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Analytic Closed Solution for the Heat Conduction with Time Dependent Heat Convection Coefficient at One Boundary

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Abstract: A new solution method is proposed to develop the analytic closed form solution for the one dimensional heat conduction with one mixed type boundary condition and general time dependent heat convection coefficient for the first time. The solution method is the combination of an extension of the shifting function method developed by Lee and his colleagues and a series expansion. It is shown that the solution is simple and accurate. The convergence of the present analysis is very fast. One can find that when the dimensionless Fourier number is greater than 0.2, the error for the one term approximation solution of the infinite serious solution can be less than 2%. Examples are given to reveal the solution method. Numerical results are compared with those in the existing literature.

Keywords: heat conduction, time dependent heat convection coefficient, closed analytic solution, shifting function method, series expansion

1 Introduction

The problems of heat transfer with variable heat transfer coefficients can be important in many engineering applications. The two heat transfer coefficients are the heat conduction coefficient and the heat convection coefficient. It is known that if the temperature and the heat flux are prescribed along the boundary surface, then the heat transfer system contains heat conduction coefficient only. If the boundary surface dissipates heat by convection according to Newton's law of cooling, the heat convection coefficient will be contained in the boundary term.

The studies on the problems of heat transfer with space and temperature dependent heat conduction coefficient are tremendous [?zisik (1968); Dul'kin and Garas'ko

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(2002); Sladek, Sladek and Atluri (2004); Liu, Liu and Hong (2007); Borukhov, Tsurko and Zayats (2009)].

For the heat conduction with mixed type boundary condition and time dependent heat convection coefficient, the problem can not be solved by the method of separation of variable. The problem was studied by various approximated and numerical methods. By introducing a new variable, Ivanov and Salomatov [1965, 1966] and Postol'nik [1970] transform the governing differential equation into to a nonlinear one. After neglecting the nonlinear term, they obtained an approximated solution which was claimed to be valid for the system with Biot number being less than 0.25. Kozlov [1970] used the Laplace transformation method to study the problems with Biot function in a rational combination of sines, cosines, polynomials and exponentials. Even through it is possible to obtain an exact series solution of the specified transformed system, there always is great difficulty while taking the inverse Laplace transform of the transformed solution in complex domain. Becker, Bivins, Hsu, Murphy, White and Wing [1983] used the finite difference method and the Laplace transformation method to study the heating of the rock adjacent to water flowing through a crevice. Different approximation methods such as the iterative perturbation method [Holya (1972)], the eigenfunction expansion method [Özisik and Murray (1974)] and the Lie point symmetry analysis [Moitsheki (2008)] were used to study the kind of problems. Recently, various inverse schemes for determining the time dependent heat convection coefficient were developed by Chantasiriwan (2000), Su and Hewitt (2004), Zueco, Alhama and Fernández (2005), Chen and Wu (2008), and Onyango, Ingham, Lesnic and Slodička (2009).

From the existing literature, due to the complexity and difficulty of the solution, it can be found that the study on the heat conduction with mixed type boundary condition and time dependent heat convection coefficient is insufficient. So far, most of the studies are limited in the problems with Biot function in a rational combination of sines, cosines, polynomials and exponentials. In addition, the way to obtain the solution is always crumbly and tedious. A simple and accurate analytic form solution for wide class of problems had never been developed before.

In this paper, a new solution method is proposed to develop the analytic closed form solution for the one dimensional heat conduction with one mixed type boundary condition and general time dependent heat convection coefficient for the first time. The solution method is an extension of the shifting function method developed by Lee and Lin (1996). By setting the Biot function in a new form and introducing a particularly chosen shifting function, the system is transformed into a partial differential equation with homogenous boundary conditions. Consequently, it is solved by a series expansion.

It is shown that the solution is simple and accurate. It is valid for a wide class of physic problems. One can find that when the dimensionless Fourier number is greater than 0.2, the error for the one term approximation solution of the infinite serious solution can be less than 2%. Examples are given to reveal the solution method. Numerical results are compared with those in the existing literature. Finally, the influence of the constants in the Biot function on the temperature is studied.

2 Mathematical Modeling

Consider the heat conduction in a slab with mixed type boundary condition at one end as shown in Figure 1. The governing differential equation of the system is

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t}, \quad 0 < x < L, \quad t > 0. \tag{1}$$

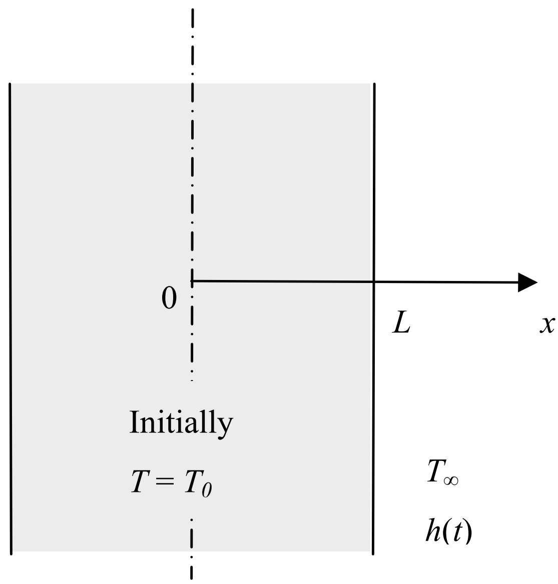

Figure 1: One-dimensional heat transfer system of a slab with a time dependent heat convection coefficient

