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Estimation of Heat-Transfer Characteristics from Fins Mounted on a Horizontal Plate in Natural Convection

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Abstract: The finite difference method in conjunction with the least-squares scheme and experimental measured temperatures is proposed to solve a two-dimensional steady-state inverse heat conduction problem in order to predict the natural-convection heat transfer coefficient under the isothermal situation $\overline{h}^{iso}$ from a three fin array mounted on a horizontal plate and fin efficiency η_f for various values of the fin spacing and fin height. The measured fin temperatures and ambient temperature are obtained from the present experimental apparatus conducted in a small wind tunnel. The heat transfer coefficient on a fin is non-uniform for the present problem, and its functional form can be difficult to be obtained. Thus the whole fin is divided into several sub-fin regions in order to predict the $\overline{h}^{iso}$ and η_f values. These two estimated values for various values of the fin spacing and fin height can be obtained using the present inverse scheme in conjunction with experimental measured temperatures. The present estimates of $\overline{h}^{iso}$ can be applied to obtain two different modified correlations of the Nusselt number and Raleigh number and compare with those obtained from the correlations recommended by current textbooks and other references. The results show that the present estimates of $\overline{h}^{iso}$ are in good agreement with those obtained from the correlations recommended by the present study, current textbooks and other references.

Keywords: heat transfer coefficient on a fin, fin efficiency, horizontal plate, hybrid inverse scheme.

Nomenclature

A_f	lateral surface area of the fin, m^2
A_j	area of the jth sub-fin region, m^2
$[A]$	global conduction matrix
C	parameter

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$[K]$	global conduction matrix
$[F]$	force matrix
g	acceleration of gravity, T_o
H	fin height, m
$h(X,Y)$	heat transfer coefficient, $W/m^2 \cdot K$
$\bar{h}$	average heat transfer coefficient, $W/m^2 \cdot K$
$\bar{h}^{iso}$	heat transfer coefficient under the isothermal situation, $W/m^2 \cdot K$
$\bar{h}_j$	average heat transfer coefficient on the jth sub-fin region, $W/m^2 \cdot K$
k_{air}	thermal conductivity of the ambient air, $\varepsilon\sigma$
k_f	thermal conductivity of the fin, $\varepsilon\sigma$
L	fin length, m
l_x	distance between two neighboring nodes in the x-directions, $L/(N_x-1)$
T_o	distance between two neighboring nodes in the y-directions, $H/(N_y-1)$
N	number of the measured temperatures on the fin or sub-fin regions
Nu	Nusselt number, $\bar{h}^{iso}S/k_{air}$
N_x	number of nodes in the x-direction
N_y	number of nodes in the y-direction
Q	total heat transfer rate dissipated from the fin, W
q_j	heat transfer rate dissipated from the jth sub-fin region, W
Ra	Rayleigh number, $g\beta(T_o-T_\infty)S^3/(\nu\alpha_{air})$
S	fin spacing, m
T	fin temperature, K
T_j	measured temperature on the jth sub-fin region, K
T_o	fin base temperature, K
T_∞	ambient air temperature, K
$[T]$	global temperature matrix
W	width of a three fin array, m
X,Y	spatial coordinates, m

Greek symbols

α_{air}	thermal diffusivity of the air, T_o
β	volumetric thermal expansion coefficient, 1/K
δ	fin thickness, m
ε	emisivity of the fin
η_f	fin efficiency
ν	kinematic viscosity of the air, T_o

Superscripts

cal calculated value
mea measured data

1 Introduction

Heat transfer augmentation techniques such as the use of extended surfaces are always applied to improve the performance of heat exchanging devices. This improvement of the convection and radiation heat transfer rates can result in the decrease of system size and weight. The properly designed fins are especially attractive for these applications because they offer a more economical solution to the problem. It is known that reliability is an important concept in engineering design. This also implies that the estimation of a more accurate heat transfer coefficient from vertical fin arrays is an important task for the device of the high-performance heat-sink. The experimental and numerical studies for heat transfer from an array of parallel rectangular finned surfaces on a horizontal surface have been studied for a long time (Rakshit and Balaji (2005); Jones and Smith (1970) ; Raithby and Hollands (1985); Kreith and Bohn (1993); Harahap and Setio (2001); Harahap and McManus (1967); Harahap, Rudianto, and Pradnyana (2005); Sobhan, Venkateshan, and Seetharamu (1989); Sobhan, Venkateshan, and Seetharamu (1990); Rammohan Rao and Venkateshan (1996); Leung, Probert, and Shilston (1985); Leung and Probert (1989); Leung and Probert (1989)) . Such problems can exhibit convection, radiation, mutual irradiation between fins and the complex three-dimensional flow and thermal fields of a chimney type for wider range of fin geometries. Thus most existing studies were empirical in natural. Even though these previous studies provided valuable results for the present problem, findings remained inconclusive especially for comparing experimental results obtained from different correlations. Moreover, these available experimental data can also remain very limited. Under the circumstances, a more accurate predictive scheme can still be needed in order to obtain a new heat transfer correlation based on the experimental data. This implies that the estimation of a more accurate heat transfer coefficient on the fin is an important task for the device of the high-performance air conditioning.

The heat transfer from vertical rectangular finned surfaces on a horizontal surface for the fin temperature higher than the ambient temperature has been measured by Jones and Smith (1970) using an experimental apparatus with seven aluminum fins extending through the top horizontal surface of a closed container. It can be found (Raithby and Hollands (1985); Kreith and Bohn (1993)) that Jones and Smith (1970) were able to correlate their measured heat transfer to within about ± 25 percent on the relation between the Nusselt number and Rayleigh number for a given fin length of 0.254 m. Under the assumption of the two-dimensional steady incompressible laminar flow, Rakshit and Balaji (2005) applied the commercially software FLUENT to obtain the correlation of the Nusellt number based on the Prandtl number, the Rayleigh number and the number of fins, etc. Harahap and Setio (2001) used 94 experimental observations to revise the correlations given by Harahap and McManus (1967) and proposed correlations for heat dissipation and natural convection heat-transfer from horizontally-based, vertically-finned arrays. Harahap, Rudianto, and Pradnyana (2005) and Harahap and Lesmana (2006) applied experimental studies to investigate the effects of miniaturizing the base plate dimensions of horizontally based straight rectangular fin arrays on the steady-state heat-dissipation performance in natural convection. Sobhan, Venkateshan, and Seetharamu (1989, 1990) applied experimental studies to investigate unsteady and steady free-convection heat-transfer problems from fin arrays. However, they only showed the one-dimensional steady heat conduction equation of the fin (Sobhan, Venkateshan, and Seetharamu (1989, 1990)). They used two copper-constantan thermocouples with 0.2 mm in diameter to measure the fin temperatures at the tip and base of the fin. Rammohan Rao and Venkateshan (1996) employed an experimental technique to investigate the interaction of free convection and radiation in an array of four vertical rectangular fins attached to a horizontal base maintained at a uniform temperature. The fin temperature profile was obtained using the measured temperatures at the fin base and fin tip. Later, they (Rammohan Rao and Venkateshan (1996)) numerically solved one-dimensional fin equation with a convecting-radiating fin array over the data range L = 0.05 m, δ = 0.0015 m, 0.03 m $\leq$ H $\leq$ 0.07 m, 0.01 m $\leq$ S $\leq$0.025 m and k = 205 W/m·k. Leung, Probert, and Shilston (1985), Leung and Probert (1989) and Leung and Probert (1989) systematically investigated the effects of the fin spacing, fin length, fin height and fin thickness on the steady-state thermal performances of rectangular fins protruding from vertical or horizontal rectangular bases under natural convection. The results of Leung, Probert, and Shilston (1985) showed that the optimal fin spacing corresponding to the maximum steady-state rate of heat dissipation to the ambient air from the fin array was 10.5 $\pm$ 1 mm.

