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A Numerical Method for Estimating the Maximal Temperature Gradients Reached in Fire-Damaged Concrete Structures Based on the Parameter Identification

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Abstract: Taking advantage of the parameter identification, a new numerical method is developed in this paper to estimate the maximal temperature gradients reached in fire-damaged concrete structures. This method can avoid the hypotheses of temperature-time curve and fire duration usually made in conventional numerical methods, availably evaluate the depth and degree of fire damage of concrete structures and consider the effects of localized fire. A material model taking into account the properties of fire-damaged concrete is firstly proposed in the present research. The least-squares estimation and the Gauss-Newton method are used to identify the material parameters of fire-damaged concrete by means of the graded finite element. Then the maximal temperature gradients reached in concrete members are estimated according to the relationship between mechanical properties and temperatures of fire-damaged concrete. Furthermore, the error tolerance and error transfer in the parameter identification are presented in detail. The effects of different polynomial expressions of the material model, the number and different thicknesses of concrete layers, element types and Young's modulus-temperature (E-T) models on the estimations are thoroughly investigated. The results of numerical examples are in good agreement with those of the experiment, which validates the feasibility and effectiveness of the developed method.

Keywords: fire; concrete structures; parameter identification; graded finite element; maximal temperature gradient

1 Introduction

Concrete structures usually suffer from serious fire disasters due to the natural or man-made factors. As a result of the incombustible nature and low thermal diffu-

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sivity of concrete, very strong temperature gradients would take place in concrete structures during the fire duration. The concrete would deteriorate gradually along the direction of heating because of the thermal effects. Structural behaviors are dominated by the thermal regime due to the close relationship between temperatures and mechanical properties of fire-damaged concrete. In order to provide a credible decision to remove or reinforce the fire-damaged concrete structures, it is very important and indispensable to investigate the material properties of fire-damaged concrete and the residual load bearing capacities of concrete structures exposed to fire. The maximal temperature gradient is a distribution of the maximal temperature in a concrete structure along the direction of heating during the fire duration. As a consequence, the estimation of the maximal temperature gradients reached in fire-damaged concrete structures is the kernel of determining the concrete degradation and evaluating the damage of concrete structures after fire [Georgali and Tsakiridis (2005), Lamont, Usmani and Gillie (2004)].

A large number of investigations in estimating the maximal temperature gradients reached in fire-damaged concrete structures are available in open literature. Based on the assumptions of the temperature-time curve and fire duration, the temperature fields in concrete structures were simulated and analyzed by means of numerical methods such as the Green's Function Method [Wang and Tan (2007)], the finite element method (FEM) [Ding and Wang (2009)] and the difference method [Hsu and Lin (2008)], and so on. Many methods of in-situ investigations and laboratory tests were also widely used to estimate the maximal temperature gradients reached in fire-damaged concrete structures. For example, the observation of surface characteristics [Guise (1997), Shortu, Purkiss and Guise (2001), Lin, Wang and Luo (2004), Arioz (2007)], the determination of carbonation [Yan, Li and Wong (2007)], the ultrasonic and rebound method [Colombo and Felicetti (2007), Dilek (2005)], the electron microscopy method [Georgali and Tsakiridis (2005), Yan, Li and Wong (2007), Jeremy (2008)], and the thermal imaging technique [Du, Zhang and Han (2001)]. However, the assumptions of the temperature-time curve and fire duration in conventional numerical methods need to be made, which generally are not in accordance with the actual situation of fire. Some methods of in-situ investigations and laboratory tests can only predict the surface temperatures of concrete structures, or can get good estimations only under the exact observation with practical experience, or can only detect the local zones of structures with destructive damage detection techniques. From the available literature, there are few methods that can be used to estimate the maximal temperature distributions from the surface to the inner of concrete members, and simultaneously can effectively consider the influences of different heating modes, the complexity of concrete and the delay of heat conduction on the estimations [Franssen and Kodur (2001)].

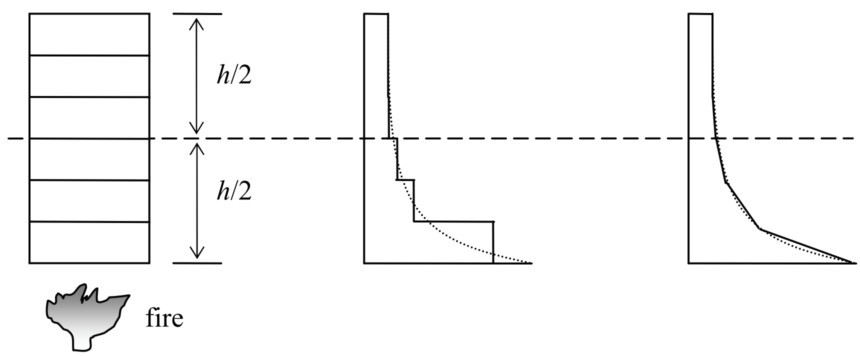
The estimation of the maximal temperature gradients reached in fire-damaged concrete structures is an inverse problem in nature. As one of nondestructive detection methods, the parameter identification (inverse method) has been widely applied in evaluating damage of structures [Gioda and Maier (1980), Resende and Martin (1988), Xiang, Snanyei et al. (1997), Swoboda and Cen (2002)] and determining model parameters of materials or structures [Skovoroda and Goldstein (2003), Liu(2006), Liu, Liu and Hong (2007), Liu(2008), Benallal et al. (2008), Daghia et al. (2009), Huang and Chung (2009), Liu and Tsai (2009)]. In the present paper, a new numerical method based on the parameter identification using the measurement displacement is developed and presented to estimate the maximal temperature gradients reached in fire-damaged concrete structures. Since the mechanical properties of fire-damaged concrete show the similarity to those of the functionally graded materials, a material model is proposed by means of the piecewise linear approximation of mechanical properties of fire-damaged concrete and the graded finite element is used to estimate the material parameters of fire-damaged concrete based on the parameter identification. The maximal temperature gradients reached in fire-damaged concrete structures would be achieved according to the relationship between mechanical properties and temperatures of fire-damaged concrete. An experimental study and comprehensive parameter analysis are done to validate the developed numerical method.

2 Numerical model of estimation

2.1 Characteristics of fire-damaged concrete

The temperature distribution in the flat concrete members such as walls, floors and ceilings is essentially one-dimensional (1D), with the maximal temperature gradient only across the thickness of the slab during a fire. In linear members, such as beams or columns, the temperature distribution is essentially two-dimensional (2D) with no variation along the length of the elements, except for the linear members subjected to 1-face heating whose temperature distributions are also one-dimensional (1D). This particular temperature distribution is taken into account in the analyses and the non-uniform temperature distribution of local zone would be neglected [Franssen, Pintea and Dotreppe (2007)].

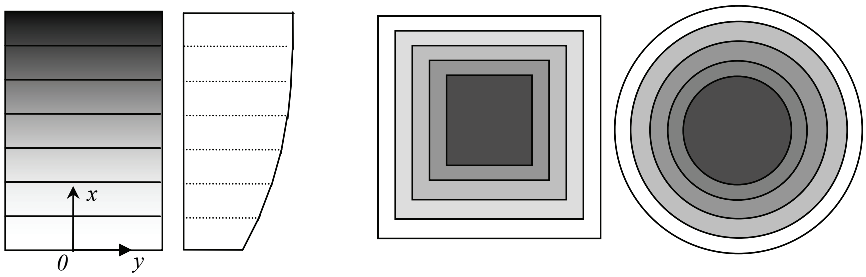
Usually, the maximal temperature gradients reached in fire-damaged concrete structures are non-linear along the direction of heating [Dwaikat and Kodur (2008), Sanad et al. (2000)]. For example, the curve of maximal temperature gradient reached in a concrete beam subjected to 1-face bottom heating can be shown as the dotted line in Fig. 1b or Fig. 1c. When the cross-section of concrete beam is divided into some layers with the same or different thicknesses (see Fig. 1a), the situation of each concrete layer can be uniformly or linearly approximated. Therefore, the curve of maximal temperature gradient reached in concrete beam can be modeled by means of piecewise constant approximation or piecewise linear approximation, as the solid line shown in Fig. 1b or Fig. 1c. Compared with the piecewise constant approximation, obviously, the piecewise linear approximation is more accurate to describe the real curve of temperature, especially when the temperature gradient changes rapidly.



