

Coupled Crack /Contact Analysis for Composite Material Containing Periodic Cracks under Periodic Rigid Punches Action

Yue-Ting Zhou¹, Xing Li², De-Hao Yu³ and Kang Yong Lee^{1,4}

Abstract: In this paper, a coupled crack/contact model is established for the composite material with arbitrary periodic cracks indented by periodic punches. The contact of crack faces is considered. Frictional forces are modeled to arise between the punch foundation and the composite material boundary. Kolosov-Muskhelisvili complex potentials with Hilbert kernels are constructed, which satisfy the continuity conditions of stress and displacement along the interface identically. The considered problem is reduced to a system of singular integral equations of first and second kind with Hilbert kernels. Bounded functions are defined so that singular integral equations of Hilbert type can be transformed to Cauchy type. Numerical analyses are conducted through two examples. The presented approach allows considering various configurations of cracks and the punches foundation. Classic results can be obtained when the basic period $a\pi \rightarrow \infty (a > 0)$.

Keywords: periodic cracks, contact, periodic punches, singular integral equation, Hilbert kernel.

1 Introduction

Through contact elements, forces are transmitted to units and mechanisms of machines, such contact phenomenon is popular found in the mechanical engineering [Babich, Guz, and Rudnitskii (2004); Liu, Liu, L. and Mahadevan (2007)].

A variety of solutions [Willis (1966); Swanson (2004); Liu, Peyronnel, Wang, and Keer (2005a, b); Wang, Li, Mai, and Shen (2008)] was obtained for the classical Hertzian contact problems of homogeneous materials. Solutions for the contact

¹ School of Mechanical Engineering, Yonsei University, Seoul 120-749, Republic of Korea

² School of Mathematics and Computer Science, Ningxia University, Yinchuan 750021, China

³ LSEC, ICMSEC, Academy of Mathematics and Systems Science, CAS, Beijing 100190, China

⁴ Corresponding author. Tel./fax: +82 2 2123 2813(K.Y. L). E-mail: zhouyueting (Y.T. Zhou), KYL2813@yonsei.ac.kr (K.Y. Lee).

problems of nonhomogeneous materials were also found. Chen, Xiong, and Shen (2008) proposed a modified method for the contact problem of a laminated composite plate indented by a rigid sphere. Functionally graded materials [Zhou, Li, and Qin (2007); Zhou, Li, and Yu (2010)] are widely used advanced composite materials due to their continuously varying properties. The contact behavior of FGM structures has received increasing research efforts in recent years. Guler and Erdogan (2004, 2006) used continuous model that the elastic modulus was assumed to vary in the depth according to a power law or an exponential function to study the contact problem of functionally graded coating. Ke and Wang (2006, 2007), and Yang and Ke (2008) used a piecewise linear multi-layer model that is capable of modeling an arbitrary material property variation in the functionally graded layer and obtained the numerical solutions for the frictionless and sliding frictional contact problems. Xiao and Yue (2009) presented a three-dimensional boundary element method for contact problems of an elastic indenter on the surface of functionally graded materials (FGMs). Xie, Lee, Hu, and Cai (2009) has considered the case of a rigid, frictionless indenter or punch pressing onto a flat surface of the elastic solids such as glass plate. The above works about contact problem neglect either crack propagating or contact of the crack faces when material is loaded.

In the engineering practice, contact fatigue damage may develop in surfaces subjected to repeated rolling and sliding contact, e.g. gear flanks and bearing surfaces, there may be a set of curvilinear cracks arise and propagate in the contact zones, and the crack faces can overlap in compression zones under the punch indentation [Bayram and Nied (2000); Dorogoy and Banks-Sills (2005); Okayasu, Chen, and Wang (2006); Pyrzanowski (2007)]. The non-periodic problem of contact of the crack faces in the field of compressive stresses was discussed in literature [Savruk (1981); Hills and Nowell (1994)]. With allowance for crack faces contact interaction, contact interaction of the crack faces for a single crack under harmonic loading [Menshykov and Guz (2007)], contact of an elliptical crack under normally incident tension-compression wave [Guz and Menshykov; Zozulya and Guz, (2007)], contact problem for a flat elliptical crack under normally incident shear wave [Guz and Zozulya (2007)] and contact problem for a penny-shaped crack with an initial opening under normally incident tension-compression wave [Menshykov, Menshykov, and Guz (2008)] have been investigated, respectively. Su, Santare, and Gazonas (2007) used the generalized self-consistent method in conjunction with a computational finite element method to calculate the anisotropic effective module of a medium containing damage consisting of microcracks with taking into account crack face contact and friction. Panasyuk, Datsyshyn, and Marchenko (2000) investigated the coupled crack/contact problems for an elastic half-plane. In this paper, Coulomb friction was assumed to exist between the punch and the half-plane, while

the crack faces were under conditions of either stick or smooth contact on contact parts. Zozulya (2009) considered different variational formulation of unilateral contact problems for cracked body with friction based on principles of virtual displacements and virtual stresses. Using boundary element based three dimensional modeling for linear fracture mechanics, Njiwa and Stebut (2004) analyzed partially closed cracking in a homogeneous medium subject to spherical rigid indenter.

Due to the periodic elasticity problems are of considerable importance to continuum mechanics and engineering, the periodic problems of various media have been received increasing research efforts. Boyadzhi, Buryshkin, and Radiollo (1988) studied the coupled crack/contact problems of a half-plane weakened by a periodic set of cracks on action of a rigid punch. Li (1999) analyzed the effect of periodic gasket on periodic contact problem. The failure characteristics of locally periodic defected composite micro-structures were investigated in terms of energy release rate predictions by Greco (2009) with considering of crack face contact. To the authors' knowledge, the coupled crack/contact problem for the composite material with periodic arbitrary cracks under periodic rigid punches action with considering of the crack contact has not been studied yet, because of the difficulty of construction of complex potential functions.

The problem under consideration is a continuation of the author's previous studies [Zhou, Li, and Yu (2008)]. This paper discusses the coupled crack/contact problem of composite material weakened by periodic cracks under periodic rigid punches action. The integral representations for Kolosov-Muskhelishvili complex potentials with the Hilbert kernels by derivatives of displacement discontinuities along the crack contours and pressure under the punches are constructed. The complex potentials satisfy the continuity conditions of stress and displacement along the interface identically. The considered problem is reduced to a system of Hilbert type singular integral equations [Yu (1993-2002); Li (2008)] of the first and second kind. Bounded functions are defined so that the singular integral equations of Hilbert type can be transformed to Cauchy type. The present model allows considering various size of the punches foundation and cracks, and also general conditions of interaction between the crack faces and between the punches and the composite material. Effective method is employed to find the numerical solution of obtained Hilbert kernel singular integral equations of composite material with periodic cracks under the periodic punches. The effect of crack configuration on the model II stress intensity factors and the pressure distribution under the punch foundation are analyzed. Classic results can be obtained when the basic period $a\pi \rightarrow \infty (a > 0)$.

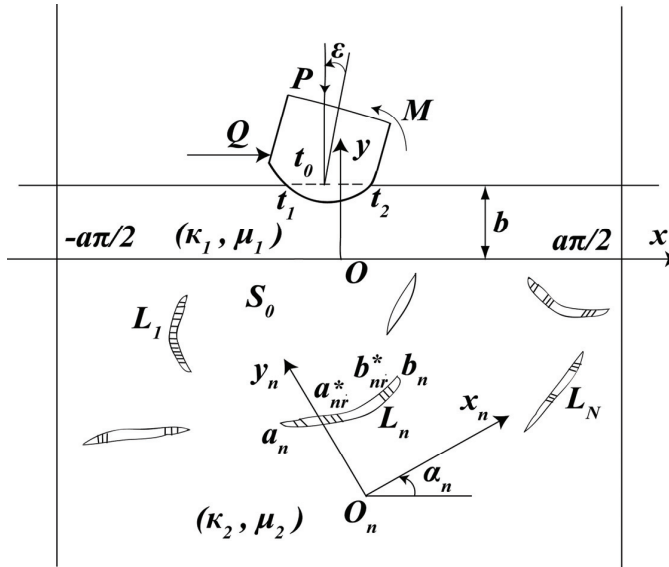


Figure 1: Composite material with periodic arbitrary cracks under periodic punches action.

