 CMES, vol.43, no.1, pp.73-90, 2009

Vibration suppression of a moving beam subjected to an active-control electrostatic force

Shueei-Muh Lin[1]

Abstract: In this study, the mathematical model of a moving beam is established. This model is composed of a governing differential equation and three homogenous boundary conditions and one non-homogenous boundary condition including a time-dependent inertia force and a nonlinear active control force. Obviously, a moving mass problem with nonlinear and time dependent boundary condition is very complicated. One solution method is here developed to derive the exact solution for this system. By taking a change of dependent variable with a shifting function the original system is transformed to be a system composed of one non-homogeneous governing differential equation and four homogeneous boundary conditions. Further, based on an orthogonality condition of the eigenfunctions the mode superposition method is used to derive the exact solution for the transformed system. It should be noted that the transformed system is proved to be non-self-adjoint. Its orthogonality condition of eigenfunctions is different to the conventional one. Based on this orthogonality condition, the mode superposition method can be used to solve the transformed problem. The effects of different accelerations of a moving beam on the dynamic positioning and vibration of beam are significant. For suppressing vibration, two simple designs of active control of electrostatic force against the vibration are presented here. These are verified to be very effective.

Keywords: moving beam; analytical solution; vibration; control

1 Introduction

In general, four kinds of moving mass problems are studied by many researchers. The first is the dynamic behavior of beam structures, such as bridges on railways, subjected to moving loads or masses. Mostly, a uniform beam is simply supported and carried a moving load [Fryba (1996), Nikkhoo et al. (2007)]. The second is

[1] Mechanical Engineering Department, Kun Shan University, Tainan, Taiwan 710-03, Republic of China. E-Mail: smlin@mail.ksu.edu.tw

the vibration characteristics of a rotating shaft subjected to a moving load or mass [Gu and Cheng (2004)]. This model can simulate dynamic behavior of a ball screw and a nut moving along it, which are the key components of a feed drive system for a machine tool. The third is the axially moving beams problem. The belt drives, high-speed magnetic tapes and fiber winding are its typical examples [Lee and Jang (2007)]. The fourth is the transverse moving beam problem. It can be used to simulate a moving scanning probe or a transversely moving spindle. This mathematical model is different to the previous ones and investigated here.

The model of the transverse moving beam is composed of a governing differential equation and a time-dependent boundary condition due to the tip mass inertia force without any external control force. The literatures investigated the time-dependent boundary condition are listed as follows:

The vibrations of uniform Bernoulli-Euler beams with classical time dependent boundary conditions can be solved by using the method of Laplace transform [Nothmann (1948)] and the method of Mindlin-Goodman (1950). In the Mindlin-Goodman method, a change of dependent variable together with four shifting polynomial functions of the fifth order is introduced. In general, by properly selecting these shifting polynomial functions, the original system will be transformed to be a system composed of a nonhomogeneous governing differential equation with four homogeneous boundary conditions. Consequently, the method of separation of variables can be used to solve the problem. Lee and Lin (1996) gave the dynamic analysis of a nonuniform Bernoulli-Euler beam with general time dependent boundary conditions. They generalized the method of Mindlin-Goodman and introduced four shifting functions with the physical meaning instead of those functions with no physical meaning given by Mindlin and Goodman (1950). The vibrations of uniform Timoshenko beams with classical time dependent boundary conditions were studied by Herrmann (1955) and Berry and Nagdhi (1956) by using the method of Mindlin-Goodman. Lee and Lin (1998) extended the previous study made by Lee and Lin (1996) and further generalized the method of Mindlin-Goodman to develop a solution procedure for studying the vibrations of a nonuniform Timoshenko beams with general time dependent boundary conditions. Lin (1998) studied the force vibration of an elastically restrained nonuniform beam with time-dependent boundary conditions. Lin (2002) investigated the forced vibration and the boundary control of the pretwisted Timoshenko beam with time dependent elastic boundary conditions. Lee *et al.* (2008) studied the large static deflection of a beam with nonlinear boundary conditions. All above studies are not for a long-distance moving beam and the dynamic positioning. So far, little literatures investigated the transverse moving beam problem.

In addition, the vibration suppression is important for engineering applications and investigated by several literatures. Lin *et al.* (2008) studied the vibration of the blade of a Horizontal-Axis Wind Power Turbine. Vadiraja and Sahasrabudhe (2008) investigated Vibration suppression of Rotating Tapered Thin-Walled Composite Beam by using Macro Fiber Composite Actuator. Lin *et al.* (2007) and Lin (2008) studied the proportional and derivative controls of vibration of a rotating beam by using a pair of piezoelectric sensor and actuator layers.

In this study, the mathematical model of a transverse moving beam is established. The exact solution for this system is derived. The effects of several geometry and material parameters on the dynamic positioning and vibration of a moving beam are investigated. Especially, the comparison of different ways of acceleration to a specific position is made. For more effective suppressing vibration, a uniform control law and a proportional control law of electrostatic force are investigated..