(a) The cross-section (b)Piecewise constant approximation (c) Piecewise linear approximation

Figure 1: Different approximations of the maximal temperature gradient reached in the cross-section of concrete beam subjected to 1-face bottom heating

Suppose that the concrete under ambient temperature is initially isotropic. The mechanical properties of fire-damaged concrete very complicated due to different heating modes and cross-section types of concrete members. The mechanical properties of concrete members with typical rectangular and circular cross-sections under different heating modes are respectively shown in Fig. 2a and Fig. 2b, where the fuscous zone denotes the little damage and the tinged zone shows the serious damage caused by a fire. The degradation tendency of concrete properties after fire is related to the distribution of maximal temperature gradient reached in concrete member because of the thermal effects. Therefore, the properties of fire-damaged concrete are similar to those of functionally graded materials, and then the graded finite element [Santare and Lambros (2000), Dolbow and Gosz (2002)] can be used to simulate the mechanical properties of fire-damaged concrete according to the piecewise linear approximation of mechanical properties (see Fig. 2a).



(a) Diagram of mechanical properties and piecewise linear approximation of fire-damaged concrete under 1-face heating

(b) Diagram of mechanical properties of fire-damaged concrete under 4-face heating

Figure 2: Diagrams of mechanical properties of fire-damaged concrete under different heating modes

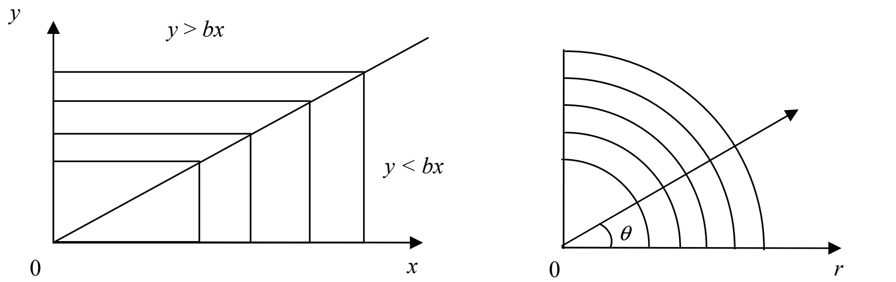
2.2 Material model of fire-damaged concrete and graded finite element

From the above analysis of mechanical properties of the concrete after fire, it is assumed that material parameters of fire-damaged concrete vary as a function of local coordinate such as the polynomial or power function. There are many methods to describe this material model of fire-damaged concrete members with different cross-section types and heating modes. For example, since the concrete is initially isotropic and local coordinate x is the direction of heating, the Young's modulus at node i of a concrete member subjected to 1-face heating (see Fig.2a) can be written as the following polynomial

$$E_i = E_0 \sum_{j=0}^{n} a_j x^j \tag{1}$$

where E_0 is the Young's modulus at the origin of local coordinate x, a_j is the coefficient of polynomial and n is the highest degree of polynomial. Obviously, E_i can be also expressed as other expressions of function according to the precision requirement of the estimation.

Thanks to the symmetrical characteristic of concrete members with rectangular cross-section and circular cross-section subjected to 4-face heating, the Young's modulus at node i of 1/4 cross-section of concrete members (see Fig.3) can be



(a) Model of 1/4 rectangular cross-section (b) Model of 1/4 circular cross-section

Figure 3: Simulations of concrete members with different cross-sections subjected to 4-face heating

shown as

$$E_i = E_0 \sum_{j=0}^{n} a_j x^j \qquad y < bx \tag{2a}$$

$$E_i = E_0 \sum_{j=0}^{n} a_j x^j \quad \text{or} \quad E_i = E_0 \sum_{j=0}^{n} c_j y^j \qquad y = bx \tag{2b}$$

$$E_i = E_0 \sum_{j=0}^{n} c_j y^j \qquad y > bx \tag{2c}$$

for concrete members with the rectangular cross-sections, and

$$E_i = E_0 \sum_{j=0}^{n} a_j r^j \tag{3}$$

for concrete members with the circular cross-sections by using the polar coordinate, in which c_j is the coefficient of polynomial, x^j, y^j and r^j are the locations of the node i, respectively.

The solution procedure of the graded finite element is similar to that of the traditional finite element. The essential difference between them is that material properties in the graded finite element formulation are interpolated from the element nodal values using shape functions. For example, the Young's modulus can be shown as

$$E_e = \sum_{i=1} N_i E_i \tag{4}$$

where N_i and E_i are respectively the shape function and the Young's modulus corresponding to node i, and the summation is done over the element nodal points.

Obviously, when the elements with the same number are used to simulated and analyze fire-damaged concrete structures, the estimations using the graded finite element with the piecewise linear approximation of material parameters would be more exact than those using the traditional finite element with the piecewise constant approximation.

2.3 Other assumptions

The influences of steel rebars, cracks and concrete spallings, etc. on the temperature fields of concrete members are neglected in the FEM simulation and analysis. Usually, the Young's modulus of steel rebars after fire almost recovers to its original status before fire. Therefore, elastic properties of steel rebars and the Poisson ratio of concrete are generally considered as invariable [Li, Hai, Lou and Jiang (2006), Kirby, Lapwood and Thomson (1986)]. In the FEM, mechanical properties of steel rebars are dispersed into the compression zone and tension zone according to the locations of steel rebars. Furthermore, the good bond is assumed and there is no slip between the concrete and steel rebars.

3 Numerical solution

3.1 Young's modulus–temperature (E-T) models

As a result of the discrete characteristics of concrete, complexities of thermal effects and different experimental conditions, etc., it's difficult to exactly describe the relationship between mechanical properties and temperatures of fire-damaged concrete. Currently, there are many different E-T models of concrete after fire [Li, Hai, Lou and Jiang (2006), Persson (2004), Aydm, Yazici and Baradan (2008), Buchanan (2000), Wang (2002)], as shown in Fig. 4. So it's important to choose the reasonable E-T model to estimate the maximal temperature gradients reached in fire-damaged concrete members according to the actual concrete and real fire.

From Fig. 4, it is shown that Wu's model (1999) [Li, Hai, Lou and Jiang (2006)] and Persson's model [Persson (2004)] are almost identical and they locate in the middle of E-T models. As a consequence, Wu's model is adopted by this paper as a standard model to estimate the maximal temperature gradients reached in concrete members, and the influences of choosing Wu's model and other E-T models on the results of estimation will be analyzed as follows. The relationship of Young's modulus-temperature in Wu's model is defined as

$$\frac{E_T}{E_C} = \begin{cases} -1.335\left(\frac{T}{1000}\right) + 1.027 & T \le 200^\circ C \\ 2.382\left(\frac{T}{1000}\right)^2 - 3.371\left(\frac{T}{1000}\right) + 1.335 & 200^\circ C \le T \le 600^\circ C \end{cases} \tag{5}$$

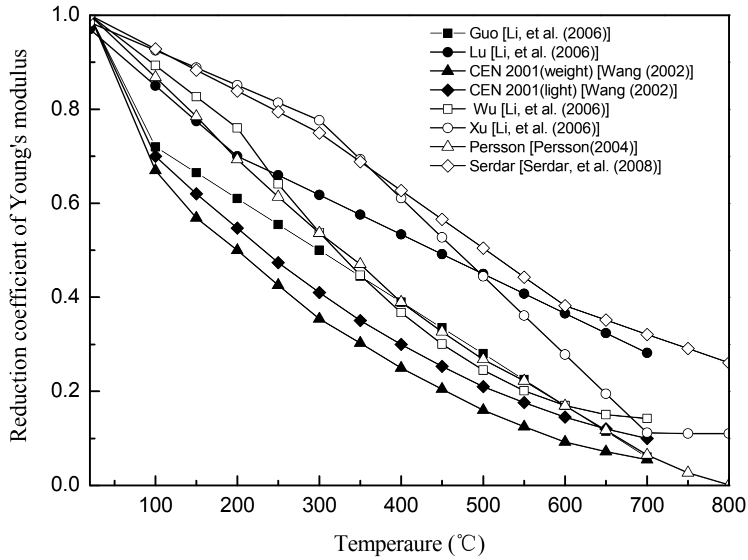


Figure 4: Different E-T models of fire-damaged concrete

where E_T and E_C are the Young's moduli at the temperature T and ambient temperature, respectively.

