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An Approach to Uncertainty Analysis of Rockfall Simulation

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Abstract: Despite the continuing advances in rockfall analysis, the mathematical modeling and simulation of rockfall phenomena continues to be significantly influenced by a large amount of aleatory and epistemic uncertainty on significant number of model parameters. This paper focuses on the representation and quantification of epistemic uncertainties in rockfall modeling and simulation by fuzzy numbers. The propagation of the epistemic uncertainties considered is then calculated by the transformation method as a practical implementation of fuzzy arithmetic. Epistemic uncertainties on the material properties, on the boulder geometry and dimensions, on the kinematics of the impact and on the contact response between boulder and slope are considered. The propagation and the influence of the uncertainties are then investigated with respect to the determination of the coefficients of restitution.

Keywords: rockfall simulation, coefficient of restitution, uncertainty analysis, fuzzy arithmetic

1 Introduction

The detachment of rock boulders from a steep slope and their subsequent downward motion may represent potentially hazardous conditions for people living in densely populated foothills of mountain areas. In order to assess the hazard zones and to design efficient protection systems, mathematical modeling and simulation procedures for rockfall problems have been developed. These procedures usually involve two main aspects: the first is the investigation of instability and failure probability of rock masses in the rockfall source area, and the second is the simulation of the trajectory of the boulders once they detach from the source area. In this paper, an approach to uncertainty analysis in modeling and simulation of rockfall

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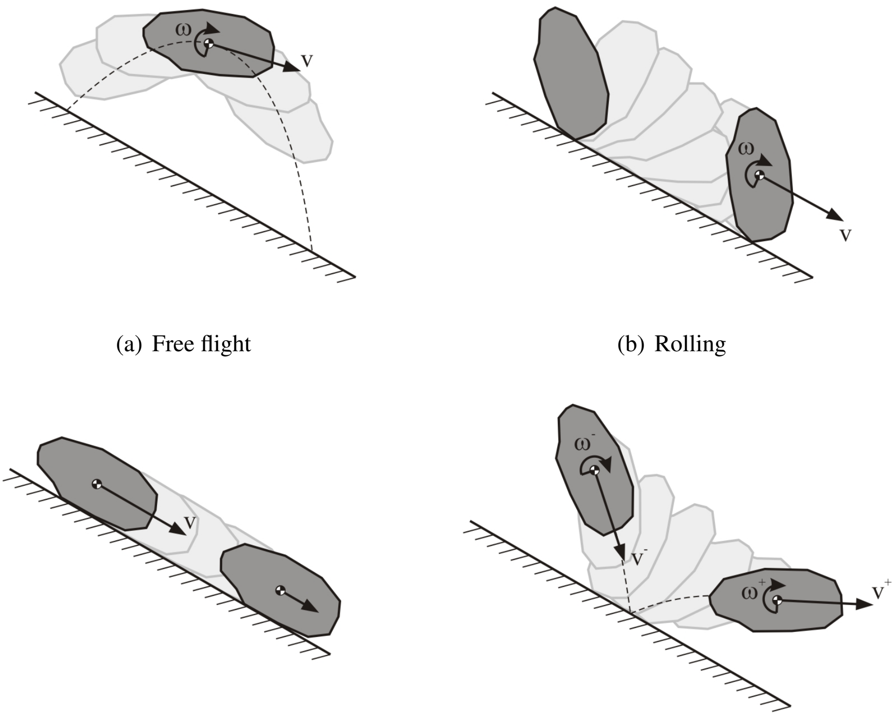
trajectories is presented. As represented in Fig. 1, the movement of a boulder may be described as a combination of four types of motion: free flight, rolling, sliding and bouncing.



(a) Free flight (b) Rolling

(c) Sliding (d) Bouncing

Figure 1: Four types of motion comprising the movement of a boulder: free flight, rolling, sliding and bouncing

Bouncing occurs when a falling boulder impacts with the slope surface. This phenomenon is the least understood and the least predictable part of the trajectory of the boulder, since it is generally characterized by fast nonlinear dynamics and by a strong dependency on the geometry and the material properties of the boulder as well as the slope [Labiouse (2004)]. The bouncing phenomenon is usually modeled in a classical way by introducing so-called coefficients of restitution (COR). The coefficients of restitution indicate the amount of kinetic energy dissipated during the impact with the slope. According to Fig. 2, for the 2-dimensional case, the coefficients of restitution are defined as:

$$R_{\mathrm{n}} = -\frac{\mathrm{v_n}^+}{\mathrm{v_n}^-} \tag{1}$$

and

$$R_{\mathrm{t}} = \frac{\mathrm{v_t}^+}{\mathrm{v_t}^-} \tag{2}$$

for the normal and tangential velocities $\mathrm{v_n}$ and $\mathrm{v_t}$ respectively, and

$$R_{\mathrm{ke}} = \frac{K_{\mathrm{tot}}^+}{K_{\mathrm{tot}}^-} = \frac{\frac{1}{2}m\left[(\mathrm{v_n}^+)^2 + (\mathrm{v_t}^+)^2\right] + \frac{1}{2}I\,(\omega^+)^2}{\frac{1}{2}m\left[(\mathrm{v_n}^-)^2 + (\mathrm{v_t}^-)^2\right] + \frac{1}{2}I\,(\omega^-)^2} \tag{3}$$

for the total kinetic energy K_{tot}. In the formulae, v and ω are the translational and rotational velocities, respectively, m is the mass of the boulder and I its moment of inertia about the z-axis. The superscripts $^-$ and $^+$ indicate the velocity or energy before and after the impact, respectively.