It can be difficult to perform the measurements of the local heat transfer coefficient on a fin under steady-state conditions because the local fin temperature and local heat flux must be required. Under the circumstances, the fin efficiency was often determined under the assumption of the uniform heat transfer coefficient. However, It was found that the measured fin efficiency was less than the calculated result assuming a uniform heat transfer coefficient (Liang, Wong and Nathan (2000)). Chen and Chou (2006) and Chen and Hsu (2007) applied the finite difference method in conjunction with the least-squares scheme and experimental temperature data to predict the average heat transfer coefficient on the vertical plate fin and vertical annular circular fin of one-tube plate finned-tube heat exchangers and fin efficiency for various values of the fin spacing in natural convection. The natural-convection heat transfer coefficient on the fin in the works of Chen and Chou (2006) and Chen and Hsu (2007) was assumed to be non-uniform. To estimate the heat transfer coefficient on the fin, the whole fin was first divided into several analysis sub-fin regions before performing the inverse calculation. Their estimated results of the heat transfer coefficient on a fin under the isothermal situation Chen and Chou (2006) and Chen and Hsu (2007) were in good agreement with those obtained from the correlations recommended by current textbooks (Raithby and Hollands (1985); Kreith and Bohn (1993)). This implies that the present inverse scheme has good accuracy and good reliability.

Quantitative studies of the heat transfer processes occurring in the industrial applications require accurate knowledge of the surface conditions and the thermal physical quantities of the material. It is well known that these physical quantities and the surface conditions can be predicted using the temperature measurements inside the material. Such problems are called the inverse heat conduction problems and have become an interesting subject recently. The main difficulty of the IHCP is that their solution can be very sensitive to changes in input data resulting from measurement errors (Özisik (1993); Kurpisz and Nowak (1995)). To date, various inverse methods, such as the implicit finite-difference, regularization, conjugate gradient, function specification, Kalman filter, group preserving scheme and hybrid scheme methods, in conjunction with the measured temperatures inside the test material have been developed for the analysis of the inverse heat conduction problems (Özisik (1993); Kurpisz and Nowak (1995); Chang, Liu, and Chang (2005)). Liu (2006) applied a hybrid inverse scheme of the Laplace transform and finite-difference methods in conjunction with the least-squares method and experimental temperature data to estimate the unknown surface temperature and surface heat flux on the top surface of the horizontal test plate for L = 0.1 m, $0.1 \leq$ H/L ≤ 0.4 and $0.06 \leq$ S/L ≤ 0.3 in natural convection. Later, the transient natural-convection heat transfer coefficient from vertical rectangular fin arrays mounted on the horizontal test plate can be determined. However, Liu (2006) did not determine the fin efficiency and the distribution of the heat transfer coefficient on the fin. Thus the present study applies the two-dimensional (2-D) inverse heat conduction model to predict the variation of the heat-transfer characteristics on the fin. Moreover, the 2-D model can provide insight of the flow and heat-transfer characteristics that was not revealed in Liu (2006). In order to enhance the overall heat transfer, the distribution of the heat transfer coefficient on the fin can be an important task for the present problem. Chen and Wang (2008) applied the finite difference method in conjunction with experimental measured temperatures given by Lin, Hsu, Chang, and Wang (2001) and least-squares method to predict the average overall heat transfer coefficients on these sub-fin regions under wet conditions. Similarly, the whole fin was also divided into several analysis sub-fin regions before performing the inverse calculation. Furthermore, the average overall heat transfer coefficient on a vertical square fin mounted on a horizontal plate under wet conditions and wet fin efficiency can be predicted for various values of the air speed and relative humidity. Their results showed that the estimates of the average overall heat transfer coefficient on a fin $\overline{h}$ obtained from the one-dimensional (1-D) and 2-D models agree well with the exact values even for the simulated temperature measurements with measurement errors. However, the estimates of the $\overline{h}$ value and the wet fin efficiency obtained from the 2-D model can slightly deviate from those obtained from the 1-D model. To the authors' knowledge, a few investigators performed the prediction of the heat transfer coefficient on the fin and fin efficiency for the present problem using the inverse heat conduction scheme in conjunction with experimental temperature data. Thus the present study applies the similar inverse scheme (Chen and Wang (2008)) in conjunction with the experimental temperature data measured from the present experimental apparatus conducted in a small wind tunnel to predict the average heat transfer coefficients on these sub-fin regions. Later, the heat transfer coefficient on a fin under the isothermal situation and fin efficiency can be obtained for various values of the fin height and fin spacing. The computational procedure for the estimates of the heat transfer coefficients on each sub-fin region is performed repeatedly until the sum of the squares of the deviations between the calculated and measured temperatures becomes minimum. In order to validate the reliability and accuracy of the present estimated results, comparison between the present estimated results of the heat transfer coefficient under the isothermal situation and those obtained from the previous correlations recommended by current textbooks (Raithby and Hollands (1985); Kreith and Bohn (1993)), Harahap, Rudianto, and Pradnyana (2005) will be made for various values of the fin spacing and fin height.

2 Mathematical formulation

A 2-D inverse heat conduction problem is introduced to estimate the unknown heat transfer coefficient from vertical fin arrays mounted on a horizontal plate and fin efficiency for various values of the fin spacing and fin height. The present heat transfer coefficient h(X, Y) is the combination of the convection and radiation heat transfer coefficients because of the fin surface temperature higher than the ambient air temperature. The experimental temperature data of the fin and ambient air temperature are measured from the experimental apparatus conducted in a small wind tunnel, as shown in Fig. 1. Fig. 2 shows the physical geometry of the 2-D thin fin with measurement locations and sub-fin regions, where L, H and δ denote the length, height and thickness of the rectangular fin, respectively. Due to the thin fin behavior, the temperature gradient in the Z-direction (the fin thickness) is small and the fin temperature varies only in the X and Y directions. The "insulated tip" assumption can be an adequate approximation provided that the actual heat transfer rate dissipated through the tip is much smaller than the total heat transfer rate drawn from the base wall. It was known that the heat transfer coefficient on the fin inside the plate fin heat exchangers under dry conditions was non-uniform (Chen and Chou (2006); Chen and Hsu (2007); Liu (2006)). Thus the heat transfer coefficient h(X, Y) in the present study is also assumed to be non-uniform. The heat transfer coefficient on a fin can be estimated provided that the fin temperatures at various measurement locations and ambient air temperature can be measured. For the direct heat conduction problems, the temperature field can be determined provided that h(X, Y) is given. However, h(X, Y) is unknown for the inverse heat conduction problems. It cannot be estimated unless additional information of the measured fin temperatures is given. In this study, the thermocouples are fixed at various measurement locations in sub-fin regions in order to record their temperatures. The IHCP investigated here involve the estimates of the unknown heat transfer coefficient and fin efficiency. Under the assumptions of the steady state and constant thermal properties, the 2-D heat conduction equation for the continuous thin fin can be expressed as

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = \frac{2h(X,Y)}{k_f \delta}(T - T_\infty) \text{ for } 0 < X < L,\ 0 < Y < H \tag{1}$$

Its corresponding boundary conditions are

$$\frac{\partial T}{\partial X} = 0 \text{ at } X = 0 \text{ and } X = L \tag{2}$$

$$T(X,0) = T_o \tag{3}$$

$$\frac{\partial T}{\partial Y} = 0 \text{ at } Y = H \tag{4}$$

where T is the fin temperature. X and Y are Cartesian coordinates. k_f is the thermal conductivity of the fin. $h(X,Y)$ is the unknown heat transfer coefficient on the fin. T_o and T_∞ respectively denote the fin base temperature and ambient air temperature.