3.2 Parameter identification method (inverse method)

Obviously, the concrete deterioration would result in additional deformation of concrete member because some mechanical properties (such as the Young's modulus or strength) of fire-damaged concrete member have been changed. Therefore, the parameter identification by means of displacement measurements can be used to estimate material parameters of the fire-damaged concrete. It is assumed that there is no residual deformation in concrete member during the fire duration.

Based on the parameter identification, the estimation of the maximal temperature gradients reached in fire-damaged concrete structures is a constrained and nonlinear optimization problem in nature. The normal least-squares estimation, which doesn't need any statistical information and weight assumption, has been widely applied in the parameter identification of composite materials, geotechnical engineering and bridge engineering, etc. Therefore, it is chosen here as the cost function $\Gamma(\mathbf{p})$ of determining the maximal temperature gradients reached in fire-damaged concrete structures.

When the cost function $\Gamma(\mathbf{p})$ is established, the estimation of the maximal temperature gradients can be considered as solving the following mathematical programming problem

$$\text{Minimize:} \quad \Gamma(\mathbf{p}) = (\mathbf{u}_s - \mathbf{S}\mathbf{u})^T (\mathbf{u}_s - \mathbf{S}\mathbf{u}) \tag{6a}$$

$$\text{Subject to:} K(\mathbf{p}, \mathbf{u}) = \mathbf{F} \tag{6b}$$

$$p \in \mathbf{D}_p \tag{6c}$$

where $\mathbf{p}$ is the identified parameter, $\mathbf{u}_s$ is the actual displacement, $\mathbf{u}$ is a state variable, $\mathbf{S}$ is the selective matrix with respect to the number and location of measurement, $\mathbf{D}_p$ is the admissible set of $\mathbf{p}$ and Eq. (6b) is the constrained condition.

It should be pointed out that the numerical models proposed in Section 2.2 are established based on fire experiments, in which the initial measurement displacements of concrete members are zero. However, since the initial measurement displacements are non-zero in practical applications, the constrained condition of Eq. (6b) can take the following form of

$$\mathbf{K}\Delta\mathbf{u} = \Delta\mathbf{F} \tag{7}$$

where Δ means the difference of random two increment steps in loading test, $\Delta\mathbf{u}$ is the displacement increment vector and $\Delta\mathbf{F}$ is the load increment vector. It is recommended that loading test of real fire-damaged concrete structures can be conducted according to the method and advice given by U. Dilek [U. Dilek (2005)].

The derivation of Eq. (6b) with respect to the identified parameter $\mathbf{p}$ is

$$\frac{\partial \mathbf{K}}{\partial \mathbf{p}}\mathbf{u} + \mathbf{K}\frac{\partial \mathbf{u}}{\partial \mathbf{p}} = 0 \tag{8}$$

$$\frac{\partial \mathbf{u}}{\partial \mathbf{p}} = -\mathbf{K}^{-1}\frac{\partial \mathbf{K}}{\partial \mathbf{p}}\mathbf{K}^{-1}\mathbf{F} \tag{9}$$

where $\partial\mathbf{K}/\partial\mathbf{p}$ is the sensitivity matrix.

In this paper, the identified parameter $\mathbf{p}$ is chosen as the Young's modulus (E) or the maximal temperature (T) of each concrete layer. In order to get a deeper understanding into the properties of fire-damaged concrete, the Young's modulus of concrete is firstly estimated through the solution of Eq. (6). Then the maximal temperature gradients reached in concrete members can be obtained according to the E-T model. As a consequence, the sensitivity matrix can be given as

$$\frac{\partial \mathbf{K}}{\partial E} = \int_V \mathbf{B}^T \frac{\partial \mathbf{D}}{\partial E}\mathbf{B}dV \tag{10}$$

where $\mathbf{B}$ is the strain-displacement matrix, $\mathbf{D}$ is the constitutive matrix and V is the domain of element.

According to Eqs. (1)-(3), the identified parameters $\mathbf{p}$ are the Young's modulus E_0 at the origin of local coordinate and coefficients a_i of the polynomial, respectively. Based on Eq. (10), it can be gained that

$$\frac{\partial \mathbf{D}}{\partial E_0} = \frac{\partial E}{\partial E_0}\frac{\partial \mathbf{D}}{\partial E} \tag{11}$$

$$\frac{\partial \mathbf{D}}{\partial a_i} = \frac{\partial E}{\partial a_i}\frac{\partial \mathbf{D}}{\partial E} \tag{12}$$

3.3 Solution of inverse problem

There are many optimum algorithms to solve the above mathematical programming problem. The Gauss-Newton method [Snanyei, et al. (1997)], whose solution is an iterative process, is used here to minimize the cost function $\Gamma(\mathbf{p})$.

Suppose that the relationship between the iterative steps k and (k-1) in the Gauss-Newton method is

$$\mathbf{p}^k = \mathbf{p}^{k-1} + \Delta\mathbf{p}^{k-1} \quad k = 1, 2, 3, ... \tag{13}$$

Since the real cost function is not known, it is locally approximated by a second order Taylor series around the current parameter values

$$\Gamma\left(\mathbf{p}^k\right) \approx q = \Gamma\left(\mathbf{p}^{k-1}\right) + \nabla_{\mathbf{p}}\Gamma\left(\mathbf{p}^{k-1}\right)\Delta\mathbf{p}^{k-1} + \frac{1}{2}\left(\Delta\mathbf{p}^{k-1}\right)^T \nabla^2_{\mathbf{p}}\Gamma\left(\mathbf{p}^{k-1}\right)\Delta\mathbf{p}^{k-1} \tag{14}$$

The gradient and the Hessian of $\Gamma(\mathbf{p})$ are calculated by

$$\nabla_p \Gamma = 2\mathbf{J}^T\mathbf{R} \tag{15}$$

$$\nabla^2_p \Gamma = 2\mathbf{J}^T\mathbf{J} + 2\mathbf{R}^T\nabla^2_p\mathbf{R} \tag{16}$$

where $\mathbf{J} = -\mathbf{S}\partial\mathbf{u}/\partial\mathbf{p}$, $\mathbf{R} = \mathbf{u}_s - \mathbf{S}\mathbf{u}$.

The necessary condition for a cost function to attain its minimum can be expressed by stating that the partial derivatives of the cost function q with respect to the identified parameters have to be zero

$$\partial q/\partial\Delta\mathbf{p}^{k-1} = 0 \tag{17}$$

Substituting Eqs. (14)-(16) into Eq. (17) and neglecting the second order term of Eq. (16) lead to

$$\Delta\mathbf{p}^{k-1} = -\left[\left(\mathbf{J}^{k-1}\right)^T\mathbf{J}^{k-1}\right]^{-1}\left(\mathbf{J}^{k-1}\right)^T R^{k-1} \tag{18}$$

Based on Eq. (13) and Eq. (18), the Gauss-Newton iterative procedure can be derived as follows

$$\mathbf{p}^k = g\left(\mathbf{p}^{k-1}\right) \tag{19}$$

$$g(\mathbf{p}) = \mathbf{p} - \left(\mathbf{J}^T\mathbf{J}\right)^{-1}\mathbf{J}^T\mathbf{R} \tag{20}$$

