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An Accurate Algorithm for Evaluating Radiative Heat Transfer in a Randomly Packed Bed

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Abstract: Motivated by Hottel's crossed-string method, this paper presents an accurate algorithm for the evaluation of the geometric view factors in a randomly packed bed of circular particles of various sizes. The radiative heat exchange can thus be predicted accurately. The solution procedure is illustrated and the solution accuracy is assessed via a numerical example.

Keywords: radiative heat transfer, geometric view factors, randomly packed bed, Hottel's crossed-string method.

1 Introduction

Heat transfer in a randomly packed bed can be found in many industrial applications. For instance, in the nuclear industry the process of a pebble bed nuclear reactor essentially involves the forced flow of gas through uranium enriched spherical pebbles that are cyclically fed through a concentric column in order to extract thermal energy. Heat transfer occurs between the moving particles and the gas in the form of conduction, convection, and radiation. As the temperature in the bed is extremely high, the radiative heat transfer is significant.

In addition to the experimental studies, mathematical modelling has become increasingly important as a powerful tool to better understand and predict the heat transfer mechanism in a packed bed. Among the approaches proposed, the continuum-based methods (see for instance, Van der Held 1952, Chen and Churchill 1963, Harmaker 1947) considered a packed bed as a pseudo homogeneous material, and the heat transfer process is described by differential equations; whilst the stochastic-based approaches such as ray tracing and Monte Carlo methods (Yang *et al* 1983, Argento and Bouvard 1996, Zedtwitz *et al* 2007) determine the radiative properties from the forward and backward fluxes measured inside a packed bed. These approaches are mainly of an approximation nature.

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One of the difficulties in evaluating the radiative heat transfer in a randomly packed bed is the determination of the geometric relations for how the solid particles view each other, or the so-called *geometric view factors*. Because of the presence of other particles that may (partially) block the two particles under consideration, straightforward analytical integration methods often become too cumbersome or impossible.

Motivated by the idea of Hottel's crossed-string method (Siegel and Howell 1992), an accurate algorithm is proposed in this work to evaluate the geometric view factors in a randomly packed bed of circular particles of various sizes so that the radiative heat exchange can be accurately predicted. This will form an integrated part of our ongoing project (Feng *et al* 2007, Feng *et al* 2008, Feng *et al* 2009, Han *et al* 2007) that aims to develop a comprehensive computational framework to model multi-physical phenomena involving fluid-thermal-particle interactions.

2 Geometric View Factors for Two Finite Surfaces

2.1 Analytical method

Consider two finite surfaces as illustrated in Fig. 1. The areas A_1 and A_2 at temperatures T_1 and T_2 are arbitrarily oriented, and have their normals at angles θ_1 and θ_2 to the line of length S joining them. The geometric view factor F_{1-2} is defined as the fraction of radiation energy leaving the diffuse surface 1 that reaches the diffuse surface 2, and is evaluated by (Siegel and Howell 1992)

$$F_{1-2} = \frac{1}{A_1}\int_{A_1}\int_{A_2}\frac{\cos(\theta_1)\cos(\theta_2)}{\pi S^2}dA_2 dA_1 \tag{1}$$

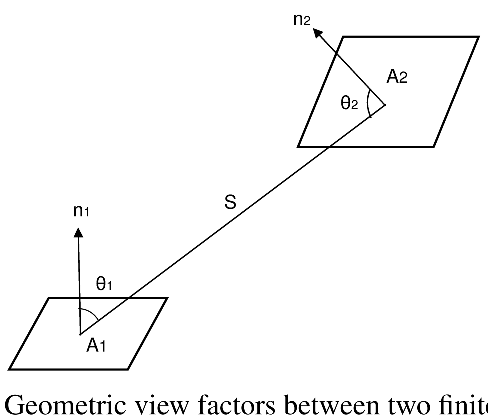


Figure 1: Geometric view factors between two finite surfaces

Similarly, the geometric view factor from surface 2 to surface 1 is calculated as

$$F_{2-1} = \frac{1}{A_2} \int_{A_1} \int_{A_2} \frac{\cos(\theta_1)\cos(\theta_2)}{\pi S^2} dA_2 dA_1 \tag{2}$$

The reciprocity relation for geometric view factors between two finite areas is found from the fact that the double integrals in Eqs.(1) and (2) are identical, therefore

$$A_1 F_{1-2} = A_2 F_{2-1} \tag{3}$$

As mentioned earlier, the straightforward integration of Eq. (1) or Eq. (2) is possible only for very few simple geometric configurations. In view of a number of mathematical techniques available for the evaluation of geometric view factors, Hottel's crossed-string method may be employed to develop an accurate solution for a randomly packed bed of circular particles.

