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Uncertainty Analysis for a Particle Model of Granular Chute Flow

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Abstract: In alpine regions human settlements and infrastructure are at risk to be hit by landslides or other types of geological flows. This paper presents a new approach that can aid the design of protective constructions. An uncertainty analysis of the flow around a debris barrier is carried out using a chute flow laboratory model of the actual debris flow. A series of discrete element simulations thereby serves to compare and assess two different barrier designs. In this study, the transformation method of fuzzy arithmetic is used to investigate the influence of epistemically uncertain model parameters. It turns out that parameter and modeling uncertainties can have a tremendous influence on the predicted efficiency of protective structures.

Keywords: debris flow, discrete element method, uncertainty analysis, fuzzy arithmetic, risk analysis, safety assessment.

1 Introduction

Landslides are prevalent geological phenomena in mountain regions. The term landslide comprises different phenomena such as debris flows, earth flows and debris avalanches. Geological flows are commonly made up of more or less unconsolidated polydisperse rock material (Easterbrook (1999)). Often, the saturation of rock material with water is the determining factor to trigger such flow events. However, water does not necessarily need to be involved, as e.g. sturzstroms demonstrate (Hsü (1975)). Geological flow events are difficult to foresee. If at all, they can be predicted with a higher precision in space than in time. Along with this quasi unpredictability, the fact that landslides can be huge dangerous events, makes them a terrible threat for human beings.

Geotechnical engineers globally cooperate with engineering geologists to design dams and barriers that are intended to protect settlements, roads and other vital

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infrastructure from the effects of geological flow phenomena. Nowadays, this engineering work is more and more aided by computer simulation. To the authors' best knowledge, all known simulation approaches for discontinuous rock material, e.g. those applied in Bourrier, Dorren, Nicot, Berger, and Darve (2009) or Chao-Lung, Jyr-Ching, Ming-Lang, Lacques, Chia-Yu, Yu-Chang, and Hao-Tsu (2009) are based on empirically derived laws that describe the dynamics of sliding material. Typically, a number of simplifying assumptions must be made to gain a simulation model with acceptable complexity. Common simplifications, such as the assumption of the rock material being homogeneous, along with the difficult choice of model parameters may impose a high degree of modeling uncertainty.

A common way to deal with uncertainties is to introduce safety factors for all crucial design variables. The computationally required cross section of a dam, e.g., is scaled with a factor greater than one to increase the likelihood that the dam holds in case of an impact that is more powerful than anticipated in the modeling assumptions. However, this kind of consideration is only feasible if the result of parameter changes is obvious. A survey on how to estimate safety factors for structures that are exposed to debris flows is given in (Proske, Kaitna, Suda, and Hübl (2008)), based on a very coarse and verbal description of the system.

But is the respective design effective in any case, e.g. for all types of rock material? This question casts into doubt the assumption that a construction is generally safe if its design is based on just a certain set of parameters and assumptions. The objective of the design process might thus be reformulated: The best design of a protective structure should not only be safe for one set of model parameters, the nominal set, but it must also be safe for all values of the input parameters that seem to be possible. Moreover, the preferable design should not be sensitive to variations of all uncertain input parameters of the design process. In order to reach these properties within the computational process, a methodology is needed, that is capable of handling uncertain model parameters. In the following, different principles in modeling uncertainty are given and the one which is used in this work, namely the transformation method of fuzzy arithmetic, is explained. Then the method is applied to a chute flow described by using the discrete element method and the results are discussed.

2 Classification, Representation and Propagation of Uncertainty

2.1 Uncertainty Classification and Representation

In general, non-determinism in numerical models may arise from different sources, motivating some categorization of uncertainties. Although other classifications are possible, the following categorization of uncertainties (Hofer (1996)) proves to be well-suited in this context: aleatory uncertainties that can be measured, such as variability or scatter caused by irregularities in fabrication, on the one side, and on the other side, epistemic uncertainties, which arise from an absence of information, rare data, vagueness in parameter definition, subjectivity in numerical implementation, or simplification and idealization processes employed in the modeling procedure. All these conditions manifest as uncertain model parameters. They entail that the results that are obtained from simulations that only use one specific set of values as the most likely ones for the model parameters cannot be considered as representative of the whole spectrum of possible model configurations. Furthermore, this fake exactness offered by the numerical simulation of models with actually uncertain, but crisply quantified parameters can make the comparison between numerical simulations and experimental testing questionable. Namely, such a comparison may be rated as unsatisfactory if the simulation results obtained with crisp, i.e., discrete and non-fuzzy values do not well match the experimental ones, even though it might be absolutely satisfactory, if the uncertainties inherent to the models would have been appropriately taken into account in the simulation procedure.