The boundary conditions are

$$\frac{\partial T}{\partial x} = 0, \text{ at } x = 0, \tag{2}$$

$$-k\frac{\partial T}{\partial x} = h(t)\,(T - T_\infty)\,, \text{ at } x = L, \tag{3}$$

and the initial condition is

$$T(x) = T_0(x), \text{ at } t = 0. \tag{4}$$

Here, T is the temperature, T_0 is the initial temperature, T_∞ is a temperature constant, α is the thermal diffusivity, k is the thermal conductivity, $h(t)$ is the time dependent heat convection coefficient, t is time and L is the thickness of the slab. In terms of the following non-dimensional quantities:

$$X = \frac{x}{L}, \quad \theta = \frac{T - T_\infty}{T_\infty}, \quad Bi(\tau) = \frac{h(t)\,L}{k}, \quad \tau = \frac{\alpha t}{L^2}, \quad \theta_0 = \frac{T_0 - T_\infty}{T_\infty}, \tag{5}$$

The associated dimensionless governing differential equation of the system is

$$\frac{\partial^2\theta}{\partial X^2} = \frac{\partial\theta}{\partial\tau}. \text{ in } 0 < X < 1, \quad \tau > 0 \tag{6}$$

The associated dimensionless boundary conditions are

$$\frac{\partial\theta}{\partial X} = 0, \text{ at } X = 0 \tag{7}$$

$$\frac{\partial\theta}{\partial X} = -Bi(\tau)\,\theta, \text{ at } X = 1 \tag{8}$$

and the associated dimensionless initial condition is

$$\theta = \theta_0(X)\,, \text{ at } \tau = 0 \tag{9}$$

To simplify the analysis and increase the accuracy of the analysis, one sets the Biot function $Bi(\tau)$ be in the form of

$$Bi(\tau) = \delta + F(\tau)\,, \tag{10}$$

where

$$\delta = Bi(0)\,, \tag{11}$$

$$F(\tau) = Bi(\tau) - Bi(0)\,. \tag{12}$$

It is obvious that $F(0) = 0$, and the boundary condition at $X = 1$, equation (8), can be re-written as

$$\frac{\partial\theta}{\partial X} + \delta\theta = -F(\tau)\theta, \text{ at } X = 1 \tag{13}$$

3 The Shifting Function Method

3.1 Change of variable

To find the solution for the partial differential equation with a time dependent heat convection coefficient at one boundary, one extends the shifting function method developed by Lee and Lin (1996), Lee, Lin, Lee, Lu and Liu (2008) and Lee, Lu, Liu and Huang (2008) by taking

$$\theta(X,\tau) = v(X,\tau) + f(\tau)g(X), \tag{14}$$

where

$$f(\tau) = -F(\tau)\theta\left(1,\tau\right), \tag{15}$$

Here $g(X)$ is a shifting function to be specified and $v(X,\ \tau)$ is the transformed function.

Substituting equations (14) into equations (6, 7, 9, 13), one has the following partial differential equation

$$\frac{\partial^2 v\left(X,\tau\right)}{\partial X^2} + f(\tau)\frac{d^2 g(X)}{dX^2} = \frac{\partial v\left(X,\tau\right)}{\partial \tau} + g(X)\frac{\partial f(\tau)}{\partial \tau}, \tag{16}$$

and the associated boundary conditions and initial condition

$$\frac{\partial v(0,\tau)}{\partial X} + f(\tau)\frac{dg(0)}{dX} = 0, \text{ at } X = 0, \tag{17}$$

$$\frac{\partial v(1,\tau)}{\partial X} + f(\tau)\frac{dg(1)}{dX} + \delta\left[v\left(1,\tau\right) + f\left(\tau\right)g\left(1\right)\right] = f(\tau), \text{ at } X = 1. \tag{18}$$

$$v\left(X,0\right) + f(0)g\left(X\right) = \theta_0\left(X\right). \tag{19}$$

It should be mentioned that equations (16-19) contain two function variables $v(X,\ \tau)$ and $\theta(1,\ \tau)$.