2 Governing equations and associated conditions

It is well known that the vibration of a high-speed moving beam will occur, as shown in Figure 1. The vibration induces the error of dynamic positioning. In general, the designs for the suppressing vibration of a moving beam include (a) increasing passive structure damping, (b) choosing a smooth acceleration and deceleration way, and (c) applying the active control of structure. These methods of suppressing vibration are investigated here. Because this mathematical model includes one non-homogenous boundary condition composed of a time- dependent inertia force and a nonlinear active control force, this system is very complicate. Without the loss of generality, a uniform beam with a tip mass is considered here. This mathematical model is established as follows:

In terms of the following dimensionless quantities

$$M = \frac{M_{tip}}{\rho AL}, \quad s(\tau) = \frac{S(t)}{L}, \quad w(\xi,\tau) = \frac{W(x,t)}{L},$$

$$\xi = \frac{x}{L}, \quad \tau = \frac{t}{L^2}\sqrt{\frac{EI}{\rho A}}, \quad f_e = \frac{F_e L^2}{EI}, \tag{1}$$

the dimensionless governing differential equation of a moving beam with a tip mass and time dependent root position $s(t)$, is expressed as

$$-\frac{\partial}{\partial\xi}\left[n\frac{\partial w}{\partial\xi} + \frac{s(\xi)}{\mu}\left(\frac{\partial w}{\partial\xi} - \Psi\right)\right] + m(\xi)\left(\frac{\partial^2 w}{\partial\tau^2} - w\alpha^2\sin^2\theta\right) = p(\xi,\tau), \tag{2}$$

where c is the dimensionless damping coefficients. $p(\xi,\tau)$ is the dimensionless inertia force due to the movement of beam, $-d^2s/d\tau^2$. $W(x,t)$ is the flexural displacement, E is the Young's modulus. x is the coordinate along the beam, t is time

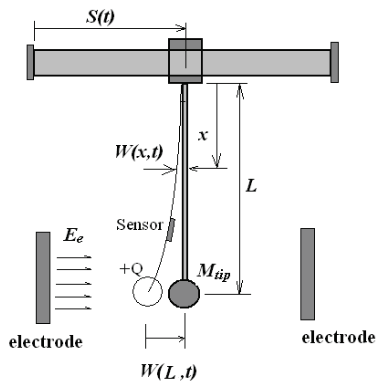


Figure 1: Geometry and coordinate system of a moving beam with a concentrate electric charge Q at the tip and a tip mass M in a electric intensity E_e.

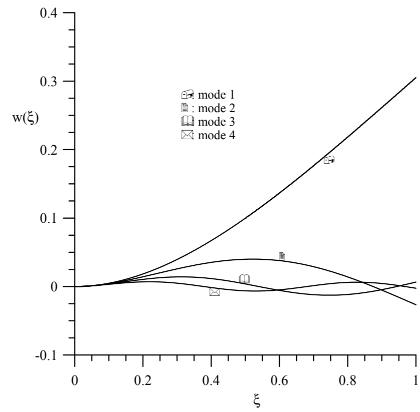


Figure 2: The first six mode shapes of a cantilever beam with a tip mass [c = 0.5, M = 0.2]

and L is the length of the beam. $I$and A denote the area moment of inertia and the cross sectional area, respectively. ρ is the mass density per unit volume and M_{tip} is the tip mass.

The associated boundary conditions are

At ξ= 0:

$$\gamma_{11}w-\gamma_{12}n\frac{\partial w}{\partial \xi}-\gamma_{12}\frac{1}{\mu}\left(\frac{\partial w}{\partial \xi}-\Psi\right)=0, \tag{3}$$

$$\gamma_{21}\Psi-\gamma_{22}\frac{\partial \Psi}{\partial \xi}=0. \tag{4}$$

At ξ= 1:

$$b\frac{\partial \Psi}{\partial \xi}+\delta_1\frac{\partial^2 \Psi}{\partial \tau^2}=f_1(\tau), \tag{5}$$

$$-\delta_2\left(\alpha^2\sin^2\theta w-\frac{\partial^2 w}{\partial \tau^2}\right)+n\frac{\partial w}{\partial \xi}+\frac{q}{\mu}\left(\frac{\partial w}{\partial \xi}-\Psi\right)=f_2(\tau). \tag{6}$$

where f_M is the inertia force due to the tip movement, $-Md^2s/d\tau^2$. f_e is the dimensionless electrostatic force, F_eL^2/EI where F_e is the adjustable electrostatic force, QE_e,in which Q is a concentrated electric charge at the tip and E_e is the electric intensity. The direction of the electric intensity is adjusted against the direction of beam displacement which can be measured by a piezoelectric sensor on the beam, as shown Figure 1. Therefore, this design can afford the restoring force to beam such that the vibration can be suppressed. In section 5, the effects of two control laws on the suppressing vibration are investigated.

The corresponding initial conditions are expressed as

$$w(\xi,0) = w_0(\xi), \tag{7}$$

$$\frac{\partial w(\xi,0)}{\partial \tau} = \dot{w}_0(\xi). \tag{8}$$

It should be noted that this system is nonlinear and non-conservative, because of the non-homogenous and nonlinear term, *f(τ)*, including the tip inertia force f_M and f_e in Eq. (6). The mode superposition method can not be used to directly solve this problem. However, after this original system is reasonably transformed, the transformed system can be solved by using this method. Therefore, the exact solution of this system can be obtained.

