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Linear Interface Crack under Plane Shear Wave

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Abstract: The study is devoted to the problem for a linear crack located between two dissimilar elastic half-spaces under normally incident time-harmonic plane shear wave. The system of boundary integral equations for displacements and tractions is derived from the dynamic Somigliana identity. The distributions of the displacements and tractions at the bonding interface and the surface of the crack are analysed. The dynamic stress intensity factors (the opening and the transverse shear modes) are computed as functions of the frequency of the incident wave for different material properties.

Keywords: Interface Crack, Plane Shear Wave, Boundary Integral Equations, Stress Intensity Factors.

1 Introduction

Understanding the mechanism of dynamic fracture in composite materials becomes more and more important with the increased use of micro- and nano-composites in modern engineering, where the components are frequently subjected to dynamic loadings, see Martin (2006); Garcia-Sanchez and Zhang (2007); Guz, Rushchitsky and Guz (2008); Zhou, Li and Yu (2008); Guz (2009); Wünsche, Zhang, Sladek, Sladek, Hirose and Kuna (2009); Mykhas'kiv, Stankevych, Zhbadynskyi and Zhang (2009); see also classical works by Comninou (1977); Sih and Chen (1980); Rice (1988); Nishimura and Kobayashi (1989); Chow and Atluri (1997).

In general, fracture mechanics problems for cracked solids under dynamic loading can be solved using advanced numerical methods, since the analytical solutions are limited to a very small number of idealized model problems corresponding to special geometrical configurations and loading conditions, see Atluri (1986); Balas, Sladek and Sladek (1989); Aliabadi and Rook (1991); Zhang and Gross (1998). Usually such problems are solved numerically using the finite element or boundary integral methods. The boundary integral approach is more preferable in dynamic

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three-dimensional problems with changing boundary conditions because this approach not only requires a relatively simple discretization of the surface, rather than the volume, but also offers the accuracy that is required for the computation of mechanical quantities such as stress intensity factors, see Nishimura and Kobayashi (1989); Mykhas'kiv, Zhang, Sladek and Sladek (2006); Garcia-Sanchez and Zhang (2007); Tan, Shiah and Lin (2009).

The two-dimensional problem for the linear interface crack under tension-compression wave was solved by Qu (1994). Linear interface cracks under action of the *antiplane* shear wave were considered by Loeber and Sih (1973), Zhang (1991).

The wave scattering from a penny-shaped interface crack in coated materials under normally incident harmonic tension-compression wave was recently considered by Guo (2009). The normal and tangential displacements of the crack faces and the opening mode of the stress intensity factor were obtained and analysed, however, the shear modes of the stress intensity factor were not considered in the study.

In papers by Guz, Menshykov and Menshykov (2006), Menshykov, Guz and Menshykov (2008), the system of boundary integral equations for the general case of an interface crack between two dissimilar elastic materials under dynamic loading was derived. Menshykov, Menshykov and Guz (2007, 2008, 2009) solved the derived integral system numerically by the method of boundary elements for the case of a penny-shaped interface crack under normally incident tension-compression and shear waves. The distributions of displacements and tractions on the bonding interface and the surface of the crack were computed for several typical materials of half-spaces. The dynamic stress intensity factors (opening and shear modes) were computed as functions of the frequency of the incident wave. It was also shown that with decreasing frequency of the loading the dynamic solution tends to the static one, and the obtained numerical results are in a very good agreement with the static solutions by Mossakovskii and Rybka (1964), Goldstein and Vainshelbaum (1976), Kilic, Madenci and Mahajan (2006).

In order to illustrate the methodology developed in [Guz, Menshykov and Menshykov (2006), Menshykov, Guz and Menshykov (2008), Mykhailova, Menshykov, Menshykova and Guz (2009)], the current paper studies the elastodynamic problem for the linear interface crack located between two dissimilar elastic half-spaces under normally incident time-harmonic *plane* shear wave.