3 Numerical analysis

In order to apply the measured fin temperatures and ambient air temperature to predict the unknown average heat transfer coefficient and fin efficiency for various

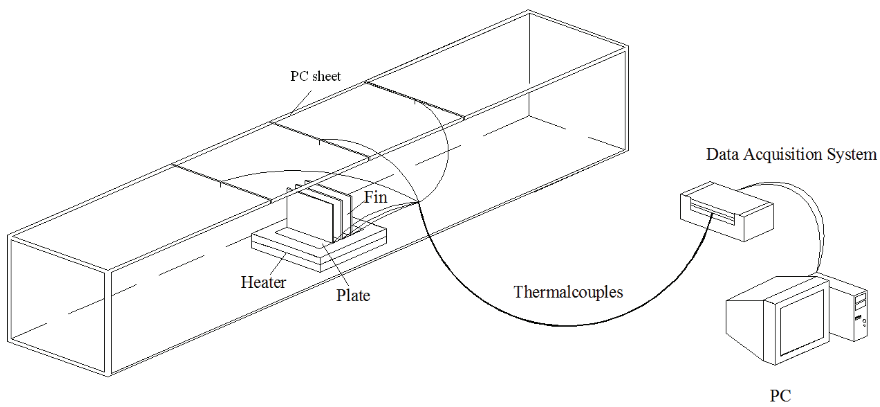

Figure 1: Experimental apparatus configuration of the present study conducted in a small wind tunnel.

values of the fin height and fin spacing, the vertical rectangular fin is divided into N sub-fin regions in the present inverse scheme and then the unknown heat transfer coefficient on each sub-fin region can be approximated by a constant value. Thus the application of the finite difference method to Eq. (1) can produce the following difference equation on the kth sub-fin region as

$$\frac{T_{i+1,j}-2T_{i,j}+T_{i-1,j}}{l_x^2}+\frac{T_{i,j+1}-2T_{i,j}+T_{i,j-1}}{l_y^2}=\frac{2\bar{h}_k}{k_f\delta}T_{i,j}$$
$$\text{for } i=1,2,\ldots,N_x,\ j=1,2,\ldots,N_y \text{ and } k=1,2,\ldots,N \quad (5)$$

where N_x and N_y are the nodal numbers in X- and Y-directions, respectively. N is the number of the sub-fin regions. l_x and l_y respectively are the distance between two neighboring nodes in the x- and y-directions and are defined as $l_x = L/(N_x - 1)$ and $l_y = H/(N_y - 1)$. $\bar{h}_k$ denotes the average heat transfer coefficient on the kth sub-fin region.

The finite difference forms of the boundary conditions (2)-(4) can be written as

$$T_{2,j} = T_{0,j} \text{ and } T_{N_x-1,j} = T_{N_x+1,j} \text{ for } j=1,2,\ldots,N_y \quad (6)$$

$$T_{i,1} = T_0 \text{ and } T_{i,N_y-1} = T_{i,N_y+1} \text{ for } I=1,2,\ldots,N_x \quad (7)$$

Substitution of Eqs. (6) and (7) into their corresponding difference equations can obtain the difference equations at the boundary surfaces as

$$\frac{2T_{2,j}-2T_{i,j}}{l_x^2}+\frac{T_{1,j+1}-2T_{1,j}+T_{1,j-1}}{l_y^2}=\frac{2\bar{h}_k}{k_f\delta}T_{1,j} \text{ for } k=1,3,\ldots,N-1 \quad (8)$$

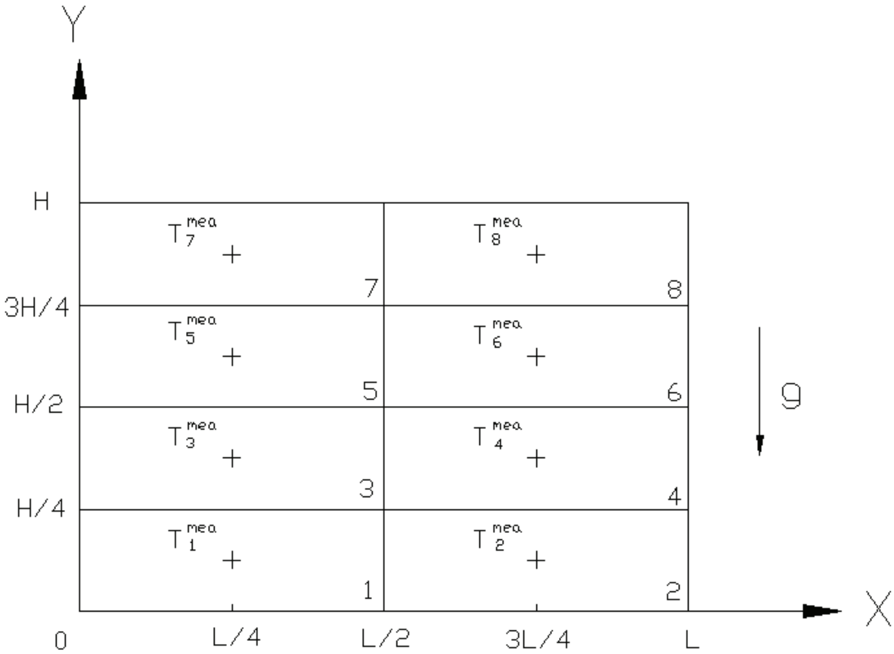

Figure 2: Physical geometry of the present problem with measurement locations and sub-fin regions.

$$\frac{-2T_{N_x,j}+2T_{N_x-1,j}}{l_x^2}+\frac{T_{N_x,j+1}-2T_{N_x,j}+T_{N_x,j-1}}{l_y^2}=\frac{2\bar{h}_k}{k_f\delta}T_{i,j} \text{ for } k=2,4,\dots,N \quad (9)$$

and

$$\frac{T_{i+1,N_y}-2T_{i,N_y}+T_{i-1,N_y}}{l_x^2}+\frac{-2T_{i,N_y}+2T_{i,N_y-1}}{l_x^2}=\frac{2\bar{h}_k}{k_f\delta}T_{i,j} \text{ for } k=N-1,N \quad (10)$$

The difference equations for the nodes at the interface between two neighboring sub-fin regions, as shown in Fig. 3, are given as

$$\frac{T_{i+1,j}-2T_{i,j}+T_{i-1.j}}{\ell_x^2}+\frac{T_{i,j+1}-2T_{i,j}+T_{i,j-1}}{\ell_y^2}-\frac{\bar{h}_k+\bar{h}_{k+1}}{k_f\delta}T_{i,j}=0$$
$$\text{for } k=1,3,\dots,N-1 \quad (11)$$

and

$$\frac{T_{i+1,j}-2T_{i,j}+T_{i-1.j}}{\ell_x^2}+\frac{T_{i,j+1}-2T_{i,j}+T_{i,j-1}}{\ell_y^2}-\frac{\bar{h}_k+\bar{h}_{k+2}}{k_f\delta}T_{i,j}=0$$

$$\text{for } k=1,2,\ldots,N-2 \quad (12)$$

The difference equations for the nodes between four neighboring sub-fin regions are given as

$$\frac{T_{i+1,j}-2T_{i,j}+T_{i-1.j}}{\ell_x^2}+\frac{T_{i,j+1}-2T_{i,j}+T_{i,j-1}}{\ell_y^2}-\frac{\bar{h}_k+\bar{h}_{k+1}+\bar{h}_{k+2}+\bar{h}_{k+3}}{2k_f\delta}T_{i,j}=0$$

$$\text{for } k=1,3,\ldots,N-3 \quad (13)$$

Rearrangement of Eqs. (5) and (8)-(13) can yield the following matrix equation as

$$[K][T]=[F] \tag{14}$$

where [K] is a global conduction matrix. [T] is a matrix representing the nodal temperatures. [F] is a force matrix. The fin temperatures at various measurement locations can be obtained from Eq. (14) using the Gauss elimination algorithm.