While aleatory uncertainties have successfully been taken into account by the use of probability theory (Loeven and Bijl (2008); Stroud, Krishnamurthy, and Smith (2002)) and, in practice, by Monte Carlo simulation, the additional modeling of epistemic uncertainties still remains a challenging topic. As a practical approach to address this issue, a special interdisciplinary methodology to comprehensive modeling and analysis of systems is presented which allows for the inclusion of uncertainties - in particular of those of epistemic type - from the very beginning of the modeling procedure. This approach is based on fuzzy arithmetic, a special field of fuzzy set theory, which will be described in the following.

A special application of the theory of fuzzy sets, which is rather different from the well-established use of fuzzy set theory in fuzzy control, is the numerical implementation of uncertain model parameters as fuzzy numbers (Kaufmann and Gupta (1991)). Fuzzy numbers are defined as convex fuzzy sets over the universal set $\mathbb{R}$ with their membership functions $\mu(x) \in [0,1]$, where $\mu(x) = 1$ is true only for one single value $x = \bar{x} \in \mathbb{R}$, the so-called center value or nominal value. For example, a fuzzy number $\widetilde{p}$ of triangular (linear) shape, expressed by the abbreviated notation (Hanss (2005))

$$\widetilde{p} = \mathrm{tfn}(\bar{x}, w_\mathrm{l}, w_\mathrm{r}) \;, \tag{1}$$

is defined by the membership function

$$\mu_{\widetilde{p}}(x) = \min\left\{ \max\left[0, 1-(\bar{x}-x)/w_\mathrm{l}\right], \max\left[0, 1-(x-\bar{x})/w_\mathrm{r}\right] \right\} \quad \forall\, x \in \mathbb{R} \;, \tag{2}$$

or, more explicitly, by

$$\mu_{\widetilde{p}}(x) = \begin{cases} 0 & \text{for} \quad x \leq \bar{x} - w_\mathrm{l} \\ 1 + (x - \bar{x})/w_\mathrm{l} & \text{for} \quad \bar{x} - w_\mathrm{l} < x < \bar{x} \\ 1 - (x - \bar{x})/w_\mathrm{r} & \text{for} \quad \bar{x} \leq x < \bar{x} + w_\mathrm{r} \\ 0 & \text{for} \quad x \geq \bar{x} + w_\mathrm{r}\,. \end{cases} \tag{3}$$

A triangular fuzzy number is shown in Fig. 1. However, any other shape of mem-

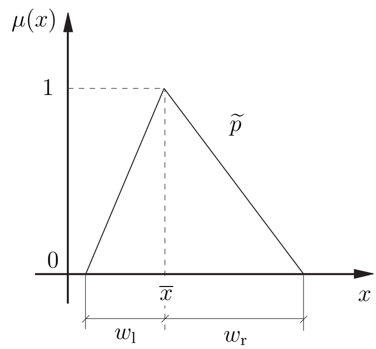


Figure 1: Triangular fuzzy number $\widetilde{p}$.

bership function may be selected if appropriate to quantify the uncertainty of a specific model parameter. The calculation with fuzzy numbers is referred to as fuzzy arithmetic and proves to be a non-trivial problem, especially with regard to the evaluation of large mathematical models with fuzzy-valued operands.

2.2 Uncertainty Propagation Based on the Transformation Method

As a successful practical implementation of fuzzy arithmetic, which allows the evaluation of arbitrary systems with uncertain, fuzzy-valued model parameters, the transformation method (Hanss (2002)) is used. Alternative methods to numerically handle uncertainties are, for example, presented in (Hanss, Herrmann, and Haag (2009)). The transformation method is available in a general, a reduced and an extended form, with the most appropriate form to be selected depending on the type of model to be evaluated (Hanss (2002, 2005, 2003)).

