

A New Method of Moments Solution Procedure to Solve Electrically Large Electromagnetic Scattering Problems

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Abstract: In this work, we present a new method of moments solution procedure for calculating acoustic/electromagnetic scattering and radiation by a metallic body whose physical dimension is very large with respect to wavelength. The specially computed basis functions and the testing procedure results in a block-diagonally-dominant moment matrix where each block along the diagonal corresponds to a portion of the structure. The new solution procedure results in considerable savings in terms of computer storage and processing times. Although the procedure is outlined in general mathematical terms, the numerical results are presented only for electromagnetic scattering from two-dimensional bodies and compared with the classical method of moments solution procedure. However, the procedure is the same for three-dimensional bodies and scattering/radiation phenomenon.

Keywords: Method of Moments, boundary element method, large scale problems, scattering, fast solution, rwg basis functions

1 Introduction

In recent times, there has been a renewed interest to develop efficient numerical methods to calculate the scattering/radiation patterns of large, complex structures related to electromagnetic or acoustic wave phenomenon. The research is focused on developing parallel processing algorithms for differential as well as integral equation techniques *viz.* finite element and boundary element methods [Takei A., Yoshimura S., and Kanayama H., (2009)], [Nie Y.F., Chang S., Fan X.K., (2007)], [Soares Jr. D, Vinagre M.D., (2008)], [Shaw A., Banerjee B., Roy D., (2008)], [Chandrasekhar B. Rao S.M., (2008)]. However, it is well-known that the Boundary Integral Equation (BIE) method (also known as the *method of moments* (MoM) in electromagnetic literature [Harrington R. F. (1968)]) is the most efficient and preferred numerical method for solving open-region electromagnetic/acoustic scattering or radiation problems [Schuster G.T., Smith L.C (1985)], [Schuster G.T.

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(1985)], [Seybert A.F., Soenarko B., Rizzo F.J., Shippy D.J. (1985)], [Malbequi P., Candel S.M., Rignot E. (1987)], [Rao, S. M., Raju, P.K. (1989)], [Chandrasekhar B., Rao S. M. (2006)], [Chandrasekhar B. (2005)]. It is because, for this method, the solution space is completely confined to the surface of the structure resulting in the fewest possible unknowns. Another advantage of this method is the exact enforcement of the radiation condition. This eliminates any need for artificial, and often erroneous, absorbing boundary conditions. In contrast, the differential equation based methods such as finite element method usually involves a grid terminating condition using Perfectly matched layers [Lan Y.Y, Zhu J., (2007)] or some other type of mesh-less schemes [Liu C.S., (2007)] and may be limited to a certain class of problems only.

One disadvantage of BIE/MoM is, however, that it generates a dense system matrix and the solution of such a system may be computationally expensive, especially when solving problems involving geometrical structures whose physical dimensions are very large with respect to the wavelength. Large problems quickly exceed the amount of available memory, forcing one to either recompute the matrix elements multiple times or store the elements on disk, which results in prohibitively large execution times. Even if the matrix can be stored entirely in the computer memory, direct solutions, such as LU decomposition, may be impractical for large matrices.

To overcome limitations imposed by expensive computer resources, the research proceeded in developing a sparse moment matrix, utilizing iterative solution schemes, and defining basis functions over larger sections of the geometry. A sparse moment matrix may be achieved in either of two ways: a) by defining a special set of basis functions to represent the unknown quantity [Chandrasekhar B., Rao S. M. (2008)] or b) by handling the influence of the kernel function in a novel way. The usage of well-known wavelet-type basis functions to provide the required sparsity belong to the former category [Steinberg B. Z., Leviatan Y. (1993)] and the application of the fast multipole method (FMM) belongs to the latter [Coifman R., Rokhlin V., Wandzura S. (1993)], [Chew W.C., Song J.M., Cui T.J. (2004)]. There is also yet another scheme, known as the impedance matrix localization scheme (IML) which achieves modest sparsity for simple problems [Canning F. X. (1993)]. Notice that the kernel function is, in general, a decaying function with respect to the distance between the source and observation points. Thus, with increasing distances, the influence of a given source becomes negligible at a sufficiently distant observation point and may be actually set to zero. The IML scheme attempts to exploit this fact. However, there is a certain degree of arbitrariness to this scheme and seems to work for simple problems only. When defining basis functions over a larger portion of the geometry, traditional methods have been valid only for canonical shapes.