2.2 Hottel's crossed-string method

To explain Hottel's crossed-string method, an expression needs to be derived first for the geometric view factor between any two of the plates in an enclosure of a triangular cross section. The enclosure is made up of three plane plates, each of finite width and infinite length, thus forming a hollow infinitely long triangular prism.

Since the plates are planer, they cannot view themselves, i.e. $F_{i-i} = 0, (i = 1, 2, 3)$. Therefore, the following relations hold

$$F_{1-2} + F_{1-3} = 1$$

$$F_{2-1} + F_{2-3} = 1$$

$$F_{3-1} + F_{3-2} = 1$$

Multiplying each equation above by the respective plate area results in

$$A_1 F_{1-2} + A_1 F_{1-3} = A_1$$

$$A_2 F_{2-1} + A_2 F_{2-3} = A_2$$

$$A_3 F_{3-1} + A_3 F_{3-2} = A_3$$

By applying the reciprocal relations to some of the terms, the three equations become

$$A_1 F_{1-2} + A_1 F_{1-3} = A_1$$

$A_1F_{1-2} + A_2F_{2-3} = A_2$

$A_1F_{1-3} + A_2F_{2-3} = A_3$

F_{1-2} is obtained by solving the above equations as

$$F_{1-2} = \frac{A_1 + A_2 - A_3}{2A_1} = \frac{L_1 + L_2 - L_3}{2L_1} \tag{4}$$

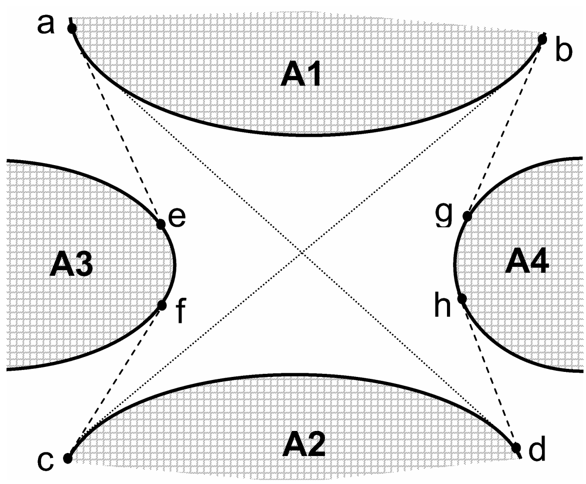


Figure 2: Hottel's crossed-string method

Now consider the geometric view factors between two surfaces A_1 and A_2 that are partially blocked by two other surfaces A_3 and A_4, as shown in Fig. 2. Draw dashed lines $aefc$ and $bghd$ that represent strings tightly stretched between the outer edges of the four surfaces with points e, f, g, h as the tangents between surfaces A_1 and A_3, A_3 and A_2, A_1 and A_4, and A_4 and A_2. Also draw two crossed strings ad and bc. The enclosure $abghda$ is composed of three sides that are either convex or planar. Then the relation in Eq.(4) can be employed to obtain the following expression

$$A_1F_{1-bghd} = \frac{A_1 + A_{bghd} - A_{ad}}{2} \tag{5}$$

Similarly, for the three-sided enclosure $aefcda$, we have

$$A_1F_{1-aefc} = \frac{A_1 + A_{aefc} - A_{bc}}{2} \tag{6}$$

Furthermore,

$$F_{1-2} + F_{1-aefc} + F_{1-bghd} = 1 \tag{7}$$

Substituting Eqs.(5) and (6) into (7) gives

$$A_1 F_{1-2} = \frac{A_{ad} + A_{bc} - A_{aefc} - A_{bghd}}{2} \tag{8}$$

This is a special case of Hottel's crossed-string method (Siegel and Howell 1992), which states that the geometric view factor from surface A_1 to surface A_2 equals the difference of the sum of the crossed string lengths and the sum of the uncrossed string lengths, divided by twice the circumference of surface 1. The significance of this method implies that the geometric view factors between two surfaces can be obtained by simply constructing the four strings, followed by calculating their lengths, which completely avoids the evaluation of the double integrals in Eq. (1).

3 Geometric View Factors for a Packed Bed of Circular Particles

Hottel's crossed-string method described above can now be applied to develop an approach for the evaluation of the geometric view factors for a packed bed of circular particles of various sizes.