Assuming the uncertain system to be characterized by n fuzzy-valued model parameters $\widetilde{p}_i$, $i = 1, 2, \ldots, n$, the major steps of the method can briefly be described as follows:

In the first step, each fuzzy number $\widetilde{p}_i$ is discretized into a number of nested intervals $X_i^{(j)} = [a_i^{(j)}, b_i^{(j)}]$, assigned to the membership levels $\mu_j = j/m$, $j = 0, 1, \ldots, m$, that result from subdividing the possible range of membership equally spaced by $\Delta\mu = 1/m$, see Fig. 2. In a second step, the input intervals $X_i^{(j)}$, $i = 1,2,\dots,n$, $j = 0,1,\dots,m$, are transformed to arrays $\widehat{X}_i^{(j)}$ that are computed from the upper and lower interval bounds after the application of a well-defined combinatorial scheme (Hanss (2002, 2005)). Each of these arrays represents a specific sample of possible parameter combinations and serves as an input parameter set to the problem to be evaluated. As a result of the evaluation of the model for the input arrays $\widehat{X}_i^{(j)}$, output arrays $\widehat{Z}^{(j)}$ are obtained which are then retransformed to the output intervals $Z^{(j)} = [a^{(j)}, b^{(j)}]$ for each membership level μ_j and finally recomposed to the fuzzy-valued output $\widetilde{q}$ of the system.

In addition to the simulation part of the transformation method described above, the analysis part of the method can be used to quantify the influence of each fuzzy-valued input parameter $\widetilde{p}_i$ on the overall fuzziness of the model output $\widetilde{q}$. For these purposes, the standardized mean gain factors κ_i and φ_i, and the normalized degrees of influence ρ_i and ω_i have been introduced (Hanss (2002, 2005); Gauger, Turrin, Hanss, and Gaul (2007)), quantifying in an absolute and in a relative character, respectively, the effect of the uncertainty of the ith model parameter $\widetilde{p}_i$ on the overall uncertainty of the model output $\widetilde{q}$. In (Hanss (2002, 2005)), a standardization with respect to the nominal values is incorporated into the computation of the standardized mean gain factors κ_i and of the normalized degrees of influence ρ_i, whereas the approach proposed in (Gauger, Turrin, Hanss, and Gaul (2007)) considers the influences of the overall input uncertainty on the overall output uncertainty.

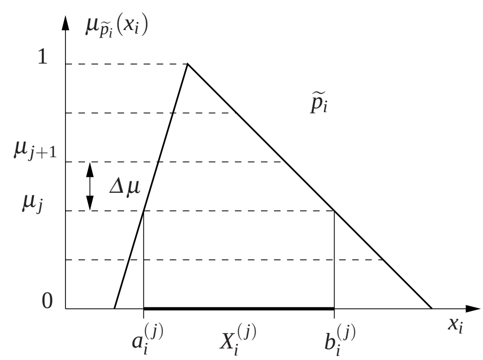


Figure 2: Decomposition of a fuzzy number $\widetilde{p}_i$ into intervals $X_i^{(j)}$, $j = 0,1,\dots,m$.

Among other advantages of the transformation method, its characteristic property of reducing fuzzy arithmetic to multiple crisp-number operations entails that the transformation method can be used with an existing software environment for system simulation even if its internals are not known or accessible (Hanss (2005)). Expensive rewriting of the program code is not required. Instead, the steps of decomposition and transformation as well as of retransformation and recomposition can be coupled to an existing software environment by a separated pre- and post-processing tool.

3 Chute Flow Studies Using Particle Methods

3.1 Laboratory Model

To demonstrate the potentially high sensitivity of numerical debris flow models, a virtual laboratory representation is generated, that serves as basis for a numerical uncertainty analysis. As debris flows tend to follow natural or artificial grooves, the flow-bed of the slide is simplified towards a prismatic chute, see Fig. 3. To allow for the simulations to be reproducible, the rock material is replaced by glass beads, thus considering only dry debris. Initially, the glass beads are stored in a reservoir at the upper end of the chute. At the beginning of the test the downstream facing barrage of the particle container is instantaneously removed. A resulting particle wave then starts to slide downstream. After a short period of free flow, the wave hits a barrier, see Fig. 3. Two types of barriers are compared, one with a single column and one with two columns. In both setups the total surface of the barrier, facing in downstream direction is equal. The barrier represents a geotechnical engineering construction that is designed to reduce the impulse of the debris flow and to smooth its peak. Such constructions are typically placed above human settlements or infrastructure. To compare the efficiency of the two barrier designs, the impulse of the flow wave is measured at a control plane behind the barrier. The geometric parameters of the chute flow laboratory setup are summarized in Tab. 1.