Current research in defining them for arbitrary bodies has focused on calculating characteristic basis functions [Mitra R., Prakash V.V.S. (2004)] in order to reduce the overall number of unknowns. It should also be mentioned here that there have been attempts to combine both FEM and BEM methods to develop a more efficient method but these methods appear to have been met with limited success [Soares Jr. D (2008)].

In this work, we have developed a domain decomposition technique that can reduce the necessary storage as well as provide a block-diagonally-dominant moment matrix that can be used in a quickly converging iterative procedure. First, the structure is divided up into smaller substructures. Next, using the classical sub-domain functions, we create entire domain basis functions spanning the given substructure. These entire domain functions are designed to produce null fields at strategically placed locations along the structure. Using these null fields, we can effectively eliminate the near-field interactions between each group and thereby replace the largest elements in the system matrix with zeros. The result is a block-diagonally-dominant matrix, which will converge very quickly within an iterative procedure. The numerical procedure is further explained in the following sections.

2 Numerical Solution Procedure

We begin with a standard MoM formulation using sub-domain basis functions. Then, we divide the complete set of basis functions into various disjoint groups. These groups may be decided by the geometry of the problem. For example, for an antenna array, each array element may be a group in the system. Otherwise, we may simply group elements which are physically close to one another. The functions within each group are then reordered if necessary so that these groups effectively partition the system matrix where each group will correspond to one block along the diagonal. Once the groups have been formed, we select a "source" group and then begin the zeroing process. We choose test points (points where we would like to produce nulls) corresponding to those elements in the MoM matrix which are the largest. Consequently, for any group that forms an immediate neighbor to the source group, we include all test points in that group. If test points on non-adjacent groups are desired, they may be chosen sparsely since their corresponding matrix elements are much smaller in magnitude. Once the test points have been chosen, we form a new set of basis functions to replace those in the source group. Each new basis function is a linear combination, with unknown weights, of a sub-domain function from the source group as well as the functions corresponding to the testing locations. Note that although we refer to locations where null fields will be produced as "testing" locations, the sub-domain functions included in this linear combination are actually the expansion functions for the unknown source quantity

such as the induced current or acoustic source distribution for electromagnetic or acoustic problems, respectively. To solve for these unknown weights for a source group of size K , we form K linear systems:

$$\begin{bmatrix} Z_{t_1 t_1} & Z_{t_1 t_2} & \cdots & Z_{t_1 t_n} \\ Z_{t_2 t_1} & Z_{t_2 t_2} & & \\ \vdots & & \ddots & \\ Z_{t_n t_1} & & & Z_{t_n t_n} \end{bmatrix} \begin{bmatrix} \alpha_{1,j} \\ \vdots \\ \alpha_{n,j} \end{bmatrix} = - \begin{bmatrix} Z_{t_1 s_j} \\ \vdots \\ Z_{t_n s_j} \end{bmatrix} \tag{1}$$

where $j = 1, \dots, K$ and there are n testing locations.

Since there are n equations and n unknowns, this system can be solved exactly. Note, we only need to invert this matrix once for each source group. If we let f_i for $i = 1, \dots, K$ be the original sub-domain functions, then we effectively create a new set of functions g_i using the following relationship:

$$\begin{bmatrix} g_{s_1} \\ g_{s_2} \\ \vdots \\ g_{s_K} \end{bmatrix} = \begin{bmatrix} f_{s_1} \\ f_{s_2} \\ \vdots \\ f_{s_K} \end{bmatrix} + \begin{bmatrix} \alpha_{1,1} & \alpha_{2,1} & \cdots & \alpha_{n,1} \\ \alpha_{1,2} & \alpha_{2,2} & & \\ \vdots & & \ddots & \\ \alpha_{1,K} & & & \alpha_{n,K} \end{bmatrix} \begin{bmatrix} f_{t_1} \\ f_{t_2} \\ \vdots \\ f_{t_n} \end{bmatrix} \tag{2}$$

We then repeat the process for each of the source groups. Within the columns associated with each source group, the new system matrix \tilde{Z} , formed by a source basis change to the newly formed functions, will have zeros for the rows corresponding to the testing locations. Since all the significant non-diagonal elements have been eliminated, the new matrix will be block-diagonally-dominant and will be suitable for iterative techniques.