In the simplest case, consider the geometric view factors between two circular particles without view blockage, as shown in Fig. 3. Let R_1 and R_2 be the radii of the two particles, and $A_1 = 2\pi R_1$ and $A_2 = 2\pi R_2$ be their circumferences, respectively. Draw two uncrossed strings, L_1 and L_4 which are the outer common tangents of the two particles, and two crossed strings, L_2 and L_3, which are the inner common tangents plus the two arcs. Based on Hottel's crossed-string rule, the geometric view factor from particle 1 to particle 2 can be expressed as

$$F_{1-2} = \frac{L_2 + L_3 - L_1 - L_4}{2A_1} \tag{9}$$

Now consider a general case in a randomly packed bed of different sized particles, as shown in Fig. 4. This example will be used to illustrate the algorithm to be developed.

For particles 1 and 2, the two uncrossed strings (outer common tangents) together with the two particles form a domain, termed the effective view domain. Obviously, any particle outside the domain has no effect on the geometric view factor of the two particles under considerations.

The first step in the evaluation of the geometric view factors is to exclude the particles outside the effective view domain. This can be achieved very efficiently by

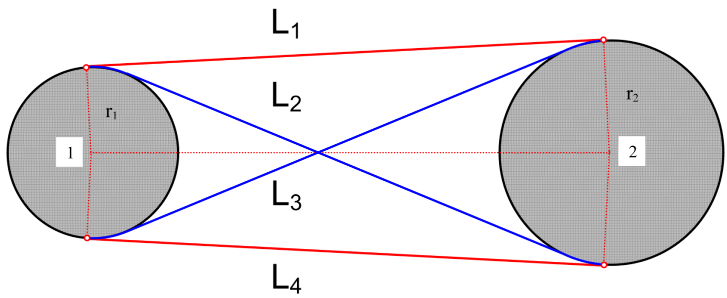


Figure 3: Hottel's method for two particles

utilising, for instance, any of the search algorithms employed in discrete element modelling (Feng *et al* 2002, Han *et al* 2007).

Particles that overlap or locate within the domain will (partially) block the view of the two particles concerned, and can be classified into three categories:

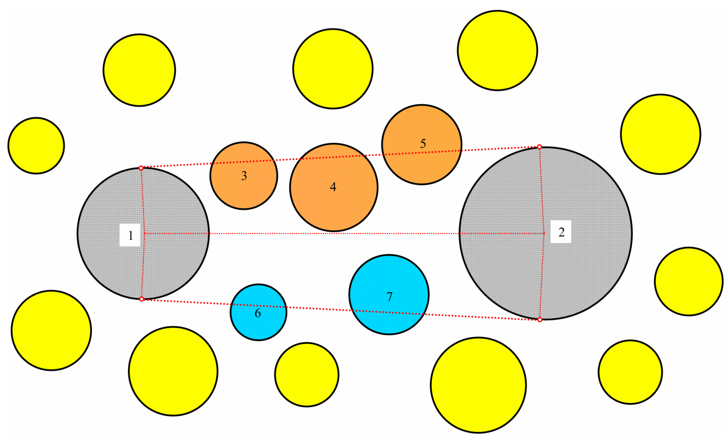


Figure 4: Two particles with blockage in a randomly packed bed of circular particles

1) Particles overlapping both upper and lower uncrossed strings. If this is the case, then particles 1 and 2 will be fully blocked, implying a zero view factor and the procedure is terminated. Note that there is no such case in the example given in Fig. 4.

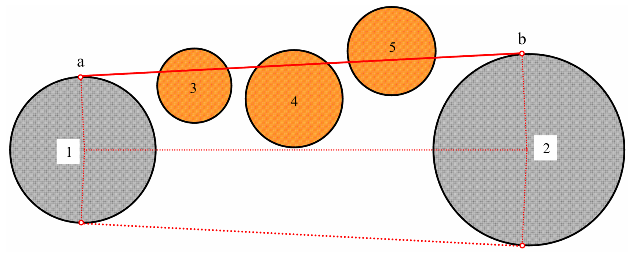
2) Particles intersecting with either the upper or the lower uncrossed string. In the given example, particles 3, 4 and 5 intersect with the upper uncrossed

string, and particles 6 and 7 intersect with the lower uncrossed string.

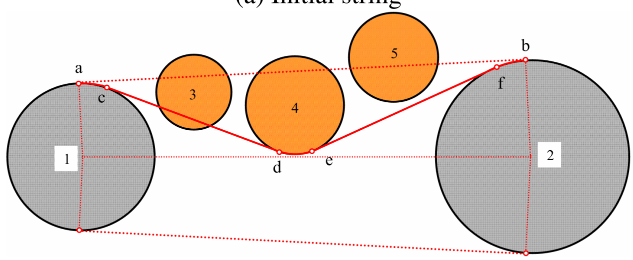
3) Particles that are embedded, i.e. completely lie within the effective view domain, and have no intersections with the upper and lower strings. Again no such particles exist in the given example.