2 Problem statement

We consider a problem of indentation of periodic punches with convex foundation into the composite material, i.e. an elastic strip with thickness b bonded to a half-plane containing periodic cracks along the entire real axis Ox (figure 1). The strip and the half-plane are characterized by their shear modulus μ_1, μ_2 and their Poisson's ratios ν_1, ν_2 , respectively. The elastic constants κ_1 and κ_2 are defined as

$$\kappa_j = \begin{cases} 3 - 4\nu_j & \text{plane stress} \\ \frac{3-\nu_j}{1+\nu_j} & \text{generalized plane strain} \end{cases} \quad j = 1, 2 \quad (1)$$

Let the basic period be $a\pi (a > 0)$. To the periodic problem, the stress and derivative of the displacement are periodic functions with basic period $a\pi (a > 0)$, the boundary conditions involved also with basic period $a\pi (a > 0)$. The periodic fundamental parallelogram is denoted by $S_0 (|x| < \frac{1}{2}a\pi, y \leq b)$. According to the periodicity, we can only consider the problem in periodic fundamental parallelogram S_0 .

It is assumed that there are N cracks of arbitrary shape in S_0 , and these cracks are smooth and non-intersecting segment $L_k (k = 1, \dots, N)$, the positive direction of L_k is from a_k to b_k . The composite material is related to the system of coordinates xOy , while the cracks are related to local systems $x_nO_ny_n$, connected with basic system

xOy by the relation $z = z_n e^{i\alpha_n} + z_n^0$ ($z = x + i \cdot y, z_n = x_n + i \cdot y_n$), z_n^0 is the affix of the point O_n in the basic system of coordinates, α_n is the angle of the $O_n x_n$ -axis with Ox .

The punch in fundamental parallelogram S_0 is subjected to following loadings, (i) a vertical force P with abscissa of the application at point t_0 , (ii) a horizontal force Q , (iii) a moment M . The contact area between the punch foundation and the composite material boundary in fundamental parallelogram S_0 is denoted by γ_0 , the edge points t_1 and t_2 of this area is determined by following conditions

$$p(t_1) = 0, \quad p(t_2) = 0, \tag{2}$$

where $p(t) = -\sigma_y(t)$ ($p(t) \geq 0, t \in \gamma_0$ for physical sense) is pressure under the punch. The positive direction of γ_0 is from t_1 to t_2 . Outside the contact area γ_0 , the boundary of composite material in fundamental parallelogram S_0 is unloaded. Frictional forces are assumed to arise between the punch foundation and the composite material boundary, which satisfy following Coulomb friction law

$$Q = \rho \cdot P \tag{3}$$

where ρ is the coefficient of friction.

Under external loadings, the punch in fundamental parallelogram S_0 fulfills following equilibrium conditions,

$$\int_{\gamma_0} p_0(t) dt = -(P + iQ) \tag{4}$$

$$\int_{\gamma_0} (t - t_0) p_0(t) dt = -M \tag{5}$$

Here, $p_0(t) = -(1 + i\rho)p(t)$, and i is the unit imaginary.

The boundary conditions of the composite material in fundamental parallelogram S_0 are following

$$\frac{dv(t)}{dt} = f'(t) + \varepsilon, \quad \tau_{xy}(t) = -\rho\sigma_y(t), \quad t \in \gamma_0 \tag{6}$$

$$\sigma_y(t) + i\tau_{xy}(t) = 0, \quad t \notin \gamma_0 \tag{7}$$

where ε denotes the small angle of rotation between the punch axis and the Oy -axis, u and v describe the displacements along the Ox - and Oy -axes of the basic coordinate system, respectively. We denote the punch foundation contour by smooth function $f(t)$.

In compression zone, the contact (overlap) of crack faces may occur under the punch indentation. Contact stress propagates in the contact parts. While the crack

faces are unstressed outside the contact parts. On the n th crack, the whole no contact parts can be denoted by

$$L_n^* = \cup_{r=1}^{R_n} a_{nr}^* b_{nr}^* \quad (n = 1, \dots, N, r = 1, \dots, R_n) \tag{8}$$

where $a_{nr}^* b_{nr}^* (r = 1, \dots, R_n)$ denotes the r th no contact part on the n th crack. It can be found $L_n^* = L_n$ for any open crack.

Generally speaking, the borders of the contact parts between the crack faces are unknown beforehand, which can be found from following equations

$$K_{1n}(t_{nr}) = 0, \quad t_{nr} = a_{nr}^*, b_{nr}^*, \quad r = 1, \dots, N, R_n, \quad a_{nr}^* \neq a_n, \quad b_{nr}^* \neq b_n \tag{9}$$

where K_{1n} denotes the Model I stress intensity factor for the n th crack.

For the stated problem, Kolosov-Muskhelishvili complex potentials can be represented as function sums [Panasyuk, Datsyshyn, and Marchenko (2000)]

$$\Phi_0(z) = \Phi_1(z) + \Phi_2(z) \quad \Psi_0(z) = \Psi_1(z) + \Psi_2(z) \tag{10}$$

The functions $\Phi_1(z)$ and $\Psi_1(z)$ are constructed as integral representations with the Hilbert kernels with respect to pressure $p_0(t)$ under the punch, which describe the stress state of the uncracked composite material with boundary loaded by the contact stress $p_0(t)$, and are given as follows

$$\Phi_1(z) = \begin{cases} -\frac{1-A}{2a\pi i} \int_{\gamma_0} p_0(t) \cot \frac{t-z}{a} dt & 0 < Imz \leq b \\ -\frac{1}{2a\pi i} \int_{\gamma_0} p_0(t) \cot \frac{t-z}{a} dt + \frac{A \cdot b}{a^2 \pi} \int_{\gamma_0} \overline{p_0(\bar{t})} \csc^2 \frac{\bar{t}-z}{a} dt & Imz < 0 \end{cases} \tag{11}$$

$$\Psi_1(z) = \begin{cases} \frac{A_0}{2a\pi i} \int_{\gamma_0} \overline{p_0(\bar{t})} \cot \frac{\bar{t}-z}{a} dt \\ + \frac{1-A}{2a\pi i} \int_{\gamma_0} p_0(t) \left[\cot \frac{t-z}{a} + \frac{\bar{t}-t+z}{a} \csc^2 \frac{t-z}{a} \right] dt, & 0 < Imz \leq b \\ \frac{1}{2a\pi i} \int_{\gamma_0} p_0(t) \left[\cot \frac{t-z}{a} + \frac{\bar{t}-t+z}{a} \csc^2 \frac{t-z}{a} \right] dt \\ + \frac{A_0-A}{2a\pi i} \int_{\gamma_0} \overline{p_0(\bar{t})} \cot \frac{\bar{t}-z}{a} dt & Imz < 0 \\ -\frac{A \cdot b}{a^2 \pi} \int_{\gamma_0} \overline{p_0(\bar{t})} \csc^2 \frac{\bar{t}-z}{a} \left[1 + \frac{2z}{a} \cot \frac{\bar{t}-z}{a} \right] dt \end{cases} \tag{12}$$

where A, A_0 are bi-material constants, which have following form

$$A = 1 - \frac{\mu_1(K_2 + 1)}{\mu_2 + K_2\mu_1} = \frac{\mu_2 - \mu_1}{\mu_2 + K_2\mu_1}, \quad A_0 = \frac{\kappa_1\mu_2(1-A) - \mu_1(A + \kappa_2)}{\mu_2 - \mu_1} \tag{13}$$

The functions $\Phi_2(z)$ and $\Psi_2(z)$ are constructed as integral representations with the Hilbert kernels with respect to derivatives of displacement discontinuities along the

crack contours, which define the composite material stress state due to displacement discontinuities along the crack contours $L_n(n = 1, \dots, N)$, and are given as follows

$$\Phi_2(z) = \begin{cases} \sum_{k=1}^N A_1 \{L_k\} g'_k(\tau_k) & 0 < Imz \leq b \\ \sum_{k=1}^N A_2 \{L_k\} g'_k(\tau_k) & Imz < 0 \end{cases} \quad (14)$$

$$\Psi_2(z) = \begin{cases} \sum_{k=1}^N B_1 \{L_k\} g'_k(\tau_k) & 0 < Imz \leq b \\ \sum_{k=1}^N B_2 \{L_k\} g'_k(\tau_k) & Imz < 0 \end{cases} \quad (15)$$

$$g'_k(\tau_k) = \frac{2\mu_2}{i(1 + \kappa_2)} \frac{d}{d\tau_k} [u_k^+(\tau_k) - u_k^-(\tau_k) + i(u_k^+(\tau_k) - u_k^-(\tau_k))] \quad (16)$$

where the operators $A_1\{L_k\}, A_2\{L_k\}, B_1\{L_k\}, B_2\{L_k\}$ are given in Appendix.