2 Problem statement

Let us consider a crack located at the bimaterial interface under external dynamic loading. For this purpose, we investigate an unbounded elastic solid which consists of two dissimilar homogeneous isotropic half-spaces $\Omega^{(1)}$ and $\Omega^{(2)}$. The interface between the half-spaces acts as the boundary $\Gamma^{(1)}$ for the upper half-space, and the boundary $\Gamma^{(2)}$ for the lower half-space. The boundaries $\Gamma^{(1)}$ and $\Gamma^{(2)}$ differ by the opposite orientation of their outer normal vectors. Henceforth, the superscript (1) refers to the upper half-space and the superscript (2) refers to the lower half-space. We assume that surfaces $\Gamma^{(m)} = \Gamma^{(m)*} \cup \Gamma^{(m)\mathrm{cr}}$ $(m = 1, 2)$ consist of the infinite part $\Gamma^{(m)*}$ and the finite part $\Gamma^{(m)\mathrm{cr}}$. Thus the bonding interface and the surface of the crack are

$$\Gamma^* = \Gamma^{(1)} \cap \Gamma^{(2)}, \quad \Gamma^{\mathrm{cr}} = \Gamma^{(1)\mathrm{cr}} \cup \Gamma^{(2)\mathrm{cr}}. \tag{1}$$

A Cartesian coordinate system with the origin located in the centre of the crack is used in order to reference the geometrical positions of the material points.

In the absence of body forces, the stress-strain state of both domains is defined by the dynamic equations of the linear elasticity for the displacement vector $\mathbf{u}^{(m)}(\mathbf{x}, t)$ (the Lamé equations)

$$(\lambda^{(m)} + \mu^{(m)}) \operatorname{grad} \operatorname{div} \mathbf{u}^{(m)}(\mathbf{x}, t) + \mu^{(m)} \Delta \mathbf{u}^{(m)}(\mathbf{x}, t) = \rho^{(m)} \partial_t^2 \mathbf{u}^{(m)}(\mathbf{x}, t),$$
$$\mathbf{x} \in \Omega^{(m)},\ t \in \mathrm{T} = [0, \infty), \tag{2}$$

where Δ is the Laplace operator, $\lambda^{(m)}$and $\mu^{(m)}$ are the Lamé elastic constants, $\rho^{(m)}$ is the specific material density.

The following conditions of continuity for displacements and stresses are satisfied at the bonding interface:

$$\mathbf{u}^{(1)}(\mathbf{x}, t) = \mathbf{u}^{(2)}(\mathbf{x}, t), \quad \mathbf{p}^{(1)}(\mathbf{x}, t) = -\mathbf{p}^{(2)}(\mathbf{x}, t), \quad \mathbf{x} \in \Gamma^*, \quad t \in \mathrm{T}, \tag{3}$$

where the known traction vectors on the crack surface, caused by the external loading, are given as

$$\mathbf{p}^{(1)}(\mathbf{x}, t) = \mathbf{g}^{(1)}(\mathbf{x}, t), \quad \mathbf{p}^{(2)}(\mathbf{x}, t) = \mathbf{g}^{(2)}(\mathbf{x}, t), \quad \mathbf{x} \in \Gamma^{(m)cr}, \quad t \in \mathrm{T}. \tag{4}$$

It was assumed that there are no initial displacements of the points of the body, and the Sommerfeld radiation-type condition, which provides a finite elastic energy of an infinite body, was imposed at infinity on the vector of displacements.

3 Boundary integral equations

The detailed procedure for deriving the system of boundary integral equations was given by Guz, Menshykov, Menshykov (2006), Menshykov, Guz, Menshykov (2008), where the components of the displacement field in the upper and the lower half-spaces were represented in terms of boundary displacements and traction using the Somigliana dynamic identity.