3.2 Shifting Function

To simplify the problem, one specifies a particular shifting function $g(X)$. One lets the shifting function $g(X)$ in equations (16-18) satisfy the differential equation

$$\frac{d^2 g\left(X\right)}{dX^2} = C, \tag{20}$$

and the following boundary conditions

$$\frac{dg(0)}{dX} = 0, \tag{21}$$

$$\frac{dg(1)}{dX} = 1, \tag{22}$$

where C is a constant. From equations (21-22), constant C is determined as $C = 1$. Therefore, $g(X)$ is

$$g(X) = \frac{1}{2}X^2 + D, \tag{23}$$

where D is a constant to be specified.
Substituting the function $g(X)$ into equation (14), it becomes

$$\theta\,(X,\tau) = v\,(X,\tau) - F\,(\tau)\,\theta\,(1,\tau)\left[\frac{1}{2}X^2 + D\right]. \tag{24}$$

Setting X= 1 in the equation above, one has

$$\theta\,(1,\tau) = v\,(1,\tau) - F\,(\tau)\,\theta\,(1,\tau)\left(\frac{1}{2} + D\right). \tag{25}$$

It is obvious that if the constant D is chosen as D = -1/2, then

$$\theta\,(1,\tau) = v\,(1,\tau)\,. \tag{26}$$

As a result, the shifting function is determined as

$$g\,(X) = \frac{1}{2}X^2 - \frac{1}{2}. \tag{27}$$

With equations (26-27), the function variables in governing differential equation (16) is reduced from two to one and expressed in terms of the function variables $v(X,\ \tau)$

$$\frac{\partial^2 v}{\partial X^2} - F(\tau)v\,(1,\tau) = \frac{\partial v}{\partial \tau} - \left(\frac{1}{2}X^2 - \frac{1}{2}\right)\left[\dot{F}\,(\tau)v\,(1,\tau) + F\,(\tau)\dot{v}\,(1,\tau)\right]. \tag{28}$$

The associated boundary conditions of transformed function $v(X,\ \tau)$, equations (17-18) are

$$\frac{\partial v(0,\tau)}{\partial X} = 0, \tag{29}$$

$$\frac{\partial v(1,\tau)}{\partial X} + \delta v(1,\tau) = 0. \tag{30}$$

Since $f(0) = -F(0)\theta\ (1, 0)$ and $F(0) = 0$, therefore, the associated initial condition, equation (19), is simplified as

$$v\,(X,0) = \theta_0\,(X)\,. \tag{31}$$

3.3 Series Expansion

To find the solution for the partial differential equation (28) with boundary conditions (29-30) and initial condition (31), one uses the method of series expansion with try functions satisfying the boundary conditions (29-30). The try functions are

$$\phi_n(X) = \cos\left(\lambda_n X\right)\ ,\ n = 1,2,3...... \tag{32}$$

where the characteristic values λ_n are the roots of the transcendental equation

$$\tan\left(\lambda_n\right) = \frac{\delta}{\lambda_n}. \tag{33}$$

The inner products of the try functions are

$$\int_0^1 \phi_m\,(X)\phi_n\,(X)\,dX = \begin{cases} 0, & \text{for } m \neq n \\ N, & \text{for } m = n \end{cases} \tag{34}$$

where

$$N = \int_0^1 \left[\phi_m\,(X)\right]^2 dX = \frac{\lambda_m + Cos\,(\lambda_m)\,Sin\,(\lambda_m)}{2\lambda_m}. \tag{35}$$

One can assume the solution takes the form of

$$v(X,\tau) = \sum_{n=1}^{\infty} \phi_n(X) q_n(\tau), \quad n = 1,2,3\ldots \tag{36}$$

Substituting solution form (36) into differential equations (28), it leads to

$$\sum_{n=1}^{\infty} \left\{\phi_n\,(X)\,\dot{q}_n\,(\tau) - g\,(X)\left[\dot{F}\,(\tau)\,\phi_n\,(1)\,q_n\,(\tau) + F\,(\tau)\,\phi_n\,(1)\,\dot{q}_n\,(\tau)\right]\right.$$
$$\left. -\phi_n''\,(X)\,q_n\,(\tau) + F\,(\tau)\,\phi_n\,(1)\,q_n\,(\tau)\right\} = 0 \tag{37}$$

One can let

$$\phi_n(X)\dot{q}_n(\tau)-g(X)\left[\dot{F}(\tau)\phi_n(1)q_n(\tau)+F(\tau)\phi_n(1)\dot{q}_n(\tau)\right] \\ -\phi_n''(X)q_n(\tau)+F(\tau)\phi_n(1)q_n(\tau)=0 \quad (38)$$

After taking the inner product with try functions $\phi_n(X)$, the resulting differential equation is

$$\dot{q}_n(\tau)\left[1-\beta_n F(\tau)\right]+q_n(\tau)\left[\lambda_n^2-\beta_n\frac{dF(\tau)}{d\tau}+\gamma_n F(\tau)\right]=0. \quad (39)$$

where

$$\beta_n=\phi_n(1)\frac{\int_0^1\phi_n(X)g(X)dX}{\int_0^1\phi_n^2(X)dX}, \quad (40)$$

$$\gamma_n=\phi_n(1)\frac{\int_0^1\phi_n(X)dX}{\int_0^1\phi_n^2(X)dX}. \quad (41)$$

The associated initial condition is

$$q_n(0)=\frac{\int_0^1\theta_0(X)\phi_n(X)dX}{\int_0^1\phi_n^2(X)dX}. \quad (42)$$