Due to the assumption of the constant heat transfer coefficient on each sub-fin region, the heat transfer rate dissipated from this sub-fin region q_i is

$$q_i=2\bar{h}_j\int_{A_j}(T-T_\infty)dA \text{ for } j=1,2,\ldots,N \tag{15}$$

The average heat transfer coefficient on the fin $\bar{h}$ can be expressed as

$$\bar{h}=\sum_{j=1}^{N}\bar{h}_jA_j/A_f \tag{16}$$

where A_f is the lateral surface area of the fin.

The actual total heat transfer rate dissipated from the rectangular fin to the ambient Q can be written as

$$Q=\sum_{j=1}^{N}q_j=2A_f(T_\infty-T_o)\bar{h}^{iso} \tag{17}$$

where $\bar{h}^{iso}$ denotes the heat transfer coefficient on the fin under the isothermal situation.

The heat transfer is due to convection and radiation from the fin to the air because of the temperature difference between the ambient air and the fin. The fin efficiency η_f is defined as the ratio of the actual total heat transfer rate from the fin to the dissipated heat from the fin maintained at the fin base temperature T_o and is expressed as

$$\eta_f = \frac{\sum_{j=1}^{N} q_j}{2A_f(T_\infty - T_o)\bar{h}} = \frac{\bar{h}^{iso}}{\bar{h}} \tag{18}$$

In order to estimate the unknown heat transfer coefficient $\bar{h}_j$ on the jth sub-fin region, additional information of measured fin temperatures is required at N interior measurement locations. The measured fin temperature taken from the jth thermocouple is denoted by T_j^{mea} , j = 1,..., N, as shown in Tables 1-3.

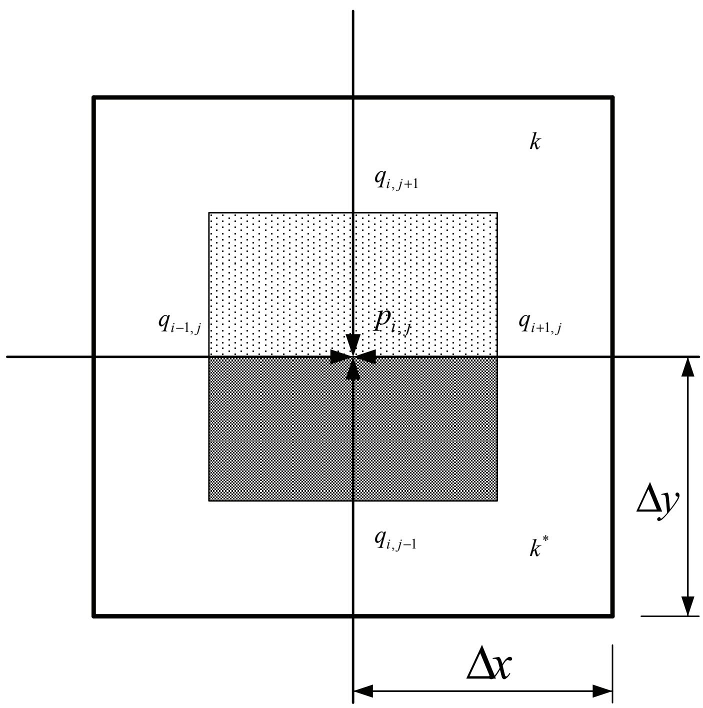

Figure 3: Nodes for the interface of two-neighboring sub-fin regions.

The least-squares minimization technique is applied to minimize the sum of the squares of the deviations between the calculated and measured fin temperatures

Table 1: Present estimates for H = 0.04 m and various S values.

	S = 0.005 m $T_0 = 360.58$K $T_\infty = 301.28$K	S = 0.010 m $T_0 = 352.99$K $T_\infty = 301.44$K	S = 0.015 m $T_0 = 351.95K$ $T_\infty = 300.28K$	S = 0.02m $T_0 = 351.77$K $T_\infty = 300.02$K
T_j^{mea} (K)	$T_1^{mea} = 355.02$K $T_2^{mea} = 355.12$K $T_3^{mea} = 347.85$K $T_4^{mea} = 347.79$K $T_5^{mea} = 343.11$K $T_6^{mea} = 343.07$K $T_7^{mea} = 340.64$K $T_8^{mea} = 340.51$K	$T_1^{mea} = 346.72$K $T_2^{mea} = 346.83$K $T_3^{mea} = 338.95$K $T_4^{mea} = 338.89$K $T_5^{mea} = 334.49$K $T_6^{mea} = 334.64$K $T_7^{mea} = 333.04$K $T_8^{mea} = 333.27$K	$T_1^{mea} = 344.98$K $T_2^{mea} = 344.87$K $T_3^{mea} = 335.98$K $T_4^{mea} = 335.86$K $T_5^{mea} = 331.52$K $T_6^{mea} = 331.51$K $T_7^{mea} = 329.94$K $T_8^{mea} = 329.89$K	$T_1^{mea} = 344.39$K $T_2^{mea} = 344.33$K $T_3^{mea} = 334.79$K $T_4^{mea} = 334.82$K $T_5^{mea} = 330.01$K $T_6^{mea} = 330.05$K $T_7^{mea} = 328.41$K $T_8^{mea} = 328.59$K
$\bar{h}_j(W/m^2K)$	$\overline{h_1} = 8.29$ $\overline{h_2} = 7.36$ $\overline{h_3} = 3.04$ $\overline{h_4} = 3.66$ $\overline{h_5} = 4.08$ $\overline{h_6} = 3.69$ $\overline{h_7} = 4.72$ $\overline{h_8} = 5.02$	$\overline{h_1} = 11.80$ $\overline{h_2} = 10.46$ $\overline{h_3} = 5.34$ $\overline{h_4} = 6.75$ $\overline{h_5} = 7.66$ $\overline{h_6} = 6.95$ $\overline{h_7} = 2.82$ $\overline{h_8} = 2.62$	$\overline{h_1} = 11.75$ $\overline{h_2} = 12.31$ $\overline{h_3} = 9.30$ $\overline{h_4} = 9.67$ $\overline{h_5} = 6.89$ $\overline{h_6} = 6.34$ $\overline{h_7} = 3.52$ $\overline{h_8} = 3.73$	$\overline{h_1} = 12.35$ $\overline{h_2} = 13.02$ $\overline{h_3} = 10.13$ $\overline{h_4} = 9.68$ $\overline{h_5} = 8.10$ $\overline{h_6} = 8.66$ $\overline{h_7} = 3.63$ $\overline{h_8} = 3.06$
Ra	463.91	3404.84	11696.98	27854.05
$\bar{h}(W/m^2K)$	4.98	6.80	7.94	8.58
$\bar{h}^{iso}(W/m^2K)$	3.94	5.17	5.8	6.11
Q (W)	1.87	2.13	2.40	2.53
η_f	0.791	0.761	0.731	0.712

at selected measurement locations. The error in the estimates E$(\bar{h}_1,\bar{h}_2,...,\bar{h}_N)$ is minimized and is defined as

$$E(\bar{h}_1,\bar{h}_2,...,\bar{h}_N) = \sum_{j=1}^{N}\left[T_j^{cal} - T_j^{mea}\right]^2 \tag{19}$$

where the calculated fin temperature at the jth thermocouple location, T_j^{cal}, is determined from Eq. (14).