Suppose $\mathbf{p}_1$, $\mathbf{p}_2 \in Q_p$, in which Q_p is the set of identified parameters. According to the intermediate value theorem, the equation can be obtained as follows

$$g(\mathbf{p}_1) - g(\mathbf{p}_2) = \frac{\partial g(\zeta)}{\partial \mathbf{p}}(\mathbf{p}_1 - \mathbf{p}_2) \tag{21}$$

where $\zeta = p_2 + \chi(p_1 - p_2), 0 < \chi < 1$.
In terms of Eq. (21), we can get the following equation

$$\|g(\mathbf{p}_1) - g(\mathbf{p}_2)\|_\infty \leq \|\Omega(\mathbf{p})\|_\infty \|(\mathbf{p}_1 - \mathbf{p}_2)\|_\infty \leq L_\Omega \|(\mathbf{p}_1 - \mathbf{p}_2)\|_\infty \tag{22}$$

where the symbol $\|\cdot\|_\infty$ means the infinity norm of the vector or matrix. According to Eq. (20), the following equation is obtained:

$$\Omega(\mathbf{p},\varepsilon) = \left(\mathbf{J}^T\mathbf{J}\right)^{-1}\frac{\partial\left(\mathbf{J}^T\mathbf{J}\right)}{\partial \mathbf{p}}\left(\mathbf{J}^T\mathbf{J}\right)^{-1}\left[\mathbf{J}^T\mathbf{R}\right] - \left(\mathbf{J}^T\mathbf{J}\right)^{-1}\frac{\partial \mathbf{J}^T}{\partial \mathbf{p}}\mathbf{R} \tag{23}$$

and

$$L_\Omega = Max\|\Omega(\mathbf{p})\|_\infty \tag{24}$$

Obviously, when $L_\Omega < 1$, the uniqueness of solution of the Gauss-Newton method can be assured according to the contraction mapping theorem.
The convergence criteria in the present study are:

$$\left\|\mathbf{u}^k - \mathbf{u}^{k-1}\right\| / \left\|\mathbf{u}^k\right\| < \varepsilon_1 \tag{25}$$

$$Max\left\{\left|\frac{\Delta p_i^{k-1}}{p_i^k}\right|\right\} < \varepsilon_2, i = 1,\ldots N_p \tag{26}$$

where ε_1 and ε_2 are the error tolerances, and N_p is the total number of identified parameters.

4 Error analysis

The error transfer in the Gauss-Newton method is investigated here, and the error tolerances of the estimations resulting from the measurement error and the chosen E-T model are also presented in detail as follows.

Suppose that there is a relative measurement error ε, which can be written as

$$\mathbf{u}_s^* = \mathbf{S}\mathbf{u}^* + \varepsilon \tag{27}$$

where $\mathbf{u}_s^*$ is the measurement displacement and $\mathbf{u}^*$ is the calculated displacement. The difference $\mathbf{h}^k$ between the identified parameter $\mathbf{p}^k$ and the real parameter $\mathbf{p}^*$ in the iterative step k is defined as

$$\mathbf{h}^k = \mathbf{p}^k - \mathbf{p}^* \tag{28}$$

So

$$\begin{aligned} \mathbf{h}^k - \mathbf{h}^{k-1} &= \mathbf{p}^k - \mathbf{p}^{k-1} \\ &= \psi(\mathbf{p}^{k-1}, \varepsilon) \\ &= \psi(\mathbf{p}^*, 0) + \frac{\partial \psi(\mathbf{p}^{\xi_{k-1}}, \varepsilon^{\xi_{k-1}})}{\partial \mathbf{p}} h^{k-1} + \frac{\partial \psi(\mathbf{p}^{\xi_{k-1}}, \varepsilon^{\xi_{k-1}})}{\partial \varepsilon} \varepsilon \end{aligned} \tag{29}$$

where $\mathbf{p}^{\xi_{k-1}} = \mathbf{p}^* + \gamma(\mathbf{p}^{k-1} - \mathbf{p}^*)$, $0 < \gamma < 1$, $\varepsilon^{\xi_{k-1}} \in (0, \varepsilon)$, and

$$\psi(\mathbf{p}, \varepsilon) = -\left(\mathbf{J}^T\mathbf{J}\right)^{-1} \mathbf{J}^T (\mathbf{S}\mathbf{u}^* + \varepsilon - \mathbf{S}\mathbf{u}),$$

in which $\mathbf{J}$ is the Jacobin matrix.

So

$$\psi(p, 0) = 0 \tag{30a}$$

$$\frac{\partial \psi(\mathbf{p}, \varepsilon)}{\partial \mathbf{p}} = \Omega(\mathbf{p}, \varepsilon) - \mathbf{I} \tag{30b}$$

$$\frac{\partial \psi(\mathbf{p}, \varepsilon)}{\partial \varepsilon} = -\left(\mathbf{J}^T\mathbf{J}\right)^{-1} \mathbf{J}^T \tag{30c}$$

The real E-T model after fire can be expressed as $T = f(E)$. ω^k and ω_1^k respectively denote the standard errors of T and E in the iterative step k. The relationship of ω^k and ω_1^kcan be expressed as

$$\omega^k = \frac{\partial f}{\partial E} \omega_1^k \tag{31}$$

where ω_1^k is $\mathbf{h}^k$ when the identified parameter is the Young's modulus.

Suppose that there is a difference e between the chosen E-T model and the real E-T model. The error of the chosen E-T model in the iterative step k is

$$\omega^k = \left(\frac{\partial f}{\partial E} + \frac{\partial e}{\partial E}\right)\omega_1^k \tag{32}$$

Obviously, the error resulting from the chosen E-T model obeys the additive rule. Therefore, the error of the chosen E-T model in the Gauss-Newton method can be written as

$$\begin{aligned} &\omega^k - \omega^{k-1} \\ &= \left(\frac{\partial f}{\partial E} + \frac{\partial e}{\partial E}\right)(\mathbf{h}^k - \mathbf{h}^{k-1}) \\ &= \left(\frac{\partial f}{\partial E} + \frac{\partial e}{\partial E}\right)\left[\psi(\mathbf{p}^*, 0) + \frac{\partial \psi(\mathbf{p}^{\xi_{k-1}}, \varepsilon^{\xi_{k-1}})}{\partial \mathbf{p}}\mathbf{h}^{k-1} + \frac{\partial \psi(\mathbf{p}^{\xi_{k-1}}, \varepsilon^{\xi_{k-1}})}{\partial \varepsilon}\varepsilon\right] \end{aligned} \tag{33}$$

Substituting Eq. (30) into Eq. (33) leads to

$$\omega^k = \Omega(p^{\xi_{k-1}}, \varepsilon^{\xi_{k-1}})\omega^{k-1} + \left(\frac{\partial f}{\partial E} + \frac{\partial e}{\partial E}\right)\frac{\partial \psi(\mathbf{p}^{\xi_{k-1}}, \varepsilon^{\xi_{k-1}})}{\partial \varepsilon}\varepsilon \tag{34}$$

When$L_1 = Max\,\|\partial\Omega(\mathbf{p},\varepsilon)\|_\infty < 1$ and mark $L_2 = \left(\frac{\partial f}{\partial E} + \frac{\partial e}{\partial E}\right) Max \left\|\left(\frac{\partial\psi(\mathbf{p},\varepsilon)}{\partial\varepsilon}\varepsilon\right)\right\|_\infty$, then

$$\begin{aligned} \|\omega^k\|_\infty &\leq L_1\|\omega^{k-1}\|_\infty + L_2 \\ &\leq L_1^2\|\omega^{k-2}\|_\infty + (1+L_1)L_2 \\ &\cdots \\ &\leq L_1^k\|\delta\mathbf{p}\|_\infty + (1+L_1+kL_1^{k-1})L_2 \end{aligned} \tag{35}$$

Since $L_1 < 1$, the following equation is obtained:

$$\lim_{k\to\infty}\|\omega^k\|_\infty \leq \frac{1}{1-L_1}L_2 \approx L_2(1+L_1) \tag{36}$$

So the upper limit of the parameter identification is $L_2(1+L_1)$ when $L_1 < 1$.

5 Experimental verification and parameter analysis

The experimental investigation of concrete slab after fire is analyzed here to validate the proposed method. Parameter analysis is also carried out to investigate the influences of some factors on the precisions of estimating the maximal temperature gradients reached in fire-damaged concrete structures.

5.1 Experimental verification

The fire-damaged concrete continuous slabs were tested in Tongji University of China by Yu [Yu (2007)], in 2007. There were totally four two-span concrete continuous slabs in the experiment, in which one concrete slab was tested under ambient temperature and the others were subjected to 1-face bottom heating.