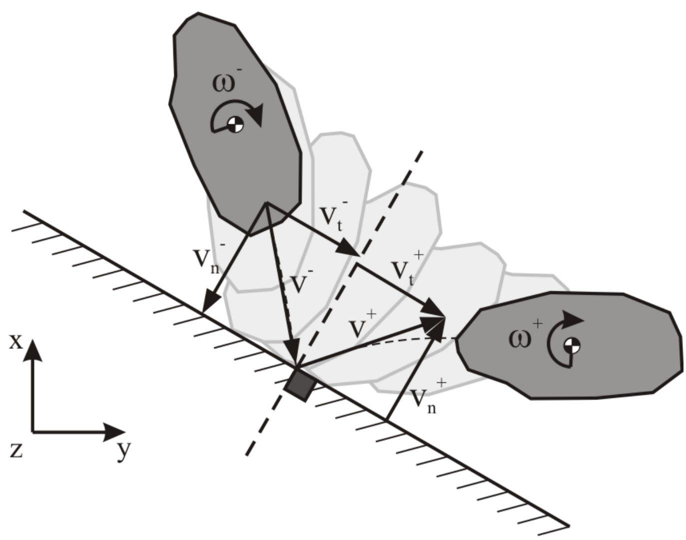


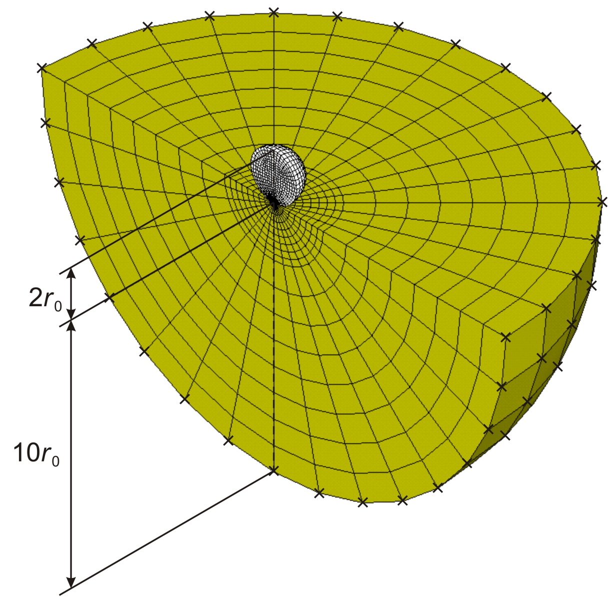
Figure 2: Velocity of a boulder before $^-$ and after $^+$ the impact with a slope surface

In general, the coefficients of restitution used to calculate the boulder trajectory are estimated based on a rough description of the slope material (e.g. rock, soil, sand) sometimes supplemented by information regarding its roughness and the vegetation cover; this results in a poor calculation of the actual values of the coefficients of restitution. In fact, as demonstrated by several authors [Giani (1992); Azzoni and de Freitas (1995); Hunger and Evans (1998)], the bouncing phenomenon (and consequently the coefficients of restitution) is strongly dependant on the kinematics of the impact, the geometry and mechanical properties of the boulder and the topography of the slope. In conclusion, bouncing models based on coefficients of restitution that depend only on some significant slope material properties are not able to correctly reproduce the actual trajectory of the boulder. In this paper, the impact between the boulder and the slope surface is numerically reproduced by a finite element model in order to improve the simulation of the bouncing phenomenon and, consequently, to achieve a better estimate of the actual trajectory of the boulder after the impact.

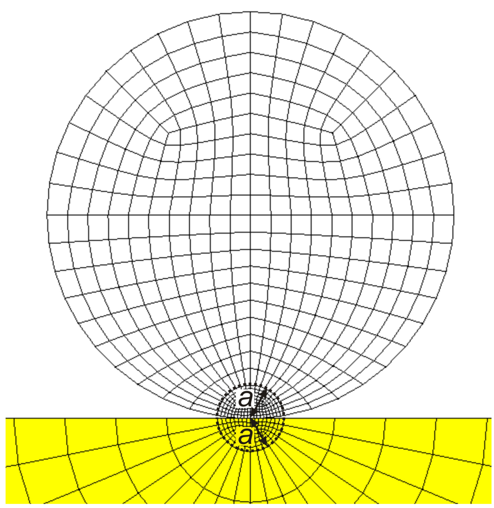
2 Finite element model

The finite element model used in this paper is shown in Fig. 3(a) for the case of the impact of a spherical boulder (radius r_0) with a plane semi-infinite slope. In this study, the simulation of the impacts is performed using the finite element software Dytran. In Fig. 3(a), for the sake of clarity, only one half of the model is shown. The plane semi-infinite slope is represented by a hemi-sphere, whose points on the external spherical surface, labeled by a cross in the figure, are fully constrained. The finite element model consists of 11096 eight-node solid elements with 12136 nodes. As mentioned by [Wu, Li, and Thornton (2005)], the appropriate modeling of the semi-infinite slope is of crucial importance for an accurate reproduction of the bouncing phenomenon. The objective is to select an appropriate dimension for the part of the slope to be modeled, in order to eliminate the influence of the boundary conditions without excessively increasing the computational cost. In other words, the slope section should be just large enough so that the stress waves propagating in the slope material as a consequence of the impact are not reflected by the boundaries during the whole impact phenomenon. If the selected slope section is too small, a reflection of stress waves by the boundaries will result in an overestimation of the actual values of the kinetic energy and velocity of the boulder after the impact. According to [Wu, Li, and Thornton (2005)], a half-sphere with radius $10r_0$ is able to efficiently represent a semi-infinite slope, avoiding stress wave reflection for impact velocities up to 150 m/s. This limit, however, should not be regarded as a general rule; such a general limit must be based on the size of the domain to one of the classical wave speeds.

As shown in Fig. 3(b), a fine mesh is used to model the impact area near the ini-



(a) Finite element model



(b) Impact area

Figure 3: Finite element model for the case of the impact of a spherical boulder with a plane semi-infinite slope

tial contact point, in order to accurately capture the localized deformation around the contact point [Wu, Li, and Thornton (2005); Zhang and Vu-Quoc (2002)]. The radius a of the impact area is given by [Hertz (1881)] for an elastic impact as

$$a = \left(15r^2 m_\mathrm{b} \mathrm{v}^2 \frac{X_\mathrm{b} + X_\mathrm{s}}{16}\right)^{\frac{1}{5}}, \tag{4}$$

with

$$X_\mathrm{b} = \frac{1 - \nu_\mathrm{b}{}^2}{E_\mathrm{b}}$$

and

$$X_\mathrm{s} = \frac{1 - \nu_\mathrm{s}{}^2}{E_\mathrm{s}},$$

where r is the radius of the spherical boulder, m_b the mass of the boulder, v the impact velocity, $E_{\mathrm{b,\,s}}$ and $\nu_{\mathrm{b,\,s}}$ the elastic modulus and the Poisson's ratio of the boulder and the slope materials, respectively. In the finite element model the dimension a of the impact area is parameterized as a function of the impact velocity v according to Eq. 4. In this way, for each impact velocity the fine mesh is able to capture the local deformation around the contact point with a relatively small number of elements, reducing the necessary computational costs. The finite element model is validated comparing the results obtained (in terms of maximum contact force, maximum compression and duration of contact) with the analytical solution proposed by [Hertz (1881)] for the impact between an elastic sphere and an elastic surface. The values of the coefficients of restitution obtained using the finite element model are then compared with the analytical results obtained by [Hunter (1957)] and [Reed (1985)], revealing a satisfactory agreement between the numerical and analytical results.