Furthermore, if we construct an initial guess that is close to the actual solution, the iterative process will converge more quickly. To do this, we simply invert the new self-blocks for each source group. Equivalently, we set all non-diagonal blocks to zero and solve the following system:

$$\begin{bmatrix} \tilde{Z}_{G_1} & 0 & \cdots & 0 \\ 0 & \tilde{Z}_{G_2} & & \\ \vdots & & \ddots & \\ 0 & & & \tilde{Z}_{G_M} \end{bmatrix} \begin{bmatrix} \tilde{I}_{G_1} \\ \vdots \\ \tilde{I}_{G_M} \end{bmatrix} = \begin{bmatrix} V_{G_1} \\ \vdots \\ V_{G_M} \end{bmatrix} \tag{3}$$

where M is the number of groups for the structure.

Solving Eq. (3) will give a good approximation to the unknown quantity which can be used as an initial guess to an iterative procedure, such as the Jacobi or Gauss-Seidel algorithms in block form, to achieve the exact solution. Since the

new system matrix \tilde{Z} is highly block-diagonally-dominant and the initial guess is a good approximation, an iterative solver will converge very quickly, mostly in one or two iterations, as demonstrated in the numerical results section.

2.1 A simple example

We will illustrate the method with a simple example. In the example, there are two 2D strips, each with three pulse functions as shown in Fig. 1. We form two groups of functions $G_1 = \{f_1, f_2, f_3\}$ and $G_2 = \{f_4, f_5, f_6\}$.

The conventional MoM system matrix is given by:

$$\begin{aligned}
 Z &= \begin{bmatrix} \begin{bmatrix} Z_{1,1} & Z_{1,2} & Z_{1,3} \\ Z_{2,1} & Z_{2,2} & Z_{2,3} \\ Z_{3,1} & Z_{3,2} & Z_{3,3} \end{bmatrix} & \begin{bmatrix} Z_{1,4} & Z_{1,5} & Z_{1,6} \\ Z_{2,4} & Z_{2,5} & Z_{2,6} \\ Z_{3,4} & Z_{3,5} & Z_{3,6} \end{bmatrix} \\ \begin{bmatrix} Z_{4,1} & Z_{4,2} & Z_{4,3} \\ Z_{5,1} & Z_{5,2} & Z_{5,3} \\ Z_{6,1} & Z_{6,2} & Z_{6,3} \end{bmatrix} & \begin{bmatrix} Z_{4,4} & Z_{4,5} & Z_{4,6} \\ Z_{5,4} & Z_{5,5} & Z_{5,6} \\ Z_{6,4} & Z_{6,5} & Z_{6,6} \end{bmatrix} \end{bmatrix} \\
 &= \begin{bmatrix} [Z_{G_1 G_1}] & [Z_{G_1 G_2}] \\ [Z_{G_2 G_1}] & [Z_{G_2 G_2}] \end{bmatrix} \tag{4}
 \end{aligned}$$

where $Z_{G_i G_j}$ is the partition consisting of all the subdomain MoM matrix elements with sources from group G_j and test points on group G_i . In order for Z to be block-diagonally dominant, the elements in $Z_{G_1 G_1}$ and $Z_{G_2 G_2}$ should be much larger in magnitude than the elements in $Z_{G_1 G_2}$ and $Z_{G_2 G_1}$. To eliminate $Z_{G_2 G_1}$, we form 3 new basis functions on group 1, which we will refer to as a source group.

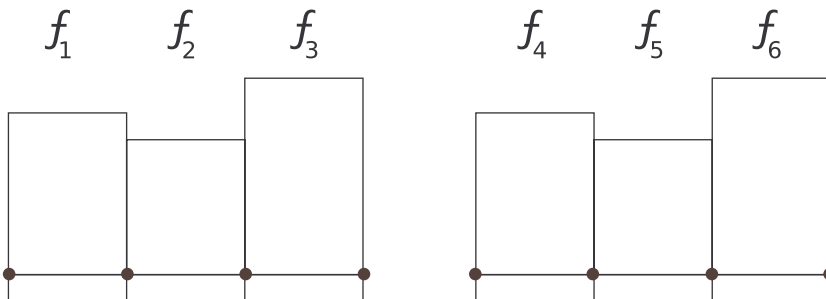


Figure 1: Pulse basis as defined for the example problem.