For the case of harmonic loading with the frequency $\omega = 2\pi/T$, which is considered in the paper, tractions and displacements are harmonic functions and can be presented as follows:

$$f(\bullet,t) = Re(f(\bullet)e^{i\omega t}), \quad f(\bullet) = \frac{\omega}{2\pi}\int_0^T f(\bullet,t)e^{-i\omega t}dt, \tag{5}$$

Then the following representation of the displacement vector at the boundary interface can be obtained from the Somigliana dynamic identity:

$$\frac{1}{2}u_j^{(m)}(\mathbf{x}) = \int_{\Gamma^{(m)}} p_i^{(m)}(\mathbf{y})U_{ij}^{(m)}(\mathbf{x},\mathbf{y},\omega)d\mathbf{y} - \int_{\Gamma^{(m)}} -u_i^{(m)}(\mathbf{y})W_{ij}^{(m)}(\mathbf{x},\mathbf{y},\omega)\,d\mathbf{y},$$
$$\mathbf{x} \in \Gamma^{(m)},\ t \in T,\ j = \overline{1,2} \tag{6}$$

Here $p_i^{(m)}(\mathbf{x})$ and $u_i^{(m)}(\mathbf{x})$ are components of complex-valued magnitudes of tractions and displacements at the interface. The physical values of tractions and displacements are obtained by multiplying the magnitudes by the exponential function, Eq. (5).

The fundamental solution in the frequency domain $W_{ij}^{(m)}(\mathbf{x},\mathbf{y},\omega)$ can be obtained from the Green fundamental displacement tensor $U_{ij}^{(m)}(\mathbf{x},\mathbf{y},\omega)$ by applying the following differential operator:

$$P_{ik}[\bullet,(\mathbf{y})] = \lambda n_i(\mathbf{y})\frac{\partial[\bullet]}{\partial y_k} + \mu\left[\delta_{ik}\frac{\partial[\bullet]}{\partial \mathbf{n}(\mathbf{y})} + n_k(\mathbf{y})\frac{\partial[\bullet]}{\partial y_i}\right]. \tag{7}$$

The tensor $U_{ij}^{(m)}(\mathbf{x},\mathbf{y},\omega)$ has the form:

$$U_{ij}^{(m)}(\mathbf{x},\mathbf{y},\omega) = \frac{1}{2\pi\mu^{(m)}}\left(\psi^{(m)}\delta_{ij} - \chi^{(m)}\frac{(y_i - x_i)}{r}\frac{(y_j - x_j)}{r}\right), \tag{8}$$

where δ_{ij} is the Kronecker delta, r is the distance between the observation point and the load point, and functions $\psi^{(m)}$ and $\chi^{(m)}$ are

$$\psi^{(m)} = K_0(l_2^{(m)}) + \frac{1}{l_2^{(m)}}\left[K_1(l_2^{(m)}) - \frac{c_2^{(m)}}{c_1^{(m)}}K_1(l_1^{(m)})\right], \tag{9}$$

$$\chi^{(m)} = K_2(l_2^{(m)}) - \left(\frac{c_2^{(m)}}{c_1^{(m)}}\right)^2 K_2(l_1^{(m)}), \tag{10}$$

where $K_n(\bullet)$ is the modified Bessel function of the second kind and order n [Abramowitz, Stegun (1964)]; $l_1^{(m)} = i\omega\, r/c_1^{(m)}, l_2^{(m)} = i\omega\, r/c_2^{(m)}$, $c_1^{(m)} = \sqrt{(\lambda^{(m)} + 2\mu^{(m)})/\rho^{(m)}}$ and $c_2^{(m)} = \sqrt{\mu^{(m)}/\rho^{(m)}}$ are the velocities of the longitudinal and the transversal waves in the material.

Thus, the expression for $W_{ij}^{(m)}(\mathbf{x},\mathbf{y},\omega)$ is:

$$W_{ij}^{(m)}(\mathbf{x},\mathbf{y},\omega) = \lambda^{(m)} n_i^{(m)}(\mathbf{y}) \frac{\partial}{\partial y_k} U_{kj}^{(m)}(\mathbf{x},\mathbf{y},\omega) + \mu^{(m)} n_k^{(m)}(\mathbf{y}) \left[\frac{\partial}{\partial y_k} U_{ji}^{(m)}(\mathbf{x},\mathbf{y},\omega) + \frac{\partial}{\partial y_i} U_{jk}^{(m)}(\mathbf{x},\mathbf{y},\omega)\right]. \tag{11}$$