3 Solution method

3.1 Change of variable

By taking a change of dependent variable with a shifting function the original system can be transformed to be one system composed of one non-homogeneous governing differential equation and four homogeneous boundary conditions. The relation among variables is assumed to be

$$w(\xi,\tau) = \bar{w}(\xi,\tau) + g(\xi) f(\tau) \tag{9}$$

where $g(x)$ is the shifting function and chosen to satisfy the following conditions

$$g(0) = 0, \quad \frac{dg(0)}{dx} = 0, \quad g(1) = 0,$$

$$\frac{d^2g(1)}{d\xi^2} = 0, \quad \frac{d^3g(1)}{d\xi^3} = -1, \quad \frac{d^4g(1)}{d\xi^4} = 0. \tag{10}$$

If the shifting function is

$$g(\xi) = \alpha_0 + \alpha_1\xi + \alpha_2\xi^2 + \alpha_3\xi^3 + \alpha_4\xi^4 + \alpha_5\xi^5, \tag{11}$$

it can be found based on the conditions (10) as follows:

$$g(\xi) = -\frac{1}{3}\xi^2 + \frac{2}{3}\xi^3 - \frac{5}{12}\xi^4 + \frac{1}{12}\xi^5. \tag{12}$$

Substituting Eqs. (9) and (12) into Eqs. (2-8), the transformed differential equation and corresponding boundary conditions are

$$\frac{\partial^4 \bar{w}}{\partial \xi^4} + c\frac{\partial \bar{w}}{\partial \tau} + \frac{\partial^2 \bar{w}}{\partial \tau^2} = \bar{p}(\xi,\tau) \tag{13}$$

where $\bar{p}(\xi,\tau) = -d^2s/d\tau^2 - \frac{d^4g}{d\xi^4}f(\tau) - cg(\xi)\frac{df}{d\tau} - g(\xi)\frac{d^2f}{d\tau^2}$.
At $\xi = 0$:

$$\bar{w}(0,\tau) = 0, \tag{14}$$

$$\frac{\partial \bar{w}(0,\tau)}{\partial \xi} = 0, \tag{15}$$

At $\xi = 1$:

$$\frac{\partial^2 \bar{w}(1,\tau)}{\partial \xi^2} = 0, \tag{16}$$

$$-\frac{\partial^3 \bar{w}(1,\tau)}{\partial \xi^3} + M\frac{\partial^2 \bar{w}(1,\tau)}{\partial \xi^2} = 0. \tag{17}$$

The transformed initial conditions (7) – (8), become

$$\bar{w}(\xi,0) = w_0(\xi) - g(\xi)f(0), \tag{18}$$

$$\frac{\partial \bar{w}(\xi,0)}{\partial \tau} = \dot{w}_0(\xi) - g(\xi)\frac{df(0)}{d\tau}. \tag{19}$$

So far, all the transformed boundary conditions are homogenous. In order to solve the transformed system by using the mode superposition method, the orthogonal condition of eigenfunctions must be found.

3.2 Orthogonal condition of eigenfunctions

Consider the free vibration of an undamped beam. Its governing equation is

$$\frac{\partial^4 w}{\partial \xi^4} + \frac{\partial^2 w}{\partial \xi^2} = 0. \tag{20}$$

The associated boundary conditions are
At $\xi = 0$:

$$w = 0, \tag{21}$$

$$\frac{\partial w}{\partial \xi} = 0, \tag{22}$$

At $\xi = 1$:

$$\frac{\partial^2 w}{\partial \xi^2} = 0, \tag{23}$$

$$-\frac{\partial^3 w}{\partial \xi^3} + M\frac{\partial^2 w}{\partial \tau^2} = 0. \tag{24}$$

Assume $w(\xi,\tau) = \bar{w}(\xi)\cos\omega\tau$. Substituting it into Eqs. (20-24), Eq. (20) becomes

$$\frac{d^4\bar{w}}{d\xi^4} - \omega^2\bar{w} = 0. \tag{25}$$

The boundary conditions (21-24) become
At $\xi = 0$:

$$\bar{w} = 0, \tag{26}$$

$$\frac{d\bar{w}}{d\xi} = 0. \tag{27}$$

At $\xi = 1$:

$$\frac{d^2\bar{w}}{d\xi^2} = 0, \tag{28}$$

$$\frac{d^3\bar{w}}{d\xi^3} + \omega^2 M\bar{w} = 0. \tag{29}$$

Multiplying $d^4\bar{w}_i/d\xi^4$ by $\bar{w}_j$ and integrating it from 0 to 1, and based on the boundary conditions (26-29), the following relation is obtained

$$\int_0^1 \bar{w}_j \frac{d^4\bar{w}_i}{d\xi^4} d\xi = (\omega_j^2 - \omega_i^2)M\bar{w}_i(1)\bar{w}_j(1) \int_0^1 \bar{w}_i \frac{d^4\bar{w}_j}{d\xi^4} d\xi. \tag{30}$$

Due to Eq. (25), the equation (30) becomes

$$(\omega_j^2 - \omega_i^2)\left[\int_0^1 \bar{w}_i\bar{w}_j d\xi + M\bar{w}_i(1)\bar{w}_j(1)\right] = 0. \tag{31}$$

Because $\omega_j^2 \neq \omega_i^2$, $i \neq j$, the following orthogonal condition is obtained

$$\int_0^1 \bar{w}_i\bar{w}_j d\xi + M\bar{w}_i(1)\bar{w}_j(1) = 0. \tag{32}$$

Obviously, because there exists '$M\bar{w}_i(1)\bar{w}_j(1)$' in Eq. (32), the system is not self-adjointness. Further, the orthogonal condition can be generalized as

$$\int_0^1 \bar{w}_i\bar{w}_j d\xi + M\bar{w}_i(1)\bar{w}_j(1) = \begin{cases} \varepsilon_{ij} = 0, & i \neq j \\ \varepsilon_{ii} \neq 0 & i = j \end{cases}. \tag{33}$$

Based on this condition and using the mode superposition method, the transformed system can be decoupled into several independent subsystems which are easily solved.