The fire experiment of three concrete slabs was respectively conducted under the conditions of different heating time (50 min, 70 min and 120 min) according to the ISO-834 standard fire curve. The other experimental conditions such as the material parameters, sizes, boundary constraints and loading modes of concrete slabs and so on were the same, as shown in Fig. 5. There were four usable points of displacement measurement and twelve points of thermocouple measurement in each concrete slab. The load was applied in four cube cement blocks ($150 \times 150 \times 150$ mm) above the middle of span. One of the ends of concrete slab was simply supported and the other was sliding support, the middle beam was hinged at both ends. In the experiment, the material performances, temperature fields, deformations and limit load-carrying capacities of fire-damaged concrete slabs were investigated.

The temperature distribution in the concrete continuous slab subjected to the bottom heating is essentially one-dimensional (1D). Based on the material model proposed in Section 2.2, the graded finite element is used to estimate the maximal temperature gradients reached in fire-damaged concrete slabs. According to Eq. (1), the Young's modulus can be expressed as follows

$$E_i = E_0(1 - bx^2) \tag{37}$$

where E_0 is the identified Young's modulus at the top of concrete slab, and b is also the identified parameter that shows the variety of Young's modulus along the direction of heating. The quadratic function model is used here because there are only few usable points of displacement measurement. The total number of measurement information in the parameter identification is generally demanded to be more than or equal to the number of identified parameters to get a good estimation. Thereby, this material model proposed in this paper would be helpful to the practical applications because it can greatly reduce the requirement of measurement numbers.

The first concrete slab (the heating time is 50 min) in the fire experiment was not quite successful. So the maximal temperature gradients reached in the second and third fire-damaged concrete slabs (the heating time is 70 min and 120 min, respectively) are estimated in this paper. The slabs are divided into 6 layers with different thicknesses (15 mm, 15 mm, 15 mm, 15 mm, 30 mm and 30 mm, respectively) along the direction of heating. Material parameters of the middle concrete

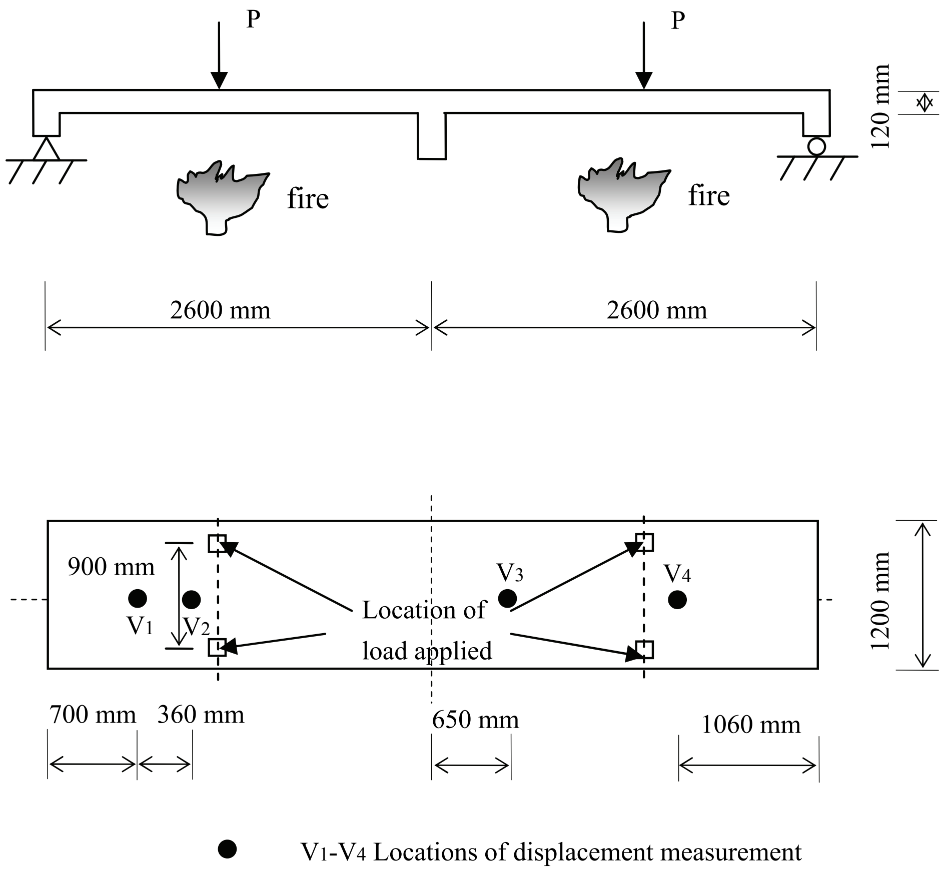


Figure 5: The schematic diagram of concrete slab in the fire experiment

beam are the same as those of the layer at the bottom of concrete slab because of the beam subjected to 3-face heating. Material parameters of the concrete under ambient temperature are used as the initial iterative values in the parameter identification. For example, the Young's modulus of concrete is 2.945E4 MPa. The identified parameters E_0 and b of each concrete slab can be obtained according to Eq. (6), and the results of parameter identification are shown in Table 1. After the Young's modulus of each layer of the concrete slab is identified, the maximal temperature gradients are estimated based on Wu's model in Section 3.1 because this E-T model agrees well with the actual concrete and the real fire in this experiment. The results of estimation are shown in Fig. 6 and Fig. 7, respectively. The curves of estimation are made through the straight line connection with temperatures of the middle of each layer of fire-damaged concrete slab.

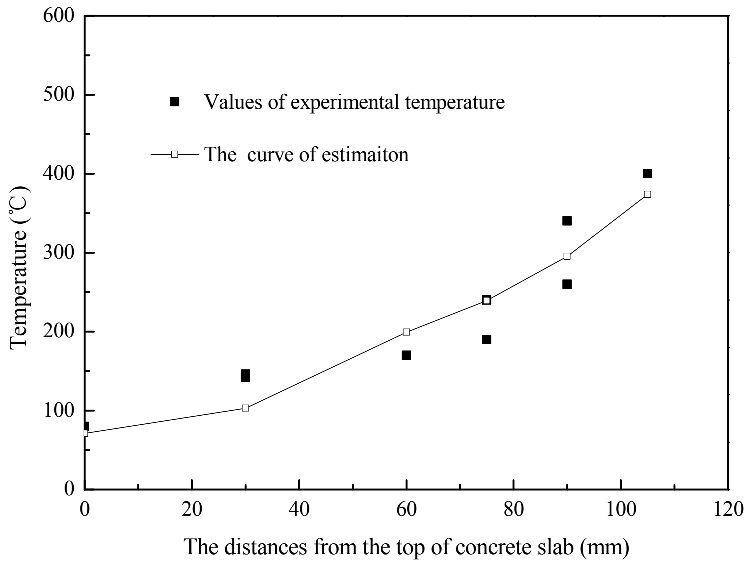


Figure 6: Estimation of the maximal temperature gradient reached in the second concrete slab

Table 1: Results of the parameter identification of two fire-damaged concrete slabs

NO. of slab	Displacements of slabs		Results of identified parameters	
	Measurement displacement (mm)	Calculation displacement (mm)	E_0 (MPa)	b
2	V_2=0.70	V_2=0.686	27625	5.145E-05
	V_4=0.67	V_4=0.681		
3	V_2=0.60	V_2=0.623	25036	6.594E-05
	V_3=0.29	V_3=0.282		

The results of numerical example and experiment show that the deformations of concrete structures are sensitive to high temperature. Using the displacement measurements, the estimations of the proposed method agree well with the experimental results, especially the tendency of maximal temperature gradients reached in fire-damaged concrete structures. Although the lower degree polynomial is used in Eq. (37), the proposed method can availably estimate the maximal temperature gradients reached in fire-damaged concrete structures. The average errors of ex-