For the considered case, when $\mathbf{n}(\mathbf{x}) = \mathbf{n}(\mathbf{y}) = (0,1)$, the kernels become:

$$U_{12}^{(m)}(\mathbf{x},\mathbf{y},\omega) = U_{21}^{(m)}(\mathbf{x},\mathbf{y},\omega) = 0, \tag{12}$$

$$U_{11}^{(m)}(\mathbf{x},\mathbf{y},\omega) = \frac{1}{2\pi\mu^{(m)}} \left[K_0(l_2^{(m)}) + \frac{1}{l_2^{(m)}}\left(K_1(l_2^{(m)}) - \frac{c_2^{(m)}}{c_1^{(m)}} K_1(l_1^{(m)})\right)\right], \tag{13}$$

$$U_{22}^{(m)}(\mathbf{x},\mathbf{y},\omega) = \frac{1}{2\pi\mu^{(m)}} \left[\left(\frac{c_2^{(m)}}{c_1^{(m)}}\right)^2 K_2(l_1^{(m)}) - K_2(l_2^{(m)}) + K_0(l_2^{(m)}) + \frac{1}{l_2^{(m)}}\left(K_1(l_2^{(m)}) - \frac{c_2^{(m)}}{c_1^{(m)}} K_1(l_1^{(m)})\right)\right], \tag{14}$$

$$W_{11}^{(m)}(\mathbf{x},\mathbf{y},\omega) = W_{22}^{(m)}(\mathbf{x},\mathbf{y},\omega) = 0, \tag{15}$$

$$W_{12}^{(m)}(\mathbf{x},\mathbf{y},\omega) = \frac{1}{2\pi r}\frac{\partial r}{\partial y_1}\left[l_2^{(m)} K_1(l_2^{(m)}) - 2K_2(l_2^{(m)}) + 2\left(\frac{c_2^{(m)}}{c_1^{(m)}}\right)^2 K_2(l_1^{(m)})\right], \tag{16}$$

$$W_{21}^{(m)}(\mathbf{x},\mathbf{y},\omega) = \frac{1}{2\pi r}\frac{\partial r}{\partial y_1}\left[-\frac{\lambda^{(m)}\mu^{(m)}}{(\lambda^{(m)} + 2\mu^{(m)})^2} l_1^{(m)} K_1(l_1^{(m)}) - 2K_2(l_2^{(m)}) + 2\left(\frac{c_2^{(m)}}{c_1^{(m)}}\right)^2 K_2(l_1^{(m)})\right]. \tag{17}$$

From Eq. (6) the following system of boundary integral equations for the coefficients of displacements and tractions at the bonding interface and the crack faces can be obtained [see also Mykhailova, Menshykov, Menshykova and Guz (2009)]:

$$-\int_{\Gamma^{(1)\mathrm{cr}}} g_i^{(1)}(\mathbf{y})U_{ij}^{(1)}(\mathbf{x},\mathbf{y},\omega)\,d\mathbf{y} = -\frac{1}{2}u_i^{(1)}(\mathbf{x}) - \int_{\Gamma^{(1)\mathrm{cr}}} u_i^{(1)}(\mathbf{y})W_{ij}^{(1)}(\mathbf{x},\mathbf{y},\omega)\,d\mathbf{y} + \int_{\Gamma^*} u_i^*(\mathbf{y})W_{ij}^{(1)}(\mathbf{x},\mathbf{y},\omega)\,d\mathbf{y} - \int_{\Gamma^*} p_i^*(\mathbf{y})U_{ij}^{(1)}(\mathbf{x},\mathbf{y},\omega)d\mathbf{y}, \quad \mathbf{x}\in\Gamma^{(1)\mathrm{cr}}, \tag{18}$$