3.3 Mode superposition

The solution of the transformed system composed of Eqs. (13-19) can be expressed in the following eigenfunction expansion form

$$\bar{w}(\xi,\tau) = \sum_{i=0}^{\infty} \bar{w}_i(\xi) T_i(\tau) \tag{34}$$

where $\bar{w}_i(\xi)$ is the ith eigenfunctions of the undamped beam derived in section 3.2. Substituting Eq. (34) back to the transformed governing equation (13) and the conditions (14-19), multiplying by '$\bar{w}_k(\xi)[1+M\delta(\xi-1)]$' and integrating in accordance with the orthogonality condition (33), one obtains

$$\frac{d^2T_k(\tau)}{d\tau^2} + c\frac{dT_k(\tau)}{d\tau} + \omega_k^2 T_k(\tau) = F_k(\tau), \tag{35a}$$

where

$$F_k(\tau) = \frac{1}{\varepsilon_{kk}} \int_0^1 \bar{w}_k(\xi)[1+M\delta(\xi-1)]\bar{p}(\xi,\tau)d\xi, \tag{35b}$$

Moreover, the corresponding initial conditions are

$$T_k(0) = \frac{1}{\varepsilon_k} \int_0^1 \bar{w}_k(\xi)[1+M\delta(\xi-1)][w_0(\xi) - g(\xi)f(0)]d\xi, \tag{36}$$

$$\frac{dT_k(0)}{d\tau} = \frac{1}{\varepsilon_k} \int_0^1 \bar{w}_k(\xi)[1+M\delta(\xi-1)]\left[\dot{w}_0(\xi) - g(\xi)\frac{df(0)}{d\tau}\right]d\xi. \tag{37}$$

Further, the solution of Eq. (35) is derived as follows:
Letting $x_1 = T_k$ and $x_2 = dx_1/d\tau$, Eq. (35a) can be written as

$$\frac{dX}{d\tau} = AX + Bu \tag{38a}$$

where

$$X=\begin{bmatrix}x_1\\x_2\end{bmatrix},\quad A=\begin{bmatrix}0 & 1\\-\omega_k^2 & -2\zeta_k\omega_k\end{bmatrix},\quad B=\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix},\quad u=\begin{bmatrix}0\\F_k(\tau)\end{bmatrix}. \tag{38b}$$

The solution of Eq. (38) is easily derived [Kailath (1980)]

$$X(\tau)=e^{A(\tau-\tau_0)}X(\tau_0)+\int_{\tau_0}^{\tau}e^{A(\chi-\tau_0)}Bu(\chi)\,d\chi \tag{39}$$

where the transfer function can be expressed in the polynomial form

$$e^{A(\tau-\tau_0)}=\sum_{k=0}^{\infty}\frac{1}{k!}A^k(\tau-\tau_0)^k. \tag{40}$$

But the calculation of the transfer function is complicate. For simplicity, the following relation is applied and proved in Appendix

$$e^{A(\tau-\tau_0)}=\begin{bmatrix}V_1(\tau-\tau_0) & V_2(\tau-\tau_0)\\\dot{V}_1(\tau-\tau_0) & \dot{V}_2(\tau-\tau_0)\end{bmatrix}, \tag{41}$$

where V_i are the two fundamental solutions of Eq. (35a)

$$V_1(\tau)=e^{-\zeta_k\omega_k\tau}\left[\cos\omega_{dk}\tau+\frac{\zeta_k\omega_k}{\omega_{dk}}\sin\omega_{dk}\tau\right],$$

$$V_2(\tau)=\frac{1}{\omega_{dk}}e^{-\zeta_k\omega_k\tau}\sin\omega_{dk}\tau,\quad \omega_{dk}=\omega_k\sqrt{1-\zeta_k^2}. \tag{42}$$

Substituting Eq. (41) into Eq. (39), Eq. (39) becomes

$$T_k(\tau)=V_1(\tau-\tau_0)T_k(\tau_0)+V_2(\tau-\tau_0)\frac{dT_k(\tau_0)}{d\tau}+\int_{\tau_0}^{\tau}V_2(\tau-\chi)F_k(\chi)\,d\chi, \tag{43}$$

$$\frac{dT_k(\tau)}{d\tau}=\frac{dV_1(\tau-\tau_0)}{d\tau}T_k(\tau_0)+\frac{dV_2(\tau-\tau_0)}{d\tau}\frac{dT_k(\tau_0)}{d\tau}+\int_{\tau_0}^{\tau}\frac{dV_2(\tau-\chi)}{d\tau}F_k(\chi)\,d\chi.$$

Conclusively, substituting the solutions $\{T_k(\tau),\bar{w}_k(\xi),g(\xi)\}$ back into Eqs. (34) and (9) sequentially, the exact general solution $w(\xi s\tau)$ is obtained.

4 Numerical results and discussion

At first, the orthogonal condition (33) is numerically verified here. Consider the dimensionless tip mass constant M= 0.2. The mode shapes are shown in Figure 2. The dimensionless natural frequencies and the parameter ε_{ij} are calculated and listed below:

$$[\omega_i]_{1\times 6} = \begin{bmatrix} 2.6127 & 18.208 & 53.559 & 108.193 & 182.431 & 273.336 \end{bmatrix}, \tag{44}$$

$$[\varepsilon_{ij}]_{6\times 6} = \begin{bmatrix} 0.04128 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.0007895 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.0000893 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.0000217 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.0000076 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.0000033 \end{bmatrix},$$

which satisfies the orthogonality condition (33).