3 Uncertainty in Rockfall Simulation

Despite the continuing advances in rockfall analysis, the mathematical modeling and simulation of rockfall phenomena remains affected by a large amount of variability and lack-of-knowledge of significant model parameters. The variability stems from the randomness of environmental conditions, such as daily or seasonal temperature fluctuations or precipitation variations, and boundary conditions, such as the seismic activity in the mountain area being considered. Uncertainty due to random parameters is also referred to as *aleatory uncertainty* and may be efficiently represented by traditional probability theory. On the other hand, lack-of-knowledge

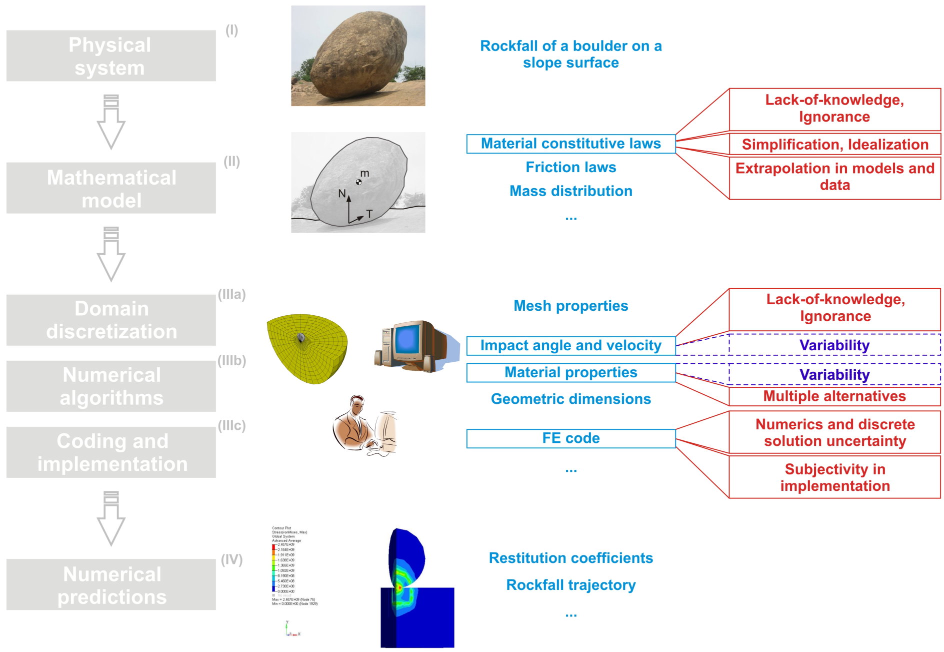


Figure 4: Aleatory (blue dashed boxes) and epistemic (orange boxes) uncertainty in modeling and simulation of rockfall trajectories

can stem from the difficulty in directly measuring or estimating some fundamental parameters and characteristics of rockfall phenomena. In addition, due to the dimensions of the phenomenon, a full-scale experimental program to calibrate or update the model parameters is generally not practical. As a consequence, a major lack-of-knowledge is encountered in the constitutive material laws, in the geometry and dimensions of the rock blocks, in the topography of the slope surface, in the contact and frictional response between blocks and slope and so on. In the literature, lack-of-knowledge is also referred to as *epistemic uncertainty*. In Fig. 4 a description of the numerical modeling and simulation procedure used in this paper to simulate the bouncing phenomenon is reported, together with some significant sources of aleatory and epistemic uncertainty. Aleatory and epistemic uncertainty analyses of different types of systems und structures have been addressed in literature [Moens, De Munck, and Vandepitte (2007); Gao, Song, and Tin-Loi (2009); Panda and Manohar (2008); Manjuprasad and Manohar (2007); Jiang and Han (2007); Parussini and Pediroda (2007); Parussini and Pediroda (2008); Loeven and Bijl (2008); Stroud, Krishnamurthy, and Smith (2002)]. This paper focuses on the representation and quantification of epistemic uncertainty in rockfall modeling and simulation by fuzzy numbers. The propagation of the considered epistemic uncertainty is then calculated using fuzzy arithmetic. For a practical implementation of fuzzy arithmetic the transformation method is used. The transformation method offers a powerful simulation tool to trace the propagation of the uncertainty through the system and also a sensitivity analysis tool to determine the influences of the uncertain input parameters on the uncertain outputs. In particular, epistemic uncertainties on the material properties and constitutive laws, on the boulder geometry and dimensions, on the kinematics of the impact and on the contact response between the boulder and slope are considered in this paper. The propagation and influence of such uncertain parameters are investigated with respect to the determination of the coefficients of restitution.

