n/x\right)\xi_2^{(1)}\left(K_{n+1}(\xi_2^{(1)}x)+K_{n-1}(\xi_2^{(1)}x)\right)\right. \\ &\left.+\left(n/x\right)\xi_2^{(1)}\left(K_{n+1}(\xi_2^{(1)}x)+K_{n-1}(\xi_2^{(1)}x)\right)-\left(2n/x^2\right)K_n(\xi_2^{(1)}x)\right], \end{aligned}$$

$$a_{25n}^{(1)}(\xi_3^{(1)}x) = -b_{25n}^{(1)}(\xi_3^{(1)}x) = a_{23n}^{(1)}(\xi_3^{(1)}x),$$
$$a_{26n}^{(1)}(\xi_3^{(1)}x) = -b_{26n}^{(1)}(\xi_3^{(1)}x) = a_{24n}^{(1)}(\xi_3^{(1)}x),$$

$$\begin{aligned} a_{21n}^{(0)}\left(\xi_1^{(0)}x\right) &= b_{21n}^{(0)}\left(\xi_1^{(0)}x\right) = \\ 2\mu^{(0)}&\left[\left(\xi_1^{(0)}\right)^2\left(I_{n+2}(\xi_1^{(0)}x)+2I_n(\xi_1^{(0)}x)+I_{n-2}(\xi_1^{(0)}x)\right)/2\right. \\ &\left.+\,2\left(n^2/x^2\right)I_n(\xi_1^{(0)}x)-\left(\left(\xi_1^{(0)}\right)^2/x\right)\left(I_{n+1}(\xi_1^{(0)}x)+I_{n-1}(\xi_1^{(0)}x)\right)\right], \end{aligned}$$

$$a_{22n}^{(0)}\left(\xi_2^{(0)}x\right) = -b_{22n}^{(0)}\left(\xi_2^{(0)}x\right) =$$
$$2\mu^{(0)}\left[-(n/x)\,\xi_2^{(0)}\left(I_{n+1}(\xi_2^{(0)}x)+I_{n-1}(\xi_2^{(0)}x)\right)\right.$$
$$\left.-(n/x)\,\xi_2^{(0)}\left(I_{n+1}(\xi_2^{(0)}x)+I_{n-1}(\xi_2^{(0)}x)\right)-\left(2n/x^2\right)I_n(\xi_2^{(0)}x)\right],$$

$$a_{23n}^{(0)}(\xi_3^{(0)}x) = -b_{23n}^{(0)}(\xi_3^{(0)}x) = a_{22n}^{(0)}(\xi_3^{(0)}x),$$
$$a_{31n}^{(1)}\left(\xi_1^{(1)}x\right) = -b_{31n}^{(1)}\left(\xi_1^{(1)}x\right) = -2\mu^{(1)}\,(2n/x)\,I_n(\xi_1^{(1)}x),$$
$$a_{32n}^{(1)}\left(\xi_1^{(1)}x\right) = -b_{32n}^{(1)}\left(\xi_1^{(1)}x\right) = -2\mu^{(1)}\,(2n/x)\,K_n(\xi_1^{(1)}x)$$
$$a_{33n}^{(1)}\left(\xi_2^{(1)}x\right) = b_{33n}^{(1)}\left(\xi_2^{(1)}x\right) = 2\mu^{(1)}\left(1+c_2^{(1)}\right)\xi_2^{(1)}\left(I_{n+1}(\xi_2^{(1)}x)+I_{n-1}(\xi_2^{(1)}x)\right),$$
$$a_{34n}^{(1)}\left(\xi_2^{(1)}x\right) = b_{34n}^{(1)}\left(\xi_2^{(1)}x\right) = -2\mu^{(1)}\left(1+c_2^{(1)}\right)\xi_2^{(1)}\left(K_{n+1}(\xi_2^{(1)}x)+K_{n-1}(\xi_2^{(1)}x)\right),$$
$$a_{35n}^{(1)}(\xi_3^{(1)}x) = b_{35n}^{(1)}(\xi_3^{(1)}x) = a_{33n}^{(1)}(\xi_3^{(1)}x), \quad a_{36n}^{(1)}(\xi_3^{(1)}x) = b_{36n}^{(1)}(\xi_3^{(1)}x) = a_{34n}^{(1)}(\xi_3^{(1)}x),$$
$$a_{31n}^{(0)}\left(\xi_1^{(0)}x\right) = -b_{31n}^{(0)}\left(\xi_1^{(0)}x\right) = -2\mu^{(0)}\,(2n/x)\,I_n(\xi_1^{(0)}x),$$
$$a_{32n}^{(0)}\left(\xi_2^{(0)}x\right) = b_{32n}^{(0)}\left(\xi_2^{(0)}x\right) = \varepsilon_n 2\mu^{(0)}\xi_2^{(0)}\left(1+c_2^{(0)}\right)\left(I_{n+1}(\xi_2^{(0)}x)+I_{n-1}(\xi_2^{(0)}x)\right),$$
$$a_{33n}^{(0)}(\xi_3^{(0)}x) = b_{33n}^{(0)}(\xi_3^{(0)}x) = a_{32n}^{(0)}(\xi_3^{(0)}x),$$
$$a_{41n}^{(1)}\left(\xi_1^{(1)}x\right) = -b_{41n}^{(1)}\left(\xi_1^{(1)}x\right) = -(2n/x)\,I_n(\xi_1^{(1)}x),$$
$$a_{42n}^{(1)}\left(\xi_1^{(1)}x\right) = -b_{42n}^{(1)}\left(\xi_1^{(1)}x\right) = -(2n/x)\,K_n(\xi_1^{(1)}x),$$
$$a_{43}^{(1)}\left(\xi_2^{(1)}x\right) = b_{43}^{(1)}\left(\xi_2^{(1)}x\right) = \xi_2^{(1)}\left(I_{n+1}(\xi_2^{(1)}x)+I_{n-1}(\xi_2^{(1)}x)\right),$$
$$a_{44n}^{(1)}\left(\xi_2^{(1)}x\right) = b_{44n}^{(1)}\left(\xi_2^{(1)}x\right) = -\xi_2^{(1)}\left(K_{n+1}(\xi_2^{(1)}x)+K_{n-1}(\xi_2^{(1)}x)\right),$$
$$a_{45n}^{(1)}\left(\xi_3^{(1)}x\right) = b_{45n}^{(1)}\left(\xi_3^{(1)}x\right) = a_{43n}^{(1)}\left(\xi_3^{(1)}x\right),$$
$$a_{46n}^{(1)}\left(\xi_3^{(1)}x\right) = b_{46n}^{(1)}\left(\xi_3^{(1)}x\right) = a_{44n}^{(1)}\left(\xi_3^{(1)}x\right),$$
$$a_{41n}^{(0)}\left(\xi_1^{(0)}x\right) = -b_{41n}^{(0)}\left(\xi_1^{(0)}x\right) = -(2n/x)\,I_n(\xi_1^{(0)}x),$$
$$a_{42n}^{(0)}\left(\xi_2^{(0)}x\right) = b_{42n}^{(0)}\left(\xi_2^{(0)}x\right) = \varepsilon_n\xi_2^{(0)}\left(I_{n+1}(\xi_2^{(0)}x)+I_{n-1}(\xi_2^{(0)}x)\right),$$
$$a_{43n}^{(0)}(\xi_3^{(0)}x) = b_{43n}^{(0)}(\xi_3^{(0)}x) = a_{42n}^{(0)}(\xi_3^{(0)}x),$$