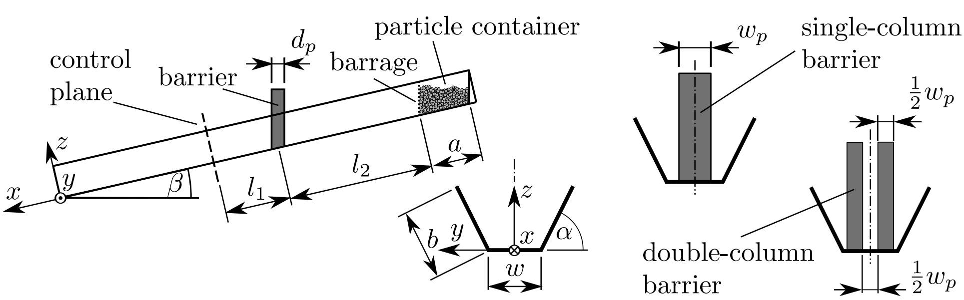


Figure 3: Setup of the chute flow laboratory model.

Table 1: Geometric parameters of the chute flow laboratory model.

parameter		value
chute wall height	b [m]	0.15
chute base width	w [m]	0.05
chute wall slope	α [-]	60°
chute slope	β [-]	20°
distance barrier-control plane	l_1 [m]	0.17
container length	a [m]	0.153

parameter		value
distance barrage-barrier	l_2 [m]	0.5
barrier width	w_p [m]	0.03
barrier length	d_p [m]	0.03
number of spheres	n [-]	5000
sphere diameter	d [mm]	6

3.2 Numerical Simulations

Using a computational model of the chute-flow laboratory model, one is able to perform a sensitivity analysis. To describe the motion of the particles, the Discrete Element Method (DEM) (Cundall (1971)) is chosen. This approach models the glass spheres as very stiff bodies with unconstrained dynamics. The spheres exchange impulse through surface contacts. Following the DEM approach, the particle dynamics is time-integrated on force-acceleration level (Fleissner and Eberhard (2008)). This requires a suitable model for the particle contact forces. In a chute flow, particle contact forces are relatively small as they only result from inertial forces due to the absence of external compression. Therefore, a linear elastic contact model is well suited to compute the elastic contact force $\mathbf{f}_e^{ij}$ acting on a sphere i at the position $\mathbf{r}^i$ in case of a contact with another sphere j at position $\mathbf{r}^j$. This force depends on the depth of spheres' common surface interpenetration δ^{ij} and the sphere radii R^i and R^j as

$$\triangle \mathbf{r}^{ij} = \mathbf{r}^i - \mathbf{r}^j, \tag{4}$$

$$\mathbf{n}^{ij} = \frac{\triangle \mathbf{r}^{ij}}{||\triangle \mathbf{r}^{ij}||}, \tag{5}$$

$$\delta^{ij} = R^i + R^j - \mathbf{n}^{ij} \cdot \triangle \mathbf{r}^{ij}, \tag{6}$$

$$\mathbf{f}_e^{ij} = k_n \delta^{ij} \mathbf{n}^{ij}. \tag{7}$$

The parameter k_n is the stiffness of the normal contact. The coefficient of restitution for contacts between glass spheres is usually quite close to one. The resulting dissipation is modeled through a linear damper that is introduced in parallel to the linear elastic contact force element. The resulting dissipative force acting on sphere i is computed from the velocities of the spheres $\mathbf{v}^i$ and $\mathbf{v}^j$ as

$$\mathbf{f}_d^{ij} = d_n \mathbf{n}^{ij} \cdot (\mathbf{v}^j - \mathbf{v}^i)\mathbf{n}^{ij}. \tag{8}$$

Though this approach is widely used, the damping parameter d_n is usually determined by matching experiments and simulations. For alpine debris flows, such experiments are hardly possible. Contact damping is thus a highly epistemically uncertain parameter which, for this study, was estimated by educated guess.

Contacts between dry glass spheres cannot be modeled adequately without considering slipping and sticking friction forces $\mathbf{f}_f^{ij}$. Thereby, sticking friction requires a special treatment as it is not an applied force. The tangential elasticity of the contact region that results from surface roughness is modeled via a linear elastic tangential element (Cundall and Strack (1979); Brendel and Dippel (1998)). However, due to a lack of knowledge about the tribologic conditions in a real debris flow, the stiffness of the respective tangential element is another epistemically uncertain parameter.