The three new basis functions are given by the following:

$$\begin{aligned}
 g_1 &= f_1 + \alpha_1 f_4 + \beta_1 f_5 + \gamma_1 f_6 \\
 g_2 &= f_2 + \alpha_2 f_4 + \beta_2 f_5 + \gamma_2 f_6 \\
 g_3 &= f_3 + \alpha_3 f_4 + \beta_3 f_5 + \gamma_3 f_6
 \end{aligned} \tag{5}$$

The weights $\alpha_i, \beta_i,$ and γ_i are chosen such that their net effect is to produce nulls at every point on the second strip (Here we refer to group 2 as the test group). For example, we form g_1 using the following criteria:

$$\begin{aligned}
 Z_{4,1} + \alpha_1 Z_{4,4} + \beta_1 Z_{4,5} + \gamma_1 Z_{4,6} &= 0 \\
 Z_{5,1} + \alpha_1 Z_{5,4} + \beta_1 Z_{5,5} + \gamma_1 Z_{5,6} &= 0 \\
 Z_{6,1} + \alpha_1 Z_{6,4} + \beta_1 Z_{6,5} + \gamma_1 Z_{6,6} &= 0
 \end{aligned} \tag{6}$$

This implies that

$$\begin{bmatrix} \alpha_1 \\ \beta_1 \\ \gamma_1 \end{bmatrix} = - \begin{bmatrix} Z_{4,4} & Z_{4,5} & Z_{4,6} \\ Z_{5,4} & Z_{5,5} & Z_{5,6} \\ Z_{6,4} & Z_{6,5} & Z_{6,6} \end{bmatrix}^{-1} \begin{bmatrix} Z_{4,1} \\ Z_{5,1} \\ Z_{6,1} \end{bmatrix} \tag{7}$$

Solving Eq. (7), we obtain $\alpha_1, \beta_1,$ and γ_1 and generate the new basis function g_1 . In a similar fashion, we form g_2 and g_3 . Note that the inverse matrix is the same for $g_1, g_2,$ and g_3 so that an inverse is taken only once per group. Likewise, we replace $f_4, f_5,$ and f_6 with new functions $g_4, g_5,$ and g_6 to produce nulls on the first strip thereby eliminating $Z_{G_1 G_2}$. We form a new system matrix \tilde{Z} by a source basis change to the newly formed g'_i s. In this case, the elements in \tilde{Z} are simply linear combinations of the original elements of Z . For example, if we wish to generate $\tilde{Z}_{2,1}$, we compute

$$\tilde{Z}_{2,1} = Z_{2,1} + \alpha_1 Z_{2,4} + \beta_1 Z_{2,5} + \gamma_1 Z_{2,6} \tag{8}$$

We are left with a system matrix for the new basis functions:

$$\tilde{Z} = \begin{bmatrix} \tilde{Z}_{G_1 G_1} & 0 \\ 0 & \tilde{Z}_{G_2 G_2} \end{bmatrix} \tag{9}$$

where the off-diagonal blocks are zero by design. In this example, the system may be solved by simply inverting the sub-blocks $\tilde{Z}_{G_1 G_1}$ and $\tilde{Z}_{G_2 G_2}$. In a problem where there are many more groups, it is not necessary to produce nulls on each element of every test group. Only the test points meeting a nearest neighbor criteria must have

nulls since their corresponding matrix elements are large in magnitude. Test points in the other groups may be chosen more sparsely if any are chosen at all. The only requirement is that the group blocks corresponding to the group self terms dominate the remaining terms in their respective block-columns. Since \tilde{Z} can be made highly block-diagonally-dominant, the system will converge quickly when used in an iterative solver.