$$a_{51n}^{(1)}\left(\xi_1^{(1)}x\right)=b_{51n}^{(1)}\left(\xi_1^{(1)}x\right)=\xi_1^{(1)}\left(I_{n+1}(\xi_1^{(1)}x)+I_{n-1}(\xi_1^{(1)}x)\right),$$

$$a_{52n}^{(1)}\left(\xi_1^{(1)}x\right)=b_{52n}^{(1)}\left(\xi_1^{(1)}x\right)=-\xi_1^{(1)}\left(K_{n+1}(\xi_1^{(1)}x)+K_{n-1}(\xi_1^{(1)}x)\right),$$

$$a_{53n}^{(1)}\left(\xi_2^{(1)}x\right)=-b_{53n}^{(1)}\left(\xi_2^{(1)}x\right)=-\left(2n/x\right)I_n(\xi_2^{(1)}x),$$

$$a_{54n}^{(1)}\left(\xi_2^{(1)}x\right)=-b_{54n}^{(1)}\left(\xi_2^{(1)}x\right)=-\left(2n/x\right)K_n(\xi_2^{(1)}x),$$

$$a_{55n}^{(1)}\left(\xi_3^{(1)}x\right)=-b_{55n}^{(1)}\left(\xi_3^{(1)}x\right)=a_{53n}^{(1)}\left(\xi_3^{(1)}x\right),$$

$$a_{56n}^{(1)}\left(\xi_3^{(1)}x\right)=-b_{56n}^{(1)}\left(\xi_3^{(1)}x\right)=a_{54n}^{(1)}\left(\xi_3^{(1)}x\right),$$

$$a_{51n}^{(0)}\left(\xi_1^{(0)}x\right)=b_{51n}^{(0)}\left(\xi_1^{(0)}x\right)=\xi_1^{(0)}\left(I_{n+1}(\xi_1^{(0)}x)+I_{n-1}(\xi_1^{(0)}x)\right),$$

$$a_{52n}^{(0)}\left(\xi_2^{(0)}x\right)=-b_{52n}^{(0)}\left(\xi_2^{(0)}x\right)=-\left(2n/x\right)I_n(\xi_2^{(0)}x),$$

$$a_{53n}^{(0)}(\xi_3^{(0)}x)=-b_{53n}^{(0)}(\xi_3^{(0)}x)=a_{52n}^{(0)}(\xi_3^{(0)}x),$$

$$a_{61n}^{(1)}\left(\xi_1^{(1)}x\right)=b_{61n}^{(1)}\left(\xi_1^{(1)}x\right)=0.0,$$

$$a_{62n}^{(1)}\left(\xi_1^{(1)}x\right)=b_{62n}^{(1)}\left(\xi_1^{(1)}x\right)=0.0,$$

$$a_{63n}^{(1)}\left(\xi_2^{(1)}x\right)=b_{63n}^{(1)}\left(\xi_2^{(1)}x\right)=\varepsilon_n 2c_2^{(1)}I_n(\xi_2^{(1)}x),$$

$$a_{64n}^{(1)}\left(\xi_2^{(1)}x\right)=b_{64n}^{(1)}\left(\xi_2^{(1)}x\right)=\varepsilon_n 2c_2^{(1)}K_n(\xi_2^{(1)}x),$$

$$a_{65n}^{(1)}\left(\xi_3^{(1)}x\right)=b_{65n}^{(1)}\left(\xi_3^{(1)}x\right)=a_{63n}^{(1)}\left(\xi_3^{(1)}x\right),$$

$$a_{66n}^{(1)}\left(\xi_3^{(1)}x\right)=b_{66n}^{(1)}\left(\xi_3^{(1)}x\right)=a_{64n}^{(1)}\left(\xi_3^{(1)}x\right),$$

$$a_{61n}^{(0)}\left(\xi_1^{(0)}x\right)=b_{61n}^{(0)}\left(\xi_1^{(0)}x\right)=0.0,$$

$$a_{62n}^{(0)}\left(\xi_2^{(0)}x\right)=\varepsilon_n 2c_2^{(0)}I_n(\xi_2^{(0)}x),\quad a_{63n}^{(0)}(\xi_3^{(0)}x)=a_{62n}^{(0)}(\xi_3^{(0)}x),$$

$$\varepsilon_0=0.5,\quad \varepsilon_n=1.0 \text{ for } n\geq 1,$$

$$c_2^{(k)}=\left[\left(\lambda^{(k)}/\mu^{(k)}+2\right)\left(\xi_2^{(k)}\right)^2-1-\sigma_{33}^{(k),0}/\mu^{(k)}\right]\times\mu^{(k)}/\left(\lambda^{(k)}+\mu^{(k)}\right),\ k=0,1. \tag{39}$$

Thus, with the foregoing, we have prepared all needed mathematical calculations to satisfy the contact conditions at $r_0 = R_0$.

Now we consider satisfaction of the boundary conditions in (22). For this purpose we must present the terms written in the coordinates (r_0, θ_0) which enter the expressions of $\sigma_{rr}^{(1),1}$, $\sigma_{r\theta}^{(1),1}$ and $\sigma_{rz}^{(1),1}$ in the system of coordinates (r, θ) . These expressions are presented in series form, the terms of which are $(unknown\, const)_n \times K_{n+k}(cr_0)\cos n\theta_0$ or $(unknown\, const)_n \times K_{n+k}(cr_0)\sin n\theta_0$ $(k = 0,1,2,3)$. It should be noted that for presentation of these terms in the system of coordinates (r, θ) we cannot employ the summation theorem for functions $K_{n+k}(cr_0)\cos n\theta_0$ and $K_{n+k}(cr_0)\sin n\theta_0$, because, in the considered case, the inequality $R > R_{10}$ occurs, i.e. the condition in type (34) does not satisfy in the satisfaction of the boundary conditions at $r = R$ (22). Therefore, we propose here the following algorithm for this satisfaction. To simplify the consideration below we will make all discussions for the stress $\sigma_{rr}^{(1),1}$. These discussions can be easily transformed for the stresses $\sigma_{r\theta}^{(1),1}$ and $\sigma_{rz}^{(1),1}$.

First we present the expressions of the stress $\sigma_{rr}^{(1),1}$ as follows.

$$
\begin{aligned}
\sigma_{rr}^{(1),1} = \sin\alpha z \Bigg\{ & \sum_{n=0}^{+\infty} X_{n1}^{(1)} a_{12n}^{(1)}(\xi_1^{(1)}\alpha r_0)\cos n\theta_0 + \sum_{n=0}^{+\infty} T_{n1}^{(1)} a_{11n}^{(1)}(\xi_1^{(1)}\alpha r)\cos n\theta \\
& + \sum_{n=0}^{+\infty} Y_{n2}^{(1)} a_{14n}^{(1)}(\xi_2^{(1)}\alpha r_0)\cos n\theta_0 + \sum_{n=0}^{+\infty} Z_{n2}^{(1)} a_{13n}^{(1)}(\xi_2^{(1)}\alpha r)\cos n\theta \\
& + \sum_{n=0}^{+\infty} Y_{n3}^{(1)} a_{16n}^{(1)}(\xi_3^{(1)}\alpha r_0)\cos n\theta_0 + \sum_{n=0}^{+\infty} Z_{n3}^{(1)} a_{15n}^{(1)}(\xi_3^{(1)}\alpha r)\cos n\theta \Bigg\} \\
+ \sin\alpha z \Bigg\{ & \sum_{n=1}^{+\infty} Y_{n1}^{(1)} b_{12n}^{(1)}(\xi_1^{(1)}\alpha r_0)\sin n\theta_0 + \sum_{n=1}^{+\infty} Z_{n1}^{(1)} b_{11n}^{(1)}(\xi_1^{(1)}\alpha r)\sin n\theta \\
& + \sum_{n=1}^{+\infty} X_{n2}^{(1)} b_{14n}^{(1)}(\xi_2^{(1)}\alpha r_0)\sin n\theta_0 + \sum_{n=1}^{+\infty} T_{n2}^{(1)} b_{13n}^{(1)}(\xi_2^{(1)}\alpha r)\sin n\theta \\
& + \sum_{n=1}^{+\infty} X_{n3}^{(1)} b_{16n}^{(1)}(\xi_3^{(1)}\alpha r_0)\sin n\theta_0 + \sum_{n=1}^{+\infty} T_{n3}^{(1)} b_{15n}^{(1)}(\xi_3^{(1)}\alpha r)\sin n\theta \Bigg\}.
\end{aligned} \tag{40}
$$