As a result, the solution for $q_n(\tau)$ is

$$q_n(\tau)=q_n(0)e^{\int_0^\tau\frac{\lambda_n^2+\gamma_n F(\tau)-\beta_n\dot{F}(\tau)}{\beta_n F(\tau)-1}d\tau}. \quad (43)$$

After substituting the transformed function $v(X,\tau)$, equations (36, 32, and 43) and the shifting function $g(X)$, equation (27) back to equation (14), one has the analytic solution

$$\theta(X,\tau)=\sum_{n=1}^{\infty}q_n(\tau)\left\{\cos\lambda_n X-\left(\frac{1}{2}X^2-\frac{1}{2}\right)F(\tau)\cos\lambda_n\right\},\quad n=1,2,3 \quad (44)$$

3.4 Constant Heat Convection Coefficient

When the heat convection coefficient h is a constant, the Biot function is a constant δ and $F(\tau)=0$. The infinite serious solution, equation (44), is reduced to

$$\theta(X,\tau)=\sum_{n=1}^{\infty}q_n(0)e^{-\lambda_n^2\tau}\cos\lambda_n X. \quad (45)$$

The solution is exactly the same as that obtained via the method of separation of variable [Özisik, 1968].

3.5 Verification and Examples

To illustrate the previous analysis and the accuracy the one term approximation solution, one examines the following two cases. The problem of heat conduction with the types of Biot functions considered has never been studied before.

Case 1: Consider the heat conduction in a slab as prescribed in the previous sections. The Biot function considered is

$$Bi(\tau) = a - be^{-s\tau}\cos\omega\tau. \tag{46}$$

This Biot function form is quite general. It covers most of the physic cases.

When $b = 0$ or both s and ω are zeros, the Biot function is a constant.

When b and ω are not zeros and $s = 0$, the Biot function is in cosine function form.

When b and s are not zeros and $\omega = 0$, the Biot function is in exponential function form.

This Biot function is a generalization of the one discussed by Ivanov and Salomatov (1965) and Postol'nik (1970).

According to equations (10-12, 15),

$$F(\tau) = b(1 - e^{-s\tau}\cos\omega\tau) \tag{47}$$

and

$$f(\tau) = -F(\tau)\theta(1,\tau). \tag{48}$$

The shifting function $g(X)$ is known as equation (27). The try functions for the transformed function $v(X, \tau)$ are

$$\phi_n(X) = \cos\lambda_n X\ ,\ n = 1,2,3...... \tag{49}$$

where λ_n are the roots of the transcendental equation

$$\tan\lambda_n = \frac{a-b}{\lambda_n}. \tag{50}$$

Consequently, the temperature distribution is

$$\theta(X,\tau) = \sum_{n=1}^{\infty} q_n(\tau)\left[\phi_n(X) - b(1-e^{-s\tau}\cos\omega\tau)\phi_n(1)\left(\frac{1}{2}X^2 - \frac{1}{2}\right)\right], \tag{51}$$

where

$$q_n(\tau) = q_n(0)\,e^{\int_0^\tau \frac{\lambda_n^2 - \beta_n be^{-s\tau}(s\cos\omega\tau - \sin\omega\tau) + \gamma_n b(1-e^{-s\tau}\cos\omega\tau)}{\beta_n b(1-e^{-s\tau}\cos\omega\tau)-1}d\tau}, \tag{52}$$

and $q_n(0)$ is the same as that given in equation (42).

When a = 1.2,b = 1, s = 1, ω = 0 and θ_0 = - 0.664, the case is exactly the same as the one discussed by Ivanov and Salomatov (1965) and Postol'nik (1970), who calculated the heating of an infinite plate. In Tables 1-2, the temperatures of the slab at both ends, X= 0 and X= 1, and various time points evaluated via the present analysis are compared with those in the existing literature. It shows the results are very consistent.

Table 1: Temperature of the slab at X = 0 and various time [a = 1.2, b = 1, s = 1, ω = 0, θ_0 = - 0.664]

τ	$\frac{T(0,\tau)}{T_\infty}$					
	A	B	C(1)	C(2)	C(10)	C(20)
0			0.315	0.341	0.336	0.336
0.1			0.322	0.331	0.331	0.331
0.2			0.336	0.339	0.339	0.339
0.3			0.354	0.355	0.355	0.355
0.4			0.376	0.376	0.376	0.376
0.5	0.400	0.390	0.400	0.401	0.401	0.401
1	0.538	0.516	0.541	0.541	0.541	0.541
1.5	0.662	0.638	0.672	0.672	0.672	0.672
2	0.770	0.738	0.774	0.774	0.774	0.774
2.5	0.833	0.814	0.848	0.848	0.848	0.848
3	0.878	0.868	0.899	0.899	0.899	0.899
3.5	0.917	0.907	0.933	0.933	0.933	0.933
4	0.944	0.935	0.956	0.956	0.956	0.956
A: given by Ivanov and Salomatov (1965) B: given by Postol'nik (1970) C (n): present analysis; n: number of term.						