The resulting overall force on a sphere i is computed as an accumulation of the applied forces that result from contacts with other spheres as

$$\mathbf{f}^i = \sum_j \mathbf{f}^{ij} = \sum_j \mathbf{f}_e^{ij} + \mathbf{f}_d^{ij} + \mathbf{f}_f^{ij}. \tag{9}$$

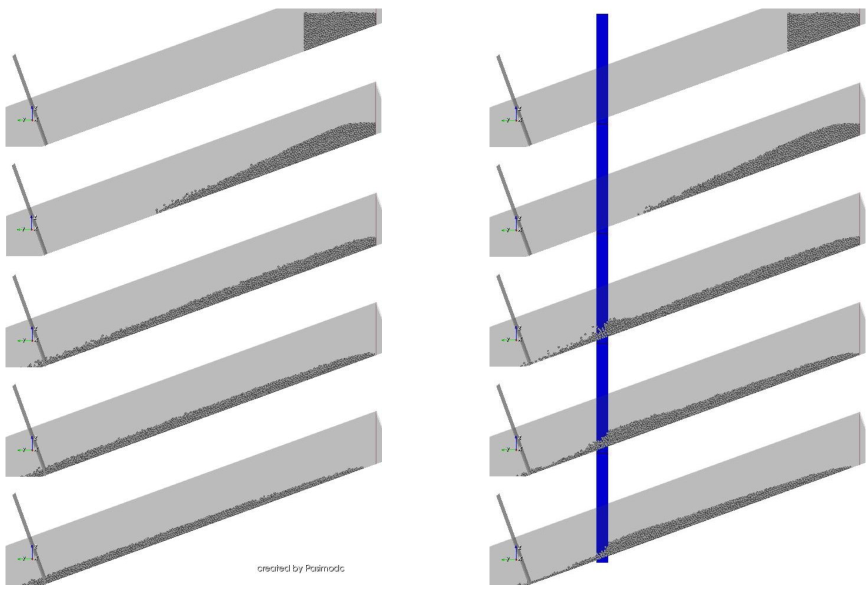
All contact forces $\mathbf{f}^{ij}$ on sphere i are thereby also considered as counter forces acting on spheres j. To resolve contacts between the glass spheres and the boundary geometry of chute and barrier, the entire boundary geometry is considered as rigid, following the approach introduced in (Fleissner, Gaugele, and Eberhard (2007)).

Another parameter that is regarded as epistemically uncertain is the material density of the debris. Only the density of the glass beads, used for this study, is measurable. Therefore, it is chosen as the nominal value for the stiffness uncertainty analysis. All important material and contact parameters of the simulation model are listed in Tab. 2.

Figures 4 and 5 depict the particle motion in the chute. The displayed snapshots exhibit how the barrier causes a stagnation of the overall particle motion above the barrier. At the barrier the spheres are deflected from their initial trajectory, which causes a loss of kinetic energy and impulse of the particle ensemble. The particle wave is thus effectively smoothed by the barrier. This becomes even more evident if the particle ensemble motion is compared to the motion of the same ensemble in case the barrier is omitted, see Figs. 4(a) and 4(b).

Table 2: Parameters of the sphere contact model.

parameter		value
material density (nominal)	ρ_{mass} [kg/m^3]	2500
slipping friction coefficient	μ [-]	0.6
sticking friction coefficient	μ_0 [-]	0.8
normal stiffness	k_n [10^5 N/m]	7.85
normal damping (nominal)	d_n [10^{-1} Ns/m]	1.4
tangential stiffness (nominal)	k_t [10^4 N/m]	1



(a) Free flow (b) Single-column barrier

Figure 4: Snapshots from chute flow particle simulations.