The estimated values of $\bar{h}_j$, j = 1, 2,..., N, are determined until the value of E$(\bar{h}_1,\bar{h}_2,...,\bar{h}_N)$ is minimum. The detailed computational procedures for estimating the unknown value $\bar{h}_j$ can be found in Liu (2006). In order to avoid repetition, they are not shown in this manuscript. The computational procedures of the present

Table 2: Present estimates for H = 0.06 m and various S values.

	S=0.005 m $T_0 = 350.84$K $T_\infty = 301.49$K	S=0.01 m $T_0 = 349.38$K $T_\infty = 300.70$K	S=0.015 m $T_0 = 347.57$K $T_\infty = 298.71$K	S=0.02 m $T_0 = 349.94$K $T_\infty = 300.67$K
T_j^{mea} (K)	$T_1^{mea} = 345.97$K $T_2^{mea} = 346.02$K $T_3^{mea} = 337.77$K $T_4^{mea} = 337.79$K $T_5^{mea} = 332.57$K $T_6^{mea} = 332.64$K $T_7^{mea} = 330.87$K $T_8^{mea} = 330.81$K	$T_1^{mea} = 339.38$K $T_2^{mea} = 339.27$K $T_3^{mea} = 329.55$K $T_4^{mea} = 329.61$K $T_5^{mea} = 324.34$K $T_6^{mea} = 324.46$K $T_7^{mea} = 321.87$K $T_8^{mea} = 321.52$K	$T_1^{mea} = 325.83$K $T_2^{mea} = 335.79$K $T_3^{mea} = 325.03$K $T_4^{mea} = 325.15$K $T_5^{mea} = 320.18$K $T_6^{mea} = 320.23$K $T_7^{mea} = 318.47$K $T_8^{mea} = 318.39$K	$T_1^{mea} = 337.51$K $T_2^{mea} = 337.32$K $T_3^{mea} = 325.92$K $T_4^{mea} = 325.89$K $T_5^{mea} = 321.84$K $T_6^{mea} = 321.63$K $T_7^{mea} = 320.08$K $T_8^{mea} = 319.66$K
$\bar{h}_j(W/m^2K)$	$\overline{h_1} = 1.37$ $\overline{h_2} = 1.18$ $\overline{h_3} = 2.76$ $\overline{h_4} = 2.93$ $\overline{h_5} = 4.21$ $\overline{h_6} = 3.86$ $\overline{h_7} = 1.57$ $\overline{h_8} = 1.80$	$\overline{h_1} = 13.56$ $\overline{h_2} = 14.19$ $\overline{h_3} = 3.84$ $\overline{h_4} = 3.69$ $\overline{h_5} = 3.81$ $\overline{h_6} = 2.65$ $\overline{h_7} = 3.77$ $\overline{h_8} = 4.99$	$\overline{h_1} = 17.59$ $\overline{h_2} = 18.03$ $\overline{h_3} = 5.70$ $\overline{h_4} = 5.19$ $\overline{h_5} = 4.98$ $\overline{h_6} = 4.87$ $\overline{h_7} = 2.52$ $\overline{h_8} = 2.82$	$\overline{h_1} = 18.15$ $\overline{h_2} = 19.18$ $\overline{h_3} = 8.91$ $\overline{h_4} = 8.17$ $\overline{h_5} = 2.68$ $\overline{h_6} = 2.81$ $\overline{h_7} = 2.93$ $\overline{h_8} = 3.59$
Ra	413.48	3314.02	11518.38	26736.09
$\bar{h}(W/m^2K)$	2.45	6.31	7.71	8.30
$\bar{h}^{iso}(W/m^2K)$	1.71	4.26	5.05	5.33
Q (W)	1.01	2.49	2.96	3.15
η_f	0.694	0.675	0.655	0.642

study are repeated until the values of $\left|\frac{T_j^{mea} - T_j^{cal}}{T_j^{mea}}\right|$ for j = 1, 2,. . . , N are all less than 10^{-5}. Once the $\bar{h}_j$ values, j = 1, 2,. . . , N, are determined, the average heat transfer coefficient $\bar{h}$, heat transfer coefficient under the isothermal situation $\bar{h}^{iso}$, total heat transfer rate Q and fin efficiency η_f can be obtained from Eqs. (16)-(18).

4 Experimental apparatus

An experimental configuration of the small wind tunnel with 3.2 m in length, 0.215 m in width and 0.215 m in height used for the present problem is shown in Fig. 1. This wind tunnel is made of acrylic-plastic sheets with 0.007 in thickness. The left and right surfaces of this wind tunnel are open. It can be observed from Fig. 1 that the array of three vertical rectangular fins with 0.1 m in length and 0.001 m in

Table 3: Present estimates for $S \to \infty$ and various H values.

	H = 0.04 m $T_0 = 351.82$K $T_\infty = 298.74$K	H = 0.05 m $T_0 = 350.36$K $T_\infty = 298.13$K	H = 0.06 m $T_0 = 350.11$K $T_\infty = 300.43$K	H = 0.08 m $T_0 = 349.91$K $T_\infty = 299.77$K
T_j^{mea} (K)	$T_1^{mea} = 344.23$K $T_2^{mea} = 344.17$K $T_3^{mea} = 334.41$K $T_4^{mea} = 334.43$K $T_5^{mea} = 330.09$K $T_6^{mea} = 330.08$K $T_7^{mea} = 328.30$K $T_8^{mea} = 328.23$K	$T_1^{mea} = 340.03$K $T_2^{mea} = 340.09$K $T_3^{mea} = 328.33$K $T_4^{mea} = 328.35$K $T_5^{mea} = 323.55$K $T_6^{mea} = 323.53$K $T_7^{mea} = 321.05$K $T_8^{mea} = 321.25$K	$T_1^{mea} = 337.31$K $T_2^{mea} = 337.53$K $T_3^{mea} = 325.67$K $T_4^{mea} = 325.64$K $T_5^{mea} = 321.84$K $T_6^{mea} = 321.70$K $T_7^{mea} = 320.04$K $T_8^{mea} = 319.99$K	$T_1^{mea} = 333.16$K $T_2^{mea} = 333.11$K $T_3^{mea} = 318.91$K $T_4^{mea} = 318.88$K $T_5^{mea} = 313.84$K $T_6^{mea} = 313.82$K $T_7^{mea} = 311.25$K $T_8^{mea} = 311.15$K
$\bar{h}_j(W/m^2K)$	$\overline{h_1} = 12.20$ $\overline{h_2} = 12.82$ $\overline{h_3} = 12.12$ $\overline{h_4} = 11.70$ $\overline{h_5} = 5.13$ $\overline{h_6} = 5.09$ $\overline{h_7} = 4.39$ $\overline{h_8} = 4.59$	$\overline{h_1} = 14.87$ $\overline{h_2} = 14.56$ $\overline{h_3} = 11.07$ $\overline{h_4} = 11.00$ $\overline{h_5} = 2.82$ $\overline{h_6} = 3.57$ $\overline{h_7} = 5.54$ $\overline{h_8} = 4.80$	$\overline{h_1} = 19.09$ $\overline{h_2} = 17.89$ $\overline{h_3} = 9.36$ $\overline{h_4} = 9.79$ $\overline{h_5} = 1.97$ $\overline{h_6} = 2.40$ $\overline{h_7} = 3.16$ $\overline{h_8} = 2.95$	$\overline{h_1} = 16.66$ $\overline{h_2} = 16.81$ $\overline{h_3} = 7.70$ $\overline{h_4} = 7.71$ $\overline{h_5} = 2.19$ $\overline{h_6} = 2.03$ $\overline{h_7} = 4.40$ $\overline{h_8} = 4.69$
$\bar{h}(W/m^2K)$	8.5	8.53	8.33	7.77
$\bar{h}^{iso}(W/m^2K)$	6.24	5.60	5.33	4.11
Q (W)	2.60	2.92	3.18	3.30
η_f	0.712	0.656	0.641	0.528