Along the interface (x-axis) of the composite material, the continuity conditions of the normal and shear stress components, as well as continuity of displacements must be satisfied, which can be written as [Ioakimidis and Theocaris (1979)]

$$\Phi_j^+(x) + \overline{\Phi_j^+(x)} + x\Phi_j'^+(x) + \Psi_j^+(x) = \Phi_j^-(x) + \overline{\Phi_j^-(x)} + x\Phi_j'^-(x) + \Psi_j^-(x) \quad (17)$$

$$\frac{1}{\mu_1} \left[\Phi_j^+(x) - \kappa_1 \overline{\Phi_j^+(x)} + x\Phi_j'^+(x) + \Psi_j^+(x) \right] = \frac{1}{\mu_2} \left[\Phi_j^-(x) - \kappa_2 \overline{\Phi_j^-(x)} + x\Phi_j'^-(x) + \Psi_j^-(x) \right] \quad (18)$$

where $j=0,1,2$, x denotes the points of x -axes. The superscripts (+) or (-) denote the boundary values of quantities when approaching the crack contours from left or from the right. The positive direction of Φ_j coincides with that of tracing the region boundary when the region stays all the time on the left [Muskhelishvili(1975)].

It can be checked that the complex potentials $\Phi_{1,2}(z)$ and $\Psi_{1,2}(z)$ we construct satisfy the continuity conditions Eqs. (17) and (18) identically. Then the complex potentials $\Phi_0(z)$ and $\Psi_0(z)$ satisfy boundary conditions Eqs. (17) and (18) identically.

When putting $a \rightarrow \infty$, we obtain the complex potential functions $\Phi_0(z)$ and $\Psi_0(z)$ of composite material with arbitrary cracks indented by a rigid punch, which are same with those of Zhou, Li, and Yu (2008).

When putting $\mu_1 = 0, b = 0$, then $A_0 = 0, A = B = 1$, and putting $a \rightarrow \infty$, we obtain the complex potential functions $\Phi_0(z)$ and $\Psi_0(z)$ of the half-plane with arbitrary

cracks indented by a rigid punch, which are same with those of Panasyuk, Datsyshyn, and Marchenko (2000).

Following two limiting cases will be investigated in this paper: (i) stick contact of the crack faces, (ii) smooth contact of the crack faces. Singular integration equations will be obtained for each case.

3 Stick contact of the crack faces

In case of stick of crack faces in contact, the boundary conditions on the crack faces in S_0 are given as follows

$$N_n^\pm(t_n) + iT_n^\pm(t_n) = 0, \quad t_n \in L_n^*, \quad n = 1, \dots, N \tag{19}$$

$$u_n^+(t_n) - u_n^-(t_n) + i[v_n^+(t_n) - v_n^-(t_n)] = 0, \quad t_n \in L_n \setminus L_n^*, \quad n = 1, \dots, N \tag{20}$$

$$N_n^+(t_n) - N_n^-(t_n) + i[T_n^+(t_n) - T_n^-(t_n)] = 0, \quad t_n \in L_n \setminus L_n^*, \quad n = 1, \dots, N \tag{21}$$

where N_n and T_n are the normal and tangential stresses on the n th crack faces along the $O_n x_n$ - and $O_n y_n$ -axes in the local coordinate system $x_n O_n y_n$, v_n the normal component of displacements of the crack faces. Considering the boundary condition (20) on the crack faces in view of (16), we rewrite $\Phi_2(z)$ and $\Psi_2(z)$ as

$$\Phi_2(z) = \begin{cases} \sum_{k=1}^N A_1 \{L_k^*\} g'_k(\tau_k) & 0 < Imz \leq b \\ \sum_{k=1}^N A_2 \{L_k^*\} g'_k(\tau_k) & Imz < 0 \end{cases} \tag{22}$$

$$\Psi_2(z) = \begin{cases} \sum_{k=1}^N B_1 \{L_k^*\} g'_k(\tau_k) & 0 < Imz \leq b \\ \sum_{k=1}^N B_2 \{L_k^*\} g'_k(\tau_k) & Imz < 0 \end{cases} \tag{23}$$

With respect to t , the derivative of complex combination of displacements expressed by Kolosov-Muskhelishvili complex potentials on the composite material edge takes the form [Muskhelishvili (1975)]

$$2\mu_1 \left[\frac{du(t)}{dt} + i \frac{dv(t)}{dt} \right] = \kappa_1 \Phi_0^-(t) - \overline{\Phi_0^-(t)} - [t \overline{\Phi_0^-(t)} + \overline{\Psi_0^-(t)}] \tag{24}$$

Substituting the complex potentials (11), (12), (22), (23) into Eq. (24), we can rewrite Eq. (24) as

$$2\mu_1 \left[\frac{du(t)}{dt} + i \frac{dv(t)}{dt} \right] = \frac{k_1}{2} (1-A)p_0(t) + \frac{1}{2a\pi} W_1(t) + \frac{i}{2a\pi} W_2(t) \tag{25}$$

where

$$W_1(t) = \sum_{k=1}^N \Gamma\{L_k^*\} g'_k(\tau_k) \tag{26}$$

$$W_2(t) = \int_{\gamma_0} \left[k_1(1-A) \cot \frac{\zeta-t}{a} - A_0 \cot \frac{\zeta-\bar{t}}{a} \right] p_0(\zeta) d\zeta \tag{27}$$

Here, the operator $\Gamma\{L_k^*\}$ takes the following form

$$\Gamma\{L_k^*\} g'_k(\tau_k) = \int_{L_k^*} [K_{11}(\tau_k, t) \psi_k(\tau_k) d\tau_k + K_{12}(\tau_k, t) \overline{\psi_k(\tau_k)} d\bar{\tau}_k] \tag{28}$$

where $K_{11}(\tau_k, t)$ $K_{12}(\tau_k, t)$ are given in Appendix.

Considering the first of the boundary conditions (6), we obtain following second kind singular integral equation with Hilbert kernel

$$ImW_1(t) + ReW_2(t) - a\pi \cdot k_1 \cdot (1-A) \cdot \rho \cdot p(t) = 4a\pi\mu_1 [f'(t) + \varepsilon], \quad t \in r_0 \tag{29}$$

The stress distribution formula along the n th crack faces expressed by Kolosov-Muskhelishvili complex potentials takes the form [Muskhelishvili (1975)]

$$2Re\Phi_0^\pm(T'_n) + \frac{dT'_n}{dT'_n} [T'_n \overline{\Phi_0^\pm(T'_n)} + \overline{\Psi_0^\pm(T'_n)}] = N_n^\pm(T'_n) + iT_n^\pm(T'_n), \quad T'_n \in L_n \tag{30}$$

We define

$$N_n + iT_n = \frac{1}{2} [N_n^+ + N_n^- + i(T_n^+ + T_n^-)] \tag{31}$$

Substituting complex potentials (11), (12), (22) and (23) into Eq. (31) in view of Eq. (30), we obtain

$$N_n(t_n) + iT_n(t_n) = \frac{1}{a\pi} [E_n(t_n) + H_n(t_n)], \quad t_n \in L_n \tag{32}$$

where

$$E_n(t_n) = \sum_{k=1}^N \Omega_n\{L_k\} g'_k(\tau_k) \tag{33}$$

$$H_n(t_n) = \int_{\gamma_0} [K_{21}(\zeta, t_n) - K_{22}(\zeta, t_n)] p(\zeta) d\zeta \tag{34}$$

where the operator $\Omega_n\{L_k\}$ is defined as,

$$\Omega_n\{L_k\}\psi_k(\tau_k) = \int_{L_k} [X_{nk}(\tau_k, t_n)\psi_k(\tau_k)d\tau_k + Y_{nk}(\tau_k, t_n)\overline{\psi_k(\tau_k)}d\bar{\tau}_k] \tag{35}$$

$$\begin{cases} X_{nk} = K_{31}(\tau_k, t_n) - K_{32}(\tau_k, t_n) \\ Y_{nk} = K_{41}(\tau_k, t_n) - K_{42}(\tau_k, t_n) \end{cases} \tag{36}$$

where $K_{21}(\zeta, t_n)$, $K_{22}(\zeta, t_n)$, $K_{31}(\tau_k, t_n)$, $K_{32}(\tau_k, t_n)$, $K_{41}(\tau_k, t_n)$ and $K_{42}(\tau_k, t_n)$ are given in Appendix.