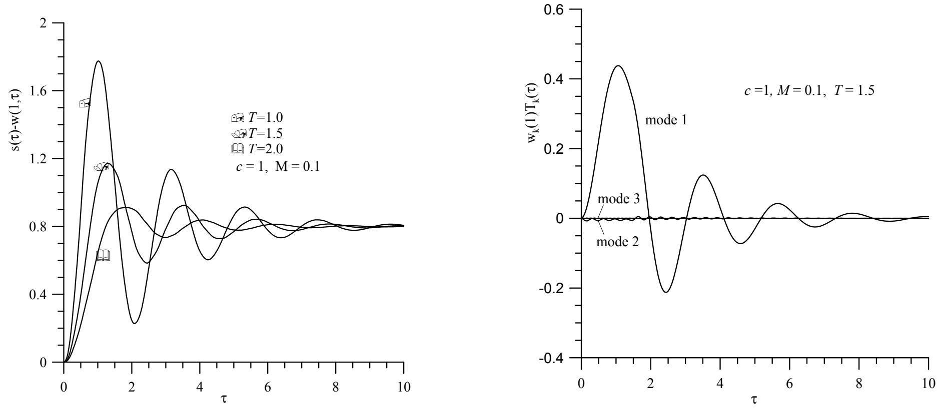
Secondly, the influences of the acceleration time T in that the root of beam is moved to a specified position, the tip mass M, the damping constant c and the way of acceleration on the vibration of a moving beam are investigated here. Assume the initial displacement and velocity to be zero, $w_0(\xi) = \dot{w}_0(\xi)$= 0. The beam is moved from the origin to some specified position in the acceleration of unit function which is expressed as

$$\frac{d^2 s}{d\tau^2}(\tau) = \begin{cases} 0, & \tau < 0 \\ a, & 0 < \tau < T/2 \\ -a, & T/2 < \tau < T \\ 0, & \tau > T \end{cases} \tag{45}$$

where a is the acceleration. The effect of the control electrostatic force is neglected in Figures 3-7.

Figure 3a shows the influence of the acceleration time T on the vibration of beam. In general, the tip of beam will overshoot when the root of beam moves to the specific position, s = 0.8 and is fixed. Further, the amplitude of vibration decays due to the effect of structure damping. Moreover, it is obvious that the shorter the acceleration time T is, the larger the tip overshooting. Meanwhile, Figure 3b shows that the first mode dominates the vibration response.

Figure 4 shows the influence of the damping coefficient c on the vibration of beam. The decaying of vibration after the acceleration time T depends greatly



(a) (b)

Figure 3: (a) Influence of the time of the acceleration of unit function T on the vibration of a moving beam. (b) Effects of each mode on the vibration of a moving beam with the acceleration of unit function.

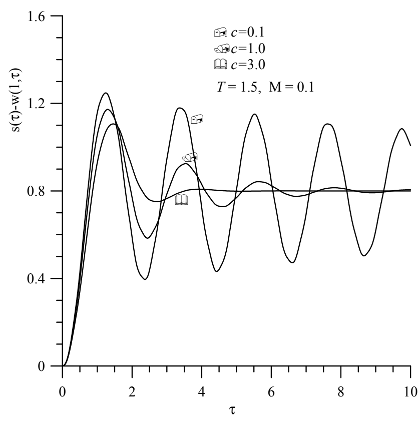


Figure 4: Influence of the damping coefficient c on the vibration of a moving beam with the acceleration of unit function.

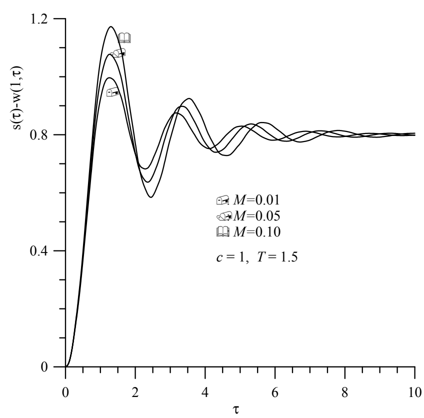


Figure 5: Influence of the tip mass M on the vibration of a moving beam with the acceleration of unit function.

on the damping coefficient. However, increasing the damping coefficient decreases slightly the tip overshooting. It reveals that the influence of the damping coefficient c on the accuracy of dynamic positioning is small.

Figure 5 shows the influence of the tip mass M on the vibration of beam. It is

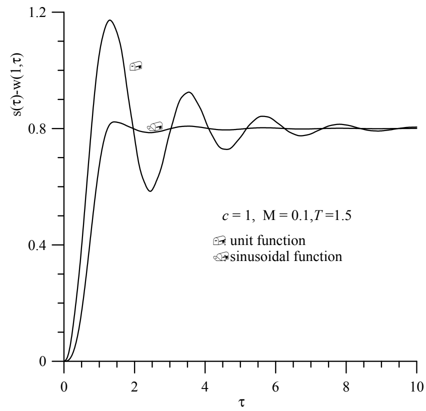


Figure 6: Two kinds of accelerations on the vibration of a moving beam.