4 Fuzzy numbers and the Transformation Method

In this section some general information about fuzzy sets, fuzzy numbers and the transformation method as a powerful and practical implementation of fuzzy arithmetic are reported. The theory of fuzzy sets as introduced by Zadeh [Zadeh (1965)] is an extension of classical set theory. In a classical set A, the membership of an element x can be defined by the characteristic function μ_A, which is a mapping of the form

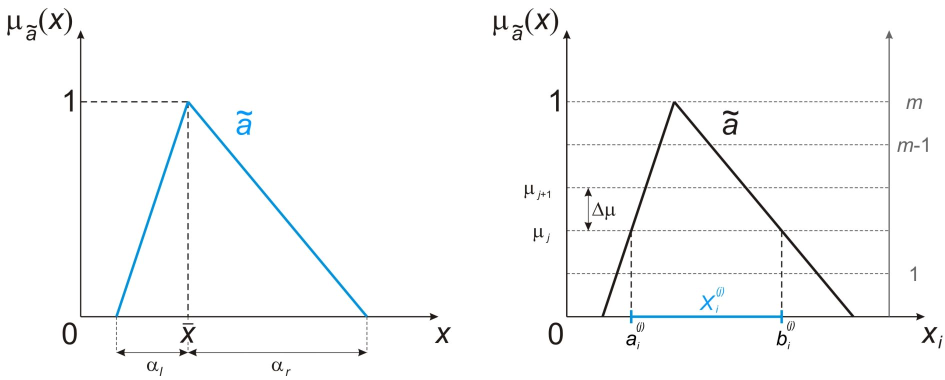
$$\mu_A : X \mapsto \{0,1\}.$$

For elements that belong to A, $x \in A$, the characteristic function takes the value $\mu_A(x) = 1$ (full membership), otherwise it takes $\mu_A(x) = 0$, which corresponds to $x \notin A$. Contrary to classical (crisp) sets, the membership $\mu_{\widetilde{A}}$ of an element x of a fuzzy set $\widetilde{A}$ can take any real value between zero and one. A fuzzy set $\widetilde{A}$ is defined by

$$\widetilde{A} = \{(x, \mu_{\widetilde{A}}(x)) \mid x \in X, \mu_{\widetilde{A}}(x) \in [0,1]\}. \tag{5}$$

A classical set is, in fact, a special case of a fuzzy set.
A fuzzy set is called a fuzzy number $\widetilde{a}$ if it is normal and convex and if there is only one element $\bar{x} \in \mathbb{R}$ with the degree of membership $\mu_{\widetilde{a}}(\bar{x}) = 1$. In addition, the membership function $\mu_{\widetilde{a}}(x), x \in \mathbb{R}$, has to be at least piecewise continuous. A fuzzy number $\widetilde{a}$ is a special case of a fuzzy set $\widetilde{A}$ and it appears to be an ideal way of describing uncertain parameters. It complies with the human perception of quantifying imprecision and it is able to represent different types of uncertainty. Using fuzzy numbers results in a possibilistic approach to the quantification of epistemic uncertainties. Certain types of fuzzy numbers, either symmetric or asymmetric, are

commonly used, such as the triangular (or linear) fuzzy numbers (Fig. 5(a)). They are defined by their membership functions $\mu_{\widetilde{a}}(x)$ and their left-hand and right-hand worst-case deviations α_{l} and α_{r}. Triangular fuzzy number are usually given in the form $\widetilde{a} = \mathrm{tfn}\,(\bar{x},\ \alpha_{\mathrm{l}},\ \alpha_{\mathrm{r}})$. The definition of fuzzy numbers leads to the development



(a) Triangular fuzzy number $\widetilde{a}$ (b) Decomposition of $\widetilde{a}$

Figure 5: Triangular fuzzy number $\widetilde{a}$ with mean value $\bar{x}$ and left-hand and right-hand worst-case deviations α_{l} and α_{r} and its decomposition into intervals $X_i^{(j)} = [a_i^{(j)}, b_i^{(j)}]$

of a mathematical concept for calculating with these numbers – fuzzy arithmetic. The extension principle, the fundamental arithmetical principle of fuzzy arithmetic, can be adapted to fuzzy numbers, thus enabling the evaluation of mathematical operations like addition, multiplication etc. Different approaches for practical implementation of fuzzy arithmetic have been developed [Dubois and Prade (1978); Kaufmann and Gupta (1991); Moore (1966)]. For the evaluation of systems with uncertain, fuzzy-valued model parameters, the transformation method (TM) can be used, proving to be a powerful and practical implementation of fuzzy arithmetic. The method is available in a general, a reduced, and an extended form [Hanss (2005)]. The reason for using the TM is the incorporation of a fuzzy arithmetical approach for the description of the uncertain parameters. For the computation itself, crisp parameter values from the interval range of the decomposed fuzzy input parameters are chosen and combined in parameter combinations at which the problem is evaluated. The results obtained are used to establish the fuzzy output. Assuming the uncertain system to be characterized by n fuzzy-valued model parameters $\widetilde{p}_i$, $i = 1, 2, \ldots, n$, the five major steps of the method can briefly be described as follows, :

1. Decomposition: Each fuzzy number $\widetilde{p}_i$ is decomposed into a number of intervals $X_i^{(j)} = [a_i^{(j)}, b_i^{(j)}]$, assigned to the membership levels μ_j, $j = 0, 1, \ldots, m$. The membership levels μ_j result from the subdivision of the axis of membership, into equally spaced intervals of length $\Delta\mu = 1/m$ (Fig. 5(b)).

2. Transformation: The input intervals $X_i^{(j)}$ are transformed to arrays $\widehat{X}_i^{(j)}$. When using the reduced TM, the parameter values $c_{l,i}^{(j)}$ in these arrays are obtained from the upper and lower interval bounds, $a_i^{(j)}$ and $b_i^{(j)}$. If the general and extended TM are used, the parameter values $c_{l,i}^{(j)}$ incorporate further values in between the interval bounds. The arrays are created by applying a well-defined combinatorial scheme.

3. Evaluation: If each array $\widehat{X}_i^{(j)}$ represents a column of a parameter matrix, then all parameter combinations can be found in the rows of this matrix. These parameter combinations feature only crisp parameter values, for each of which the model has to be evaluated. In practice, only the well-known concepts of crisp arithmetic are needed for these evaluations.

4. Retransformation: As a result of the evaluation of the model, an output array $\widehat{Z}^{(j)}$ is obtained. In the fourth step, the resulting values of the array are retransformed to the output intervals $Z^{(j)} = [a^{(j)}, b^{(j)}]$ for each membership level μ_j, where $Z^{(j)}$ represents the fuzzy output $\widetilde{q}$ of the model in its decomposed form.

5. Recomposition: The intervals $Z^{(j)}$ are recomposed to the fuzzy-valued output $\widetilde{q}$ of the system.

In addition to the described simulation, the TM offers a sensitivity analysis and an uncertainty analysis [Gauger, Turrin, Hanss, and Gaul (2008)]. Both analyses give an insight into the effect of the uncertain input parameters $\widetilde{p}_i$ on the uncertain model output $\widetilde{q}$.