In what follows, the second and third cases will be discussed separately.

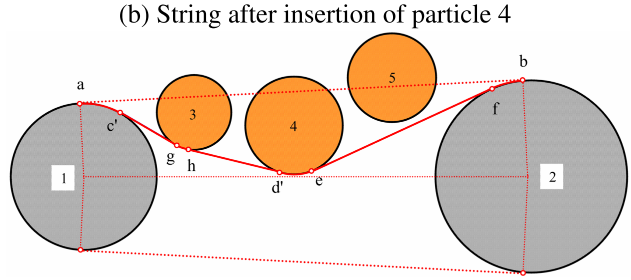
3.1 Geometric view factors between two particles without blockage of embedded particles



(a) Initial string



(b) String after insertion of particle 4



(c) final string

Figure 5: Generation of the upper uncrossed string between two particles with blockage

For the second case, it is essential to numerically define each string, uncrossed and crossed, so that their lengths can be accurately evaluated.

The generation of the upper uncrossed string is taken as an example. The procedure for generating the other three strings can be formulated in a similar manner.

Step 1 (Fig. 5a): Construct the initial upper uncrossed string, i.e. the common outer tangent $a \rightarrow b$, of particles 1 and 2 defined by the two touch points, a and b , and the unit vector (the two direction cosines).

Step 2 (Fig. 5b): Loop over the particles intersecting with the string. Particle 4 is identified as the one that has the maximum overlapping/penetrating distance with the string $a \rightarrow b$. Then this string should be re-generated with two sub-strings, one from particle 1 to particle 4, and another one from particle 4 to particle 2. Consequently the newly generated string comprises three arcs and two straight lines, i.e. $a \curvearrowright c \rightarrow d \curvearrowright e \rightarrow f \curvearrowright b$.

Note that the maximum penetration of the particles with the original string needs to be determined. If the particles are simply inserted to overlap the original string, some may not touch the string at later stages.

Step 3 (Fig. 5c): Repeat steps 1 and 2 for each of the two subsections of the new string, i.e. loop over the particles (except for the particles already included, e.g. particle 4) to identify a particle with the maximum overlapping/penerating distance (particle 3 for the first sub-string, for instance). If such a particle exists, this subsection of the string is further replaced by another two sub-strings. The above procedures are then repeated. Otherwise, go to the next subsection of the string using the same technique. The entire procedure is terminated if no particles intersect with all the sections of the string.

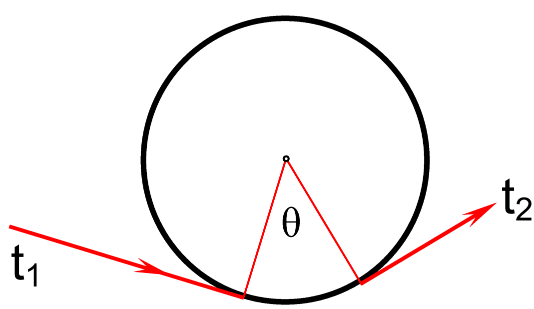


Figure 6: Evaluation of an arc length

It is worth pointing out that when a new particle is inserted to split a string

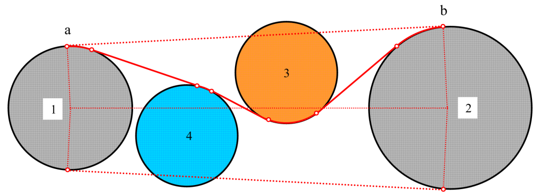


Figure 7: Full blockage: zero geometric view factor

into two sub-strings, the two touch points of the string with its two associated particles will be changed. For instance, particle 3 overlaps with the string section between particle 1 and particle 4, thereby the touch points c and d are moved to c' and d', as shown in Fig. 5c.

The final upper uncrossed string in the given example consists of four arcs and three straight lines: $a \curvearrowright c' \rightarrow g \curvearrowright h \rightarrow d' \curvearrowright e \rightarrow f \curvearrowright b$.