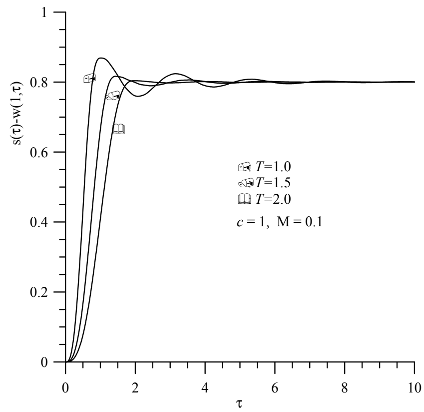


Figure 7: Influence of the time of the acceleration of sinusoidal function T on the vibration of a moving beam.

found that increasing the tip mass greatly increases the overshooting of the tip. It reveals that the influence of the tip mass M on the accuracy of dynamic positioning is significant.

It is concluded from Figures 3-5 that the acceleration of unit function results in a large overshooting. In other words, this acceleration causes a large error of dynamic positioning. Based on this fact, a sinusoidal acceleration is considered and listed as follows:

$$\frac{d^2 s}{d\tau^2}(\tau) = \begin{cases} 0, & \tau < 0 \\ \beta \sin(2\pi\tau/T), & 0 < \tau < T \\ 0, & \tau > T \end{cases} \tag{46}$$

Figure 6 shows the effects of different accelerations on the vibration of a moving beam. Obviously, the overshooting in the sinusoidal acceleration is much smaller than that in the unit-function acceleration. Obviously, the sinusoidal acceleration is more suitable for the dynamic positioning.

Further, Figure 7 shows the influence of the sinusoidal acceleration time T on the vibration of beam. If the acceleration time T = 1, the overshooting is significant. In other words, although the sinusoidal acceleration is effective for suppressing vibration, the overshooting will occurs due to too short acceleration time. For overcoming this fault an active control for suppressing vibration is studied next.

Thirdly, the influence of active control law on the vibration is investigated here. In closed-loop control for suppressing vibration, the control electrostatic force is designed by the uniform control law and expressed as '$F_e = QE_e$' where the concentrate electric charge Q is constant and the electric intensity is

$$E_e(\tau) = -sign(w(1,\tau_0))E_0, \quad \tau_0 < \tau < \tau_0 + \Delta\tau, \tag{47}$$

in which the E_0 is constant and the direction of the electric field is adjusted against the direction of beam displacement. $\Delta\tau$ is the sampling time of the piezoelectric sensor measuring the tip displacement. Therefore, the dimensionless electric force can be expressed as

$$f_e(\tau) = -k_e sign(w(1,\tau_0)), \quad \tau_0 < \tau < \tau_0 + \Delta\tau. \tag{48}$$

Because the dimensionless electric force $f_e$depends on the displacement $w(1,\tau_0)$, the system is nonlinear. In general, it is hard to solve Eq. (35). However, when $\tau_0 < \tau < \tau_0 + \Delta\tau$, f_e is constant. Eq. (35) can be solved step by step as follows:

Substituting Eq. (48) into Eq. (35), one obtains

$$\begin{aligned}\frac{d^2T_k(\tau)}{d\tau^2} + c\frac{dT_k(\tau)}{d\tau} + \omega_k^2 T_k(\tau) &= F_k(\tau)\\ &= F_{ele} + F_c\cos(2\pi\tau/T) + F_s\sin(2\pi\tau/T),\ \tau_0 < \tau < \tau_0 + \Delta\tau\end{aligned} \tag{49}$$

where

$$\begin{aligned}F_{ele}(\tau) &= \frac{-k_e sign(w(1,\tau_0))}{\varepsilon_{kk}}\int_0^1 \bar{w}_k(\xi)(-10+10\xi)\,d\xi,\\ F_c &= \frac{2\pi Mc\beta}{T\varepsilon_{kk}}\int_0^1 \bar{w}_k(\xi)g(\xi)\,d\xi\\ F_s &= \frac{\beta}{\varepsilon_{kk}}\int_0^1 \bar{w}_k(\xi)\left[[(-10+10\xi)M-1] - Mg(\xi)\left(\frac{2\pi}{T}\right)^2\right]d\xi + \frac{\beta M}{\varepsilon_{kk}}\bar{w}_k(1).\end{aligned} \tag{50}$$

Further, substituting Eq. (50) into Eq. (43), the solutions $T_k(\tau)$ and $dT_k(\tau)/d\tau$, $\tau_0 < \tau < \tau_0 + \Delta\tau$, are obtained. Substituting these solutions back into Eqs. (34) and (9), the next initial conditions $\{w(1,\tau_0+\Delta\tau), \partial w(1,\tau_0+\Delta\tau)/\partial\tau\}$ are derived. In the similar way, the overall solution of this system is obtained.

Figure 8 shows that the overshooting is greatly decreased by using this control law, especially for the case with the gain factor $k_e = 0.3$. However, there exists small oscillation. The first reason is that when the beam is restoring, there is a constant force applied to push the beam back in spite of small or large displacement. The second reason is that the sampling time is too large. The second will be discussed in Figure 10.

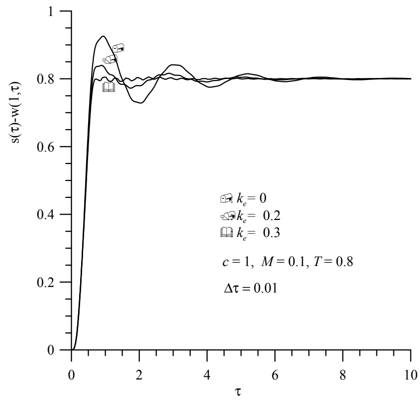


Figure 8: Influence of the uniform electric control law on the vibration suppression of a moving beam.