Due to the specific properties of the TM, the method can be coupled to any software code, such as finite element software, without expensive re-coding; examples for various industrial problems can be found in [Hanss, Gauger, and Turrin (2006)]. The steps of decomposition and transformation as well as retransformation and recomposition can be coupled to any existing (commercial) software by a separate pre- and postprocessing tool without changes to the source code of the software.

5 Effect of epistemic uncertainties on the coefficients of restitution

As an application of the suggested approach, the effect of epistemic uncertainties on the determination of the coefficients of restitution was studied. The impact between an elastic boulder and an elastic slope was considered. Epistemic uncertainties in the material properties, the boulder geometry and dimensions, the kinematics of the impact and the contact response between the boulder and slope were taken into account. The uncertainties are represented and quantified by fuzzy numbers and their propagation and influence on the coefficients of restitution were calculated using the transformation method. Two different impact configurations were considered: a *normal impact*, with vertical impact velocity and horizontal slope, and an *inclined impact*, with vertical impact velocity and inclined slope (three different inclinations of 30°, 45° and 60°, respectively, were considered).

5.1 Normal impact

5.1.1 Influence of uncertain material properties and boulder dimension

The normal impact of an elastic spherical boulder on an elastic slope was simulated using the finite element model reported in Section 2 in order to determine the effect of epistemic uncertainties on the coefficients of restitution. The material of both boulder and slope, was assumed to be linear elastic with the following properties: density $\rho_0 = 2.6 \cdot 10^{-6}$ kg/mm^3, elastic modulus $E_0 = 68.2$ GPa and Poisson's ratio $\nu_0 = 0.25$ (which correspond to the material properties of granite). The boulder was modeled as a sphere of radius $r_0 = 1000$ mm. As discussed above, rockfall modeling and simulation is characterized by epistemic uncertainties stemming from the difficulty in directly measuring or estimating some fundamental parameters and characteristics of rockfall phenomena. In order to take into account such epistemic uncertainties, the model nominal parameters ρ_0, E_0, ν_0 and r_0 were replaced by symmetric triangular fuzzy numbers, with left- and right-hand worst-case deviations equal to 5% of the respective nominal value. The fuzzy inputs used in this study are shown in Fig. 6. The epistemic uncertainty of the radius of the spherical boulder, for example, is represented by the triangular fuzzy number $\widetilde{r} = \mathrm{tfn}\,(r_0,\ 0.05r_0,\ 0.05r_0)$.

For this first uncertainty analysis, a vertical impact velocity v equal to 10 m/s was considered. The effects of the epistemic uncertainties on the maximum compression, the maximum contact force, the duration of contact and the coefficients of restitution for the normal velocity and for the kinetic energy were calculated using the transformation method. The results are shown in Fig. 7. As is evident from Fig. 7, the effect of the epistemic uncertainties on the maximum compression, the

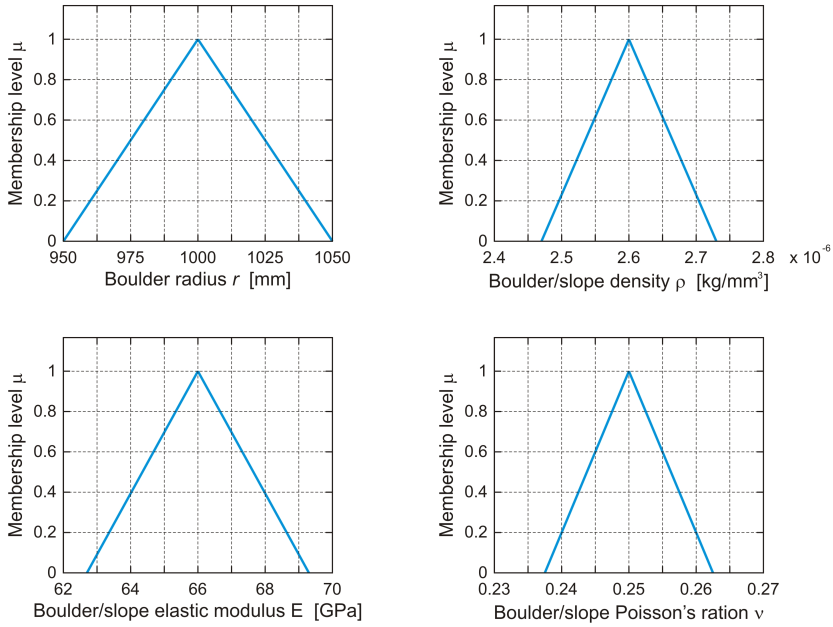


Figure 6: Uncertain input parameters: boulder radius, boulder/slope density, boulder/slope elastic modulus and boulder/slope Poisson's ratio

maximum contact force and the duration of contact is much stronger than that on the two coefficients of restitution. For example, in the case of the maximum contact force, the left- and right-hand worst case deviation are equal to 14.5% and 16.1% of the nominal value ($F_0 = 8.96 \cdot 10^4$ kg mm/ms^2), respectively. Otherwise, for the coefficient of restitution of the normal velocity, the worst-case deviations are only equal to 0.21% and 0.25% of the nominal value ($R_{n0} = 0.9856$). It is therefore possible to conclude that, for a normal impact, the coefficients of restitution are not sensitive to the considered epistemic uncertainties that were considered.