3.3 Uncertainty and Sensitivity Analysis

The sensitivity of a parametric system is characterized by the magnitude of the variation of specific output parameters with respect to variations of specific input parameters. Three epistemically uncertain input parameters are investigated in a sensitivity analysis. The chosen parameters are the material density ρ_{mass}, the tangential stiffness k_t of the sticking friction model and the normal contact damping parameter d_n. The density is chosen as an example for a parameter that is uncertain

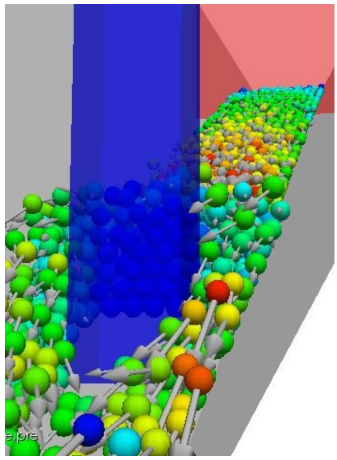


Figure 5: Debris deflection at the single-column barrier

as it is very difficult to estimate for a large amount of real debris. The latter two parameters are examples for the type of epistemically uncertain parameters that are part of a simplified physical model. This type of parameters is usually determined by curve fitting of experimental data, a method that is inherently subjective and thus uncertain. Even a curve fitting requires a number of experiments and simulations. As the resulting overhead is often unacceptable, parameters of the respective type are determined by educated guess. In Tab. 3, the parameters of the triangular fuzzy input parameters $\widetilde{p}_i$ that are used for the uncertain simulations are given.

Table 3: Input parameters for the uncertainty analysis.

parameter $\widetilde{p}_i = \text{tfn}(\bar{x}_i, w_{\text{l},i}, w_{\text{r},i})$		$\bar{x}_i$	$w_{\text{l},i}$	$w_{\text{r},i}$
material density (nominal)	ρ_{mass} [kg/m^3]	2500	250	250
normal damping (nominal)	d_n [10^{-1} Ns/m]	1.4	0.7	0.7
tangential stiffness (nominal)	k_t [10^4 N/m]	1	0.5	0.5

A uniform grid is used to generate sets of parameters from the three dimensional parameter space of ρ_{mass}, k_t and d_n. This sampling process results in an array $\widehat{X}_i^{(j)}$ with 189 sets of input parameters. All simulations are carried out using the particle simulation program Pasimodo (Fleissner (2009)), which is developed at the Institute of Engineering and Computational Mechanics of the University of Stuttgart. One simulation run of the chute flow using Pasimodo on a 3.2 GHz Pentium IV takes approximately three hours. Thus the overall computation time is about 24 days. As the computations for different parameter sets are independent, the overall simulation process is accelerated by distributing the simulations to several processors on a computer cluster. The results of all simulations are gathered to serve as input for a post processing that reassembles the actual uncertainty analysis.

4 Results

From a practical point of view, it is interesting to consider the impulse that affects a structure that is hit by a debris flow. In this work, the debris flow is approximated by the previously described chute flow and the affected structure is represented by a virtual control plane that is located below the debris flow barrier and oriented perpendicular to the flow. The quantity that is used to assess the potential damage of the structure is the accumulated impulse that acts on the control plane. Figure 6 shows the accumulated impulse on the control plane for three nominal simulations, one without any barrier and the other two with a single-column or a double-column barrier, respectively. The accumulated impulse is reduced by about 90% through the barriers which shows their effectiveness.

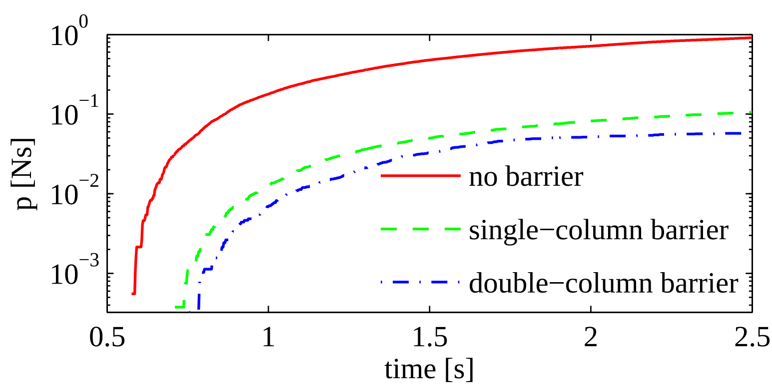
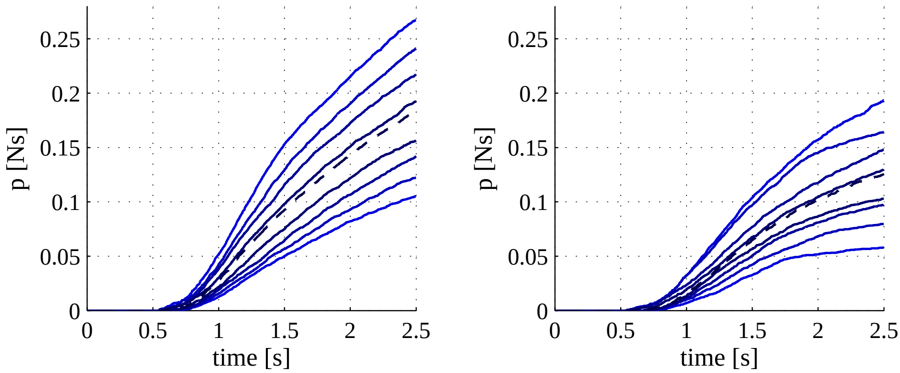