In Tables 3-4, the convergence of the solutions evaluated by the present analysis is shown. The temperatures of the slab, with physic parameters being a = 1.2, b = 1, s = 2, ω = 2 and θ_0 = - 0.664, at both ends and different time points are listed. From Tables 1-4, one can find that the convergence of the present analysis is very fast. When the dimensionless Fourier number τ is greater than 0.2, the error for the one term approximation solution can be less than 2%.

With various combinations of the physic parameters, the temperatures of the slab at both ends are evaluated and listed in Tables 5-6. In figure 2, the influence of b/a in Biot function on the temperature variation of the slab at X = 1 is shown. It

Table 2: Temperature of the slab at X = 1 and various time [a = 1.2, b = 1, s = 1, ω = 0, θ_0 = - 0.664]

τ	$\frac{T(1,\tau)}{T_\infty}$					
	A	B	C (1)	C (2)	C (10)	C (20)
0			0.379	0.353	0.339	0.338
0.1			0.410	0.401	0.401	0.401
0.2			0.443	0.439	0.439	0.439
0.3			0.475	0.474	0.474	0.474
0.4			0.507	0.507	0.507	0.507
0.5	0.526	0.545	0.538	0.538	0.538	0.538
1	0.678	0.680	0.676	0.676	0.676	0.676
1.5	0.769	0.778	0.780	0.780	0.780	0.780
2	0.847	0.848	0.853	0.853	0.853	0.853
2.5	0.883	0.894	0.902	0.902	0.902	0.902
3	0.921	0.925	0.936	0.936	0.936	0.936
3.5	0.947	0.948	0.958	0.958	0.958	0.958
4	0.964	0.964	0.972	0.972	0.972	0.972

A: given by Ivanov and Salomatov (1965)
B: given by Postol'nik (1970)
C (n): present analysis; n: number of term.

can be found that the temperatures associated with high b/awill lower than those associated with low b/a. As time increases, all the temperature will reach to the same limiting value.

Figures 3 and 4 illustrate the influence of the s parameter and the ω parameter on the temperature variation of the slab at X = 1, respectively. It can be observed that as time goes, at the beginning, the temperatures associated with high s or ω parameter will be more sensitive than those associated with low s or ω parameter. Finally, all the temperature will reach to the same limiting value. In figure 5, as time increases, the temperature variation along the slab is illustrated.

Case 2: Consider the heat conduction in a slab as prescribed in the previous sections. The Biot number and the initial relative temperature considered are

$$Bi(\tau) = 1 - 0.5\frac{1}{(1+\tau)^3}, \tag{53}$$

and θ_0 = 5, respectively.

Table 3: Temperature of the slab at X = 0 and various time [a = 1.2, b = 1, s = 2, ω = 2, θ_0 = - 0.664]

τ	$\theta(0,\tau)$			
	1 term	2 terms	10 terms	20 terms
0	-0.621	-0.647	-0.661	-0.662
0.1	-0.569	-0.578	-0.579	-0.579
0.2	-0.517	-0.521	-0.521	-0.521
0.3	-0.468	-0.469	-0.469	-0.469
0.4	-0.423	-0.424	-0.424	-0.424
0.5	-0.383	-0.383	-0.383	-0.383
1	-0.237	-0.237	-0.237	-0.237
2	-0.100	-0.100	-0.100	-0.100
3	-0.043	-0.043	-0.043	-0.043
4	-0.018	-0.018	-0.018	-0.018
5	-0.008	-0.008	-0.008	-0.008
6	-0.003	-0.003	-0.003	-0.003
7	-0.001	-0.001	-0.001	-0.001
8	-0.001	-0.001	-0.001	-0.001
9	0	0	0	0

According to equations (10-12, 15),

$$F(\tau) = 0.5 - 0.5\frac{1}{(1+\tau)^3}. \tag{54}$$

Following the same procedures as those revealed, one has the analytic solution

$$\theta(X,\tau) = \sum_{n=1}^{\infty} q_n(\tau)\left[\phi_n(X) - \left(0.5 - 0.5\frac{1}{(1+\tau)^3}\right)\phi_n(1)\left(\frac{1}{2}X^2 - \frac{1}{2}\right)\right]. \tag{55}$$

where

$$q_n(\tau) = q_n(0)\,e^{\int_0^\tau \frac{\lambda_n^2 - \beta_n \frac{1.5}{(1+\tau)^4} + \gamma_n\left(0.5-0.5\frac{1}{(1+\tau)^3}\right)}{\beta_n\left(0.5-0.5\frac{1}{(1+\tau)^3}\right)-1}d\tau}, \tag{56}$$

$$q_n(0) = 5\frac{\int_0^1 \phi_n(X)\,dX}{\int_0^1 \phi_n^2(X)\,dX}, \tag{57}$$

$$\phi_n(X) = \cos\lambda_n X, \tag{58}$$

Table 4: Temperature of a slab at X = 1 and various time [a = 1.2, b = 1, s = 2, ω = 2, θ_0 = - 0.664]