As mentioned above, the transformation method offers a sensitivity analysis to give an insight into the relative and absolute influence of each uncertain input parameter on each uncertain model output. For the coefficients of restitution, the result of the sensitivity analysis is shown in Fig. 8 in terms of relative measures of influence ω_i, where i is the i-th uncertain input parameter. In the case of a normal impact of a sphere, the measures of influence for the two coefficients of restitution are obviously the same since the coefficient of restitution for the kinetic energy is simply the square of the coefficient of restitution for the normal velocity (the tangential

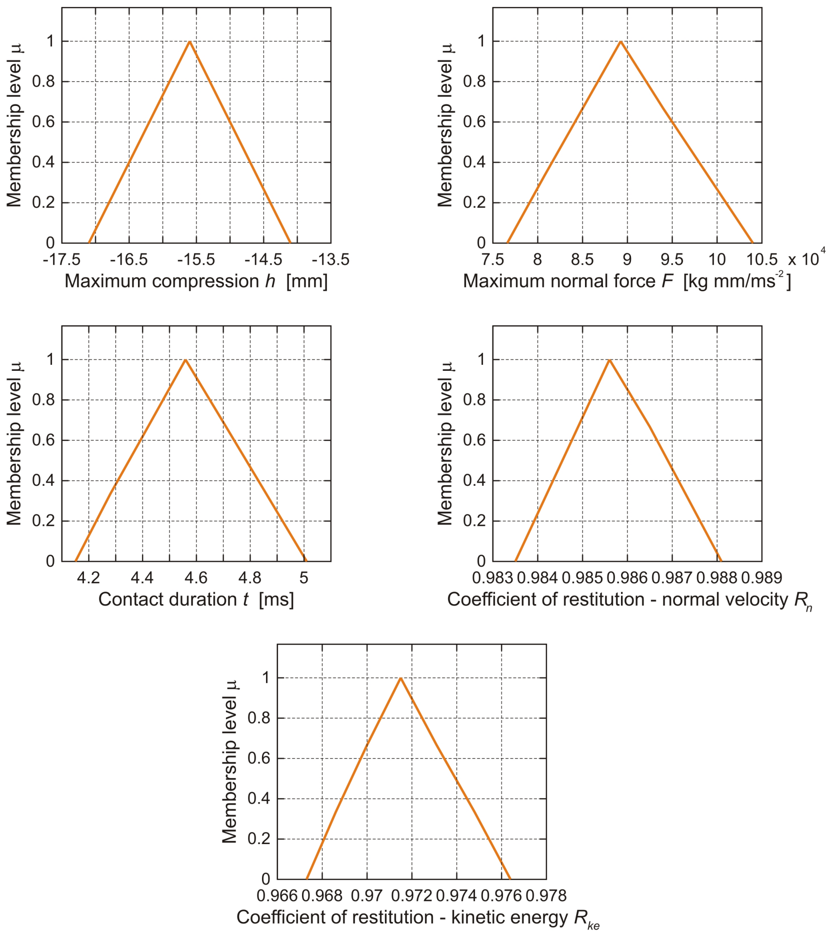


Figure 7: Uncertain model outputs: maximum compression, maximum normal force, contact duration, coefficient of restitution for the normal velocity and coefficient of restitution for the kinetic energy

velocity before and after the impact is 0). It is evident, from Fig. 8, that the uncer-

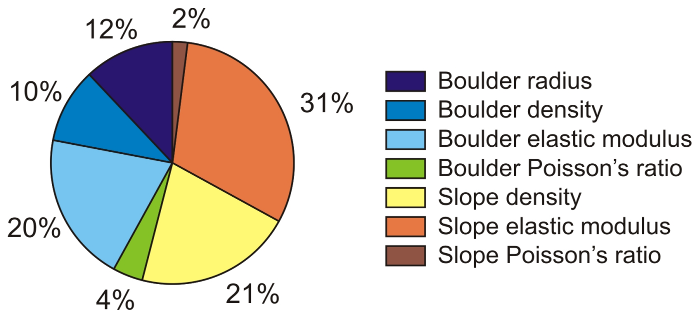


Figure 8: Relative measures of influence for the coefficient of restitution (normal velocity and kinetic energy).

tainty of the slope elastic modulus is the most important input for the uncertainty of the coefficients of restitution (approx. 31% of the overall uncertainty). On the other hand, the influence of the uncertainties of the boulder and slope Poisson's ratio on the coefficients of restitution are relatively small (approx. 4% and 2% respectively) in comparison to the influences of the other input parameters. For this reason, in the following, the uncertainties of the Poisson's ratio will not be taken into account.

The same uncertainty analysis, with the same input parameters (except for the boulder and slope Poisson's ratio), was then performed for a vertical impact velocity v = 1 m/s and v = 30 m/s in order to investigate the influence of the impact velocity on the uncertainty propagation. The corresponding results are shown in Fig. 9. As predicted by [Hunter (1957)] and [Reed (1985)] the coefficients of restitution decrease with increasing impact velocity. From Fig. 9 it is also possible to observe that the uncertainties of the coefficients of restitution increase with increasing impact velocity, even if the amount of uncertainty of the input parameters does not vary. As shown in Fig. 10 the relative measures of influence vary, varying the impact velocity. For low impact velocities (v = 1 m/s) the uncertainty of the boulder radius has the predominant influence on the uncertainty of the coefficients of restitution. With increasing impact velocity, the uncertainty of the boulder radius becomes less important and the predominant role is now played by the uncertainty of the slope elastic modulus. This implies that the importance of the input parameters on the determination of the uncertainty of the coefficients of restitution is dependant on the impact velocity. Consequently a preliminary estimation of the impact velocity within the boulder trajectory is fundamental for an efficient representation of the uncertain input parameters that have a strong influence on the uncertainty of

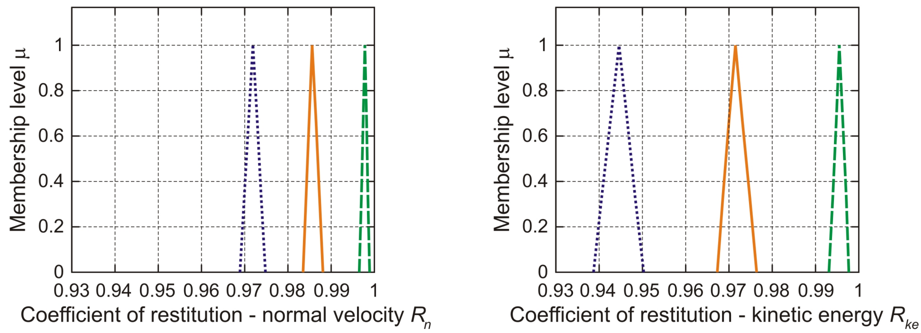


Figure 9: Coefficients of restitution for vertical impact velocities v = 1 m/s (green dashed line), v = 10 m/s (orange solid line) and v = 30 m/s (blue dotted line)

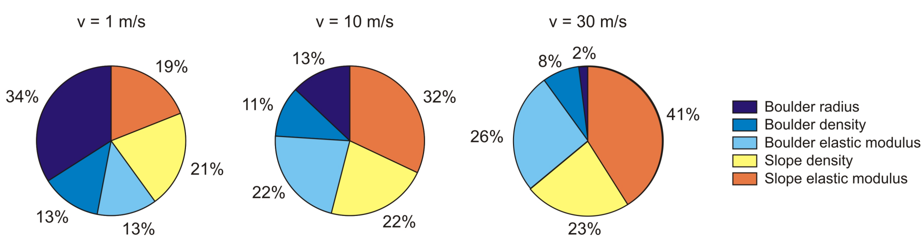


Figure 10: Relative measures of influence for the coefficient of restitution (normal velocity and kinetic energy) for vertical impact velocities v = 1, 10 and 30 m/s

the boulder trajectory itself.