thickness are vertically mounted on the top surface of the horizontal plate with 0.1 m in length, 0.1 m in width and 0.006 m in thickness. In order to heat three parallel rectangular fins, a square heater with 0.08 m in length is fixed on the bottom of this plate using the adhesive tapes (Nitto Denko Co., Ltd). Later, the test fins and horizontal plate enclosed the insulated material are placed in a small wind tunnel and then is heated about 7600 seconds using the 40W heater. Fig. 4 shows the experimental configuration of three vertical fins mounted on a horizontal plate in natural convection. The test fin is made of AISI 304 stainless material. It can be found from Arpaci , Kao, and Selamet (1999) that the thermal conductivity of AISI 304 stainless material is 14.9 W/m·k. Its emissivity ε measured by using FT-IR Spectrum 100 (Perkin Elmer Co., Ltd) is 0.18. The ambient air temperature and test fin temperature are measured using T-type thermocouples. The limit of error of the T-type thermocouple is $\pm0.4\%$ for 0 $^oC \leq$ T $\leq$ 350 ^oC. In order to reduce the effect of the thermal contact resistance between the fin and the horizontal plate on the present estimates, their gap is filled with the cyanoacrylate (Satlon, D-3). In addition, four thermocouples placed in the gap between the fin and the horizontal plate are fixed at (L/5, 0), (2L/5, 0), (3L/5, 0) and (4L/5, 0). The average of these four measured temperatures is taken as the fin base temperature T_o. Thus the thermal contact resistance between the fin and horizontal plate can be neglected in the present study. In order to measure the ambient air temperature T_∞, two thermocouples penetrated the central line of two lateral surfaces are positioned at 0.025 m away from the test fin. One thermocouple is positioned at the top surface, as shown in Fig.1. The average of these three measured temperatures is taken as the ambient air temperature T_∞. In order to estimate the average heat transfer coefficient on the fin, the vertical rectangular fin is divided into eight regions in the present study, i.e., N = 8. Thus eight thermocouples are fixed at (L/4, H/8), (3L/4, H/8), (L/4, 3H/8), (3L/4, 3H/8), (L/4, 5H/8), (3L/4, 5H/8), (L/4, 7H/8) and (3L/4, 7H/8), as shown in Fig. 2.

5 Results and discussion

The ratio of the surface area of the fin tip to the total fin surface area can be written as $\frac{(L+2H)\delta}{2LH+(L+2H)\delta}$. The "insulated tip" assumption in the present study will be reasonable provided that this ratio is very small. For simplicity, the average heat-transfer coefficient on the tip surface can be assumed to be the same as that on the lateral surfaces of the fin. Under the condition of L = 0.1 m and δ = 0.001 m, the ratio of the surface area of the fin tip to the total fin surface area, $\frac{(L+2H)\delta}{2LH+(L+2H)\delta}$, is about 1.6% for H = 0.08 m and 2.2% for H = 0.04 m. This implies that the assumptions of Eqs. (2) and (4) should be reasonable for the present study.

Chen and Wang (2008) have applied the present inverse scheme in conjunction with simulated temperature data and experimental temperature data given by Lin, Hsu, Chang, and Wang (2001) to estimate heat-transfer characteristics on a rectangular fin under wet conditions. It can be found from the illustrative examples of Chen and Wang (2008) that the present inverse scheme can obtain good estimates for the simulated temperature measurements with measurement errors, as shown in Tables 2 and 3 of this reference (Chen and Wang (2008)). In addition, the initial guesses cannot be very sensitive to the estimated results. Thus its detailed discussion will be shown in this paper. All the physical properties are evaluated at the film temperature or the average of the fin base temperature and ambient air temperature for the present problem. All the computations are performed with N_x = 21 and N_y = 17. The unknown heat transfer coefficients $\bar{h}_j$ for j = 1, 2,..., N used to begin the iterations are taken as unity. The measured temperatures, T^{mea} (L/4, H/8), T^{mea} (3L/4, H/8), T^{mea} (L/4, 3H/8), T^{mea} (3L/4, 3H/8), T^{mea} (L/4, 5H/8), T^{mea} (3L/4,

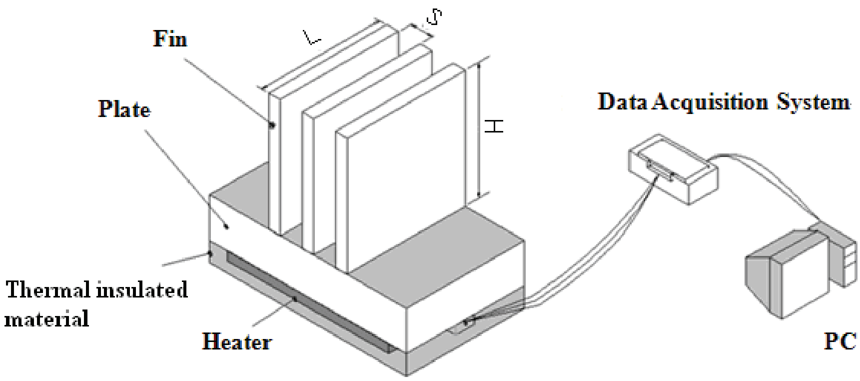

Figure 4: Schematic diagram of three parallel rectangular fins vertically mounted on the top surface of a horizontal test plate.

5H/8), T^{mea} (L/4, 7H/8) and T^{mea} (3L/4, 7H/8) are respectively denoted as T_1^{mea}, T_2^{mea}, T_3^{mea}, T_4^{mea}, T_5^{mea}, T_6^{mea}, T_7^{mea} and T_8^{mea}. It can be found from Tables 1-3 that the measured fin temperatures for the measurement locations at X = L/4 are not equal to those for the measurement locations at X = 3L/4 for various H and S values. The average heat transfer coefficient on the jth sub-fin region $\bar{h}_j$ slightly deviates from $\bar{h}_{j+1}$ for j = 1, 2, 3 and 4. This phenomenon can result from the measurement error of the temperature or the random motion of the flow pattern, etc. It can be observed that the formation of irregular chimney flow patterns between two vertical rectangular fins for the present problem from Fig. 4 of Rakshit and Balaji (2005) and Fig. 4 of Sobhan, Venkateshan, and Seetharamu (1990). On the other hand, there exists a complex three-dimensional flow and thermal fields within two vertical rectangular fins. It is worth mentioning that it may be difficult to keep a fixed fin base temperature and a fixed ambient air temperature for the present problem. Tables 1 and 2 respectively show the effect of the fin spacing S on the average heat transfer coefficient on the jth sub-fin region $\bar{h}_j$, total heat transfer rate from the whole fin Q, average heat transfer coefficient $\bar{h}$, heat transfer coefficient under the isotherm condition $\bar{h}^{iso}$ and fin efficiency η_f for H = 0.04 m and 0.06 m. Table 3 shows the present estimates for a single fin (S $\rightarrow \infty$) and various H values. It can be observed some interesting results from Tables 1-3 that there is a considerable temperature drop in the neighborhood of the fin base or from the fin base (Y = 0) to Y = 3H/8 for S $\geq$ 0.005 m and 0.04 m $\leq$ H $\leq$ 0.08 m. In addition, there is also a more fin temperature drop at the same measurement location for 0.005 m $\leq$ S $\leq$ 0.01 m and 0.04 m $\leq$ H $\leq$ 0.06 m. However, the variation of the fin temperature at the same measurement location can gradually become small for S $\geq$ 0.01 m and H = 0.04 m and 0.06 m. This implies that the optimum fin spacing can be about 0.01 m for H = 0.04 m and 0.06 m. It can be observed from Leung, Probert, and Shilston (1985) that the value of the optimum fin spacing for δ/L = 3/190, H/L = 6/19 and L = 0.19 m is $(10.5 \pm 1) \times 10^{-3}$m. Fig. 6 in Jones and Smith (1970) also showed that the value of the optimum fin spacing for δ/L = 0.0125, 0.05 $\leq$ H/L $\leq$ 0.144, L = 0.025 m (10 inch) and various $(T_o - T_\infty)$ values is about 0.013 m (0.5 inch).