3 Memory Requirements

There are a few points to note. First, if we only use one iteration for the solver, then the matrix generation and iteration scheme can be streamlined in such a way that the matrix elements are computed only once and then discarded. If the matrix is too large to store in system memory, another approach is to generate one block-row of \tilde{Z} at a time while proceeding through the iterative algorithm. This has the extra computational load of filling the matrix once per iteration. However, if only a few iterations are used, then this method is still much more economical than a full LU decomposition. Also, when generating the coefficients for each group, a matrix inversion is required. If there are n test points, then this matrix will be an $n \times n$ matrix. Thus, the maximum storage necessary is equal to the requirements for the group coefficients plus the largest of the following three quantities in terms of memory usage:

- a) The largest Z_{G_i} given in Eq. (3).
- b) The largest matrix inverted for calculating group coefficients.
- c) If multiple iterations are necessary, the largest block-row in \tilde{Z} .

Note that each one of the quantities requires far less storage than the full system matrix.

4 Analysis of Operations

There are four computationally significant steps to the algorithm.

1. Inverting each matrix in (1)
2. Inverting the group matrices in (3)
3. Creating \tilde{Z}
4. The iterative solution

The cost of steps 1 and 2 depends on the number of testing points as well as the number of groups. If we assume all the group sizes are equal and we let K be the number of basis functions per group and M be the total number of groups, then we have $N = KM$ where N is the total number of basis functions for the structure. For efficiency, step 2 requires that each group matrix be smaller than the system matrix requiring $K \ll N$. The number of test points is determined by the number of groups in the near field range of a source group. This will typically consist of only a few groups. Since all the group sizes are small with respect to N , inverting an individual matrix in step 1 will therefore not be computationally significant unless it is performed too many times. In order to prevent this, we should also force the number of groups to be as small as possible giving us $M \ll N$. Although the optimal choice of K and M will vary from problem to problem (and actually appears to be very flexible), $K = M = \sqrt{N}$ will satisfy the above requirements and seems to be a good rule of thumb since $\sqrt{N} \ll N$ for large bodies.

The number of operations will then be as follows. Inverting each matrix in (1) is $O(K^3)$. Inverting each group matrix in (3) is also $O(K^3)$. The most expensive operation is creating \tilde{Z} . Since this can be viewed as a source basis change, we can write $\tilde{Z} = ZR$ where each column of R has the coefficients for a new source basis in \tilde{Z} . However, R is a sparse matrix and since the group sizes are small with respect to the total number of unknowns, creating \tilde{Z} is an $O(N^2)$ operation. Finally, although the iterative method may vary, here we have used the Gauss-Seidel algorithm, which is computationally insignificant for just a few iterations.

5 Numerical Results

The following two-dimensional examples using both Transverse Magnetic (TM) and Transverse Electric (TE) polarization show the effectiveness of the technique. All closed body solutions utilize the combined-field formulation while open bodies use the electric field formulation for each polarization. Also, both codes use pulse functions for both the source basis and test basis. Finally, the block form of the Gauss-Seidel iterative solver has been used.

Consider a square cylinder with 50λ sides illuminated by a plane wave as shown in Fig. 2. The total contour length for this case is 200λ . Figs. 3 and 4 show the real and imaginary currents for faces 1 and 2 for the TM case. Figs. 5 and 6 show the real and imaginary currents for faces 1 and 2 for the TE case. The amplitude of the incident magnetic field H_o is 1 Amp/m. The angle of incidence for both cases is 35° with respect to x -axis. The contour of the cylinder is divided into 2000 divisions using a 10 divisions per wavelength criterion. Furthermore, the basis functions are collected into 40 groups with 50 basis functions per group. For each source group, the testing is carried out on the two adjacent groups. The initial guess, obtained by

computing currents using only the individual groups, is refined with one iteration of the Gauss-Seidel procedure. The currents are shown to be in agreement with the standard MoM solution.

For the next example, consider a highly complex shaped contour with concavity as shown in Fig. 7. The total circumference is 100λ . The incident wave is at 45° with respect to x -axis with $H_o = 1$. There are 25 groups with 40 sub-domain basis functions in each group. Figs. 8 and 9 show the real and imaginary currents for TM and TE cases, respectively. The numerical results, after one iteration, are compared with standard MoM solution and show good agreement for each case.