τ	$\theta(1,\tau)$			
	1 term	2 terms	10 terms	20 terms
0	-0.685	-0.659	-0.664	-0.664
0.1	-0.683	-0.675	-0.675	-0.675
0.2	-0.669	-0.666	-0.666	-0.666
0.3	-0.644	-0.643	-0.643	-0.643
0.4	-0.612	-0.611	-0.611	-0.611
0.5	-0.575	-0.575	-0.575	-0.575
1	-0.386	-0.386	-0.386	-0.386
2	-0.160	-0.160	-0.160	-0.160
3	-0.068	-0.068	-0.068	-0.068
4	-0.029	-0.029	-0.029	-0.029
5	-0.012	-0.012	-0.012	-0.012
6	-0.005	-0.005	-0.005	-0.005
7	-0.002	-0.002	-0.002	-0.002
8	-0.001	-0.001	-0.001	-0.001
9	0	0	0	0

where λ_n are the roots of the transcendental equation

$$\tan\lambda_n = \frac{0.5}{\lambda_n}. \tag{59}$$

In Tables 7-8, the temperatures of the slab at both ends and different time points evaluated via the present analysis, using different number of terms in the series, are shown. The conclusion about the convergence of the present analysis is the same as that given in Example1. It is When the dimensionless Fourier number τ is greater than 0.2, the error for the one term approximation solution can be less than 2%. In figure 6, the temperature variation of the slab at both ends is shown. It can be found that the temperature variation of the slab is a monotonic function of τ. The temperatures at X = 0 are higher those at X = 1.

4 Conclusions

In this paper, a new solution method is proposed to develop the analytic closed form solution for the one dimensional heat conduction with one mixed type boundary condition and general time dependent heat convection coefficient for the first time.

Table 5: Temperature of a slab, with various s, ω and τ parameters, at X = 0 [a = 1.2, b = 1, θ_0 = - 0.664]

τ	$\theta(0,\tau)$								
	$s=0$			$s=1$			$s=2$		
	$\omega=0$	$\omega=1$	$\omega=2$	$\omega=0$	$\omega=1$	$\omega=2$	$\omega=0$	$\omega=1$	$\omega=2$
0	-0.664	-0.664	-0.664	-0.664	-0.664	-0.664	-0.664	-0.664	-0.664
0.5	-0.623	-0.622	-0.617	-0.599	-0.598	-0.593	-0.581	-0.579	-0.575
1	-0.568	-0.527	-0.443	-0.459	-0.443	-0.407	-0.411	-0.404	-0.386
2	-0.471	-0.253	-0.135	-0.226	-0.196	-0.158	-0.182	-0.174	-0.160
3	-0.390	-0.080	-0.059	-0.101	-0.081	-0.067	-0.078	-0.074	-0.068
4	-0.324	-0.023	-0.042	-0.044	-0.034	-0.029	-0.033	-0.031	-0.029
5	-0.268	-0.008	-0.013	-0.019	-0.014	-0.012	-0.014	-0.013	-0.012
6	-0.222	-0.005	-0.005	-0.008	-0.006	-0.005	-0.006	-0.006	-0.005
7	-0.184	-0.004	-0.004	-0.003	-0.003	-0.002	-0.003	-0.002	-0.002
8	-0.153	-0.002	-0.001	-0.001	-0.001	-0.001	-0.001	-0.001	-0.001
9	-0.127	-0.001	-0.001	-0.001	-0.001	0	-0.001	0	0
10	-0.105	0	0	0	0	0	0	0	0

Table 6: Temperature of a slab, with various s, ω and τ parameters, at X = 1 [a = 1.2, b = 1, θ_0 = - 0.664]

τ	$\theta(1,\tau)$								
	$s=0$			$s=1$			$s=2$		
	$\omega=0$	$\omega=1$	$\omega=2$	$\omega=0$	$\omega=1$	$\omega=2$	$\omega=0$	$\omega=1$	$\omega=2$
0	-0.662	-0.662	-0.662	-0.662	-0.662	-0.662	-0.662	-0.662	-0.662
0.5	-0.566	-0.535	-0.464	-0.462	-0.448	-0.412	-0.410	-0.402	-0.383
1	-0.515	-0.396	-0.245	-0.324	-0.295	-0.243	-0.268	-0.258	-0.237
2	-0.427	-0.140	-0.070	-0.147	-0.120	-0.096	-0.114	-0.109	-0.100
3	-0.354	-0.038	-0.053	-0.064	-0.050	-0.042	-0.049	-0.046	-0.043
4	-0.294	-0.012	-0.025	-0.028	-0.021	-0.018	-0.021	-0.020	-0.018
5	-0.244	-0.006	-0.007	-0.012	-0.009	-0.008	-0.009	-0.008	-0.008
6	-0.202	-0.005	-0.004	-0.005	-0.004	-0.003	-0.004	-0.004	-0.003
7	-0.167	-0.003	-0.003	-0.002	-0.002	-0.001	-0.002	-0.002	-0.001
8	-0.139	-0.001	-0.001	-0.001	-0.001	-0.001	-0.001	-0.001	-0.001
9	-0.115	0	0	0	0	0	0	0	0
10	-0.095	0	0	0	0	0	0	0	0