Under satisfying the boundary condition (22) at $r = R$ with the use of equation (40), instead of $r_0$and θ_0 we write the following expressions which are obtained from equation (1).

$$r_0 = r_0(r,\theta) = R\sqrt{1 - 2\frac{R_{10}}{R}\cos\theta + \left(\frac{R_{10}}{R}\right)^2},$$

$$\theta_0 = \theta_0(r,\theta) = \arg\cos\left(\left(\cos\theta - \frac{R_{10}}{R}\right)/\sqrt{1 - 2\frac{R_{10}}{R}\cos\theta + \left(\frac{R_{10}}{R}\right)^2}\right). \tag{41}$$

Taking equation (41) into account, we expand the terms $a_{1jn}^{(1)}(cr_0)\cos n\theta_0$ and $b_{4jn}^{(1)}(cr_0)\sin n\theta_0$ with the following cosine and sine Fourier series, respectively.

$$a_{1jn}^{(1)}(cr_0)\cos n\theta_0 = \sum_{k=0}^{+\infty}\alpha_{1jnk}^{(1)}(cR)\cos k\theta, \quad b_{1jn}^{(1)}(cr_0)\sin n\theta_0 = \sum_{k=1}^{+\infty}\beta_{1jnk}^{(1)}(cR)\sin k\theta, \tag{42}$$

where

$$\alpha_{1jn0}^{(1)}(cR) = \frac{2}{\pi}\int_0^{\pi} a_{1jn}^{(1)}(cr_0)\cos n\theta_0 d\theta$$

$$\alpha_{1jnk}^{(1)}(cR) = \frac{1}{\pi}\int_0^{\pi} a_{1jn}^{(1)}(cr_0)\cos n\theta_0 \cos k\theta d\theta \text{ for } k \geq 1,$$

$$\beta_{1jnk}^{(1)}(cR) = \frac{1}{\pi}\int_0^{\pi} b_{1jn}^{(1)}(cr_0)\sin n\theta_0 \sin k\theta d\theta. \tag{43}$$

Substituting expression (42) into equation (40) and doing some mathematical transformations we obtain the following expression for $\sigma_{rr}^{(1)}$ at $r = R$.

$$
\begin{aligned}
\sigma_{rr}^{(1),1}\Big|_{r=R} = \sin\alpha z \Bigg\{ & \sum_{n=0}^{+\infty} \Bigg[\sum_{k=0}^{+\infty} X_{k1}^{(1)}\ \alpha_{12nk}^{(1)}(\xi_1^{(1)}\alpha R) \\
& + T_{n1}^{(1)} a_{11n}^{(1)}(\xi_1^{(1)}\alpha R) + \sum_{k=0}^{+\infty} Y_{k2}^{(1)} \alpha_{14nk}^{(1)}(\xi_2^{(1)}\alpha R) + Z_{n2}^{(1)} a_{13n}^{(1)}(\xi_2^{(1)}\alpha R) \\
& + \sum_{k=0}^{+\infty} Y_{k3}^{(1)} \alpha_{16nk}^{(1)}(\xi_3^{(1)}\alpha R) + Z_{n3}^{(1)} a_{15n}^{(1)}(\xi_3^{(1)}\alpha R) \Bigg] \cos n\theta \\
& + \sum_{n=1}^{+\infty} \Bigg[\sum_{k=1}^{+\infty} Y_{k1}^{(1)} \beta_{12nk}^{(1)}(\xi_1^{(1)}\alpha R) + Z_{n1}^{(1)} b_{11n}^{(1)}(\xi_1^{(1)}\alpha R) \\
& + \sum_{k=1}^{+\infty} X_{k2}^{(1)} \beta_{14nk}^{(1)}(\xi_2^{(1)}\alpha R) + T_{n2}^{(1)} b_{13n}^{(1)}(\xi_2^{(1)}\alpha R) \\
& + \sum_{k=1}^{+\infty} X_{k3}^{(1)} \beta_{16nk}^{(1)}(\xi_3^{(1)}\alpha R) + T_{n3}^{(1)} b_{15n}^{(1)}(\xi_3^{(1)}\alpha R) \Bigg] \sin n\theta \Bigg\}.
\end{aligned} \tag{44}
$$

In a similar manner we obtain the following expressions for the stresses $\sigma_{r\theta}^{(1),1}$ and $\sigma_{rz}^{(1),1}$ at $r = R$.

$$
\begin{aligned}
\sigma_{r\theta}^{(1),1}\Big|_{r=R} = \sin\alpha z \Bigg\{ & \sum_{n=1}^{+\infty} \Bigg[\sum_{k=1}^{+\infty} X_{k1}^{(1)}\ \alpha_{22nk}^{(1)}(\xi_1^{(1)}\alpha R) \\
& + T_{n1}^{(1)} a_{21n}^{(1)}(\xi_1^{(1)}\alpha R) + \sum_{k=1}^{+\infty} Y_{k2}^{(1)} \alpha_{24nk}^{(1)}(\xi_2^{(1)}\alpha R) + Z_{n2}^{(1)} a_{23n}^{(1)}(\xi_2^{(1)}\alpha R) \\
& + \sum_{k=1}^{+\infty} Y_{k3}^{(1)} \alpha_{26nk}^{(1)}(\xi_3^{(1)}\alpha R) + Z_{n3}^{(1)} a_{25n}^{(1)}(\xi_3^{(1)}\alpha R) \Bigg] \sin n\theta \\
& + \sum_{n=0}^{+\infty} \Bigg[\sum_{k=0}^{+\infty} Y_{k1}^{(1)} \beta_{22nk}^{(1)}(\xi_1^{(1)}\alpha R) + Z_{n1}^{(1)} b_{21n}^{(1)}(\xi_1^{(1)}\alpha R) \\
& + \sum_{k=0}^{+\infty} X_{k2}^{(1)} \beta_{24nk}^{(1)}(\xi_2^{(1)}\alpha R) + T_{n2}^{(1)} b_{23n}^{(1)}(\xi_2^{(1)}\alpha R) \\
& + \sum_{k=0}^{+\infty} X_{k3}^{(1)} \beta_{26nk}^{(1)}(\xi_3^{(1)}\alpha R) + T_{n3}^{(1)} b_{25n}^{(1)}(\xi_3^{(1)}\alpha R) \Bigg] \cos n\theta \Bigg\},
\end{aligned}
$$