Step 4 : Evaluate the total length of the string based on its final configuration. The only issue is how to evaluate the arc lengths, or the angles of the arcs involved, effectively. Suppose a particle is part of the string, touching two sub-strings, as shown in Fig. 6. As the unit vectors, $\mathbf{t}_1$ and $\mathbf{t}_2$, of the two sub-strings are already known, the angle θ of the arc can be computed as

$$\theta = \cos^{-1}(\mathbf{t}_1 \cdot \mathbf{t}_2)$$

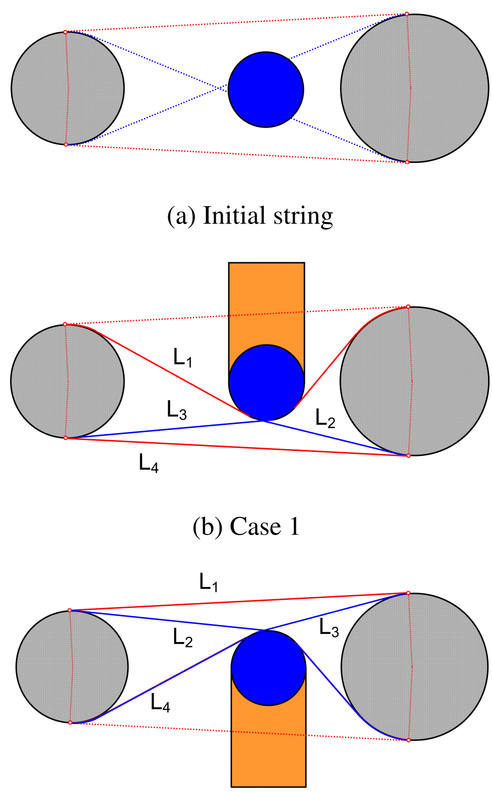
In the above procedure, the particles that intersect the lower uncrossed string (i.e. particles 6 and 7) are not checked for the intersection with the upper uncrossed string only for simplicity of illustration. They nevertheless should be included in the check procedure since the upper string may be affected by their presence, as is the case shown in Fig. 7. In this case, however, particles 1 and 2 are fully blocked, thus leading to a zero view factor. In general, when a particle that intersects the lower (upper) uncrossed string is involved in the formation of the upper (lower) uncrossed string, the corresponding view factor will be zero. Nevertheless, both types of particles overlapping with the upper or lower uncrossed string can alter the two crossed strings without necessarily leading to a zero view factor.

Besides the two cases identified above that lead to a full blockage, or a zero view factor, more complex full blockage situations may be exploited and identified, which may not be computationally beneficial as it generally involves more checks.

The above algorithm can be easily implemented by employing a linked data structure and recursive operations.

3.2 Geometric view factors between two particles with blockage of embedded particles

For the third case where some particles are embedded in the computational domain of particles 1 and 2, the algorithm presented above for the second case should be modified to accommodate this more complex situation.



(a) Initial string

(b) Case 1

(c) Case 2

Figure 8: Blockage with one embedded particle

Consider a particle in Fig. 8a that is embedded in the effective view domain of particles 1 and 2. In this case Hottel's crossed-string method cannot be applied in a straightforward manner. Instead, two cases need to be considered separately. In the first instance, the embedded particle is assumed to be extended upwards beyond the upper uncrossed string of the two particles, as shown in Fig. 8b. In other words, the particle is treated as a long obstacle. Then Hottel's rule can be applied to generate four strings, L_1^1, L_2^1, L_3^1 and L_4^1. The corresponding geometric view factor from particle 1 to particle 2 can be evaluated as

$$F_{1-2}^{1} = (L_3^1 + L_4^1 - L_1^1 - L_2^1)/A_1 \tag{10}$$

In the second instance, the embedded particle is assumed to be extended downwards beyond the lower uncrossed string of the two particles, as shown in Fig. 8c. The four strings, L_1^2, L_2^2, L_3^2 and L_4^2, can then be generated. According to Hottel's rule, the corresponding geometric view factor in this case can be evaluated as

$$F_{1-2}^{2} = (L_3^2 + L_4^2 - L_1^2 - L_2^2)/A_1 \tag{11}$$

As a result, the geometric view factor between particle 1 and particle 2 is the combination of Eqs. (10) and (11),

$$F_{1-2} = F_{1-2}^{1} + F_{1-2}^{2}$$

In a randomly packed bed more than one particle can be embedded in the effective view domain of particles 1 and 2. Fig. 9 depicts such a case where two particles are embedded. Similar to the above discussion, two cases need to be considered for each embedded particle, thereby giving rise to a total of 2^m cases, where m represents the total number of embedded particles. The geometric view factor can then be obtained by the combination of these 2^m results. However, for a particle assembly with a wide range of particle size distribution, m can be large and this 'brute force' approach is thus not computationally efficient, although the implementation is fairly straightforward.