Considering boundary conditions (19) on the crack faces in view of Eq. (32), we obtain a system of N singular integral equations of the first kind with Hilbert kernel

$$E_n(t_n) + H_n(t_n) = 0, \quad t_n \in L_n^*, \quad n = 1, \dots, N \tag{37}$$

where,

$$E_n(t_n) = \sum_{k=1}^N \Omega_n\{L_k^*\}g'_k(\tau_k) \tag{38}$$

To complete the equations system of the present problem, the solution of singular integral equations (29), (37) must satisfy the single-valued conditions at the ends of the no contact parts

$$\int_{a_{nr}^*b_{nr}^*} g'_n(\tau_n)d\tau_n = 0, \quad n = 1, \dots, N, r = 1, \dots, R_n \tag{39}$$

and the conditions of punch equilibrium (4), (5).

We can find the unknown functions ($n = 1, \dots, N$), $p(t)$ and ε through solving singular integral equations (29), (37) and conditions (4), (5), (39).

On the basis of the solution of the complete equation system constructed, we can calculate the stress intensity factors (SIF) at the crack tips and at the ends of the no contact parts by

$$K_{In}(t_{nr}) - iK_{IIIn}(t_{nr}) = \mp \lim_{t_n \rightarrow t_{nr}} \left[\sqrt{2\pi(t_n - t_{nr})}g'_n(\tau_n) \right] \tag{40}$$

$$t_{nr} = a_{nr}^*b_{nr}^*, \quad n = 1, \dots, N, r = 1, \dots, R_n$$

The upper sign (-) conforms to the points a_{nr}^* , while the lower sign (+) conforms to the points b_{nr}^* .

We can determine the contact stresses on the contact parts of the crack faces when considering the obtained solution and Eqs. (21), (32), by following formulae

$$N_n(t_n) + iT_n(t_n) = \frac{1}{a\pi} [E_n(t_n) + H_n(t_n)], \quad t_n \in L_N \setminus L_N^* \tag{41}$$

4 Smooth contact of the crack faces

In case of smooth contact of the crack faces, the boundary conditions on the crack faces in S_0 are as follows

$$N_n^\pm(t_n) + iT_n^\pm(t_n) = 0, \quad t_n \in L_n^*, \quad n = 1, \dots, N \tag{42}$$

$$T_n^\pm(t_n) = 0, \quad v_v^+(t_n) - v_v^-(t_n) = 0, \quad t_n \in L_n \setminus L_n^*, \quad n = 1, \dots, N \tag{43}$$

$$N_n^+(t_n) - N_n^-(t_n) = 0, \quad t_n \in L_n \setminus L_n^*, \quad n = 1, \dots, N \tag{44}$$

Following Savruk (1981) and Panasyuk, Datsyshyn, and Marchenko (2000), the unknown functions $g'_n(t_n)$ ($n = 1, \dots, N$) in Eq. (16) can be represented as sums of two functions

$$g'_n(t_n) = g'_{1n}(t_n) + g'_{2n}(t_n) \tag{45}$$

where

$$g_{1n}(t_n) = \frac{2\mu_2}{1+k_2} [v_v^+(t_n) - v_v^-(t_n)] \frac{dt_n}{ds_n} \tag{46}$$

$$g_{2n}(t_n) = -\frac{2i\mu_2}{1+k_2} [v_s^+(t_n) - v_s^-(t_n)] \frac{dt_n}{ds_n} \tag{47}$$

where v_v and v_s are normal and tangential components of displacement vector of the crack contour L_n , s_n is the arc abscissa of a point t_n on L_n .

Considering the second group of the boundary conditions (43), we can rewrite the complex potentials $\Phi_2(z)$, $\Psi_2(z)$, as

$$\Phi_2(z) = \begin{cases} \sum_{k=1}^N [A_1 \{L_k^*\} g'_{1k}(\tau_k) + A_1 \{L_k\} g'_{2k}(\tau_k)] & 0 < Imz \leq b \\ \sum_{k=1}^N [A_2 \{L_k^*\} g'_{1k}(\tau_k) + A_2 \{L_k\} g'_{2k}(\tau_k)] & Imz < 0 \end{cases} \tag{48}$$

$$\Psi_2(z) = \begin{cases} \sum_{k=1}^N [B_1 \{L_k^*\} g'_{1k}(\tau_k) + B_1 \{L_k\} g'_{2k}(\tau_k)] & 0 < Imz \leq b \\ \sum_{k=1}^N [B_2 \{L_k^*\} g'_{1k}(\tau_k) + B_2 \{L_k\} g'_{2k}(\tau_k)] & Imz < 0 \end{cases} \tag{49}$$

Substituting the complex potentials (11), (12), (48), (49) into Eq. (24) in view of the boundary condition (6), (43), we obtain a singular integral equation as follows

$$ImW_1(t) + ReW_2(t) = 4\pi\mu_1[f'(t) + \varepsilon], \tag{50}$$

where

$$W_1(t) = \sum_{k=1}^N [u \{L_k^*\} g'_{1k}(\tau_k) + u \{L_k\} g'_{2k}(\tau_k)] \tag{51}$$

and $W_2(t)$ is determined by Eq. (27)

Substituting the complex potentials (11), (12), (48), (49) into Eq. (30) in view of the boundary condition (42), (43), we obtain a system of $2N$ singular integral equations as follows

$$Re [E_n(t_n) + H_n(t_n)] = 0, \quad t_n \in L_n^*, n = 1, \dots, N \tag{52}$$

$$Im [E_n(t_n) + H_n(t_n)] = 0 - t_n \in L_n, n = 1, \dots, N \tag{53}$$

Here

$$E_n(t_n) = \sum_{k=1}^N [\Omega_n \{L_k^*\} g'_{1k}(\tau_k) + \Omega_n \{L_k\} g'_{2k}(\tau_k)] \tag{54}$$

The quantity $H_n(t_n)$ is determined by Eq. (34), and the operator $\Omega_n \{L_k\}$ is determined by Eq. (35).

To complete the system of equations (50), (52) and (53), it should add equations

$$Im \left[g_{1n}(t_n) \frac{d\bar{t}_n}{dS_n} \right] = 0, \quad t_n \in L_n^*, n = 1, \dots, N \tag{55}$$

$$Re \left[g_{2n}(t_n) \frac{d\bar{t}_n}{dS_n} \right] = 0, \quad t_n \in L_n, n = 1, \dots, N \tag{56}$$

and displacement continuity conditions at the ends of the open parts

$$\int_{a_{nr}^* b_{nr}^*} g'_{1n}(\tau_n) d\tau_n = 0, \quad n = 1, \dots, N, \quad r = 1, \dots, R_n \tag{57}$$

$$\int_{L_n} g'_{2n}(\tau_n) d\tau_n = 0, \quad n = 1, \dots, N \tag{58}$$

The singular integral equations (50), (52), (53) and conditions (4), (5), (55), (56), (57), (58) allow to find the functions along the open parts ($n = 1, \dots, N, r = 1, \dots, R_n$) of the cracks, the functions $g'_{2n}(\tau_n)$ along the contours ($n = 1, \dots, N$) and the pressure $p(t)$ in S_0 .

Using the solutions of the obtained equations in view of Eqs. (32) and (42), we can determine normal stress on the contact parts of the crack face by the relation

$$N(t_n) = \frac{1}{a\pi} Re [E_n(t_n) + H_n(t_n)], \quad t_n \in L_n \setminus L_n^* \tag{59}$$

The SIF can be calculated by Eq. (40).