Figure 6: Accumulated impulse on the control plane for the nominal input parameter set.

In Figs. 7(a) and 7(b), the accumulated impulse is shown for a debris flow barrier with a single column and a double column, respectively. The dashed lines depict the results of nominal simulations, i.e., the results that emerge if no uncertainty is considered. The other lines reflect the ranges of possible outputs for different membership levels or, equivalently, for different degrees of possibility. It is evident, in fact, that it makes sense to consider uncertainty, as the worst-case output deviates



(a) Single-column barrier (b) Double-column barrier

Figure 7: Uncertain accumulated impulse over time for two debris-flow barrier designs.

about ±50% from the nominal configuration. For assessing the risk of an exposed structure, this variance definitely has to be taken into account.

In order to reduce the output uncertainty and to achieve a better understanding of the system under consideration, it is important to know which uncertain input parameter causes which amount of output uncertainty. This question is answered by the so-called measures of influence which are a by-product of the transformation method. Figure 8 shows the values of ρ_i for the accumulated impulse over time. The influence measure ρ_i reflects the relative effect of an unitary percentaged variation of the input $\widetilde{p}_i$ on the output. Recalling the definition of the impulse as the product of velocity and mass, it appears plausible that the uncertainty of the impulse is predominantly governed by the uncertainty of the material density, but it is also clear that the parameters of the contact model do have a non-negligible effect on the dynamic behavior of the particle set. Figure 8 quantifies these competing influences and reveals that the uncertainty of the density makes up about 80% of the output uncertainty while the two parameters of the contact model make up about 10% each. The short-term fluctuations that can be observed for points in time which are smaller than about 0.8s are attributed to the normalization with very small values and cannot be considered as reliable.

The use of a unitary percentaged variation of the input parameters is indicated when a rather theoretical analysis of a system is performed. In the previous paragraph, this is done for the accumulated impulse, and the acquired influence measures are compared to an estimation based on the underlying mathematical equations.

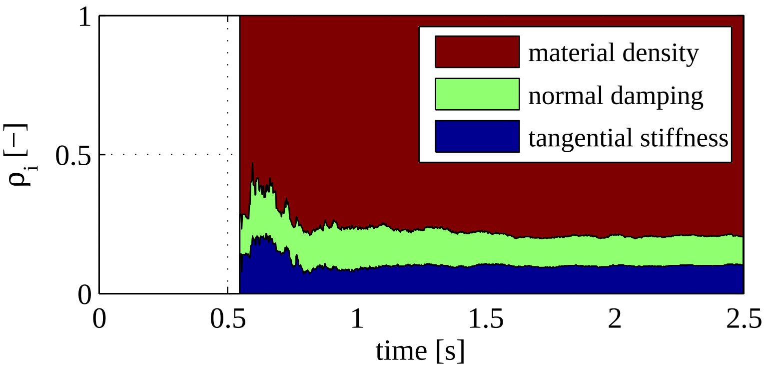
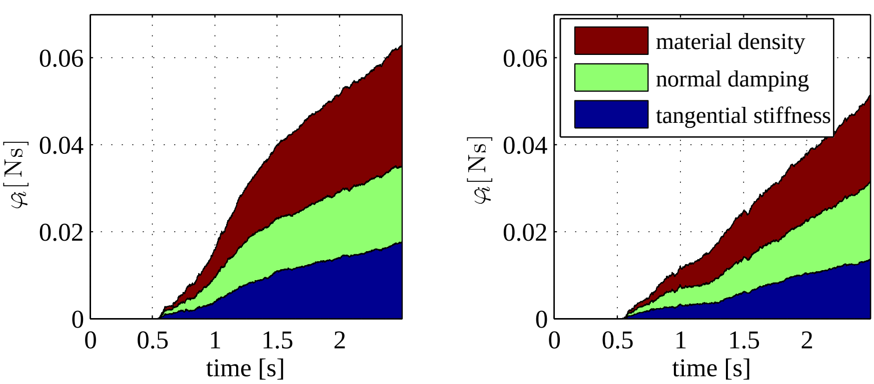