$$
\begin{aligned}
\sigma_{rz}^{(1),1}\Big|_{r=R} = & \cos\alpha z \left\{ \sum_{n=0}^{+\infty} \left[\sum_{k=0}^{+\infty} X_{k1}^{(1)}\ \alpha_{32nk}^{(1)}(\xi_1^{(1)}\alpha R) \right. \right. \\
& + T_{n1}^{(1)} a_{31n}^{(1)}(\xi_1^{(1)}\alpha R) + \sum_{k=0}^{+\infty} Y_{k2}^{(1)} \alpha_{34nk}^{(1)}(\xi_2^{(1)}\alpha R) + Z_{n2}^{(1)} a_{33n}^{(1)}(\xi_2^{(1)}\alpha R) \\
& \left. + \sum_{k=0}^{+\infty} Y_{k3}^{(1)} \alpha_{36nk}^{(1)}(\xi_3^{(1)}\alpha R) + Z_{n3}^{(1)} a_{35n}^{(1)}(\xi_3^{(1)}\alpha R) \right] \cos n\theta \\
& + \sum_{n=1}^{+\infty} \left[\sum_{k=1}^{+\infty} Y_{k1}^{(1)} \beta_{32nk}^{(1)}(\xi_1^{(1)}\alpha R) + Z_{n1}^{(1)} b_{31n}^{(1)}(\xi_1^{(1)}\alpha R) \right. \\
& + \sum_{k=1}^{+\infty} X_{k2}^{(1)} \beta_{34nk}^{(1)}(\xi_2^{(1)}\alpha R) + T_{n2}^{(1)} b_{33n}^{(1)}(\xi_2^{(1)}\alpha R) \\
& \left. \left. + \sum_{k=1}^{+\infty} X_{k3}^{(1)} \beta_{36nk}^{(1)}(\xi_3^{(1)}\alpha R) + T_{n3}^{(1)} b_{33n}^{(1)}(\xi_3^{(1)}\alpha R) \right] \sin n\theta \right\}.
\end{aligned}
\tag{45}
$$

where

$$
\begin{aligned}
& \alpha_{2jn0}^{(1)}(cR) = \frac{2}{\pi}\int_0^{\pi} a_{2jn}^{(1)}(cr_0)\cos n\theta_0 d\theta, \\
& \alpha_{2jnk}^{(1)}(cR) = \frac{1}{\pi}\int_0^{\pi} a_{2jn}^{(1)}(cr_0)\cos n\theta_0 \cos k\theta d\theta \text{ for } k \geq 1, \\
& \beta_{2jnk}^{(1)}(cR) = \frac{1}{\pi}\int_0^{\pi} b_{2jn}^{(1)}(cr_0)\sin n\theta_0 \sin k\theta d\theta, \\
& \beta_{2jn0}^{(1)}(cR) = \frac{2}{\pi}\int_0^{\pi} b_{2jn}^{(1)}(cr_0)\cos n\theta_0 d\theta \\
& \beta_{2jnk}^{(1)}(cR) = \frac{1}{\pi}\int_0^{\pi} b_{2jn}^{(1)}(cr_0)\cos n\theta_0 \cos k\theta d\theta \text{ for } k \geq 1, \\
& \alpha_{2jnk}^{(1)}(cR) = \frac{1}{\pi}\int_0^{\pi} a_{2jn}^{(1)}(cr_0)\sin n\theta_0 \sin k\theta d\theta.
\end{aligned}
\tag{46}
$$

Thus, using equations (37) and (38) we can satisfy the contact conditions in (26), and using equations (44) and (45) we can satisfy boundary condition (22). In this way we attain from these boundary and contact conditions the infinite system's linear algebraic equations for determination of the unknown constants in the foregoing expressions. The analyses of these expressions show that the mentioned unknowns can be divided into two uncrossing sets (groups): the first of them (denoted by S_I) is

$$S_I = \left\{ X_{n1}^{(1),1},\quad T_{n1}^{(1),1},\quad Y_{n2}^{(1),1},\quad Y_{n3}^{(1),1},\quad Z_{n2}^{(1),1},\quad Z_{n3}^{(1),1},\quad X_{n1}^{(0),1},\quad Y_{n2}^{(0),1},\quad Y_{n3}^{(0),1} \right\}, \tag{47}$$

but the second of them (denoted by S_{II}) is

$$S_{II} = \left\{ Y_{n1}^{(1),1},\quad Z_{n1}^{(1),1},\quad X_{n2}^{(1),1},\quad X_{n3}^{(1),1},\quad T_{n2}^{(1),1},\quad T_{n3}^{(1),1},\quad Y_{n1}^{(0),1},\quad X_{n2}^{(0),1},\quad X_{n3}^{(0),1} \right\}. \tag{48}$$

From the foregoing contact and boundary conditions we attain the system of equations for each group of unknowns separately.

The equations for the S_I (47) group of unknowns are as follows.

$$\sum_{k=0}^{+\infty} X_{k1}^{(1)} \alpha_{i2nk}^{(1)}(\xi_1^{(1)}\alpha R) + T_{n1}^{(1)} a_{i1n}^{(1)}(\xi_1^{(1)}\alpha R) + \sum_{k=0}^{+\infty} Y_{k2}^{(1)} \alpha_{i4nk}^{(1)}(\xi_2^{(1)}\alpha R)$$
$$+ Z_{n2}^{(1)} a_{i3n}^{(1)}(\xi_2^{(1)}\alpha R) + \sum_{k=0}^{+\infty} Y_{k3}^{(1)} \alpha_{i6nk}^{(1)}(\xi_3^{(1)}\alpha R) + Z_{n3}^{(1)} a_{i5n}^{(1)}(\xi_3^{(1)}\alpha R) = 0,$$

$$X_{n1}^{(1)} a_{j2n}^{(1)}\left(\xi_1^{(1)}\alpha R_0\right) + \sum_{m=1}^{+\infty} T_{m1}^{(1)} \lambda_{nm}^{-}(\xi_1^{(1)}\alpha R_{10}) a_{j1m}^{(1)}\left(\xi_1^{(1)}\alpha R_0\right)$$
$$+ Y_{n2}^{(1)} a_{j4n}^{(1)}\left(\xi_2^{(1)}\alpha r_0\right) + \sum_{m=0}^{+\infty} Z_{m2}^{(1)} \lambda_{nm}^{+}(\xi_2^{(1)}\alpha R_{10}) a_{j3m}^{(1)}\left(\xi_2^{(1)}\alpha R_0\right)$$
$$+ Y_{n3}^{(1)} a_{j6n}^{(1)}\left(\xi_3^{(1)}\alpha R_0\right) + \sum_{m=0}^{+\infty} Z_{m3}^{(1)} \lambda_{nm}^{+}(\xi_3^{(1)}\alpha R_{10}) a_{j5m}^{(1)}\left(\xi_3^{(1)}\alpha R_0\right)$$
$$= \delta_j^6 \delta_1^n 2\pi \left(\sigma_{33}^{(1),0} - \sigma_{33}^{(2),0}\right)\cos\beta, \quad n = 0,1,2,...,\infty;\ i = 1,2,3;\ j = 1,2,3,4,5,6. \tag{49}$$

The equations for the S_{II} (48) group of unknowns are as follows.