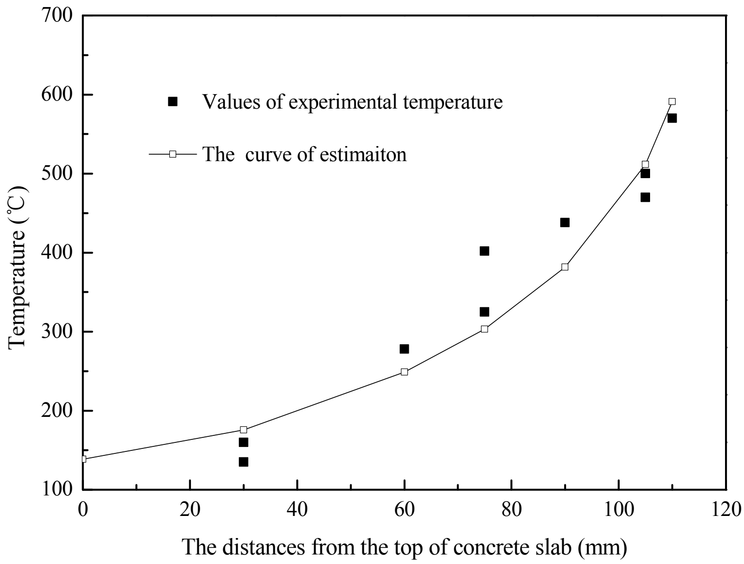


Figure 7: Estimation of the maximal temperature gradient reached in the third concrete slab

perimental results and numeral estimations in the second and third fire-damaged concrete slabs are approximate 15.57% and 12.15%, respectively.

The differences between experiment and numerical results are mainly from the effects of several factors, i.e., cracks and concrete spallings, different polynomial expressions of the material model, the measurement error, element types and E-T models. The local effects of cracks and concrete spallings presented in fire-damaged concrete structures on the estimation can be neglected, when the cracks and concrete spallings are not serious. The influences of the others factors on the precisions of estimation are investigated in detail as follows.

5.2 Parameters analysis

Two examples are conducted here to analyze the influences of some main factors such as different polynomial expressions of the material model, the number and different thicknesses of concrete layers, element types and E-T models on the precisions of estimation. Firstly, the normal analysis of concrete members in fire is carried out to get the temperature field according to the ISO-834 standard fire curve. Then, mechanical behaviors of concrete members after fire are simulated based on Wu's model. The displacements resulting from the normal analysis are used as the initial input parameters of estimation. Finally, parameter identification is done to achieve the maximal temperature gradients reached in fire-damaged concrete members. The material parameters of concrete under ambient temperature are used as follows: the Young's modulus E_C = 2.95E4 MPa, the Poisson's ratio μ_C= 0.23 and the mass density ρ_C= 2500 kg/m^3.

(1) Concrete column subjected to 4-face heating

A concrete column is subjected to 4-face heating with the heating time of 70 min, as shown in Fig. 8. The size of concrete column is $a \times a \times L = 400 \times 400 \times 3000$ mm, the thickness of covering layer is 20 mm and the main steel bars are located as 4ϕ20. The top of concrete column is subjected to the axial compression force P= 320 KN. The temperature distribution in concrete column subjected to 4-face heating is essentially two-dimensional (2D) with no variation along the length of the member. As a result of the symmetrical characteristics of concrete column subjected to 4-face heating, 1/4 cross-section of concrete column are simulated and analyzed in the FEM, in which the discrete elements are divided as shown in Fig. 8.

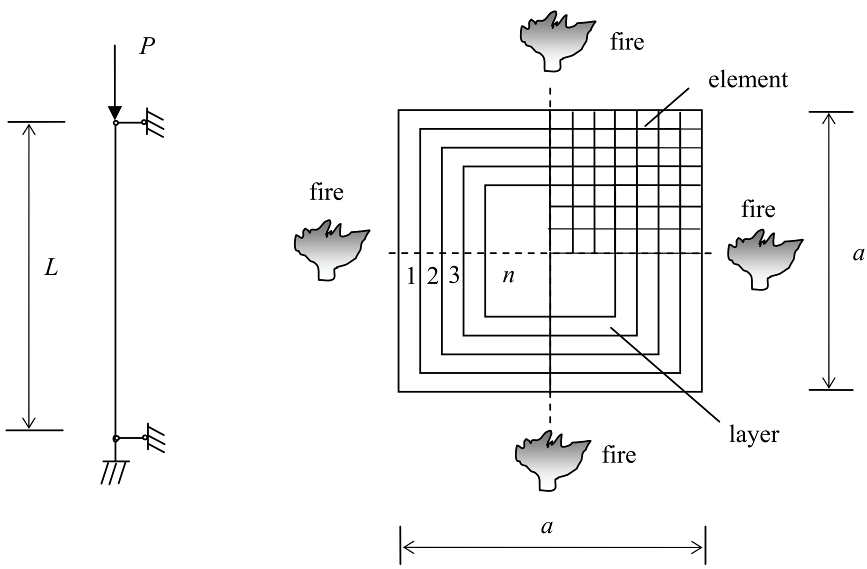


Figure 8: The model of concrete column subjected to 4-face heating

As mentioned in Section 2.2, the Young's modulus of fire-damaged concrete along the direction of heating can be expressed as a higher degree polynomial. When the material models are expressed as the different degrees of polynomial, the results

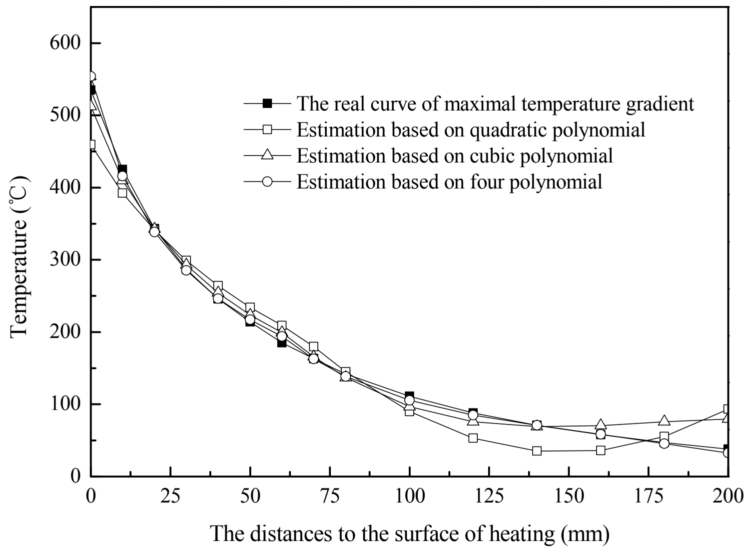


Figure 9: Influences of different degrees of polynomial expressions of the Young's modulus on the results of estimation

of estimation are given in Fig. 9. The highest degree n in Eq. (2) is respectively chosen as 2, 3 and 4, so there are $(n + 1)$ identified parameters in these above inverse problems. Obviously, the precision of estimation is higher as the highest degree of polynomial increases. However, the influences of different degrees of polynomial on the precisions of estimation are slight when the highest degree of polynomial is greater than or equal to 2. Unless otherwise stated, the highest degree n of polynomial is 3 in the following examples.

The influences of concrete layers with different numbers and thicknesses are investigated as follows. The results of estimating the maximal temperature gradient reached in fire-damaged concrete column are shown in Fig. 10, in which the concrete member is divided into 7 layers with different thicknesses. The symbol "+" means the addition of thickness of each concrete layer. For example, '20 + 20 + 20 + 20 + 20 + 40 + 60 mm' denotes that the thicknesses of 7 layers in 1/4 concrete cross-section are respectively 20 mm, 20 mm, 20 mm, 20 mm, 20 mm, 40 mm and 60 mm. The sum of thicknesses of 7 layers is equal to the width of 1/4 cross-section. Fig. 11 gives the estimations of 1/4 concrete cross-section divided into 5, 7 and 10 layers. From Fig. 10 and Fig. 11, it's shown that the estimations are more accurate to agree with the real curve of maximal temperature gradient as the number of concrete layers increases. Moreover, it is also shown that there are no evident influences of concrete layers with the same number and different thicknesses on the precisions of estimation.