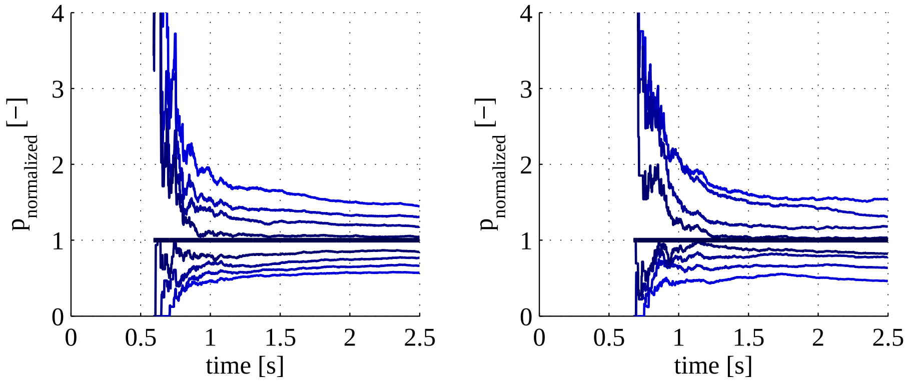
Figure 8: Normalized relative measure of influence ρ_i for the accumulated impulse on the control plane behind the single-column barrier.

In engineering applications, however, some parameters allow for a much larger relative worst-case deviation than others. As a consequence, it is indicated to consider the direct contribution φ_i of the input uncertainty to the uncertainty of the output on an absolute scale. In Fig. 9, these absolute measures of influence φ_i are shown for the accumulated impulse for both designs. As the estimation of the uncertainties of the inputs is done to the authors' best knowledge, it becomes clear that the influence of the uncertainty of the material density loses its dominant role and in reality only makes up about a third of the uncertainty of the output and thus is not



(a) Single-column barrier (b) Double-column barrier

Figure 9: Absolute measure of influence φ_i for the accumulated impulse.



(a) Single-column barrier (b) Double-column barrier

Figure 10: Relative deviation of the accumulated impulse for two different designs.

more important than the two uncertain contact parameters tangential stiffness and normal damping. For the single-column design, the uncertainty increases faster at the beginning while it increases linearly for the double-column design.

Besides the computation of worst-case bounds and influence measures, the uncertainty analysis based on the transformation method offers the possibility to compare different models and designs on a much broader basis than it can be done by only crisp-valued simulations. For the debris flow under consideration, two different designs are compared with each other. The first design contains one column in the middle of the particle flow while the second design consists of two columns with the same total cross-sectional area as the first design. A comparison of the two designs reveals that the nominal value of the second design is significantly smaller than the nominal value of the first design, see Fig. 7. Thus, the potential damage of a structure is expected to be smaller and the design with two columns is preferable to the other design. Another aspect that has to be compared before making final design decisions is the reliability of the computational results. For this reason, Figs. 7(a) and 7(b) are normalized with respect to their nominal value and compared in Fig. 10. It can be seen that the relative deviation of the output uncertainty is approximately the same for both designs. Moreover, both designs are equally sensitive to the input uncertainties that are considered in this work. Thus, from an uncertainty point of view, the preference of the double-column design, which originally was based on simulations with nominal values only, can be confirmed.

5 Conclusions

For the particle-flow problem considered in this work, it is strongly recommended to consider uncertainties in the simulation process as precise values for the contact parameters as well as for the mass properties of the system are unknown for the most part and these uncertainties have an enormous effect on the output of interest. With respect to the issue of risk analysis and safety assessment, the inclusion of uncertainties in analysis and design of protective structures using a fuzzy arithmetical approach represents a comprehensive and well-defined strategy. Moreover, advanced and better-founded safety factors can be derived which are based on a numerical analysis of the problem, rather than on educated guesses only.

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