$$\sum_{k=1}^{+\infty} Y_{k1}^{(1)} \beta_{i2nk}^{(1)}(\xi_1^{(1)}\alpha R) + Z_{n1}^{(1)} b_{i1n}^{(1)}(\xi_1^{(1)}\alpha R) + \sum_{k=1}^{+\infty} X_{k2}^{(1)} \beta_{i4nk}^{(1)}(\xi_2^{(1)}\alpha R)$$
$$+ T_{n2}^{(1)} b_{i3n}^{(1)}(\xi_2^{(1)}\alpha R) + \sum_{k=1}^{+\infty} X_{k3}^{(1)} \beta_{i6nk}^{(1)}(\xi_3^{(1)}\alpha R) + T_{n3}^{(1)} b_{i3n}^{(1)}(\xi_3^{(1)}\alpha R) = 0,$$

$$Y_{n1}^{(1)} b_{j2n}^{(1)}\left(\xi_1^{(1)}\alpha R_0\right) + \sum_{m=0}^{+\infty} Z_{m1}^{(1)} \lambda_{nm}^{+}(\xi_1^{(1)}\alpha R_{10}) b_{j1m}^{(1)}\left(\xi_1^{(1)}\alpha R_0\right)$$
$$+ X_{n2}^{(1)} b_{j4n}^{(1)}\left(\xi_2^{(1)}\alpha R_0\right) + \sum_{m=1}^{+\infty} T_{m2}^{(1)} \lambda_{nm}^{-}(\xi_2^{(1)}\alpha R_{10}) b_{j3m}^{(1)}\left(\xi_2^{(1)}\alpha R_0\right)$$
$$+ X_{n3}^{(1)} b_{j6n}^{(1)}\left(\xi_3^{(1)}\alpha R_0\right) + \sum_{m=0}^{+\infty} T_{m3}^{(1)} \lambda_{nm}^{+}(\xi_3^{(1)}\alpha R_{10}) b_{j5m}^{(1)}\left(\xi_3^{(1)}\alpha R_0\right)$$
$$= \delta_j^6 \delta_1^n 2\pi \left(\sigma_{33}^{(1),0} - \sigma_{33}^{(2),0}\right) \sin\beta. \quad n = 0,1,2,\ldots,\infty;\; i = 1,2,3;\; j = 1,2,3,4,5,6. \tag{50}$$

Consequently, as $S_I \cap S_{II} = \emptyset$ the system of equations in (49) and (50) can be solved separately. But for this purpose the contact and boundary conditions must be also separated into two parts with respect to the presentation of the corresponding terms by the $\sin n\theta$ and $\cos n\theta$.

With the foregoing we have exhausted the consideration of the solution method as well as the computer modeling algorithm for obtaining the corresponding numerical results. It should be noted that the main difference in the method and algorithm developed from the methods and algorithms used in the papers by Akbarov and Kosker (2003a, 2003b), Guz and Lapusta (1988), Lapusta (1988) and others listed in the references of these papers is the expansion of the sought values in the cosine and sine Fourier series under satisfaction of the boundary conditions at $r = R$ for which the summation theorem for the cylindrical functions is not applicable.

The values of the second and subsequent approximations in (12) can also be determined as the values of the first approximation by taking the obvious changes into account. The large number of numerical investigations carried out and detailed in the monograph by Akbarov and Guz (2000) show that the second and subsequent approximations give only insignificant quantitative improvement over the numerical results. Therefore we here restrict ourselves to consideration of the zeroth and first approximation only.

The approach proposed above with the corresponding stability loss criterion (with infinitesimal initial imperfection criterion) can also be applied for investigation of the fiber micro-buckling which is near the convex cylindrical bounded surface under compression of the considered body in the direction of the Oz axis (Fig.1). Note that the corresponding micro-buckling problems for the fiber which is near the concave cylindrical surface (cavity) as well as the plane-surface were investigated in the papers by Guz and Lapusta (1986, 1988).

4 Numerical results and discussions

We introduce the dimensionless parameters $\chi = 2\pi R_0/\ell$, $\delta = \sigma_{33}^{(1),0}/\mu^{(1)}$, $L = R_{10}/R_0$, $h = H/R_0$, $(R_{10} + H = R)$, $e = E^{(0)}/E^{(1)}$, where $E^{(0)}(E^{(1)})$ is a modulus of elasticity of the fiber (matrix) material, ℓ is a period of the form of the fiber's curvature and R_0 is a radius of the fiber's circular cross section. The meaning of the other geometrical parameters is shown in Fig. 1. Nevertheless, we note that here the dimensionless parameters h and L characterize the distance between the fiber and convex cylindrical surface and the distance between the fiber and the center of the cylinder which contains this fiber.

In obtaining the numerical results, the infinite series in equations (49) and (50) must be replaced by the corresponding finite series, i.e., for example,

$$\sum_{k=0}^{+\infty} X_{k1}^{(1)} a_{i2nk}^{(2)} \left(\xi_1^{(1)} \alpha R\right) \approx \sum_{k=0}^{+M} X_{k1}^{(1)} a_{i2nk}^{(2)} \left(\xi_1^{(1)} \alpha R\right). \tag{51}$$

From the comparison of the corresponding numerical results obtained for various M in (51), the final value of M is determined. For example, if

$$\left| \left| \sigma_{rr}^{(1),1} \right| \right|_M - \left| \sigma_{rr}^{(1),1} \right| \Big|_{M-1} \Big| / \left| \sigma_{rr}^{(1),1} \right| \Big|_{M-1} \leq 10^{-3},$$

then the increase in the number M or the increase in the numbers in series (50) is stopped. It should be noted that for the validity of the replacement of the infinite series with the finite ones, i.e. for the validity of equation (51), it is necessary to prove that the determinant of coefficients of the unknowns in equations (49) and (50) is a normal type determinant (Kantarovich and Krilov (1962)).

Similar proofs were also presented in the papers by Akbarov and Kosker (2003a, 2003b) and others, therefore we will not stop here to show the procedure for this proof; rather, we begin to analyze the numerical results attained for the self-balanced normal stress acting on the interface between the fiber and surrounding medium and for the critical compression deformation under which the micro-buckling of the fiber occurs.

In the numerical investigation, the integrals (43) and (46) are calculated by the use of the Gauss integration algorithm. In this case the interval $[0, \pi]$ is divided into a certain number of short intervals. The number of these intervals is determined from the numerical convergence of the values of the integrals.

4.1 Numerical results related to the self-balanced normal stress

Assuming that $\beta = 0$, $\varepsilon = 0.015$, $e = E^{(0)}/E^{(1)} = 50$, $\nu^{(1)} = \nu^{(0)} = 0.3$ and considering the values of $\sigma_{nn} = \varepsilon\, \sigma_{rr}^{(0)}\Big|_{r_0=R_0}$ calculated under $\alpha z = \pi/2$ and $\theta_0 = 0$, we investigate the dependence between $\left|\sigma_{nn}/\sigma_{33}^{(1),0}\right|$ and χ for various values of L and h. Through the parameter $\delta = \sigma_{33}^{(1),0}/\mu^{(1)}$ we will characterize the influence of the geometrical non-linearity on the values of $\left|\sigma_{nn}/\sigma_{33}^{(1),0}\right|$. First, we assume that $\delta \sim O\left(10^{-5}\right)$ (i.e. we assume that $\beta = 10^{-5}$), for which the effect of the mentioned geometrical non-linearity on the values of $\left|\sigma_{nn}/\sigma_{33}^{(1),0}\right|$ can be ignored with very high accuracy, and we analyze the convergence of the numerical results obtained for various values of M in (51). For this purpose, consider the case where $L = 2.0$. According to the well known mechanical consideration, this convergence must improve with h, because the influence of the free cylindrical boundary on the self-balanced stress σ_{nn} decreases with h, and primarily because this influence causes the sought values to be presented in series form. Thus, consider the graphs given in Figs. 2a, 2b and 2c which show the dependence between $\left|\sigma_{nn}/\sigma_{33}^{(1),0}\right|$ and χ under $h = 3.0$, 5.0 and 9.0, respectively, for various M and n, where n indicates the number of the equations from which the unknown constants $S_I$are determined. It follows from these graphs that, as has been predicted above, the convergence of the numerical results improves with h. Taking the results illustrated in Figs. 2a, 2b and 2c into account, below we will discuss the results attained in the case where $\{M = 5; n = 51\}$.