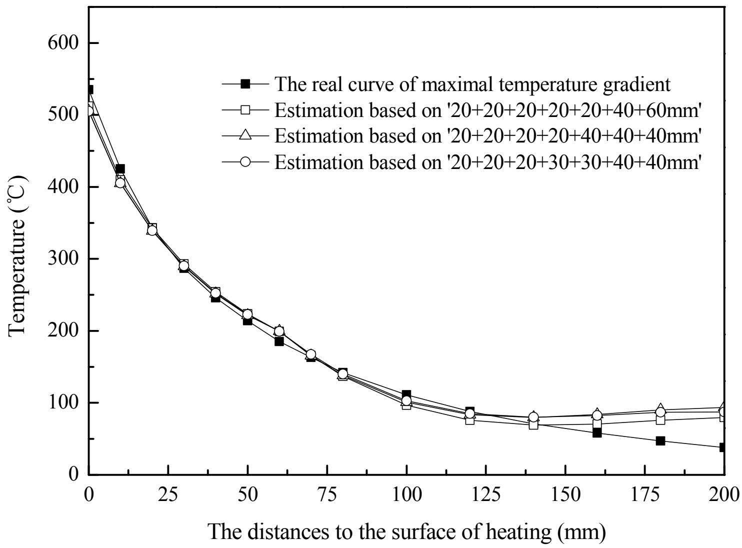


Figure 10: Estimations of the maximal temperature of 7 concrete layers with different thicknesses

By means of the graded finite solid element and traditional solid element, Fig. 12 presents the results of estimating the maximal temperature gradient reached in fire-damaged concrete column. Obviously, numerical results of the graded finite element using piecewise linear approximation would be more exact to agree with the real curve of maximal temperature gradient than those of the traditional finite element using piecewise constant approximation. Moreover, the requirement of measurement data based on the graded finite element is greatly reduced due to polynomial expressions of the material model. Simultaneously, the computational efficiency of the graded finite element is apparently improved in the parameter identification as a result of the decrease of the identified parameters.

The influences of different E-T models such as Wu's model [Li, Hai, Lou and Jiang (2006)], Guo's model [Li, Hai, Lou and Jiang (2006)] and Persson's model [Persson (2004)] on the estimations are great, as shown in Fig. 13. For example, the maximal error of estimating the temperature is more than 120 $^\circ$ between Wu's model and Guo's model. As a consequence, it's very important to choose the reasonable E-T model to estimate the maximal temperature gradients reached in concrete members according to the actual concrete and real fire.

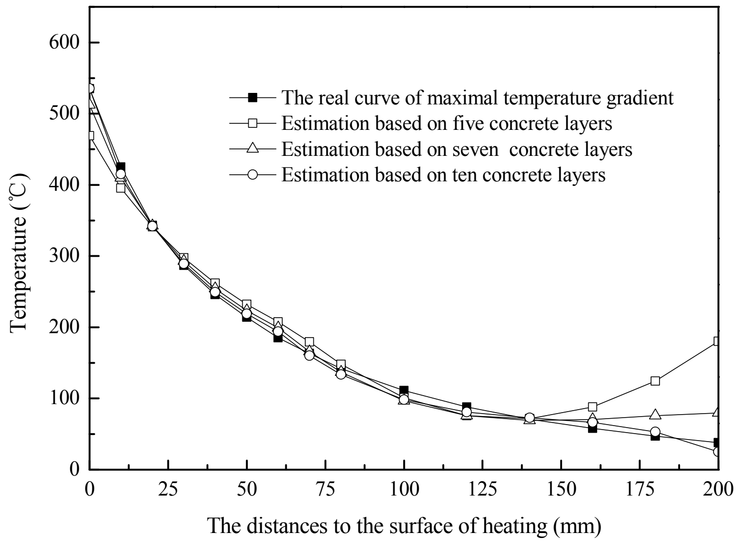


Figure 11: Estimations of the maximal temperature of concrete member with different layers

(2) Continuous concrete beam subjected to 1-face bottom heating

Three-span continuous concrete beam subjected to 1-face bottom heating is analyzed here, as shown in Fig. 14. Three single-span beams (called the left beam, the middle beam and the right beam) are respectively suffered from different heating modes. The size of beam is $b \times a \times (L_1 + L_2 + L_3) = 200$ mm $\times$ 120 mm $\times$(3000 mm + 3000 mm + 3000 mm). The thickness of covering layer is 10 mm and the main steel bars are located as 4Φ12. The middle of each single-span beam is subjected to the concentrated force P= 10 KN.

The effect of local heat conduction is neglected in the FEM simulation and analysis. The cross-section of continuous concrete beam is divided into 10 layers along the direction of heating. The Young's modulus of concrete is expressed as Eq. (1), in which n=3. Therefore, there are 4 identified parameters in the inverse problem.

The heating modes of continuous concrete beam are defined as that each single-span beam is under different heating time, i.e., 0 min, 70 min and 100 min, as shown in Table 2. The heating time of 0 min means that the single-span beam is under ambient temperature. The results of the parameter identification are shown in Table 3. Average errors between the estimated temperatures (by parameter identification) and real temperatures (through normal analysis) at the interfaces of each concrete layer are also presented in Table 3. Estimations of the maximal temper-

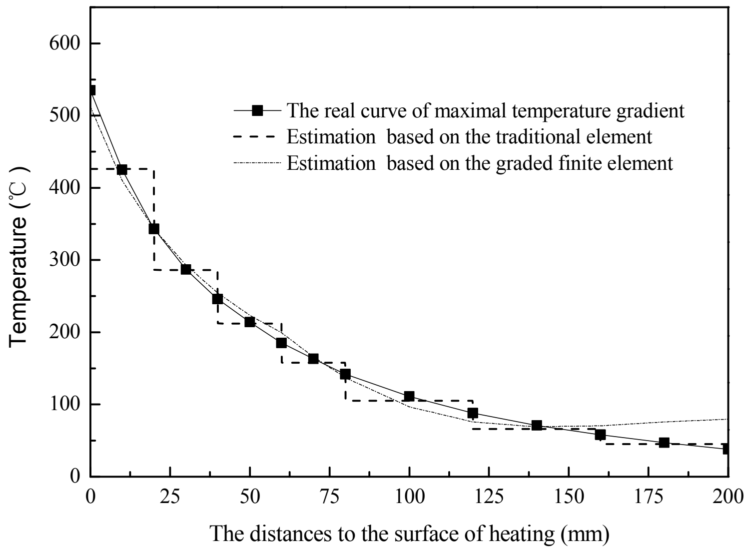


Figure 12: Estimations of the maximal temperature of concrete column based on the graded finite element and traditional element

ature gradients reached in fire-damaged continuous concrete beam under the third heating mode are shown in Figs. 15a-15c.

Table 2: Different heating modes of continuous concrete beam

NO. of heating modes	Heating time of different single-span beams (min)		
	The left beam	The middle beam	The right beam
1	0	0	0
2	0	70	0
3	100	70	0

Numerical results of continuous concrete beam show that the maximal temperature gradients reached in different zones of the same concrete member can be estimated based on the proposed method. It means this method can availably evaluate the entire zone of concrete member suffered from the localized fire.