5.1.2 Influence of uncertain boulder geometry

Fuzzy numbers can also be used to represent lack-of-knowledge or imprecision on the actual geometry of boulder and slope. In this section, an approach that takes into account epistemic uncertainty of the boulder geometry is presented. The boulder is modeled as a spheroid with horizontal axis c and vertical axis d, as shown in Fig. 11. For $c = d = 1000$ mm the spheroid is simply a sphere of radius $r_0 = 1000$ mm corresponding to the nominal geometry of the previous analyses. Representing the two axes c and d by fuzzy numbers is a practical method to introduce uncertainties stemming from an approximate knowledge of the actual boulder geometry in the numerical model.

The influence of the uncertain boulder geometry on the coefficients of restitution

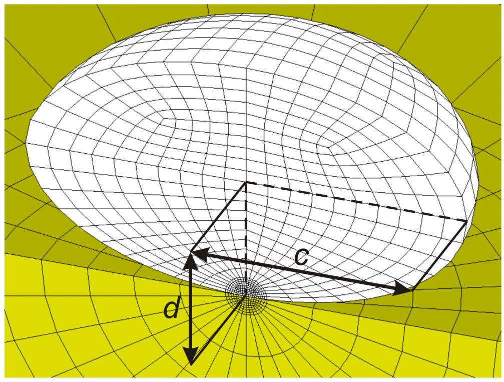


Figure 11: Spheroidal boulder

can now be studied. The horizontal axis c was represented by the symmetric triangular fuzzy number $\widetilde{c} = \mathrm{tfn}(1000, 100, 100)$. The values of the vertical axis d are given by the equation for the volume of a spheroid:

$$V = \frac{4}{3}\pi \hat{c}^2 d \tag{6}$$

where $V = \frac{4}{3}\pi {r_0}^3$ is the volume of the nominal boulder (i.e. a sphere) and $\hat{c}$ is the actual value of the horizontal axis within the fuzzy number $\widetilde{c}$. In this way the volume and, consequently, the mass of the boulder is kept constant and the resulting uncertainty of the coefficients of restitutions results only from uncertainty of the boulder geometry. In Fig. 12, the fuzzy number $\widetilde{c}$ is shown with the boulder geometries corresponding to its left- and right-hand worst-case points and to the nominal point. The effect of the uncertainty of the boulder geometry on the coefficient of restitution for the normal velocity is shown in Fig. 13 (blue solid line). As evident from the Fig. 13, the effect of the uncertain geometry on the coefficient of restitution is much bigger than the effect of uncertain material properties and boulder dimension. That means that, for a normal impact, a good knowledge of the actual boulder geometry or a convenient representation of its uncertainty are indispensable for a correct calibration of the coefficients of restitution and, consequently, the boulder trajectory.

5.2 Inclined impact

The effect of epistemic uncertainties on the coefficients of restitution for an inclined impact was studied. The impact velocity of the boulder is vertical and equal

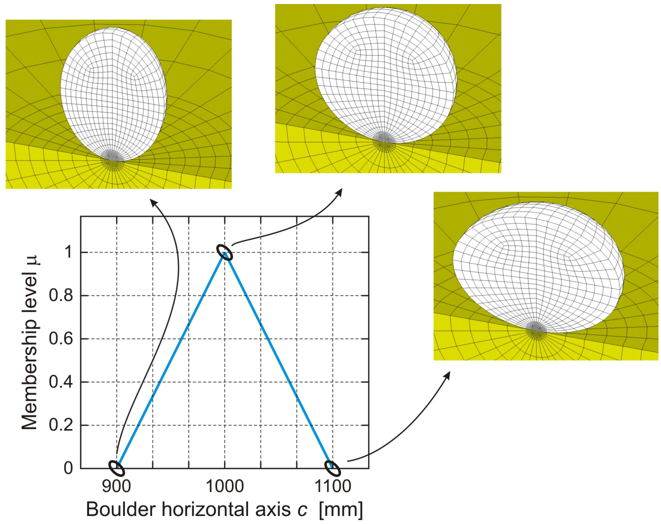


Figure 12: Fuzzy horizontal axis and boulder geometry corresponding to its left- and right-hand worst-case point and to its nominal point

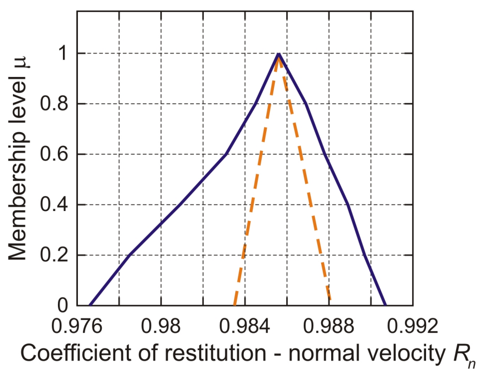


Figure 13: Influence of uncertain boulder geometry (blue solid line) and influence of uncertain material properties and boulder dimension (orange dashed line) on the coefficient of restitution of the normal velocity

to 10 m/s. Three different slope inclinations with respect to the horizontal plane were considered: 30°, 45° and 60°. As uncertain parameters, the same input fuzzy numbers as in Subsection 5.1.1 were considered, except for the Poisson's ratio of