Next, we consider a large two-dimensional strip array. Each element is 2λ length and there are 100 collinear elements in the array spaced 0.1λ apart. Each element is a group with 20 segments per group resulting in 100 groups. Testing for a given source group is done on the two adjacent elements. If the element is at either end of the array, then testing is done only on the single adjacent element. The incident wave is normal to the array and has magnitude $H_o = 1$. Figs. 10 (a) and (b) show the bistatic radar cross section with respect to the azimuthal angle for both TM and TE cases, respectively. The results are compared with conventional MoM solution and good agreement is evident in each case.

Finally, we show the real-time results for a 700λ circumference two-dimensional circular cylinder using the new procedure and compare with the conventional MoM method. The scattering case is solved for a 600 MHz incident wave at 180° with respect to x -axis with $H_o = 1$ and TM polarization. The cross section of the cylinder is divided into 7000 edges using a 10 divisions per wavelength criterion. For the conventional MoM solution, the matrix fill and execution times are 12 minutes and 189 minutes, respectively. For the new procedure, 7000 unknowns are divided into 70 groups of 100 unknowns each. In the following table, we provide the computational time for each step involved. The processor for the test machine is a 2.4 GHz Pentium 4.

1	Matrix fill	12 minutes 20 seconds
2	Construction of new basis	16.1 seconds
3	Construction of new system matrix	15.7 seconds
4	Iterative solution (single iteration)	15.8 seconds
5	New method total solution time (Add 1,2,3, and 4)	13 minutes 8 seconds

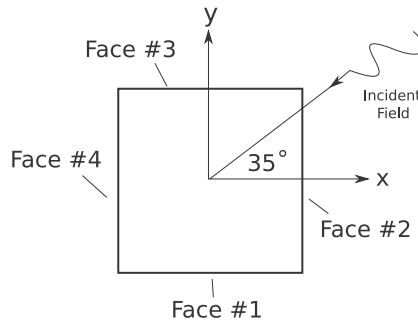


Figure 2: Cross section of the 50λ square cylinder.

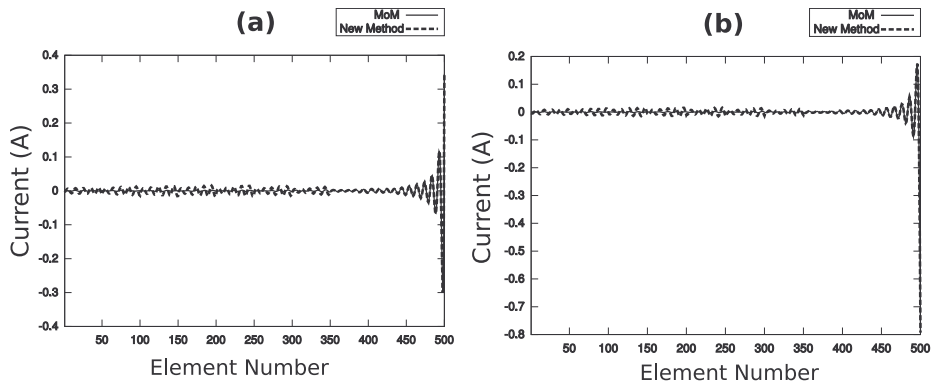


Figure 3: Currents in the shadow region (Face#1) for square cylinder with 50λ sides with TM incident wave $H_0=1$ at 35° . (a) Real part and (b) Imaginary part.

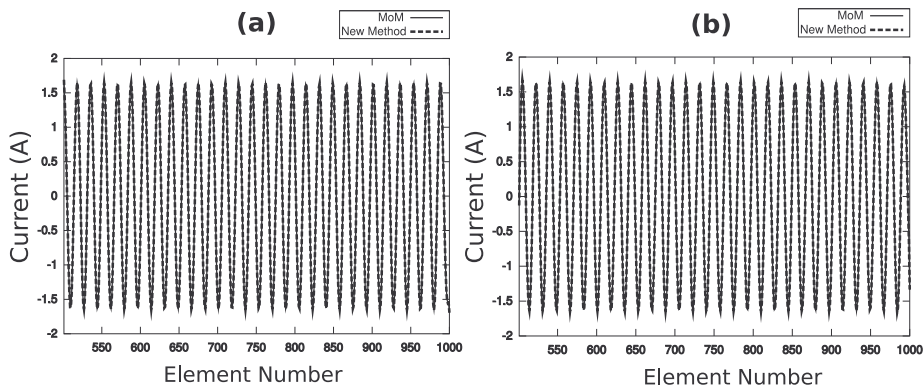


Figure 4: Currents in the lit region (Face #2) for square cylinder with 50λ sides with TM incident wave $H_0=1$ at 35° . (a) Real part and (b) Imaginary part.