The average heat transfer coefficient on each sub-fin region can be sensitive to the measured fin temperatures. It can be found from Tables 1 and 2 that the average heat transfer coefficient decreases from the fin base to the fin tip for S $\geq$ 0.015 m and H = 0.04 m and 0.06 m. The average heat transfer coefficients on the sub-fin regions 1 and 2 in the neighbour of the fin base, $\bar{h}_1$ and $\bar{h}_2$, are higher than those on other sub-fin regions for S $\geq$ 0.01 m and 0.04 m $\leq$ H $\leq$ 0.06 m. The ratio of the average heat transfer coefficients on the sub-fin regions 1 and 2 in the neighbour of the fin base to those on the sub-fin regions 7 and 8 in the neighbour of the fin tip, $(\bar{h}_1 + \bar{h}_2)/(\bar{h}_7 + \bar{h}_8)$, for S = 0.005 m, 0.01 m, 0.015 m and 0.02 m, respectively, is about 1.6 times, 4.1 times, 3.3 times and 3.8 times as H = 0.04 m and is about 0.8 times, 3.2 times, 6.7 times and 5.7 times as H = 0.06 m. Tables 1 and 2 also show that the discrepancy of the average heat transfer coefficient on each sub-fin region is small for S = 0.005 m and H = 0.04 m and 0.06 m.

The average heat transfer coefficient $\bar{h}$ and heat transfer coefficient under the isotherm condition $\bar{h}^{iso}$ for H = 0.04 m is higher than that for H = 0.06 m for various S values. Rammohan Rao and Venkateshan (1996) stated that increase in the fin height, likewise, reduced both the convection and radiation heat transfer coefficients. Thus the present results coincide with that observed by Jones and Smith (1970) and Rammohan Rao and Venkateshan (1996).

5.1 Previous heat transfer correlations

Various heat transfer correlations for the present problem have been proposed (Raithby and Hollands (1985); Kreith and Bohn (1993); Harahap and Setio (2001); Harahap and McManus (1967); Harahap, Rudianto, and Pradnyana (2005); Sobhan, Venkateshan, and Seetharamu (1989); Sobhan, Venkateshan, and Seetharamu (1990); Rammohan Rao and Venkateshan (1996); Leung, Probert, and Shilston (1985); Leung and Probert (1989); Leung and Probert (1989); Liu (2006)). However, among these heat transfer correlations, the correlations shown in Raithby and Hollands (1985); Kreith and Bohn (1993); Harahap, Rudianto, and Pradnyana (2005); Harahap and Lesmana (2006) are selected to compare with the present estimated results. The present problem has been studied by Jones and Smith (1970) using an experimental apparatus with seven aluminum fins extending through the top horizontal surface of a closed container and Rammohan Rao and Venkateshan (1996) using four aluminum fins. It can be found from Raithby and Hollands (1985); Kreith and Bohn (1993) that Jones and Smith (1970) were able to correlate their measured heat transfer to within about ± 25 percent on the relation between the Nusselt number Nu and the Rayleigh number Ra over the data range $2\times10^2 \leq$ Ra $\leq 6\times10^5$, Pr = 0.71, L = 0.254 m, 0.026 $\leq$ H/L $\leq$0.19 and 0.016 $\leq$ S/L $\leq$0.20. This correlated relationship between Nu and Ra under steady-state conditions can closely be represented as (Raithby and Hollands (1985); Kreith and Bohn (1993))

$$Nu = \left[\left(\frac{Ra}{1500}\right)^{-2} + (0.081\times Ra^{0.39})^{-2}\right]^{-1/2} \tag{20}$$

where Pr is the Prandtl number defined by Pr $= \nu/\alpha$. Nu and Ra are defined as $Nu = \bar{h}^{iso}S/k_{air}$ and $Ra = g\beta(T_o - T_\infty)S^3/(\nu\alpha_{air})$. g is the acceleration of gravity. β is the volumetric thermal expansion coefficient. k_{air}, ν and α_{air} are the thermal conductivity, kinematic viscosity and thermal diffusivity of the air, respectively.

In accordance with experimental data given by Sobhan, Venkateshan, and Seetharamu (1989, 1990), Rammohan Rao and Venkateshan (1996) proposed a new correlation of Nu, Ra and k/k_{air} with an error spread of $\pm$10% for H = 0.07 m, 0.01 m $\leq$ S $\leq$ 0.025 m and $10^3 \leq$ Ra $\leq 10^6$ as

$$Nu = 0.022(\frac{k}{k_{air}})^{0.299}Ra^{0.337} \tag{21}$$

Rammohan Rao and Venkateshan (1996) stated that there was a fair bit of scatter between their experimental data and Eq. (21) for 0.03 m $\leq$ H $\leq$ 0.07 m, 0.01 m $\leq$ S $\leq$ 0.025 m, L = 0.05 m, δ = 0.0015 m and $10^3 \leq$ Ra $\leq 10^6$. Their experimental data and Eq. (21) showed close agreement with the results given by Jones and Smith (1970) for large Rayleigh numbers. However, the discrepancy for lower Rayleigh numbers may be attributed to the neglect of heat capacity effects.

Harahap and Lesmana (2006) also proposed a correlation of Nu, Ra, S/L, W/L and S/H for the vertically based array, H = 0.0135 m, 0.003 m $\leq$ S $\leq$ 0.011 m, 0.025 m $\leq$ L $\leq$ 0.049 m, δ = 0.001 m and $2\times10^5 \leq (L/S)^3\varepsilon Ra\ le\ 5\times10^5$ as

$$Nu = 3.35Ra^{0.153}(S/L)^{0.541}(L/W)^{0.121}(S/H)^{0.605} \tag{22}$$

where the parameter W denotes the width of the fin arrays and is defined as $W = 3\delta + 2S$ in the present study. However, it can be observed from their works (Harahap and Lesmana (2006)) that Nu was proportional to $(L/W)^{0.62}$ for the horizontally based array. Harahap and Lesmana (2006) may mistake the exponent of 0.62.

Thus the comparison between the present results of the $\bar{h}^{iso}$ value and those obtained from Eq. (22) will not be made.

It can be found that their conclusions of Harahap and Lesmana (2006) were consistent with those observed by Harahap, Rudianto, and Pradnyana (2005). Harahap, Rudianto, and Pradnyana (2005) proposed the following modified correlations for H = 0.0135 m, δ = 0.001 m, 0.025 m $\leq$ L $\leq$ 0.049 m , 0.003 m $\leq$ S $\leq$ 0.01 m and 0.025 m $\leq$ W $\leq$ 0.049 m.

$$Nu = 9.209Ra^{0.241}\exp(-0.241\frac{k_{air}H}{k\delta})(\frac{S^2}{LH})^{0.9158}(\frac{L}{W})^{0.344}$$
$$\text{for } 2.72\times10^{-6} < Ra\exp(-\frac{k_{air}H}{k\delta})(\frac{S^2}{LH})^{3.8} < 9.2\times10^{-5} \quad (23)$$

and

$$Nu = 3.203Ra^{0.175}\exp(-0.175\frac{k_{air}H}{k\delta})(\frac{S^2}{LH})^{0.665}(\frac{L}{W})^{0.344}$$
$$\text{for } 2.58 < Ra\exp(-\frac{k_{air}H}{k\delta})(\frac{S^2}{LH})^{3.8} < 94.8 \quad (24)$$