$$-\int_{\Gamma^{(2)\mathrm{cr}}} g_i^{(2)}(\mathbf{y})U_{ij}^{(2)}(\mathbf{x},\mathbf{y},\omega)\,d\mathbf{y} = -\frac{1}{2}u_i^{(2)}(\mathbf{x}) - \int_{\Gamma^{(2)\mathrm{cr}}} u_i^{(2)}(\mathbf{y})W_{ij}^{(2)}(\mathbf{x},\mathbf{y},\omega)\,d\mathbf{y} - \int_{\Gamma^*} u_i^*(\mathbf{y})W_{ij}^{(2)}(\mathbf{x},\mathbf{y},\omega)\,d\mathbf{y} + \int_{\Gamma^*} p_i^*(\mathbf{y})U_{ij}^{(2)}(\mathbf{x},\mathbf{y},\omega)d\mathbf{y}, \quad \mathbf{x}\in\Gamma^{(2)\mathrm{cr}}, \tag{19}$$

$$-\int_{\Gamma^{(1)\mathrm{cr}}} g_i^{(1)}(\mathbf{y})U_{ij}^{(1)}(\mathbf{x},\mathbf{y},\omega)\,d\mathbf{y} = -\frac{1}{2}u_i^{*}(\mathbf{x}) - \int_{\Gamma^{(1)\mathrm{cr}}} u_i^{(1)}(\mathbf{y})W_{ij}^{(1)}(\mathbf{x},\mathbf{y},\omega)\,d\mathbf{y} + \int_{\Gamma^*} u_i^*(\mathbf{y})W_{ij}^{(1)}(\mathbf{x},\mathbf{y},\omega)\,d\mathbf{y} - \int_{\Gamma^*} p_i^*(\mathbf{y})U_{ij}^{(1)}(\mathbf{x},\mathbf{y},\omega)d\mathbf{y}, \quad \mathbf{x}\in\Gamma^{*}, \tag{20}$$

$$-\int_{\Gamma^{(2)\mathrm{cr}}} g_i^{(2)}(\mathbf{y})U_{ij}^{(2)}(\mathbf{x},\mathbf{y},\omega)\,d\mathbf{y} = -\frac{1}{2}u_i^{*}(\mathbf{x}) - \int_{\Gamma^{(2)\mathrm{cr}}} u_i^{(2)}(\mathbf{y})W_{ij}^{(2)}(\mathbf{x},\mathbf{y},\omega)\,d\mathbf{y} - \int_{\Gamma^*} u_i^*(\mathbf{y})W_{ij}^{(2)}(\mathbf{x},\mathbf{y},\omega)\,d\mathbf{y} + \int_{\Gamma^*} p_i^*(\mathbf{y})U_{ij}^{(2)}(\mathbf{x},\mathbf{y},\omega)d\mathbf{y}, \quad \mathbf{x}\in\Gamma^{*}. \tag{21}$$

Here $g_i^{(m)}(\mathbf{x})$, $p_i^*(\mathbf{x})$, $u_i^*(\mathbf{x})$ and $u_i^{(m)}(\mathbf{x})$ are components of complex-valued magnitudes of known tractions at the crack surface, unknown tractions and displacements at the interface and the crack surface. The physical values of tractions and displacements are obtained by multiplying the magnitudes by the exponential function.

In the present study the form of the resulting boundary integral equations system (18)–(21) differs significantly from the corresponding integral system in Guz, Menshykov, Menshykov (2006); Menshykov, Guz, Menshykov (2008). It becomes simpler and does not contain integral kernels $K_{ij}^{(m)}(\mathbf{x},\mathbf{y},\omega)$ and $F_{ij}^{(m)}(\mathbf{x},\mathbf{y},\omega)$. The additional algebraic manipulations (i.e. summation and subtraction of the integral equations) are also omitted, therefore the system of boundary integral equations does not contain residuals of all integral kernels and the corresponding matrix of linear algebraic equations becomes sparser, which results in the significant acceleration of the numerical solution of the problem. On average, the solution time decreases by 40–50% [see also Mykhailova, Menshykov, Menshykova and Guz (2009)]. The robustness of the algorithm also increases, especially for higher frequencies of the external loading.

Note that due to the presence of singular terms in the integral kernels $W_{ij}^{(m)}(\mathbf{x},\mathbf{y},\omega)$, the corresponding singular integrals of the system of boundary integral equations (18)–(21) are treated in the sense of the Cauchy principal value, see Han and Atluri (2007); Menshykov, Menshykova and Guz (2008); Vavourakis (2008); Li, Wu and Yu (2009).