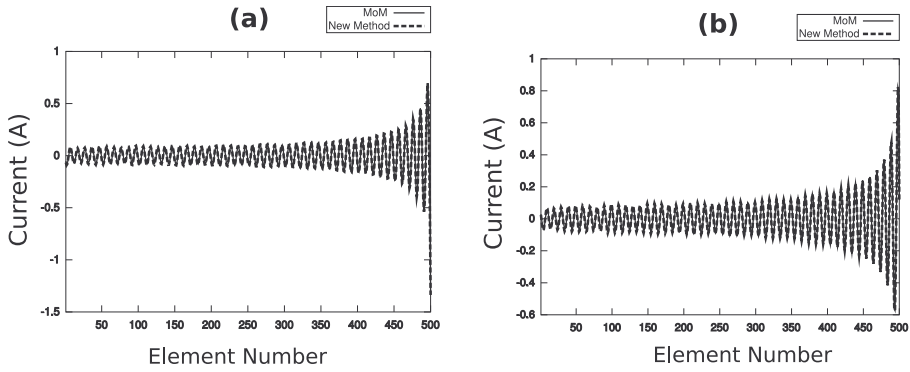


Figure 5: Currents in the shadow region (Face #1) for square cylinder with 50λ sides with TE incident wave $H_0=1$ at 35° . (a) Real part and (b) Imaginary part.

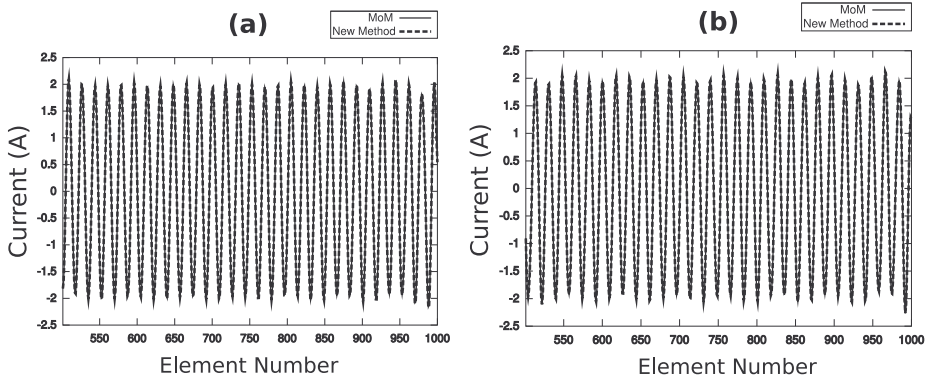


Figure 6: Currents in the lit region (Face #2) for square cylinder with 50λ sides with TE incident wave $H_0=1$ at 35° . (a) Real part and (b) Imaginary part.

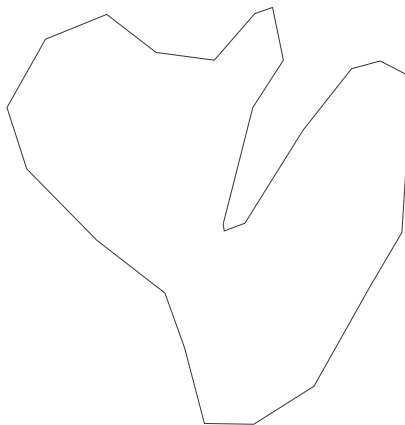


Figure 7: Object with complex shape and concavity.