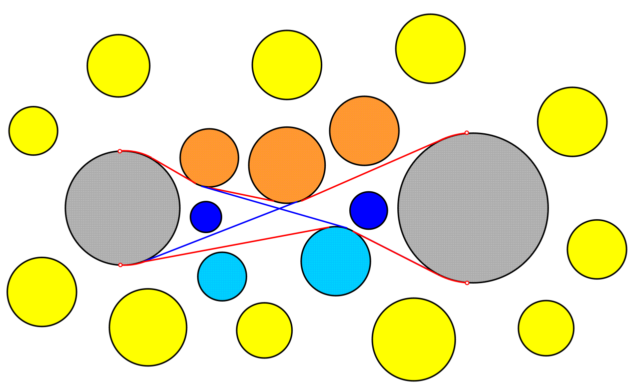


Figure 9: Blockage with more embedded particles

The numerical efficiency for evaluating the geometric view factors involving embedded particles can be improved in a number of ways. One approach, which is implemented in this work, is to firstly construct the four strings without the consideration of the embedded particles following the procedure described in the previous subsection. Then all the embedded particles are checked against the new effective view domain formed by the two uncrossed strings and the two particles, which may have three outcomes for each embedded particle:

1) The embedded particle lies completely outside the domain. Clearly this particle is excluded in the following operation.

2) The embedded particle intersects with one or two of the uncrossed strings. If the particle intersects with both strings, this is equivalent to a full blockage and the procedure is terminated; Otherwise, if the particle intersects with only one of the uncrossed strings, this string should be modified by inserting the particle. Unlike the particle insertion operation in the previous subsection, some existing particles that form the string may have to be deleted as they may lose contact with the string after the new particle is inserted.

3) The embedded particle is still embedded in the domain.

As the strings are kept updated during the checking process, all the embedded particles should be repeatedly checked against the strings until no changes occur to the strings or no embedded particles exist.

The purpose of the above checking operation is to further exploit a possible full blockage which will immediately halt the procedure, and to minimise the number of embedded particles so as to reduce the computational costs at the next stage, where all the combination cases for the remaining embedded particles are considered following the procedure outlined earlier.

3.3 Geometric view factors in a large packed bed

The preceding subsections describe the procedures for evaluating the geometric view factors between two particles with or without embedded particles. To model radiative heat transfer in a packed bed, all the nonzero view factors between one particle and the other particles as well as the problem boundaries need to be determined.

For small scale and static problems where the geometric view factors are evaluated only once, this can be achieved by looping over all the particles without significantly compromising the computational efficiency. For a packed bed with a large number of particles, however, an alternative solution strategy must be sought since the complexity of the procedure is $O(n^2)$ with n being the total number of particles.

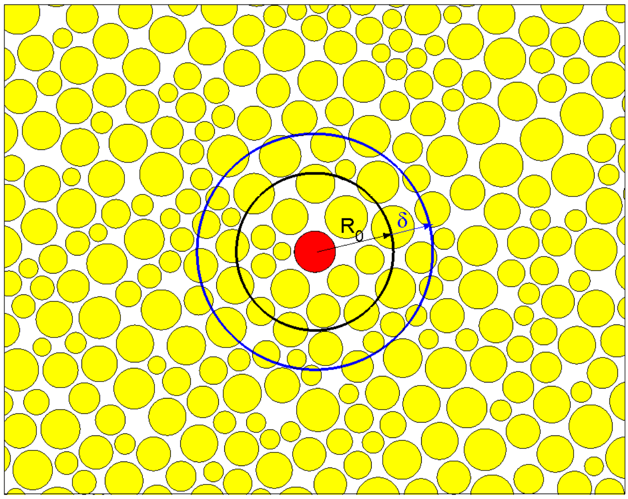


Figure 10: Definition of a circular cover domain and enlargement for a particle

A more efficient approach is to define a circular cover domain of radius R_0 for each particle in the packed bed, the black circle for the red particle in Fig. 10, for instance. Then the geometric view factors of this particle with other particles in the cover domain are evaluated. If the sum of the geometric view factors is approaching 1 within a given tolerance τ, i.e.

$$1 - \sum_j F_{i-j} < \tau \tag{12}$$

the calculation for this particle is completed. Otherwise, the radius of the cover domain is increased by a given value δ, and the evaluation of the geometric view factors with the newly included particles is performed. This procedure is repeated until condition (12) is satisfied.

Clearly it is essential to identify those particles in the cover domain. Again this can be achieved by employing some of the efficient search algorithms employed in discrete element modelling, such as the ASDT algorithm (Feng *et al* 2002). The values of R_0 and δ should be chosen based on the packing characteristics of the particles, such as packing density and size distribution, to achieve a better performance. However, those particles near the domain boundary may be treated in a slightly different way, which depends on the shape and other features of the boundary.