Thus, we consider the influence of the parameter h (for fixed L) on the values of $\left|\sigma_{nn}/\sigma_{33}^{(1),0}\right|$ attained for various χ. This influence is illustrated by the graphs given in Fig. 3. These graphs show that the values of $\left|\sigma_{nn}/\sigma_{33}^{(1),0}\right|$ approach with h the corresponding values of $\left|\sigma_{nn}/\sigma_{33}^{(1),0}\right|$ obtained for the case where the fiber is embedded within the infinite medium (Akbarov and Guz (1985b)). This statement again confirms the validity of the computational algorithm that has been developed and used here.

Now we analyze the change in the values of $\left|\sigma_{nn}/\sigma_{33}^{(1),0}\right|$ as the fiber goes away from the cylindrical free surface under fixed $R/R_0\,(= L+h = 50.0)$. The graphs illustrating this change are given in Fig. 4 for various combinations of the parameters

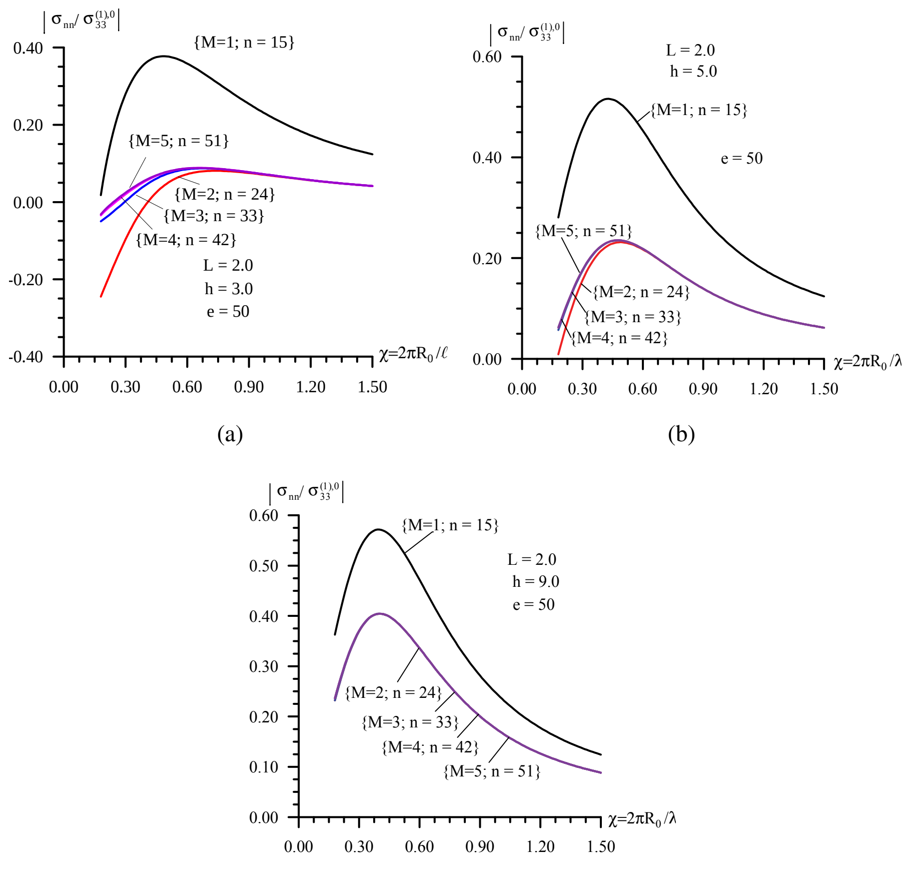

Figure 2: (a) The graphs of the dependence between the $\left|\sigma_{nn}/\sigma_{33}^{(1),0}\right|$ and parameter χ for various M and n under $h = 3.0$. (b) The graphs of the dependence between the $\left|\sigma_{nn}/\sigma_{33}^{(1),0}\right|$ and parameter χ for various M and n under $h = 5.0$. (c) The graphs of the dependence between the $\left|\sigma_{nn}/\sigma_{33}^{(1),0}\right|$ and parameter χ for various M and n under $h = 9.0$

L and h. It follows from these graphs that the values of $\left|\sigma_{nn}/\sigma_{33}^{(1),0}\right|$ increase as the fiber goes away from the cylindrical surface and, as in the previous case, approach the values obtained in the paper by Akbarov and Guz (1985b) for the single fiber contained in the infinite elastic body.

Fig. 5 shows the graphs of the dependence of the values of $\left|\sigma_{nn}/\sigma_{33}^{(1),0}\right|$ and χ

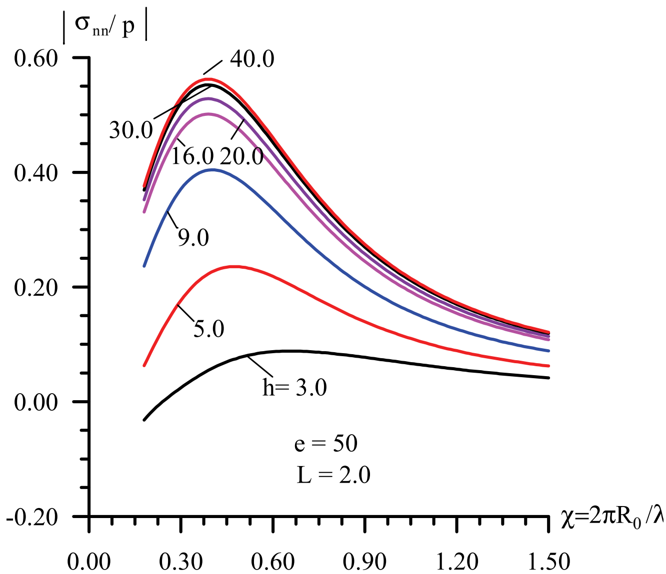

Figure 3: The influence of the distance of the fiber from the cylindrical surface (i.e. of the parameter h) on the values of $\left|\sigma_{nn}/\sigma_{33}^{(1),0}\right|$. The graph constructed in the case where $h = 40.0$ coincides with the corresponding one constructed in the paper by Akbarov and Guz (1985)

constructed for various L under fixed value of $h\,(=3.0)$, i.e. under a fixed distance of the fiber from the free cylindrical surface. The graphs show that the $\left|\sigma_{nn}/\sigma_{33}^{(1),0}\right|$ also approach certain asymptotic values with L (in other words, with R/R_0). The increasing R means the curvature radius of the cylindrical surface is increasing. Consequently, the cylindrical surface approaches the plane-surface with R/R_0 (for fixed R_0). It follows from the foregoing statement that the mentioned asymptotic values of $\left|\sigma_{nn}/\sigma_{33}^{(1),0}\right|$ correspond to the values attained for the fiber which is near to the free plane-surface. The results also show that the values of $\left|\sigma_{nn}/\sigma_{33}^{(1),0}\right|$ increase with R/R_0.