5 Numerical results

5.1 Composite material with periodic vertical cracks under periodic punches action

In basic periodic fundamental parallelogram $S_0(|x| < \frac{1}{2}a\pi, y \leq b)$, there is a frictionless flat punch be indented by a force P into an elastic strip bonded to a half-plane with a near-surface vertical crack of length $2l$ (figure 2). We assume the punch does not rotate under external loading ($\varepsilon = 0$). The composite material and the vertical crack in S_0 are placed in the system of coordinates xoy and $x_1o_1y_1$, respectively. The punch axis and the line of the action of the force P coincides with oy -axis.

Following the algorithm proposed Panasyuk, Datsyshyn, and Marchenko (2000), we find that the faces of vertical crack near the interface contact throughout its length.

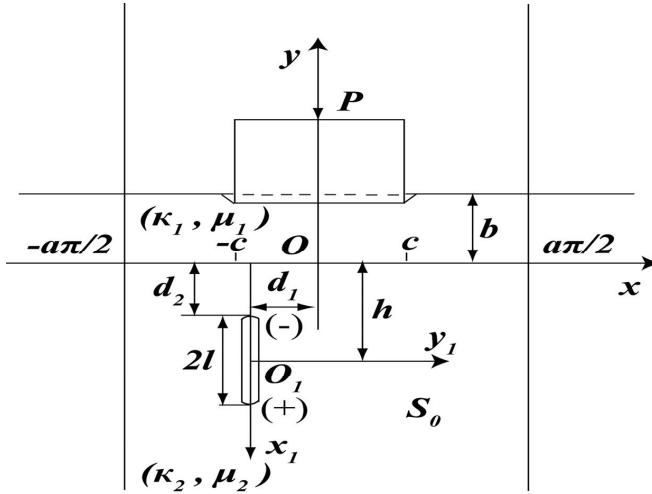


Figure 2: Composite material with periodic vertical cracks under periodic punches action.

The case of smooth contact of the crack faces is investigated. Considering Eqs. (50)-(54), we obtain the following singular integral equations

$$\int_{-l}^l V(\tau_1, x)\varphi(\tau_1)d\tau_1 + \int_{-c}^c W(\zeta_1, x)p(\zeta)d\zeta_1 = 0, \quad |x| \leq c \tag{60}$$

$$\int_{-l}^l Q_1(\tau_1, t_1)\varphi(\tau_1)d\tau_1 + \int_{-c}^c U_1(\zeta_1, t_1)p(\zeta)d\zeta_1 = 0, \quad |t_1| < L \tag{61}$$

with the kernels $V(\tau_1, x), W(\zeta_1, x), Q(\tau_1, t_1), U(\zeta_1, t_1)$

$$V(\tau_1, x) = Im \left[\kappa_1(1-B)cot \frac{T-t}{a} - (1-A)cot \frac{\bar{T}-\bar{t}}{a} \right] - 2Re \left[\frac{(1-A)ImT - b(1-B)}{a} csc^2 \frac{\bar{T}-\bar{t}}{a} \right], \quad T = d_1 - i(\tau_1 + h), \quad t = x + i \cdot b \tag{62}$$

$$W(\zeta_1, x) = A_0Re \left(cot \frac{\zeta - \bar{t}}{a} \right) - \kappa_1(1-A)cot \frac{\zeta_1 - x}{a}, \quad \zeta = \zeta_1 + i \cdot b \tag{63}$$

$$Q(\tau_1, t_1) = -\frac{\tau_1 - t_1}{a} csc^2 \frac{i(\tau_1 - t_1)}{a} + \frac{B-A}{2} \cdot i \cdot cot \frac{i \cdot \omega(\tau_1, t_1)}{a} + \frac{A(\tau_1 - t_1)}{a} csc^2 \frac{i \cdot \omega(\tau_1, t_1)}{a} - \frac{4 \cdot A \cdot i}{a^2} (\tau_1 + h)(t_1 + h) csc^2 \frac{i \cdot \omega(\tau_1, t_1)}{a} cot \frac{i \cdot \omega(\tau_1, t_1)}{a} \omega(\tau_1, t_1) = \tau_1 + t_1 + 2h \tag{64}$$

$$U(\zeta_1, t_1) = \frac{1}{2}Re \left(cot \frac{\zeta - \bar{T}'}{a} \right) - \frac{Im(\zeta - T')}{a} Im \left(csc^2 \frac{\zeta - \bar{T}'}{a} \right) + \frac{4A \cdot b \cdot ImT'}{a} Re \left(csc^2 \frac{\zeta - \bar{T}'}{a} cot \frac{\zeta - \bar{T}'}{a} \right) + \frac{A_0 - A}{2} Re \left(cot \frac{\zeta - \bar{T}'}{a} \right) - \frac{A \cdot b}{a} Im \left(csc^2 \frac{\zeta - \bar{T}'}{a} \right), \quad T' = d_1 - i(t_1 + h) \tag{65}$$

The additional conditions are needed to complete the system equations (60), (61)

$$\int_{-l}^l \varphi(\tau_1) d\tau_1 = 0 \tag{66}$$

$$\int_{-c}^c p(\zeta_1) d\zeta_1 = P \tag{67}$$

$$\int_{-c}^c \zeta_1 p(\zeta_1) d\zeta_1 = -M \tag{68}$$

Solving the equations (60), (61), (66), (67), we can determine the unknown functions

$$\varphi(t_1) = -ig_2(t_1) = -\frac{2\mu_2}{1+k_2} [u^+(t_1) - u^-(t_1)], \quad |t_1| < 1 \tag{69}$$

and $p(t)(t = x + ib, |x| \leq c)$. Besides, the moment M which restrains the punch from rotation can be determined by the condition (68).

Introducing the substitutions

$$\begin{aligned} \tau_1 &= l\xi, \zeta_1 = c\xi, |\xi| \leq 1, \quad t_1 = l\eta, x = c\eta, |\eta| < 1, \\ V_t(\xi, \eta) &= V(l\xi, c\eta), \quad W_t(\xi, \eta) = W(c\xi, c\eta), \quad Q_t(\xi, \eta) = Q(l\xi, l\eta) \\ U_t(\xi, \eta) &= U(c\xi, l\eta), \quad \phi(\xi) = l\varphi(l\xi), \quad \chi(\xi) = cp(c\xi + i \cdot b) \end{aligned} \quad (70)$$

and defining bounded functions

$$W_p(\xi, \eta) = W_t(\xi, \eta) + \frac{c\kappa_1(1-A)}{\xi - \eta} \quad (71)$$

$$Q_p(\xi, \eta) = Q_t(\xi, \eta) - \frac{l}{\xi - \eta} \quad (72)$$

we can transform a system of Hilbert singular integral equations (60), (61) to a system of Cauchy singular integral equations

$$\int_{-1}^1 \left\{ V_t(\xi, \eta)\phi(\xi) + \left[-\frac{c\kappa_1(1-A)}{\xi - \eta} + W_p(\xi, \eta) \right] \chi(\xi) \right\} d\xi = 0, \quad |\eta| < 1 \quad (73)$$

$$\int_{-1}^1 \left\{ \left[\frac{l}{\xi - \eta} + Q_p(\xi, \eta) \right] \phi(\xi) + U_t(\xi, \eta)\chi(\xi) \right\} d\xi = 0, \quad |\eta| < 1 \quad (74)$$

and conditions (67)-(68) to

$$\int_{-1}^1 \phi(\xi) d\xi = 0 \quad (75)$$

$$\int_{-1}^1 \phi(\xi) d\xi = 0 \quad (76)$$

$$c \int_{-1}^1 \xi \phi(\xi) d\xi = -M \quad (77)$$

The solution of equations (73), (74) can be written as

$$\phi(\eta) = \frac{\gamma(\eta)}{\sqrt{1-\eta^2}}, \quad \chi(\eta) = \frac{\lambda(\eta)}{\sqrt{1-\eta^2}} \quad (78)$$

Then well developed method of Gaussian quadrature is used to solve equations (73)-(77) numerically, see [Panasyuk, Datsyshyn, and Marchenko (2000); Erdogan and Gupta (1972)].