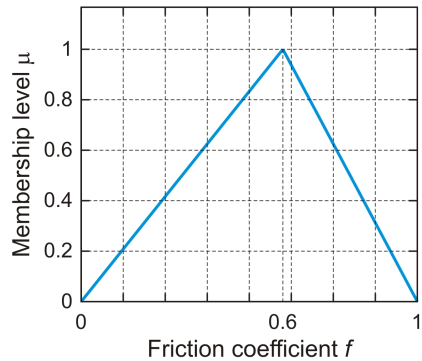
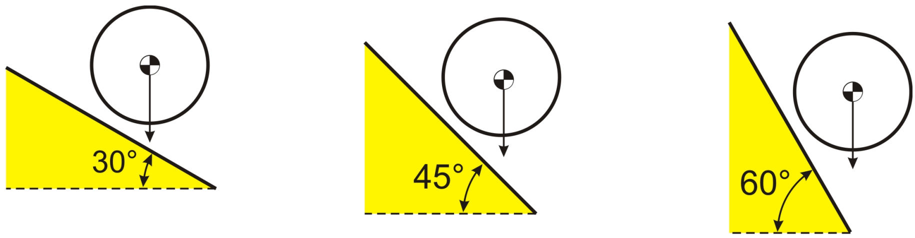


Figure 14: Uncertainty friction coefficient

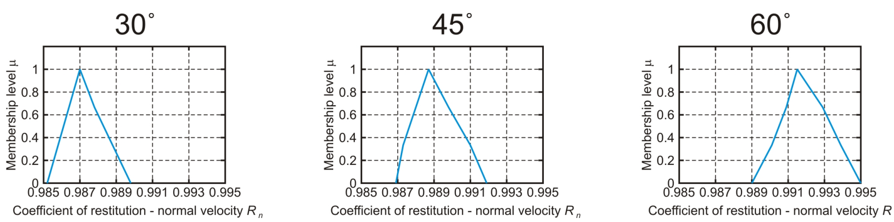
the boulder and slope material. In addition to these parameters, epistemic uncertainty on the contact response between boulder and slope was taken into account. For this reason, the friction coefficient between the boulder and slope (both static and kinetic) is represented by the fuzzy number $\widetilde{f} = (0.6, 0.6, 0.4)$, as shown in Fig. 14. The nominal value of $\widetilde{f}$ is the dry friction coefficient of granite. The left- and right-hand worst-case deviations (0.6 and 0.4, respectively) were chosen in such a way as to cover a large spectrum of possible friction coefficients and, consequently, of possible boulder-slope interface roughnesses.

The effect of the considered uncertainties on the coefficients of restitution is shown in Fig. 15 for the three different slope inclinations.

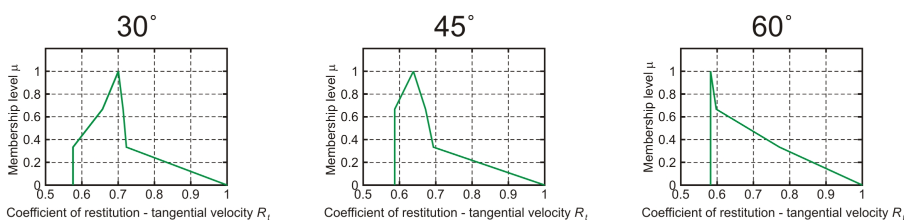
Even for an inclined impact, the effect of the uncertainties on the coefficients of restitution for the normal velocities is negligible (Fig. 15(b)). The introduction of some uncertainties in the friction coefficients does not have a noticeable influence on the uncertainty of the normal velocity after the impact. For the tangential velocity, on the contrary, the uncertainty of the friction coefficient is responsible for the large variations that characterize the coefficients of restitution for the tangential velocity (Fig. 15(c)). A sensitivity analysis (not reported in this paper) revealed that 98% of the uncertainty of the coefficients of restitution for the tangential velocity was due to uncertainty of the friction coefficient. This means that a good estimation of the actual contact response between the boulder and slope is absolutely necessary for a realistic simulation of the boulder trajectory. In addition, it was also observed that the uncertainty on the coefficient of restitution of the tangential velocity does not vary for the three different slope inclinations. In contrast, for the kinetic energy the uncertainty increases with increasing slope inclination (Fig. 15(d)). This



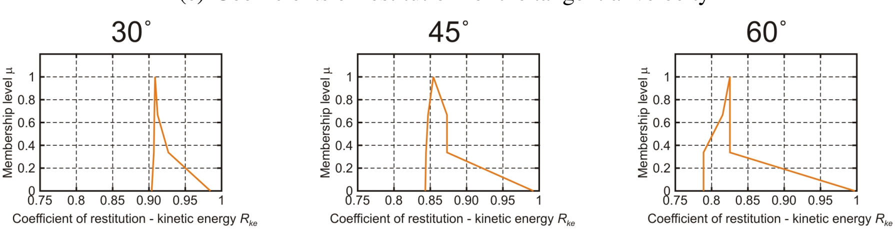
(a) Three different slope inclinations



(b) Coefficients of restitution for the normal velocity



(c) Coefficients of restitution for the tangential velocity



(d) Coefficients of restitution for the kinetic energy

Figure 15: Uncertainty analysis for inclined impact

means that a steeper slope increases the uncertainty of the rotational velocity of the boulder after the impact (not explicitly reported in this paper).

6 Conclusions

Modeling and simulation of rockfall phenomena are affected by a large amount of aleatory and epistemic uncertainty on significant model parameters. In this paper, an approach to epistemic uncertainty analysis of rockfall simulation is suggested. Uncertain input parameters were represented by fuzzy numbers and their influence on the coefficients of restitution was calculated using fuzzy arithmetic. In the case of an elastic normal impact, lack-of knowledge or imprecision in the actual geometry of the boulder results in a poor estimation of the coefficients of restitution. For an elastic inclined impact, the predominant role in calibrating the coefficients of restitution is played by the contact response between the boulder and slope. A natural continuation of this work would be the uncertainty analysis for normal and inclined elasto-plastic impact. In this case, epistemic uncertainty stemming from an imperfect knowledge of the material constitutive properties should also be considered. Finally, the inclusion of epistemic uncertainties in the calculation of the whole trajectory of the boulder may lead to a better understanding of rockfall phenomena and to a more reliable risk analysis and safety assessment.

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