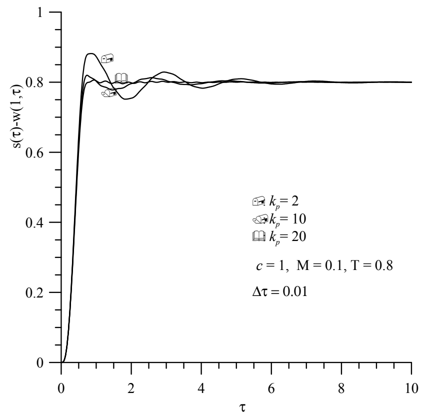


Figure 9: Influence of the proportional electric control law on the vibration suppression of a moving beam.

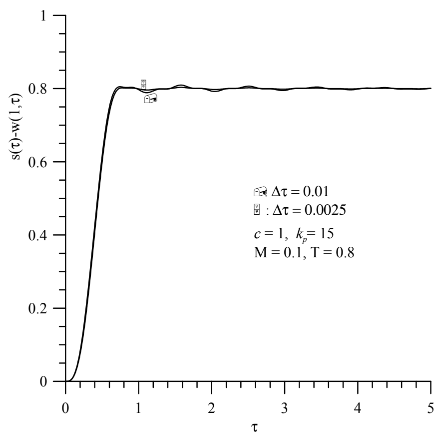


Figure 10: Influence of the sampling time $\Delta\tau$ on the vibration suppression of a moving beam with the proportional electric control law.

For overcoming the first default of the uniform control law, the proportional control law [Lin *et al.*, 2007] is designed

$$f_e(\tau) = -k_p w(1, \tau_0), \; \tau_0 < \tau < \tau_0 + \Delta\tau. \tag{51}$$

Figure 9 illustrates the influence of the gain factor k_p on the vibration. It is found that the larger the gain factor k_p is, the more effective the suppressing vibration. It is also observed from Figures 8 and 9 that at the same sampling time $\Delta\tau$ the oscillation in the proportional control law after the acceleration time is much smaller than that in the uniform control law. Moreover, the influence of the sampling time $\Delta\tau$gn the accuracy of dynamic positioning is investigated in Figure 10. It is observed that the larger the sampling time $\Delta\tau$ is, the larger the oscillation. In other words, decreasing the sampling time $\Delta\tau$ will greatly decreases the oscillation and increases the accuracy of dynamic positioning.

5 Conclusion

In this study, the mathematical model of a moving beam is established. The exact solution for this system is derived. This methodology can be applied to other moving problems, for example, moving frame and robotic arm. The active control of electric field for suppressing vibration of the moving beam is verified to be very effective. Beside, the effects of several important parameters on the vibration of a moving beam are concluded as follows:

1. The moving way of the sinusoidal acceleration induces much smaller vibration and overshooting than that of the unit-function acceleration.

2. The first mode dominates the vibration response of a moving beam.

3. Increasing the tip mass greatly increases the tip overshooting and the vibration.

4. Decreasing the acceleration time to a specified position greatly increases the tip overshooting.

5. Although the vibration decays greatly due to a large damping coefficient after the acceleration time, the influence of the damping coefficient on the tip overshooting is slight.

6. The proportional control law for suppressing vibration is better than the constant control law.

7. Decreasing the sampling time $\Delta\tau$ will greatly increases the accuracy of dynamic positioning.

Acknowledgement: The support of the National Science Council of Taiwan, R. O. C., is gratefully acknowledged (Grant number: Nsc96-2212-E168-003).

References

Berry, J. G.; Nagdhi, P. M. (1956): On the vibration of elastic bodies having time-dependent elastic boundary conditions. *Quarterly of Applied Mathemetics*, vol. 14, no. 1, pp. 43-50.

Fryba, L. (1996): *Dynamics of Railway Bridges*. Prague: Thomas Telford Services Ltd.

Gu, U.C.; Cheng, C.C. (2004): Vibration analysis of a high-speed spindle under the action of a moving mass. *Journal of Sound and Vibration*, vol. 278, pp. 1131–1146

Herrmann, G. (1955): Forced motions of Timoshenko beams. *ASME Journal of Applied Mechanics*, vol. 22, no.1, pp. 53-56.

Kailath, T. (1980): *Linear system*. Prentice-Hall, Inc.

Lee, U.; Jang, I. (2007): On the boundary conditions for axially moving beams. *Journal of Sound and Vibration*, vol. 306, pp. 675–690.

Lee, S.Y.; Lin, S.M. (1996): Dynamic analysis of non-uniform beam with time dependent elastic boundary conditions. *ASME Journal of Applied Mechanics*, vol. 63, no.2, pp. 474-478.

Lee, S.Y.; Lin, S.M. (1998): Nonuniform Timoshenko beams with time dependent elastic boundary conditions. *Journal of Sound and Vibration*, vol. 217, no. 2, pp. 223-238.

Lee, S.Y.; Lin, S.M.; Lee, C.S.; Lu, S.Y.; Liu, Y.T. (2008): Exact large deflection of beams with nonlinear boundary condition. *CMES: Computer Modeling in Engineering & Sciences*, vol. 30, no. 1, pp. 27-36.