On the basis of numerical solutions, the stress intensity factors at the crack tips can be defined as

$$K_{II}^+ = -\frac{\sqrt{\pi/l}}{K} \sum_{k=1}^K (-1)^k \cot\left(\frac{2k-1}{4N}\pi\right) \gamma(\xi_k) \tag{79}$$

$$K_{II}^- = -\frac{\sqrt{\pi/l}}{K} \sum_{k=1}^N (-1)^{k+K} \tan\left(\frac{2k-1}{4N}\pi\right) \gamma(\xi_k) \tag{80}$$

where the upper sign (+) concerns the right crack tip, while the lower sign (-) concerns the left crack tip, ξ_k are the zeros of the Chebyshev polynomials of the first kind.

The pressure in the contact area can be defined as

$$p(\eta) = -\frac{T_N^C(\eta)}{N\sqrt{1-\eta^2}} \sum_{k=1}^N (-1)^k \frac{\sqrt{1-\xi_k^2}}{\eta-\xi_k} \lambda(\xi_k), \quad |\eta| < 1 \tag{81}$$

where, $T_N^C(\xi) = \cos(N \cos^{-1} \xi)$ is the Chebyshev polynomials of the first kind.

The numerical calculations of the normalized Mode II SIF, $F_{II}^+ = k_{II}^+ \sqrt{C}/(\rho \sqrt{\pi})$, are conducted. The variations of the normalized Mode II SIF with relative distance d_1/c of the crack from the punch axis, relative distance d_2/c of the left crack tip from the x -axis, relative thickness b/c of the elastic strip and relative crack length $2l/c$ are analyzed. Plane stress state is considered here. Young’s modulus of the strip and half-plane are taken as $E_1 = 140\text{Mpa}$, $E_2 = 280\text{Mpa}$ respectively. Poisson’s ratios are taken as $\nu_1 = 0.2$, $\nu_2 = 0.4$, respectively.

The influences of relative distance d_1/c of the crack from punch axis on normalized Model II intensity factors F_{II}^\pm for $c/a = 0.8$, $b/c = 0.2$ and $2l/c = 0.1, 0.5$ are plotted in figure 3(a) and 3(b). When the cracks approach the punches edge ($d_1/c \approx 1.1$)-the Mode II stress intensity factors K_{II} reach their maximums. The factors K_{II} are equal to zero at the crack tips for the reason of problem symmetry when the cracks are placed along the punches axes. Thus, in S_0 the vertical crack placed along the punch axis are no longer stress concentrator and do not influence the pressure distribution under the punch.

Figure 4 illustrates the variations of the normalized Mode II SIF with relative distance d_2/c of the left crack tip from the x -axis for different crack length at the most dangerous crack position ($d_1/c = 1.1$). We assume $b/c = 0.1$. Both F_{II}^+ and F_{II}^- decrease with the increasing of d_2/c . For the same value of d_2/c , the values of F_{II}^\pm increase with the increasing of crack length $2l/c$.

Figure 5 depicts the variations of the normalized Mode II SIF with relative thickness b/c of the elastic strip for different d_2/c at the most dangerous crack position

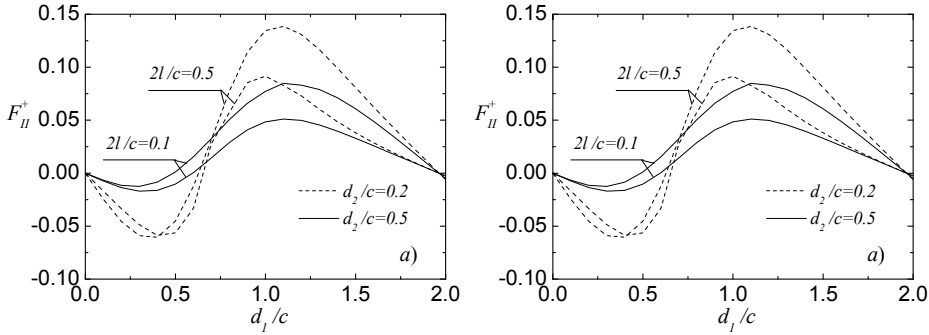


Figure 3: Influences of d_1/c on normalized Model II intensity factors F_{II}^{\pm} (b) $F_{II}^{\pm} = K_{II}^{\pm} \sqrt{c} / (P \sqrt{\pi})$ for $c/a = 0.8, b/c = 0.2$ and $2l/c = 0.1, 0.5$. (a) F_{II}^+ for the right tip, F_{II}^- for the left tip.

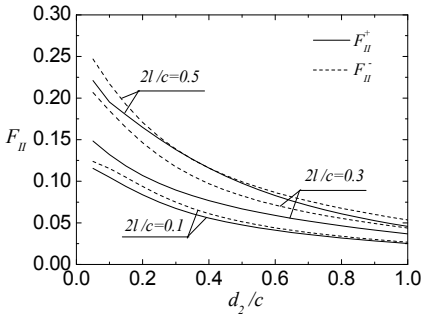


Figure 4: Influences of d_2/c on normalized Model II intensity factors F_{II}^{\pm} for $c/a = 0.8, b/c = 0.1$ and $2l/c = 0.1, 0.3, 0.5$, solid lines are for right tip, while dash lines are for left tip.

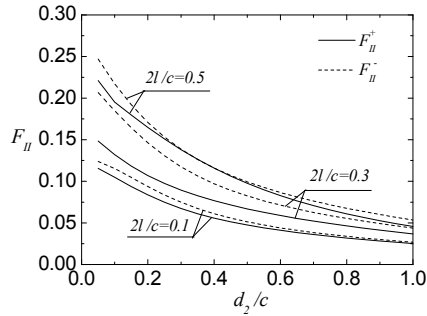


Figure 5: Influences of b/c on normalized Model II intensity factors F_{II}^{\pm} for $c/a = 0.8, 2l/c = 0.5$ and $d_2/c = 0.2, 0.3, 0.5$, solid lines are for right tip, while dash lines are for left tip.

(d_1/c). We assume $2l/c = 0.5$. Both F_{II}^+ and F_{II}^- decrease with the increasing of b/c . For the same value of b/c , the values of F_{II}^{\pm} decrease with the increasing of d_2/c .

The dimensionless pressure $cp(t)/P$ under the punch for the most dangerous crack positions are given in Table 1 when the SIF in magnitude are maximum for $c/a = 0.8$. For the pressure distributions under the periodic punches in uncracked composite material, we check our numerical method by comparison with the closed-form solution provided of Liu (1992). It is observed that the present results coincide well with those of Liu (1992), which verifies the validity of our procedure. It is seen that

Table 1: Pressure $cp(t)/P$ under a frictionless flat punch which is indented to bonded material plane with a vertical cracks in S_0 .

	$2l/c$	d_1/c	d_2/c	b/c	$x/c = -0.9$	$x/c = -0.5$	$x/c = 0$	$x/c = 0.5$	$x/c = 0.9$
Exact solution	0	-	-	-	0.6777	0.3893	0.3550	0.3893	0.6777
Numerical solution	0	-	-	-	0.6777	0.3893	0.3550	0.3893	0.6777
	0.1	1.0	0.2	0.2	0.6775	0.3893	0.3550	0.3895	0.6776
	0.1	1.1	0.5	0.2	0.6776	0.3893	0.3550	0.3894	0.6776
	0.5	1.1	0.2	0.2	0.6742	0.3885	0.3556	0.3923	0.6781
	0.5	1.1	0.5	0.2	0.6762	0.3891	0.3556	0.3905	0.6771
	0.1	1.1	0.05	0.1	0.6767	0.3889	0.3546	0.3891	0.6808
	0.3	1.1	0.05	0.1	0.6720	0.3870	0.3535	0.3899	0.6832
	0.5	1.1	0.05	0.1	0.6665	0.3852	0.3530	0.3929	0.6859
	0.5	1.1	0.2	0	0.6709	0.3891	0.3547	0.3937	0.6862
	0.5	1.1	0.3	0	0.6729	0.3880	0.3554	0.3932	0.6803
	0.5	1.1	0.5	0	0.6751	0.3888	0.3557	0.3916	0.6773

in comparison with the uncracked composite material, the exist of vertical crack in S_0 near the punch foundation has an insignificant effect on the pressure distribution under the punch.