5.3 Error transfer of the estimation

From the above parameter analysis, there are three main errors of estimating the maximal temperature gradients reached in fire-damaged concrete structures, i.e.,

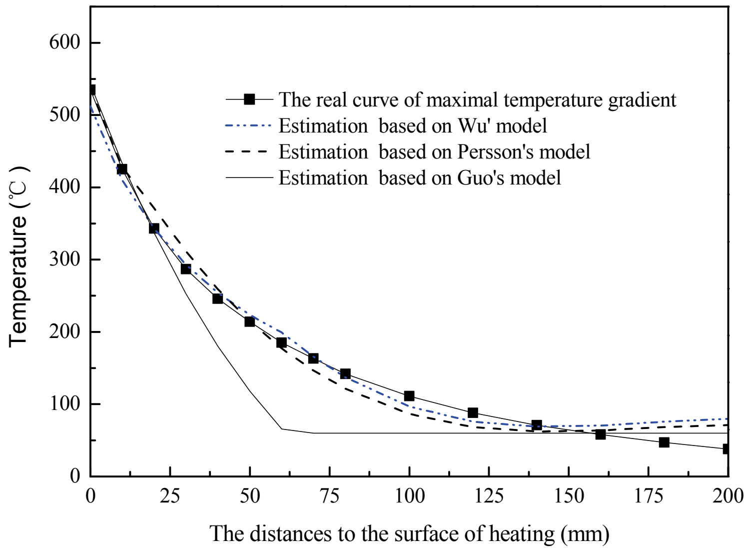


Figure 13: Estimations of the maximal temperature of concrete column based on different E-T models

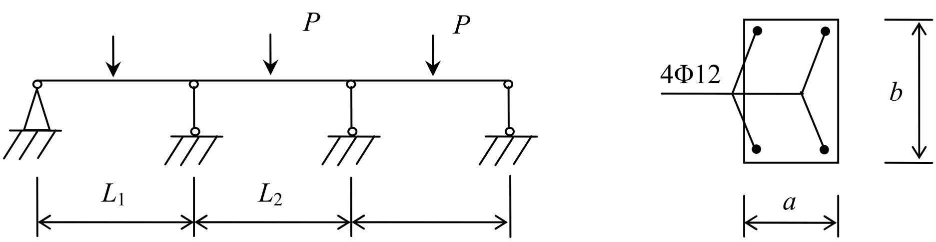
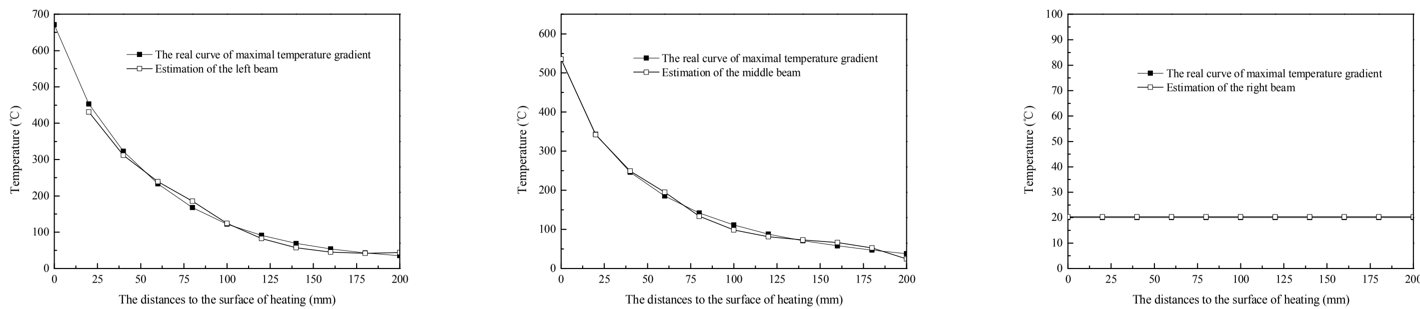


Figure 14: The diagram of three-span continuous concrete beam

the measurement error, the error of material model and the error of chosen E-T model. According to the error calculation and analysis in Section 4, the influences of these errors on the precisions of estimation are presented as follows.

Firstly, zero and random errors in the measurement error are considered here according to Eq. (27). Good results of the parameter identification can be obtained under the condition of zero error. The average error between the estimated values (by parameter identification) and the real values of Young's modulus (through normal analysis) of homogeneous concrete under ambient temperature is less than



(a) The left beam with the heating time of 100 min

(b) The middle beam with the heating time of 70 min

(c) The right beam with the heating time of 0 min

Figure 15: Estimations of the maximal temperature gradient reached in different zones of continuous concrete beam

Table 3: Results of estimation of continuous concrete beam subjected to different heating modes

NO. of heating modes	Results of parameter identification					
	The left beam		The middle beam		The right beam	
	Identified parameters	Average errors	Identified parameters	Average errors	Identified parameters	Average errors
1	a_1= 0.000 a_2= 0.000 a_3= 0.000 a_4= 2.950e4	1.758%	a_1= 6.383e-6 a_2= 1.426e-3 a_3= 8.893e-2 a_4= 2.950e4	1.179%	a_1= 0.000 a_2= 0.000 a_3= 0.000 a_4= 2.950e4	1.505%
2	a_1= 0.000 a_2= 0.000 a_3= 0.000 a_4= 2.950e4	1.505%	a_1= 6.620e-3 a_2= 2.841 a_3= 4.190e2 a_4= 6.277e3	5.515%	a_1= 0.000 a_2= 0.000 a_3= 0.000 a_4= 2.950e4	1.631%
3	a_1= 2.951e-3 a_2= 1.870 a_3= 3.859e2 a_4= 2.571e3	7.512%	a_1= 6.621e-3 a_2= 2.847 a_3= 4.198e2 a_4= 6.286e3	6.945%	a_1= 0.000 a_2= 0.000 a_3= 0.000 a_4= 2.950e4	1.695%

0.1% according to Eq. (36). Similarly, the average error of identification is about 2.0% when the random error is 0.01 mm.

The error of material model mainly comes from the piecewise linear approximation of mechanical properties and the number of concrete layers. Obviously, the precision of estimation is higher as the number of concrete layers increases. The error is defined as the difference between the estimated value and the real value of Young's modulus at the interface or the middle of each concrete layer. When 1/4 cross-section of concrete column is divided into 7 layers in above parameter analysis of different element types, the average error based on the graded finite element is about 2.073% under the condition of zero error of measurement. Similarly, the average error of the traditional solid element is about 7.785%.

The error of chosen E-T model results from the difference between the chosen E-T model and the real E-T model. It's assumed that there is no error between the chosen E-T model and the real E-T model and the error of identified Young's modulus is $\Delta\delta$. Then the average error between the estimated temperatures and the real temperatures of concrete member is $\partial f/\partial E \times \Delta\delta$ according to Eq. (36), in which $\partial f/\partial E$ is the error transfer factor of chosen E-T model. Similarly, the average error of estimated temperatures is $(\partial f/\partial E + \partial e/\partial E) \times \Delta\delta$ when there is an error e between the chosen E-T model and the real E-T model.

6 Conclusions

Thanks to the importance of estimating the maximal temperature gradients reached in fire-damaged concrete structures, a new numerical method using the parameter identification is developed in this paper to get a deeper understanding into the problem. Based on the piecewise linear approximation, a material model is proposed to describe the mechanical properties of fire-damaged concrete. The graded finite element is used to estimate the mechanical properties of fire-damaged concrete by means of the parameter identification method. The maximal temperature gradients reached in fire-damaged concrete structures are obtained according to the E-T model. Moreover, an experimental study and comprehensive parameter analysis are conducted to investigate the influences of main factors on the numerical estimations. It is shown that the estimations from the developed method agree well with the results of experiments. From our investigations, the conclusions can be drawn as follows:

(1) As a result of the stiffness degradation of fire-damage concrete structures, numeral examples and a relevant experiment show that the deformations of concrete structures are sensitive to high temperature. Therefore, the parameter identification method using displacement measurements presented in this paper provides a feasible and effective means to estimate the maximal temperature gradients reached in fire-damaged concrete structures.

(2) Numerical results show that the developed method can avoid the hypotheses of temperature-time curve and fire duration usually made in conventional numerical methods. Moreover, it can availably consider the effects of localized fire and evaluate the depth and degree of concrete damage caused by a fire.

(3) Numerical results with the graded finite element are more exact to agree with the real curve of maximal temperature gradient than those with the traditional finite element. Furthermore, the computational efficiency of the graded finite element is apparently improved in the parameter identification since the number of identified parameters decreases, and simultaneously the requirement of total number of measurement information is also availably reduced due to the polynomial expressions of mechanical properties of fire-damaged concrete.

(4) The error tolerance and error transfer in the parameter identification are presented to evaluate and analyze the influences of several main factors on the numeri-

cal estimations. It is shown that the error tolerances coming from the measurement error, the error of material model and the error of chosen E-T model can be respectively estimated based on the developed method. When there are more concrete layers, higher degree polynomial expression of mechanical properties and more reasonable E-T model in the present study, the results of estimation are more accurate to agree with the real curve of maximal temperature gradients reached in fire-damaged concrete structures.

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