Lin, S.M. (1998): Pretwisted nonuniform beams with time dependent elastic boundary conditions. *AIAA Journal*, vol.36, no.8, pp.1516-1523.

Lin, S. M. (2008): PD control of a rotating smart beam with an elastic root. *Journal of Sound and Vibration, Vol.* 312, pp. 109–124.

Lin, S.M.; Lee, S.Y. (2002): The Forced Vibration and Boundary Control of Pretwisted Timoshenko Beams with General Time Dependent Boundary Conditions. *Journal of Sound and Vibration*, vol. 254, no.27, pp. 69-90.

Lin, S.M.; Lee, S.Y.; Lin, Y.S. (2008): Modeling and Bending Vibration of the Blade of a Horizontal-Axis Wind Power Turbine. *CMES: Computer Modeling in Engineering & Sciences*, vol.23, no.2, pp.175-186.

Lin, S. M.; Mao, I.C.; Lin, J.H. (2007): Vibration of a rotating smart beam. *AIAA Journal*, Vol. 45, No. 2, pp.382-389.

Mindlin, R.D.; Goodman, L.E. (1950): Beam vibrations with time-dependent boundary conditions. *ASME Journal of Applied Mechanics*, vol. 17, no. 4, pp. 377-380.

Nikkhoo, A.; Rofooei, F.R.; Shadnam, M.R. (2007): Dynamic behavior and modal control of beams under moving mass. *Journal of Sound and Vibration,* vol. 306, pp. 712–724

Nothmann, G. A. (1948): Vibration of a cantilever beam with prescribed end motion. *ASME Journal of Applied Mechanics*, vol.15, no.2, pp. 327-334.

Vadiraja, D. N.; Sahasrabudhe, A.D. (2008): Vibration and Control of Rotating Tapered Thin-Walled Composite Beam Using Macro Fiber Composite Actuator. *CMES: Computer Modeling in Engineering & Sciences*, vol.27, no.1, pp.49-62.

Appendix

Prove the relation between the transfer function and the fundamental functions.

$$e^{A(t-t_0)} = \begin{bmatrix} V_1(t-t_0) & V_2(t-t_0) \\ \dot{V}_1(t-t_0) & \dot{V}_2(t-t_0) \end{bmatrix} \tag{A1}$$

Proof:
Consider a second-order ordinary differential equation

$$\frac{d^2T}{dt^2} + a\frac{dT}{dt} + bT = 0, \tag{A2}$$

where a and b are constants. Letting $y_1 = T$ and $y_2 = dy_1/dt$, Eq. (A2) can be expressed as

$$\frac{dY}{dt} = AY, \tag{A3}$$

where

$$Y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}, \quad A = \begin{bmatrix} 0 & 1 \\ -b & -a \end{bmatrix}.$$

If V_1 and V_2 are the fundamental solutions of Eq. (A2) and satisfy the following normalized condition

$$\begin{bmatrix} V_1(0) & V_2(0) \\ \dot{V}_1(0) & \dot{V}_2(0) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \tag{A4}$$

these solutions must satisfy Eq. (A3) and two relations can be expressed as

$$\frac{dY_1}{dt} = AY_1 \quad \frac{dY_2}{dt} = AY_2, \tag{A5}$$

where

$$Y_1 = \begin{bmatrix} V_1 \\ \frac{dV_1}{dt} \end{bmatrix}, \quad Y_2 = \begin{bmatrix} V_2 \\ \frac{dV_2}{dt} \end{bmatrix}.$$

It is well known that the solution of Eq. (A3) is

$$Y(t) = e^{A(t-t_0)} Y(t_0). \tag{A6}$$

Therefore,

$$\begin{bmatrix} V_1(t) \\ \dot{V}_1(t) \end{bmatrix} = e^{A(t-t_0)} \begin{bmatrix} V_1(t_0) \\ \dot{V}_1(t_0) \end{bmatrix} \quad \begin{bmatrix} V_2(t) \\ \dot{V}_2(t) \end{bmatrix} = e^{A(t-t_0)} \begin{bmatrix} V_2(t_0) \\ \dot{V}_2(t_0) \end{bmatrix} \tag{A7}$$

These relations can be combined into one as follows:

$$\begin{bmatrix} V_1(t) & V_2(t) \\ \dot{V}_1(t) & \dot{V}_2(t) \end{bmatrix} = e^{A(t-t_0)} \begin{bmatrix} V_1(t_0) & V_2(t_0) \\ \dot{V}_1(t_0) & \dot{V}_2(t_0) \end{bmatrix}, \tag{A8}$$

or

$$e^{A(t-t_0)} = \begin{bmatrix} V_1(t) & V_2(t) \\ \dot{V}_1(t) & \dot{V}_2(t) \end{bmatrix} \begin{bmatrix} V_1(t_0) & V_2(t_0) \\ \dot{V}_1(t_0) & \dot{V}_2(t_0) \end{bmatrix}^{-1}. \tag{A9}$$

Because these two fundamental solutions satisfy the following normalized condition (A4), one can derive the following relations via Eq. (A8)

$$e^{At} = \begin{bmatrix} V_1(t) & V_2(t) \\ \dot{V}_1(t) & \dot{V}_2(t) \end{bmatrix} \quad e^{At_0} = \begin{bmatrix} V_1(t_0) & V_2(t_0) \\ \dot{V}_1(t_0) & \dot{V}_2(t_0) \end{bmatrix}. \tag{A10}$$

Substituting Eq. (A10) back into Eq. (A8), the relation (A1) is obtained.