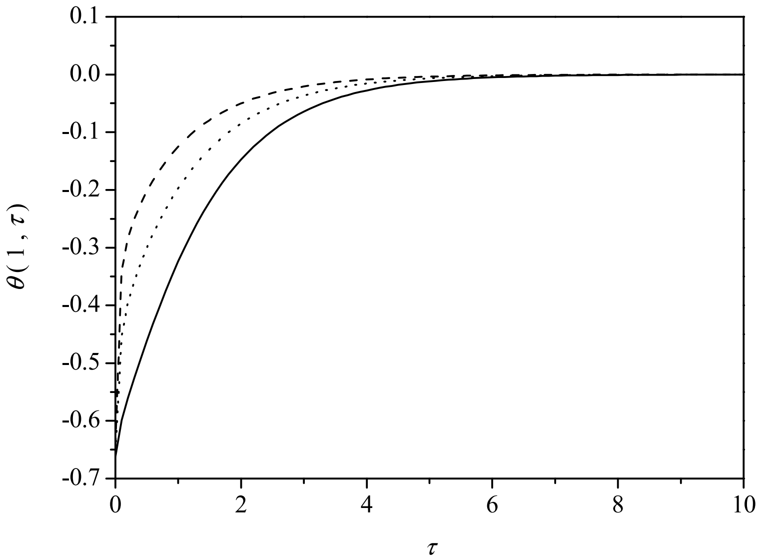

Figure 2: Influence of b/a on the temperature variation of the slab at X = 1 [a = 1.2, s = 1, ω = 0, θ_0 = - 0.664 (example 1)] [dashed line: b/a = -1; dot line: b/a = 0; real line: b/a = 5/6]

Table 7: Temperature of a slab with various τ and terms of solution in example 2 at X = 0

τ	$\theta(0,\tau)$			
	1 term	2 terms	10 terms	20 terms
0	5.350	4.914	4.997	4.999
0.1	5.167	5.028	5.030	5.030
0.2	4.934	4.889	4.889	4.889
0.3	4.676	4.662	4.662	4.662
0.4	4.410	4.406	4.406	4.406
0.5	4.145	4.144	4.144	4.144
1	2.959	2.959	2.959	2.959
2	1.440	1.440	1.440	1.440
3	0.690	0.690	0.690	0.690
4	0.329	0.329	0.329	0.329

The solution method is the combination of an extension of the shifting function method developed by Lee and his colleagues (1996, 2008, and 2008) and a series expansion. It is shown that the solution is simple accurate and fast convergent. One

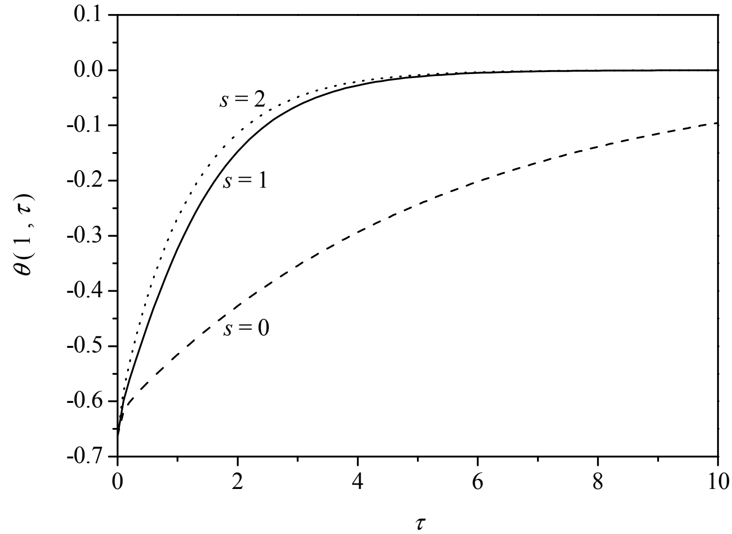

Figure 3: Influence of s parameter on the temperature variation of the slab at X = 1 [a = 1.2,b = 1, ω = 0, θ_0 = - 0.664 (example 1)]