5.2 Modified heat transfer correlations

Deviations in the predictions of the heat transfer coefficient obtained from previous correlations and the present estimated results show the need to develop a modified correlation that can yield more accurate predictions. The present estimated results show that the $\bar{h}^{iso}$ value has an obvious increase with increasing the fin spacing for S/L $\leq$0.2 and decreases with increasing the fin height. On the other hand, S/L can have a significant effect on the $\bar{h}^{iso}$ value for S/L $\leq$ 0.1 and H/L = 0.4 and 0.6. However, the effect of the S value on the $\bar{h}^{iso}$ value can gradually become weak for S/L > 0.15 and 0.4$\leq$ H/L $\leq$ 0.6. The $\bar{h}^{iso}$ value can approach the asymptotical value obtained from a single fin as S/L > 0.2. Tables 1-3 also show that the effect of the H value on the present estimated results of the $\bar{h}^{iso}$ values is not negligible. This conclusion coincides with that shown in Raithby and Hollands (1985); Kreith and Bohn (1993). The smoothing curves can be applied to match the present estimates and the estimated results of Nu given by Liu (2006) for various Ra, S and H values, the modification of Eq. (20) in conjunction with the present estimates can yield a modified correlation of Nu and Ra as

$$Nu = \left[0.65\left(\frac{Ra}{1500}\right)^{-2} + (0.081Ra^{0.39})^{-2}\right]^{-1/2} \quad (25)$$

In accordance with the present estimates of the $\bar{h}^{iso}$ value, the modified correlation of Eq. (20) for the horizontally based array, Nu, Ra, S/L, W/L and S/H can be expressed as

$$Nu = 3.35Ra^{0.153}(S/L)^{0.541}(L/W)^{0.126}(S/H)^{0.605} \tag{26}$$

The $\bar{h}^{iso}$ values obtained from Eqs. (25) and (26) are in good agreement with the present estimates for $413 \leq$ Ra ≤ 27854, $0.1 \leq$ S/L ≤ 0.2 and H/L = 0.4 and 0.6.

Table 4: Comparison of $\bar{h}^{iso}$ between the present estimates and previous results for various H and S values.

H (m)	S (m)	$\bar{h}^{iso}(W/m^2K)$				
		Present estimates	Present correlations		Eq. (21)	Eqs. (24) and (25)
			Eq. (26)	Eq. (27)		
0.04	0.005	3.94	2.01	3.56	1.67	4.39
	0.01	5.17	4.51	4.93	4.17	5.30
	0.015	5.96	5.60	6.01	5.46	6.62
	0.02	6.11	6.08	6.93	6.02	7.73
0.06	0.005	1.71	1.79	2.70	1.48	2.88
	0.01	4.26	4.41	3.82	4.07	3.98
	0.015	5.05	5.52	4.66	5.38	4.97
	0.02	5.33	5.97	5.38	5.9	5.81

In order to validate the accuracy and reliability of the present inverse method in conjunction with the present experimental method further, comparisons between the present estimates of the $\bar{h}^{iso}$ value and those obtained from the correlations recommended by current textbooks (Raithby and Hollands (1985); Kreith and Bohn (1993)), Harahap, Rudianto, and Pradnyana (2005) are shown in Table 4 for L = 0.1 m and various values of the fin spacing and fin height. Examination of Table 4 shows that the present estimates of the $\bar{h}^{iso}$ value obtained from Eq. (25) are slightly higher than those obtained from the correlations recommended by current textbooks (Raithby and Hollands (1985); Kreith and Bohn (1993)). The deviation between them is less than 7.8% for $0.1 \leq$ S/L ≤ 0.2 and $0.4 \leq$ H/L ≤ 0.6. However, their variation can be up to 17% for S/L = 0.05. Thus, in general, the present estimates of the $\bar{h}^{iso}$ value obtained from Eq. (25) agree with those obtained from the correlations recommended by current textbooks (Raithby and Hollands (1985); Kreith and Bohn (1993)), as shown in Eq. (20), for $0.05 \leq$ S/L ≤ 0.2 and $0.4 \leq$

H/L $\leq$ 0.6 even though H/L and S/L are not in range of 0.026 $\leq$ H/L $\leq$0.19 and 0.016 $\leq$ S/L $\leq$0.20. The deviation between them is less than 7.8% for 0.1 $\leq$ S/L $\leq$ 0.2 and 0.4 $\leq$ H/L $\leq$ 0.6. The deviation between the present estimates of the $\bar{h}^{iso}$ value obtained from Eq. (26) and those obtained from the correlations (23) and (24) recommended by Harahap, Rudianto, and Pradnyana (2005) is less than 11.5% for 0.01 $\leq$ S/L $\leq$ 0.2 and 0.4 $\leq$ H/L $\leq$ 0.6. However, their variation can be up to 23.3% for S/L = 0.05 and H/L = 0.4. The above comparison implies that the present estimates should have good accuracy and good reliability. The above results coincide with those observed by Rammohan Rao and Venkateshan (1996) for large Ra values.

Tables 1-3 respectively show the present estimates of the total heat transfer rate Q and fin efficiency η_f for various values of the fin spacing and fin height. The Q value for H/L = 0.4 is higher than that for H/L = 0.6 as S/L = 0.05. This phenomenon can result from that the fin base temperature for H/L = 0.4 is approximately 10 K higher than that for H/L = 0.6. It can be observed that the Q value increases with increasing the fin spacing or the fin height, the η_f value decreases with increasing the fin spacing or the fin height. The Q value increases with increasing the fin height and also gradually approaches an asymptotical value with increasing the fin spacing for a fixed H value. The ratio of the Q value for H/L = 0.5, 0.6 and 0.8 to that for H/L = 0.4, respectively, is about 1.12 times, 1.22 times and 1.27 times as $S \to \infty$. Thus the proper choice of the H/L parameter can be required in order to increase the total heat rate dissipated from the fin.

6 Conclusions

The present study proposes a numerical inverse scheme involving the finite difference method in conjunction with the least-squares method and experimental fin temperatures at ten measurement locations to estimate the natural-convection heat transfer coefficient under the isothermal situation $\bar{h}^{iso}$ from vertical fins mounted on a horizontal plate and fin efficiency η_f for various values of the fin spacing and fin height. An important finding is that the present estimated results can be applied to obtain two modified correlations of the Nusselt number and the Rayleigh number. The results show that the correlation $\bar{h}^{iso}$ values obtained from two modified correlations recommended by the present study are in good agreement with the present estimates for 413 $\leq$ Ra $\leq$ 27854, 0.1 $\leq$ S/L $\leq$ 0.2 and H/L = 0.4 and 0.6. The present estimates of the $\bar{h}^{iso}$ value obtained from the present new correlation agree with those obtained from the correlations recommended by current textbooks for 0.05 $\leq$ S/L $\leq$ 0.2 and 0.4 $\leq$ H/L $\leq$ 0.6 even though H/L and S/L are not in range of 0.026 $\leq$ H/L $\leq$0.19 and 0.016 $\leq$ S/L $\leq$0.20. In addition, the deviation between the present estimates of the $\bar{h}^{iso}$ value obtained from another present modified correlation and those given by Harahap, Rudianto, and Pradnyana (2005) is also less than 11.5% for $0.01 \leq$ S/L ≤ 0.2 and $0.4 \leq$ H/L ≤ 0.6. This implies that the present hybrid inverse method can give a good accuracy for various values of the fin height and fin spacing. It is worth noting that that S/L can have a significant effect on the present estimates of $\bar{h}^{iso}$ for S/L ≤ 0.15. In addition, the effect of H/L on the present estimates of $\bar{h}^{iso}$ for $0.4 \leq$ H/L ≤ 0.8 and a fixed fin spacing cannot be negligible, too.

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