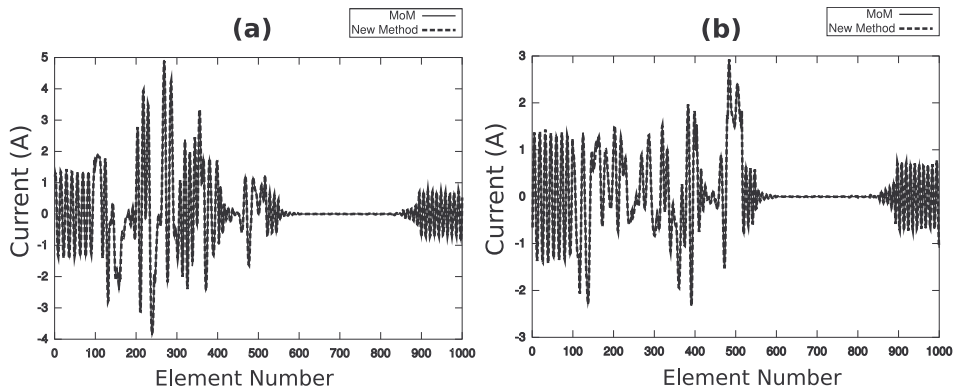


Figure 8: Currents for object with concavity with 100λ total circumference and TM incident wave with $H_0=1$ at 45° . (a) Real part and (b) Imaginary part.

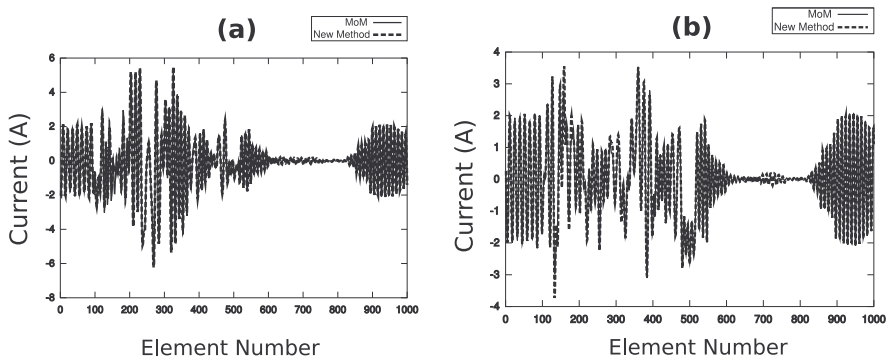


Figure 9: Currents for object with concavity with 100λ total circumference and TE incident wave with $H_0=1$ at 45° . (a) Real part and (b) Imaginary part.

6 Conclusion

The method was shown to drastically reduce the necessary computation as well as storage requirements for 2D MoM codes. Furthermore, the procedure is very simple and can be easily included with existing MoM codes. Further research includes extending the method to arbitrary conductors in three dimensions as well creating a parallel version of the code. Also, it may be worthwhile to experiment with various iterative solution techniques as well as initial guesses in order to fully optimize the overall process.

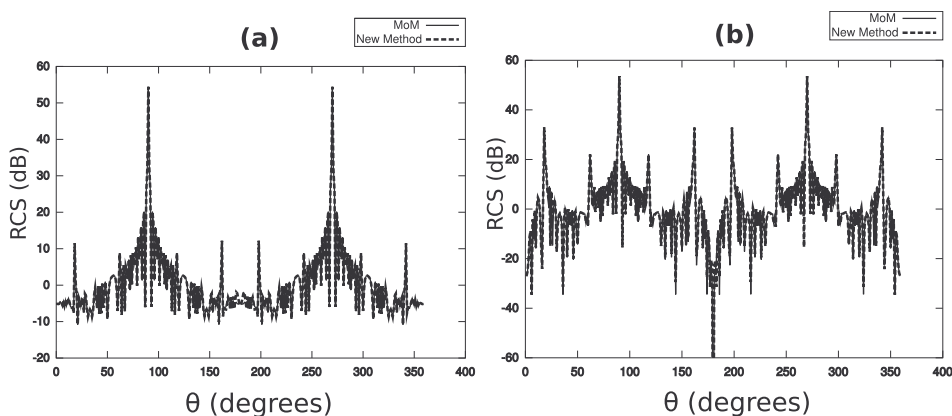


Figure 10: RCS for 2D strip array with 100 2λ elements. The incident wave is normal to the array and $H_0=1$. (a) TM case and (b) TE case.

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