4 Numerical results

As a numerical example let us consider a linear crack with length of $2R$ under the normally incident time-harmonic plane shear wave of the unit intensity.

The piecewise-constant approximation of the known and unknown functions was used to solve the problem numerically. Note that the solution is symmetric with respect to the centre of the crack, and the conditions of continuity for displacements and stresses are satisfied at the interface. The Sommerfeld radiation-type condition is satisfied at the infinity, i.e. the displacements at the bonding interface decrease gradually with increase in the distance to the crack.

The normal and tangential components of the displacement discontinuity at the crack surface, $\Delta\mathbf{u}(\mathbf{x},t) = \mathbf{u}^{(1)}(\mathbf{x},t) - \mathbf{u}^{(2)}(\mathbf{x},t)$ (maximal values within the oscillation period) are given in Figs. 1 and 2.

The given displacements discontinuities are normalised by the factor $2\mu_0/R\sigma_0$, where σ_0 is the stress amplitude of the incident wave and the factor μ_0 was specified as follows [Kilic, Madenci and Mahajan (2006)]:

$$\mu_0 = \mu^{(1)} \frac{1-\gamma_2}{1+\kappa^{(1)}}, \quad \gamma_2 = \left(\frac{a_1}{2} - a_2\right),$$
$$a_1 = \frac{\mu^{(1)} - \mu^{(2)}}{\mu^{(1)} + \kappa^{(1)}\mu^{(2)}}, \quad a_2 = \frac{\kappa^{(1)}\mu^{(2)} - \kappa^{(2)}\mu^{(1)}}{2(\mu^{(2)} + \kappa^{(2)}\mu^{(1)})}, \quad \kappa^{(m)} = 3 - 4\nu^{(m)}. \tag{22}$$

The dynamic stress intensity factors (the opening and the transverse shear modes) were computed in the vicinity of the crack tip using the following asymptotic formulas [Loeber and Sih (1973); Balas, Sladek and Sladek (1989); Aliabadi and Rook (1991)]:

$$K_I^{\max} = \max_t \lim_{r\to 0} p_n^*(R+r,t)\sqrt{2\pi r}, \tag{23}$$

$$K_{II}^{\max} = \max_t \lim_{r\to 0} p_\tau^*(R+r,t)\sqrt{2\pi r}, \tag{24}$$

$$K_I^{\max} = \max_t \lim_{r\to 0} \frac{\mu}{4(1-\nu)} \sqrt{\frac{2\pi}{r}} \Delta u_n(R-r,t), \tag{25}$$

$$K_{II}^{\max} = \max_t \lim_{r\to 0} \frac{\mu}{4(1-\nu)} \sqrt{\frac{2\pi}{r}} \Delta u_\tau(R-r,t). \tag{26}$$

Here $p_n^*(R+r,t)$, $p_\tau^*(R+r,t)$, $\Delta u_n(R-r,t)$, $\Delta u_\tau(R-r,t)$ are the normal and tangential components of the traction vector at the bonding interface, and the displacement discontinuity vector at the crack surface; r is the distance from the crack tip.

The distributions of the dynamic stress intensity factors are given in Figs. 3–8 as functions of the dimensionless wave number $k_2^{(1)}R = \omega R/c_2^{(1)}$. The ratios of the shear moduli and the material densities are used as variables. The results are normalised by the corresponding static values.

In Figs. 3 and 4 the transverse shear mode of the stress intensity factor (which is the dominant mode in the considered problem) is given for different meshes, which can be characterised by the length of the boundary element near the crack tip, $h = R/2^n$. The results were obtained using the tractions distribution at the bonding interface, Eqs. (23) and (24) (see Fig. 3), and the displacements of the crack faces, Eqs. (25) and (26) (see Fig. 4).