In all the foregoing results we observe that the dependence between $\left|\sigma_{nn}/\sigma_{33}^{(1),0}\right|$ and the parameter χ has a non-monotone character. This character of the mentioned dependence was also noted in many investigations detailed in the monograph by Akbarov and Guz (2000). However, the results given in Fig. 5 show that the non-monotone character of the dependence considered becomes weak with decreasing L, i.e. as the curvature radius of the cylindrical surface decreases.

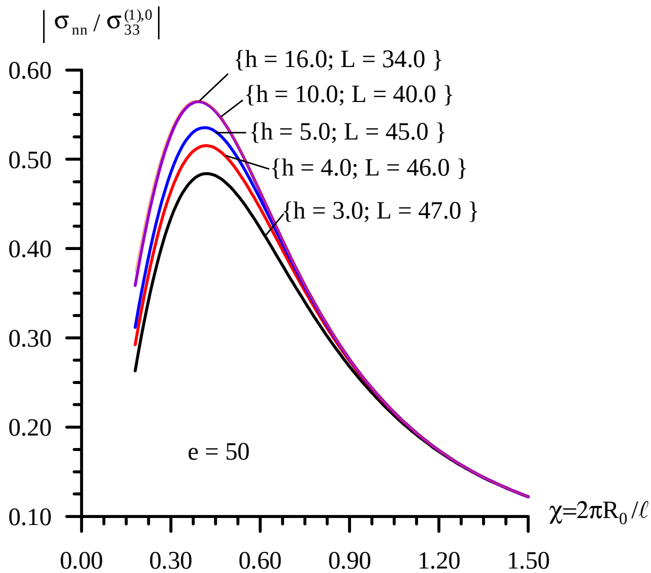

Figure 4: The change of the values of $\left|\sigma_{nn}/\sigma_{33}^{(1),0}\right|$ as the fiber goes away from the cylindrical surface under fixed $R/R_0\,(=L+h=50)$

So far, it has been assumed that $\delta \sim O\left(10^{-5}\right)$ and the influence of the geometrical non-linearity was not taken into account. Now we consider the influence of an increase in the absolute values of δ on the values of $\left|\sigma_{nn}/\sigma_{33}^{(1),0}\right|$. In this case, the distinction between the cases where the body considered is compressed and stretched must be made, i.e., between the cases where $\delta < 0.0$ and $\delta > 0.0$. The corresponding graphs are given in Fig. 6 for the case where $L = 47.0$ and $h = 3.0$. It follows from these results that under compression (tension), as a result of the geometrical non-linearity being take into account, the absolute values of $\left|\sigma_{nn}/\sigma_{33}^{(1),0}\right|$ increase (decrease). This conclusion agrees, in the quantitative sense, with the corresponding one attained in the paper by Akbarov and Kosker (2003b).

With the foregoing we have restricted ourselves to consideration of the numerical results regarding the distribution of the self-balanced normal stress which is caused namely by the periodic curving of the fiber which is near the convex cylindrical surface.

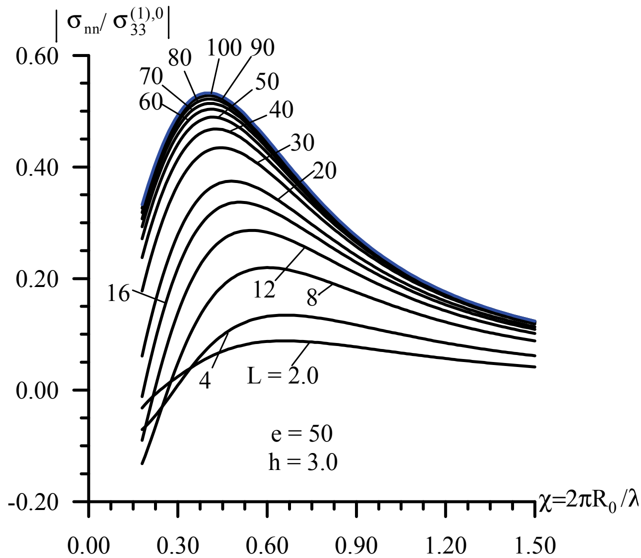

Figure 5: The graphs of the dependence between $\left|\sigma_{nn}/\sigma_{33}^{(1),0}\right|$ and the parameter χ constructed for various L under fixed $h(=3.0)$, i.e. under a fixed distance of the fiber from the free cylindrical surface

4.2 Numerical results related to the micro-buckling of the fiber

Now we examine the employment of the developed approach to investigate the micro-buckling of a fiber which is near the convex cylindrical free surface. For this purpose we use the initial imperfection criterion, which, according to the problem under consideration, is formulated as follows.

$$u_r^{(0),1}/\ell \to \infty \text{ as } \varepsilon_{33}^{(1),0}\left(=\varepsilon_{33}^{(0),0}=p/E^{(1)}\right) \to \varepsilon_{33.cr}^{(1),0}\left(=\varepsilon_{33.cr}^{(0),0}=p_{cr}/E^{(1)}\right). \quad (52)$$

Fig. 7 shows the realization of this criterion for the case where $L=6.0$, $h=2.0$, and $\chi=0.4$. At the same time, Fig. 7 shows the convergence of the critical values of $\varepsilon_{33.cr}^{(1),0}$ (52) with respect to M and n. The results confirm that the mentioned convergence is quite good.

Consider the influence of the parameters$L$and $h$on the values of $\left|\varepsilon_{33.cr}^{(1),0}\right|$. This influence is illustrated by the data given in Tables 1 and 2 which show the values of $\left|\varepsilon_{33.cr}^{(1),0}\right|$for various values L and h under $\chi=0.4$. These results show that under fixed h (i.e. under $h=2.0$) the values of $\left|\varepsilon_{33.cr}^{(1),0}\right|$ approach (along with L) the

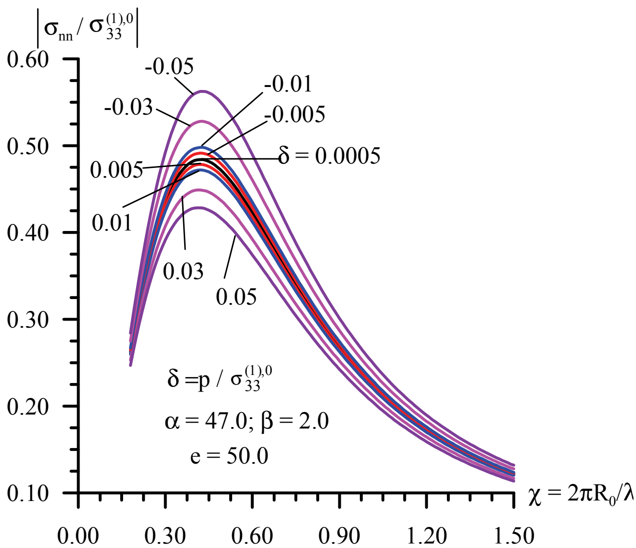

Figure 6: The influence of the geometrical non-linearity being taken into account for the values of the $\left|\sigma_{nn}/\sigma_{33}^{(1),0}\right|$

corresponding values attained in the paper by Guz and Lapusta (1986) for the fiber which is near the plane-surface, but under fixed L (i.e. under $L = 6.0$) these values approach (along with h) the corresponding values attained for the fiber embedded within the infinite elastic medium (see Babich (1973) and Akbarov and Kosker (2001)). Consequently, these results again confirm the validity and correctness of the proposed approach and developed computational algorithm for obtaining accurate numerical results. The last statement is also conformed by the graphs given in Fig. 10, which show the dependence between $\left|\varepsilon_{33.cr}^{(1),0}\right|$ and χ for various values of h under $L = 12.0$. Note that in Fig. 8 the graph corresponding to the case where the fiber is surrounded by the infinite body (i.e. the case where $h = \infty$) was also given for a comparison of the results obtained in the present investigation with those obtained in the paper by Babich (1973). It follows from these graphs that among the critical values of $\left|\varepsilon_{33.cr}^{(1),0}\right|$ there exists a min $\left|\varepsilon_{33.cr}^{(1),0}\right|$ which is taken within the scope of the bifurcation theory of the stability loss as a criterion for the near surface failure under compression of the body along the fiber. For fixed L and h the values of the mentioned failure force depend only on the values of the ratio of the mechanical constants of the materials of the fiber and surrounding body. But the value of χ which corresponds to the failure force is determined at the end of the