5.2 Composite material with periodic horizontal cracks under periodic punches action

We investigate the contact interaction of indentation of periodic frictionless flat punches into the bonded material plane. There are periodic horizontal cracks with basic period $a\pi(a > 0)$ in the half-plane, whose lengths are $2l$ and placed symmetrically about the punches axes (figure 6). Employing the algorithm proposed by Panasyuk, Datsyshyn, and Marchenko (2000), it has been established that the crack faces contact throughout their length.

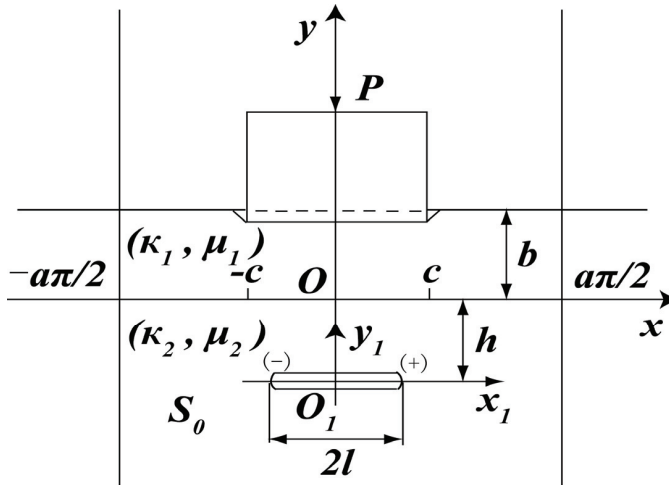


Figure 6: Composite material with periodic horizontal cracks under periodic punches action.

In the case of smooth contact of the crack faces, the singular integral equations of the problem are (60), (61), whose kernels are following

$$\begin{aligned}
 V(\tau_1, x) = & \operatorname{Re} \left[\kappa_1(1-B) \cot \frac{T-t}{a} - (1-A) \cot \frac{\bar{T}-\bar{t}}{a} \right] \\
 & - 2 \operatorname{Im} \left[\frac{(1-A) \operatorname{Im} T - b(1-B)}{a} \csc^2 \frac{\bar{T}-\bar{t}}{a} \right], \quad T = \tau_1 - i \cdot h, \quad t = x + i \cdot b \quad (82)
 \end{aligned}$$

$$W(\zeta_1, x) = A_0 \operatorname{Re} \left(\cot \frac{\zeta - \bar{i}}{a} \right) - \kappa_1 (1 - A) \cot \frac{\zeta_1 - x}{a}, \quad \zeta = \zeta_1 + i \cdot b \quad (83)$$

$$Q(\tau_1, t_1) = \cot \frac{\tau_1 - t_1}{a} - \frac{A - B}{2} \operatorname{Re} \left[\cot \frac{i \cdot \omega(\tau_1, t_1)}{a} \right] + \frac{2A \cdot h}{a} \operatorname{Im} \left[\operatorname{csc}^2 \frac{i \cdot \omega(\tau_1, t_1)}{a} \right] + A \cdot \operatorname{Re} \left[\cot \frac{\omega(\tau_1, t_1)}{a} - \frac{4h^2}{a^2} \operatorname{csc}^2 \frac{\omega(\tau_1, t_1)}{a} \cot \frac{\omega(\tau_1, t_1)}{a} \right], \quad \omega(\tau_1, t_1) = t_1 - \tau_1 + 2i \cdot h \quad (84)$$

$$U(\zeta_1, t_1) = -\frac{1}{2} \operatorname{Re} \left(\cot \frac{\bar{\zeta} - \bar{T}'}{a} \right) + \frac{\operatorname{Im}(\zeta - T')}{a} \operatorname{Im} \left(\operatorname{csc}^2 \frac{\bar{\zeta} - \bar{T}'}{a} \right) - \frac{4A \cdot b \cdot \operatorname{Im} T'}{a} \operatorname{Re} \left(\operatorname{csc}^2 \frac{\zeta - \bar{T}'}{a} \cot \frac{\zeta - \bar{T}'}{a} \right) - \frac{A_0 - A}{2} \operatorname{Re} \left(\cot \frac{\zeta - \bar{T}'}{a} \right) + \frac{A \cdot b}{a} \operatorname{Im} \left(\operatorname{csc}^2 \frac{\zeta - \bar{T}'}{a} \right), \quad T' = t_1 - i \cdot h \quad (85)$$

The equations (60), (61) must satisfy conditions (66)-(68).

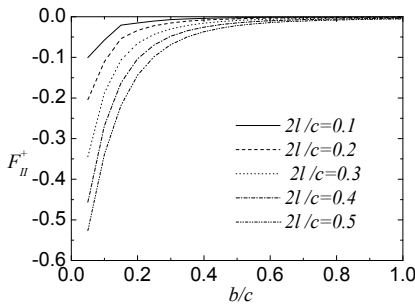


Figure 7: Influences of b/c on F_{II}^+ for $c/a = 0.8$, $b/c = 0.1$ and $2l/c = 0.1, 0.2, 0.3, 0.4, 0.5$.

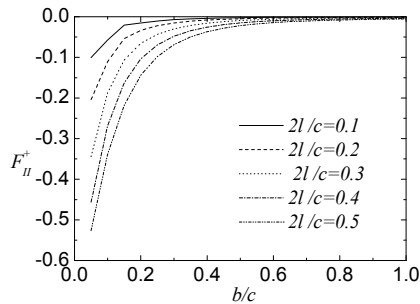


Figure 8: Influences of h/c on F_{II}^+ for $c/a = 0.8$, $b/c = 0.1$ and $2l/c = 0.1, 0.2, 0.3, 0.4, 0.5$.

As in the previous case, introducing the substitutions (70) and defining bounded functions (71), (72), we can transform a system of Hilbert type singular integral equations (60), (61) with kernels (82)-(85) to a system of Cauchy type singular integral equations and solve them numerically.

The influences of relative strip thickness b/c and relative distance h/c of the crack from the real x -axis on the normalized Mode II SIF, $F_{II}^+ = K_{II}^+ \sqrt{C} / (\rho \sqrt{\pi})$ are

shown in figure 7 and figure 8. It can be found the values of F_{II}^+ in magnitude decrease with the increasing of either b/c or h/c . The values of F_{II}^+ in magnitude increase with the increasing of crack length.

Table 2: Pressure $cp(t)/P$ under a frictionless flat punch, which is indented to composite material with a horizontal crack in S_0 .

$2l/c$	h/c	b/c	$x/c = 0$	$x/c = 0.5$	$x/c = 0.9$
0.1	0.05	0.1	0.3550	0.3893	0.6777
0.2	0.05	0.1	0.3550	0.3893	0.6777
0.3	0.05	0.1	0.3550	0.3891	0.6775
0.4	0.05	0.1	0.3550	0.3891	0.6773
0.5	0.05	0.1	0.3555	0.3884	0.6775
0.1	0.1	0.05	0.3550	0.3893	0.6777
0.2	0.1	0.05	0.3550	0.3893	0.6777
0.3	0.1	0.05	0.3550	0.3891	0.6775
0.4	0.1	0.05	0.3550	0.3891	0.6773
0.5	0.1	0.05	0.3555	0.3884	0.6775

As shown in Table 2, the presence of the horizontal crack in S_0 near the punch foundation does not influence the pressure distribution under the punch.

6 Conclusions

In this paper, the coupled crack/contact problem of composite material with periodic arbitrary cracks under periodic rigid punches action is considered. The cracks may contact, the crack faces on contact parts are assumed to be either stick or smooth. The singular integral equation method has been used to study this problem. Various configurations of cracks and the punch foundation, and also general conditions of interaction between the crack faces and between the punch and the composite material can be considered by the singular integral equation method. The singular integral equations of Hilbert type of the stated problem can be transformed to singular integral equations of Cauchy type through defining bounded functions.