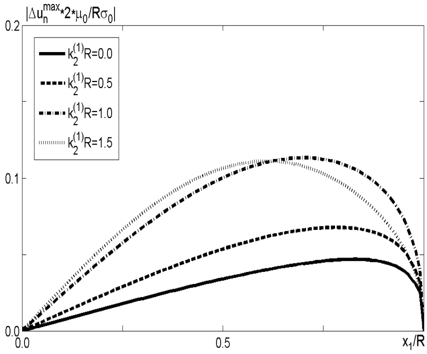

Figure 1: Normal displacement discontinuity at the crack surface, $\rho^{(1)} = \rho^{(2)}$, $\mu^{(1)} = 2\mu^{(2)}$

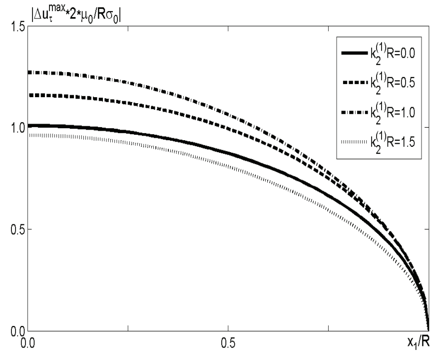

Figure 2: Tangential displacement discontinuity at the crack surface, $\rho^{(1)} = \rho^{(2)}$, $\mu^{(1)} = 2\mu^{(2)}$

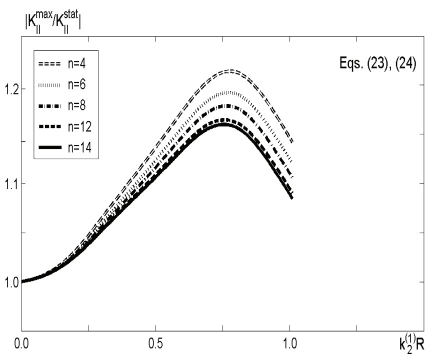

Figure 3: Stress intensity factor (the transverse shear mode) plotted against the wave number, $\rho^{(1)} = \rho^{(2)}$, $\mu^{(1)} = 2\mu^{(2)}$

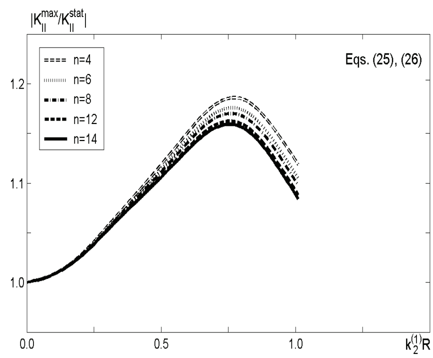

Figure 4: Stress intensity factor (the transverse shear mode) plotted against the wave number, $\rho^{(1)} = \rho^{(2)}$, $\mu^{(1)} = 2\mu^{(2)}$

As can be seen the solution of the problem demonstrates a practical convergence, i.e. with the increase in the number of boundary elements (and with the appropriate decrease in the size of the boundary elements) the distributions of the stress intensity factors tend to certain limiting distributions.

In Figs. 5-8 the stress intensity factors (the opening and the transverse shear mode) are given for different ratios of shear moduli, $\mu^{(2)}/\mu^{(1)}$. The results were obtained for the mesh with $h = R/2^{14} \approx 6.1 * 10^{-5}R$.

Note that in the considered range of the wave number, the opening and the shear modes of the stress intensity factors are unimodal functions. Stress intensity factors increase gradually with the increasing wave number, up until the maximal values are achieved, and decrease gradually after that.

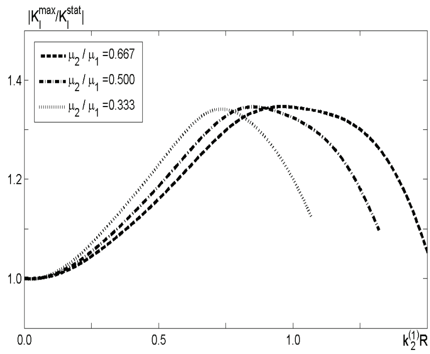

Figure 5: Stress intensity factor (the opening mode) plotted against the wave number, $\rho^{(1)} = \rho^{(2)}$, $\mu^{(2)}/\mu^{(1)} < 1$