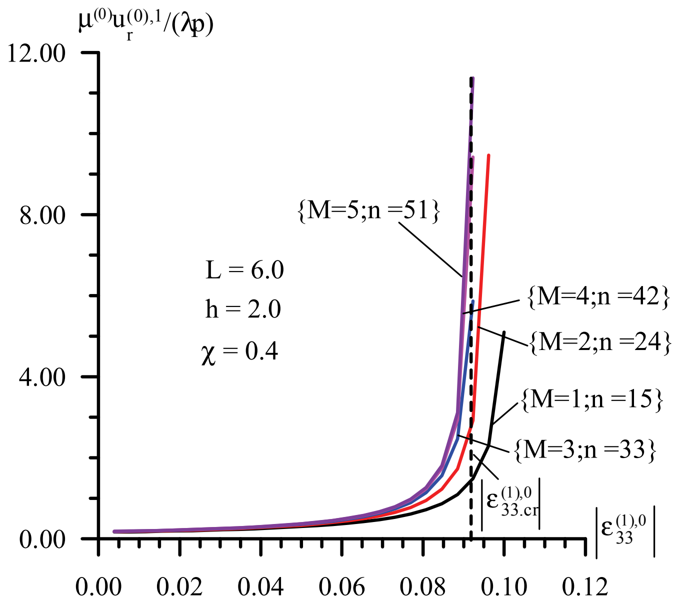

Figure 7: The convergence of the values of $\left|\varepsilon_{33.cr}^{(1),0}\right|$ with respect to numbers M and n

solution procedure. However, the value of the failure force determined within the scope of the initial imperfection criterion depends also on the values of χ because in the selection of the initial imperfection form, the value of χ is determined before the solution procedure. Consequently, the failure value of the initial force attained within the framework of the initial imperfection criterion in general, differs from that which is attained within the framework of the bifurcation approach and is more real than that determined within the scope of the bifurcation theory.

With this we have restricted ourselves to the consideration of the analyses of the numerical results related to the micro-buckling problem of the fiber which is near the convex cylindrical surface. The approach proposed and employed here can also be applied for the nano-fibers in a polymer matrix under modeling of the nano-fibers as continuous fibers with infinite length, the elasticity modulus of which are 300 – 1000 times greater than that of the polymer matrix (see: the papers by Guz, Rushchitsky and Guz (2008), Qian, Dickey, Andrews and Rantell (2000), Zhuk and Guz (2007) and others). These and other similar problems on the mechanics of micro- or nano-fibers near the convex cylindrical surface, in elastic and viscoelastic polymer materials which can be investigated within the scope of the approach proposed in the present paper will be the subject of further investigations on the part of the

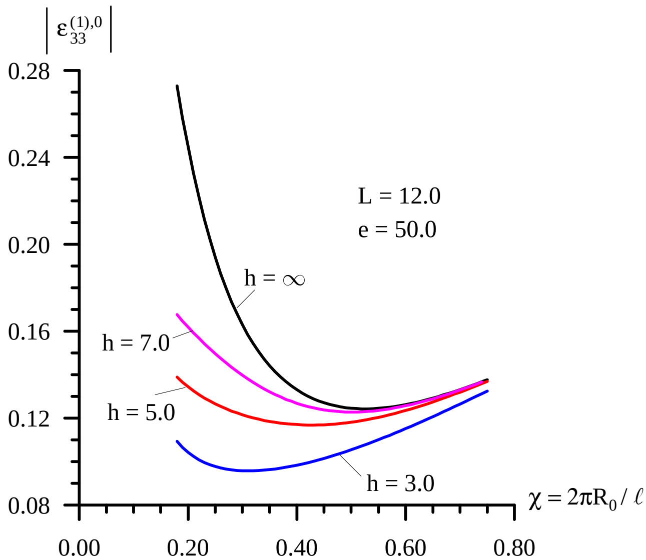

Figure 8: The graphs of the dependence between $\left|\varepsilon_{33.cr}^{(1),0}\right|$ and the parameter χ constructed for various h under $L = 12.0$. The graph constructed in the case where $h = \infty$ is the graph constructed in the papers by Babich (1973) and Akbarov and Kosker (2001)

authors.

Table 1: The values of $\left|\varepsilon_{33.cr}^{(1),0}\right|$ for various L under $h = 2.0$, $\chi = 0.4$. The case where $L = \infty$ corresponds to the paper by Guz and Lapusta (1986)

L	2.0	10.0	20.0	30.0	∞
$\left\|\varepsilon_{33.cr}^{(1),0}\right\|$	0.0884	0.0961	0.1038	0.1076	0.1077

Table 2: The values of $\left|\varepsilon_{33.cr}^{(1),0}\right|$ for various h under $L = 6.0$, $\chi = 0.4$. The case where $h = \infty$ corresponds to the papers by Babich (1973) and Akbarov and Kosker (2001)

h	2.0	6.0	10.0	20.0	∞
$\left\|\varepsilon_{33.cr}^{(1),0}\right\|$	0.0923	0.1230	0.1307	0.1307	0.1307

5 Conclusions

In the present paper, within the scope of the piecewise homogeneous body model through the use of the three-dimensional geometrically non-linear exact equations of the theory of elasticity, an approach for the investigation of problems in the micromechanics of a periodically curved fiber which is near the convex free cylindrical surface has been proposed and employed. The main difficulties in solving the corresponding problems are caused by the impossibility of employing the summation theorem of the cylindrical functions for satisfaction of the boundary conditions on the convex free cylindrical surface. For this purpose, the authors of the paper propose the use of the cosine and sine Fourier series presentation of the sought values. The coefficients of these series are calculated numerically through the integrals of the cylindrical functions whose argument depends on the integrating variable in the complicated form.

The developed approach is employed successfully for the solution to the corresponding boundary value problems in the determination of the self-balanced stresses which are caused namely by the periodic curving of the fiber and for the solution to the near surface micro-buckling problems through the use of the initial imperfection criterion.

The numerical results of the self-balanced stress distribution and of the critical strains of the micro-buckling of the fiber are presented and discussed. The validity of the developed approach and of the obtained numerical results are proven with the convergence of the numerical results with a number of algebraic equations and with the correspondence of these with the known results from the particular cases of other published investigations.

The proposed approach can easily be developed for application to more complicated cases, i.e. to those cases where two or more than two neighboring fibers are near the convex free cylindrical surface. Those mechanical problems which can be studied by the employment of the proposed approach have great significance in the micromechanics of structural members made from materials reinforced by the unidirectional fibers and bounded with convex cylindrical surfaces in the estimation of their load carrying capacity under the compression and stretching of these structural members along the fibers.

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