For the special cases of indentation of periodic frictionless punches with flat foundation into the composite material weakened by periodic vertical cracks or periodic horizontal cracks placed symmetrically about punch axes. Numerical analysis have been conducted, numerical results show: The crack length and crack position have significant influences on the Mode II stress intensity, the Mode II stress intensity at the tips of the periodic vertical cracks or periodic horizontal cracks increases with the increasing of crack length ; The thickness of the strip takes effects on the

Mode II stress intensity at the tips of both periodic vertical cracks and periodic horizontal cracks; The presence of the periodic vertical cracks and periodic horizontal cracks near the punches foundation has an insignificant influence on the pressure distribution under the punches.

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Appendix

Expressions of operators $A_1\{L_k\}$, $A_2\{L_k\}$, $B_1\{L_k\}$, $B_2\{L_k\}$ in Eqs. (14) and (15)

$$A_1\{L_k\}\psi_k(\tau_k) = \frac{1-B}{2a\pi} \int_{L_k} (\cot \frac{T_k-z}{a} e^{i\alpha_k} \psi_k(\tau_k) d\tau_k \quad (86)$$

$$A_2\{L_k\}\psi_k(\tau_k) = \frac{1}{2a\pi} \int_{L_k} (\cot \frac{T_k-z}{a} - A \cdot \cot \frac{\bar{T}_k-z}{a}) \psi_k(\tau_k) e^{i\alpha_k} d\tau_k \\ + \frac{A2ImT_k}{a^2\pi} \int_{L_k} \csc^2 \frac{\bar{T}_k-Z}{a} e^{i\alpha_k} \overline{\psi_k(\tau_k)} d\bar{\tau}_k \quad (87)$$

$$B_1\{L_k\}\psi_k(\tau_k) = \\ - \frac{1}{2a\pi} \int_{L_k} \left[(1-B) \cot \frac{T_k-z}{a} + \frac{(1-B)z - (1-A)2iImT_k}{a} \csc^2 \frac{T_k-z}{a} \right] e^{i\alpha_k} \psi_k(\tau_k) d\tau_k \\ + \frac{1-A}{2a\pi} \int_{L_k} \cot \frac{T_k-z}{a} e^{-i\alpha_k} \overline{\psi_k(\tau_k)} d\bar{\tau}_k \quad (88)$$

$$\begin{aligned}
 B_2 \{L_k\} \psi_k(\tau_k) = & \\
 \frac{1}{2a\pi} \int_{L_k} & \left[A \cdot \cot \frac{\bar{T}_k - z}{a} - \cot \frac{T_k - z}{a} + \frac{T_k - \bar{T}_k - z}{a} \csc^2 \frac{T_k - z}{a} \right] \psi_k(\tau_k) e^{i\alpha_k} d\tau_k \\
 - \frac{1}{2a\pi} \int_{L_k} & (B \cdot \cot \frac{\bar{T}_k - z}{a} - \cot \frac{T_k - z}{a}) \overline{\psi_k(\tau_k)} e^{-i\alpha_k} d\bar{\tau}_k \\
 & - \frac{2A \cdot z \cdot i}{a^3 \pi} \int_{L_k} \text{Im} T_k \csc^2 \frac{\bar{T}_k - z}{a} \cot \frac{\bar{T}_k - z}{a} \overline{\psi_k(\tau_k)} e^{-i\alpha_k} d\bar{\tau}_k \quad (89)
 \end{aligned}$$

where

$$T_k = \tau_k e^{i\alpha_k} + z_k^0, \quad B = 1 - \frac{\mu_1(k_2 + 1)}{\mu_1 + k_1\mu_2} = \frac{k_1\mu_2 - k_2\mu_1}{\mu_1 + k_1\mu_2} \quad (90)$$

Expressions of $K_{11}(\tau_k, t)$, $K_{12}(\tau_k, t)$ in Eq. (28)

$$K_{11}(\tau_k, t) = e^{i\alpha_k} \left[k_1(1 - B) \cot \frac{T_k - t}{a} - (1 - A) \cot \frac{\bar{T}_k - \bar{t}}{a} \right] \quad (91)$$

$$K_{12}(\tau_k, t) = 2 \cdot i \cdot e^{-i\alpha_k} \frac{(1 - A) \text{Im} T_k - b(1 - B)}{a} \csc^2 \frac{\bar{T}_k - \bar{t}}{a} \quad (92)$$

Expressions of $K_{21}(\zeta, t_n)$, $K_{22}(\zeta, t_n)$ in Eq. (34)

$$K_{21}(\zeta, t_n) = \text{Im} \left[(1 + i\rho) \cot \frac{\zeta - T'_n}{a} \right] - \frac{2 \cdot A \cdot b}{a} \text{Re} \left[(1 + i\rho) \csc^2 \frac{\zeta - \bar{T}'_n}{a} \right] \quad (93)$$

$$\begin{aligned}
 K_{22}(\zeta, t_n) = & \frac{i}{2} \left[(1 - i\rho) \cot \frac{\zeta - \bar{T}'_n}{a} + \frac{2i \cdot (1 - i\rho) \cdot \text{Im}(\zeta - T'_n)}{a} \csc^2 \frac{\zeta - \bar{T}'_n}{a} \right. \\
 & + \frac{8 \cdot A \cdot b \cdot (1 + i\rho) \cdot \text{Im} T'_n}{a^2} \cot \frac{\zeta - \bar{T}'_n}{a} \csc^2 \frac{\zeta - \bar{T}'_n}{a} \\
 & \left. + (A_0 - A)(1 + i\rho) \cot \frac{\zeta - \bar{T}'_n}{a} + \frac{2 \cdot A \cdot b \cdot i \cdot (1 + i\rho)}{a} \csc^2 \frac{\zeta - \bar{T}'_n}{a} \right] \frac{d\bar{t}_n}{dt_n} e^{-2i\alpha_n} \quad (94)
 \end{aligned}$$

where

$$T'_n = t_n e^{i\alpha_n} + z_n^0 \quad (95)$$

Expressions of $K_{31}(\tau_k, t_n)$, $K_{32}(\tau_k, t_n)$, $K_{41}(\tau_k, t_n)$ and $K_{42}(\tau_k, t_n)$ in Eq. (36)

$$K_{31}(\tau_k, t_n) = \frac{e^{i\alpha_k}}{2} \left(\cot \frac{T_k - T'_n}{a} - A \cdot \cot \frac{\bar{T}_k - T'_n}{a} - \frac{2 \cdot A \cdot i}{a} \text{Im} T_k \csc^2 \frac{T_k - \bar{T}'_n}{a} \right) \quad (96)$$

$$K_{32}(\tau_k, t_n) = \frac{e^{i\alpha_k}}{2} \left(B \cdot \cot \frac{T_k - \bar{T}'_n}{a} - \cot \frac{\bar{T}_k - \bar{T}'_n}{a} - \frac{2 \cdot A \cdot i}{a} \text{Im} T_k \csc^2 \frac{T_k - \bar{T}'_n}{a} \right) - \frac{8A}{a^2} \text{Im} T_n \text{Im} T'_n \csc^2 \frac{T_k - \bar{T}'_n}{a} \cot \frac{T_k - \bar{T}'_n}{a} \frac{d\bar{t}_n}{dt_n} e^{-2i\alpha_n} \quad (97)$$

$$K_{41}(\tau_k, t_n) = \frac{e^{-i\alpha_k}}{2} \left(\cot \frac{\bar{T}_k - \bar{T}'_n}{a} - A \cdot \cot \frac{T_k - \bar{T}'_n}{a} + \frac{2 \cdot A \cdot i}{a} \text{Im} T_k \csc^2 \frac{\bar{T}_k - T'_n}{a} \right) \quad (98)$$

$$K_{42}(\tau_k, t_n) = \frac{e^{-i\alpha_k}}{2} \left(-A \cdot \cot \frac{T_k - T'_n}{a} + \cot \frac{\bar{T}_k - \bar{T}'_n}{a} + \frac{2 \cdot i \cdot \text{Im}(T_k - T'_n)}{a} \csc^2 \frac{\bar{T}_k - \bar{T}'_n}{a} + \frac{2 \cdot A \cdot i}{a} \text{Im} T'_n \csc^2 \frac{T_k - \bar{T}'_n}{a} \right) \frac{d\bar{t}_n}{dt_n} e^{-2i\alpha_n} \quad (99)$$