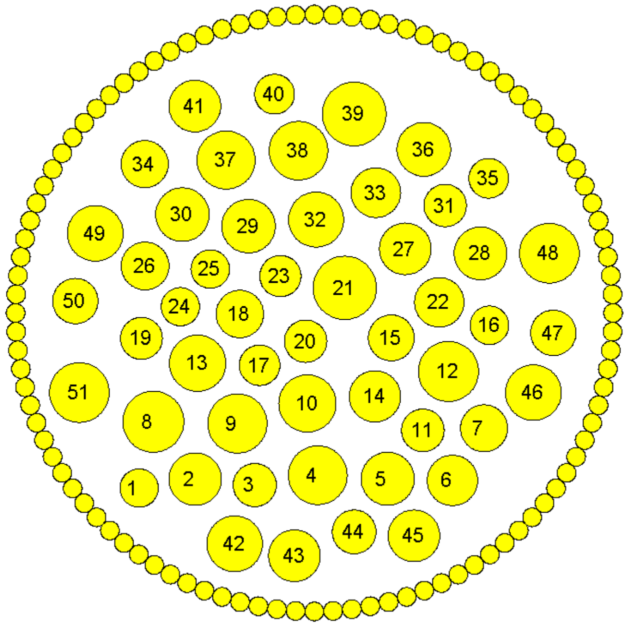


Figure 11: Example - a loosely packed bed in a circular enclosure

4 Numerical Example

A simple numerical example is provided in this section to illustrate the procedure for modelling the radiative heat transfer in a packed bed of particles within a circular enclosure, as depicted in Fig. 11. A total of 51 circular particles are loosely packed with radii in the range of [0.05,0.09]m. To assess the accuracy of the algorithm proposed in this work, the circular boundary of radius 0.6m is represented by 100 mono-sized particles. The temperatures of the boundary particles are prescribed with a linear distribution from 250°C at the leftmost particle to 500°C at the rightmost particle. The particles in the packed bed are assigned with a random initial temperature in the range of $[250, 500]$°C.

The geometric view factors associated with each particle are evaluated following the procedure described in the preceding section. As the scale of the particle assembly is small, each particle in the enclosure is checked against all the other particles including those representing the boundary for possible nonzero view factors. The sum of the geometric view factors for each particle serves as an indicator against the exact value which is 1.

Table 1 presents the results calculated for 5 selected particles at different positions ranging from the near-boundary to the interior in the packed bed. The second column of the Table lists the particles, termed 'visible' particles, that have nonzero geometric view factors with the chosen particles. The number of 'visible' particles and the partial sum of the geometric view factors excluding the boundary particles are given in the third and fourth columns, respectively, while the total sum including the boundary particles is presented in the last column. As the number of particles is small and loosely packed, the number of 'visible' particles is fairly large (around 25), and these particles are distributed across a large area of the packed bed (see particle 20). As expected, for particles that are close to the boundary, particles 1 and 40 for example, the partial sum of the geometric view factors is small (around 0.5), while for the interior particles, such as 10 and 20, the partial sum is very close to 1. The values in the last column indicate that the relative solution accuracy of the algorithm proposed is about 10^{-3}. The error stems mainly from numerical round-offs in the operations.

Note that for a pair of particles i and j only F_{i-j} is evaluated while F_{j-i} is obtained by utilising the reciprocity relation in Eq. (3). After discarding those negligible view factors with a cut-off value of 10^{-5}, the total number of the nonzero view factors is 2842 for all the particles in the packed bed. This number mainly dictates both computer memory requirement and CPU time in the next stage for simulating the temperature evolution.

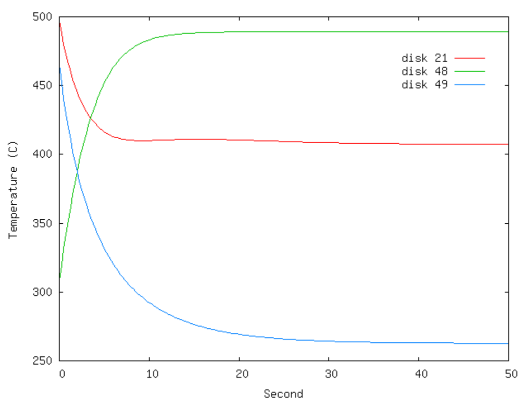


Figure 12: Temperature evolution histories for particles 21, 48 and 49

The particles in the packed bed are assumed isothermal and their temperature evolution is governed by the following transient heat transfer equations:

$$C_i\dot{T}_i(t) = \sigma \sum_j A_i F_{i-j}(T_i^4 - T_j^4) \tag{13}$$

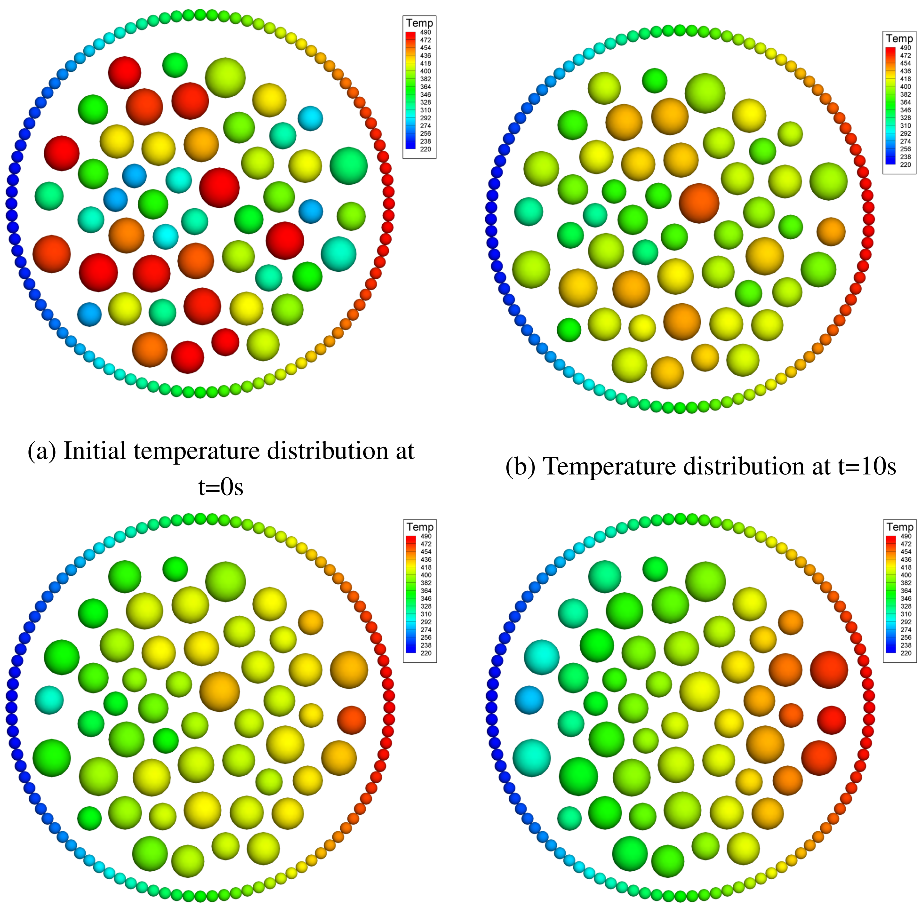
where C_i is the thermal capacity of particle i, T_i is its temperature in degrees Kelvin,

Table 1: The geometric view factors for a few selected particles

particle (i)	list of 'visible' particles (j)	no. of particle (j)	$\sum_j F_{i-j}$	$\sum_j F_{i-j}$(total)
1	2,3,8,9,13,17-19,23,42,44,45,49-51	15	0.506607	1.001501
5	3,4,6,7,9-12,14,15,20-22,32,42-45,47	17	0.928369	0.999202
10	2-6, 8,11-15,17,18,20,21,23, 27-29,35,42-45, 51	25	0.995864	0.999815
20	3,5,7,9,10,12-15,17-19 21-25, 27-30,32,38,44	24	0.999088	0.999989
40	23,29,33,34,37,38,39,41,49	9	0.554262	1.000136

$\dot{T}_i$ the time derivative of the temperature, A_i is the circumference of the particle; j represents all the 'visible' particles having nonzero view factors F_{i-j} with particle i, and $\sigma = 5.6704 \times 10^{-8}$ kg s^{-3} K^{-4} is the Stefan-Boltzmann constant. The transient equations are numerically integrated using the forward Euler time stepping scheme. In this example, the time step can be chosen in a wide range without encountering any numerical instability problems.

The time histories of the temperature evolution for three particles, 21, 48 and 49, are displayed in Fig. 12. The temperature evolutions of the system at several time instants, t=0, 10, 20, 50s, are depicted in Fig. 13a-d. Clearly the system has reached the correct steady-state, as expected.



(a) Initial temperature distribution at t=0s

(b) Temperature distribution at t=10s

(c) Temperature distribution at t=20s

(d) Temperature distribution at t=50s

Figure 13: Temperature evolution of the particle assembly at several time instants

5 Concluding Remarks

This paper has proposed an accurate algorithm for the evaluation of the geometric view factors in a randomly packed bed of circular particles of various sizes, motivated by the idea of Hottel's crossed-string method. This approach provides an appropriate basis for calculating the radiative heat exchange in a packed bed of particles encountered in many industrial applications. The procedure is illustrated and the solution accuracy is assessed through a numerical example.

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