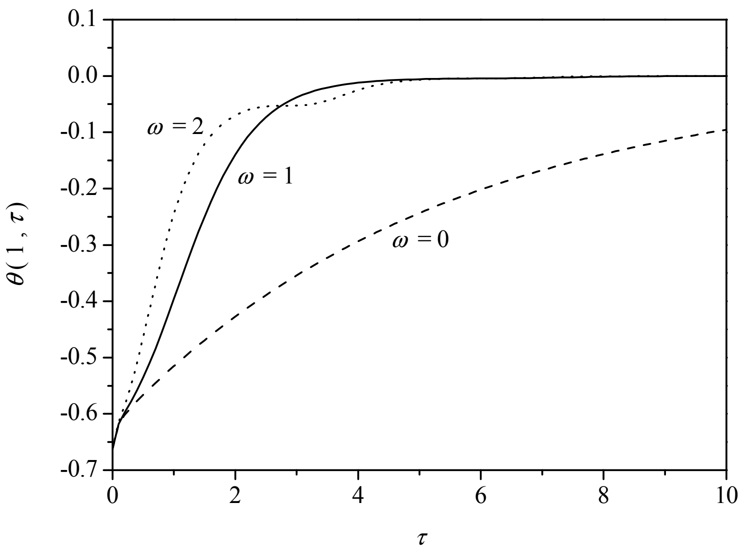

Figure 4: Temperature variation of the slab with various ω at X = 1 [a = 1.2, b = 1, s = 0, θ_0 = - 0.664 (example 1)]

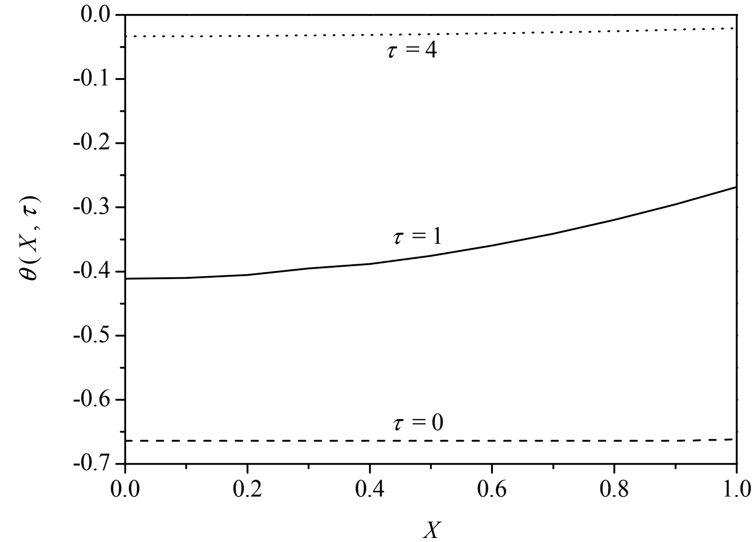

Figure 5: Temperature variation along the slab at τ = 0, 1, 4 [a = 1.2, b = 1, s = 2, $\omega = 0$, θ_0 = - 0.664 (example 1)]

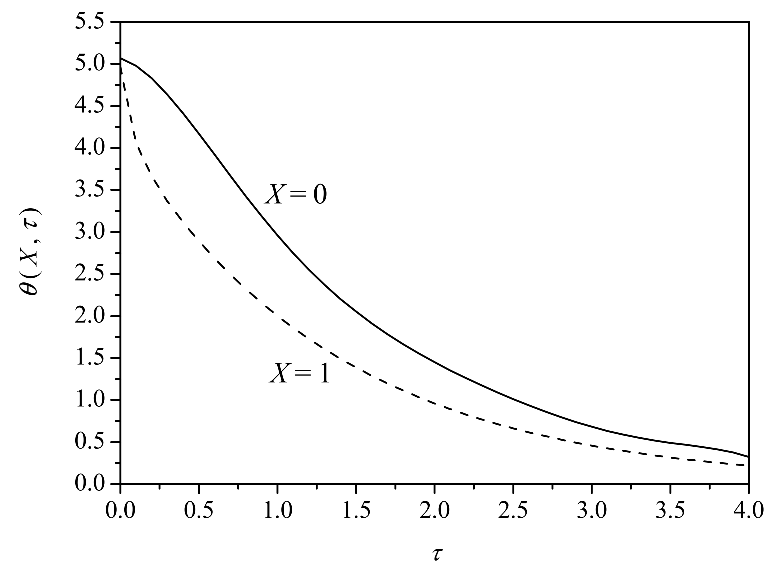

Figure 6: Temperature variation of the slab at X = 0, 1 (example 2)

Table 8: Temperature of a slab with various τ and terms of solution in example 2 at $X = 1$

τ	$\theta(1,\tau)$			
	1 term	2 terms	10 terms	20 terms
0	4.249	4.680	4.942	4.965
0.1	3.910	4.057	4.059	4.059
0.2	3.615	3.664	3.664	3.664
0.3	3.351	3.367	3.367	3.367
0.4	3.110	3.115	3.115	3.115
0.5	2.888	2.890	2.890	2.890
1	2.002	2.002	2.002	2.002
2	0.960	0.960	0.960	0.960
3	0.458	0.458	0.458	0.458
4	0.218	0.218	0.218	0.218

can find that when the dimensionless Fourier number is greater than 0.2, the error for the one term approximation solution of the infinite serious solution can be less than 2%. Numerical results are shown to be consistent with those in the existing literature. The proposed solution method can also be extended to the studies of the problems with different kinds of boundary conditions and those of multiple dimensions.

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