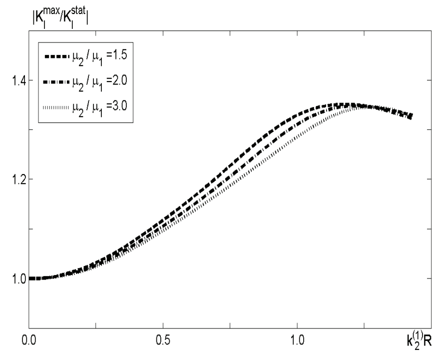

Figure 6: Stress intensity factor (the opening mode) plotted against the wave number, $\rho^{(1)} = \rho^{(2)}$, $\mu^{(2)}/\mu^{(1)} > 1$

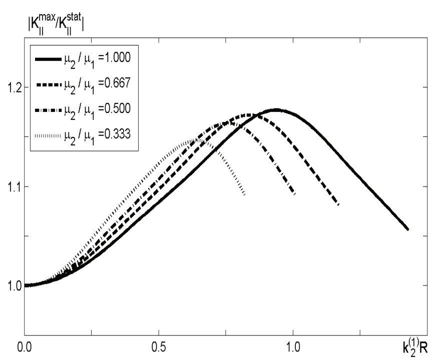

Figure 7: Stress intensity factor (transverse shear mode) plotted against the wave number, $\rho^{(1)} = \rho^{(2)}$, $\mu^{(2)}/\mu^{(1)} < 1$

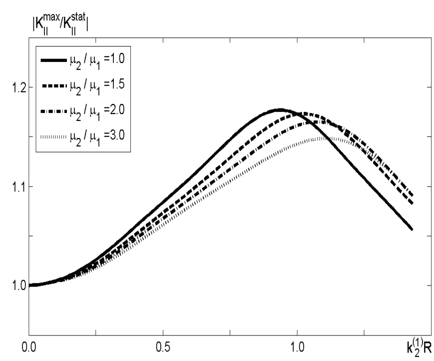

Figure 8: Stress intensity factor (transverse shear mode) plotted against the wave number, $\rho^{(1)} = \rho^{(2)}$, $\mu^{(2)}/\mu^{(1)} > 1$

Add that the peaks of all curves move towards higher wave numbers if $\mu^{(2)}/\mu^{(1)} > 1$ and toward lower wave numbers if $\mu^{(2)}/\mu^{(1)} < 1$. The magnitudes of the peaks decrease for $\mu^{(2)}/\mu^{(1)}$ diverging from unity.

The results obtained for the case of an interface crack located between identical materials (solid curves, Figs. 7 and 8) coincide with the results for a homogeneous cracked body [Atluri (1986), Aliabadi and Rook (1991)]. The results obtained here for the elastodynamics problem are in a very good agreement with the static solution. As the wave number approaches zero, dynamic stress intensity factors approach the static ones.

5 Conclusions

In this study the boundary integral equations method was applied to the problem of an interface crack between two dissimilar elastic half-spaces under harmonic loading. The distributions of the displacements and tractions at the bimaterial interface were obtained and analysed. The stress intensity factors (the opening and the transverse shear modes) were computed as functions of the wave number.

Add also that, in reality, the opposite crack faces under dynamic loading interact with each other, significantly changing the stress and strain fields near the crack tips. However, since the area of interest is hidden in the solid, the direct observation and measurement of the contact characteristics is impossible. The nature of the contact interaction between two opposite crack surfaces is very complex. Under deformation of the material, the contact area changes in time. It is unknown beforehand and must be determined as a part of solution. The complexity of the problem is further compounded by the fact that the contact behaviour is very sensitive to the material properties of two contacting surface, frequency, magnitude and direction of the external loading [Guz and Zozulya (2007); Guz, Menshykov, Zozulya and Guz (2007); Menshykov, Menshykova and Guz (2008), Zozulya (2009)]. Taking these effects into account will make the contact crack problem highly non-linear. Thus considering the crack closure effect is the natural next stage of this research.

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