Viscous Linear Instability of an Incompressible Round Jet with Petrov-Galerkin Spectral Method and Truncated Boundary

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Abstract: A Fourier-Chebyshev Petrov-Galerkin spectral method is described for computation of temporal linear stability in a circular jet. The outer boundary of unbounded domains is truncated by large enough diameter. The mathematical formulation is presented in detail focusing on the analyticity of solenoidal vector field used for the approximation of the flow. The scheme provides spectral accuracy in the present cases studied and the numerical results are in agreement with former works.

Keywords: hydrodynamic stability; circular jet; spectral method; finite element method;

1   Introduction

Jets are important in many practical applications, e.g., related to combustion, propulsion, mixing, and aeroacoustic. As one of the generic flow of fluid mechanics, jets have been of scientific interest for over 100 years. The round jet results when fluid is emitted, with a given initial momentum, out of a circular orifice into a large space. At sufficiently high Reynolds numbers this jet will be turbulent. The stability properties of the flow play a fundamental role in the transition to turbulence and the formation of coherent vortex structures in a turbulent fluid (Schmid & Henningson, 2000; Drazin & Reid, 2004).

Frequently, the choice of independent variables is motivated by the symmetry of circular jet, then cylindrical coordinates are likely most appropriate. However, the choice of a particular set of independent variable might inadvertently introduce

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mathematically allowable but physically unrealistic terms, e.g., singularities, although the flow is continuous and regular at the axis (Lewis & Bellan, 1990). These nonphysical terms must be eliminated by the imposition of physical constraints on the mathematical solutions. The strategy to deal with this difficulty in analytical approaches is commonly that of discarding the singular solutions among all the admissible ones (Batchelor & Gill, 1962).

The treatment of the geometrical singularity in cylindrical and spherical coordinates has been a difficulty in the development of accurate finite difference (FD) and pseudo-spectral (PS) schemes for many years (Mohseni & Colonius, 2000). A variety of numerical procedures for dealing with the singularity have been suggested. The use of a spectral representation is often to be preferred for the accurate solution of problems with simple geometry (Chan *et al.*, 2009; Xie *et al.*, 2008, 2009a, 2009b, 2009c, 2009d). For spectral methods, Lopez *et al.* (2002) derived a regularity conditions by using the properties at the axis of the functions chosen to expand velocity and pressure along the radial direction. Pole conditions for Poisson-type equations in the physical space were derived by Huang and Sloan (1993). But the time step restriction problem that arises for advection problems due to the increased resolution near the coordinate singularity can't be avoided. One way to avoid the time step restriction is to use a Fourier filter in the azimuthally direction as used by Fornberg (1995). Priymak and Miyazakiy (1998) presented a robust numerical technique for incompressible Navier Stokes equations in cylindrical coordinates. Lin and Atluri (2000, 2001) proposed several up-winding Meshless Local Petrov-Galerkin (MLPG) schemes to solve steady convection-diffusion problems in one and two dimensions. It shows that the MLPG method is very promising to solve the convection-dominated flow and fluid mechanics problems. Meseguer and Trefethen (2003) described a Fourier-Chebyshev Petrov-Galerkin spectral method for high accuracy computation of linearized dynamics flow in finite circular pipe in the light of Chapman's analysis. Bierbrauer & Zhu (2007) present three analytical solutions, the Bounded Creeping Flow, Solenoidal and Conserved Solenoidal Solutions, which are both continuous, incompressible, retain as much of the original mathematical formulation as possible and provide a physically reasonable initial velocity field. For hydrodynamic stability problems, the linearized Navier-Stokes equations are a general eigenvalues equation, the conditioner of the matrix usually is an ill-conditioned system arising from very-high order polynomial interpolations. Liu & Atluri (2008, 2009) developed a highly accurate technique based on modified Trefftz method to deal with the ill-posed linear problems. A link between these structures is discussed by van Doorne & Westerweel (2009). Theoretical approaches to extract perturbations that are efficient in triggering turbulence are presented by Biau & Bottaro (2009) and Cohen et al. (2009), respectively. Re-

cently, MLPG methods mainly are used in the bounded domain for fluid mechanics problems. Shan *et al.* (2008) used local MLPG method to solve 3D incompressible viscous flows with curved boundary. Sellountos and Sequeira developed a hybrid multi-region MLPG velocity-vortices scheme for the 2D Navier stokes equations. To improve the accuracy and stability of MLPG methods, Mohammadi (2008) and Orsini *et al.* (2008) proposed a meshless radial basis function techniques, and Sellountos & Sequeira (2009) applied the radial basis function networks to transient viscous flows.

This paper considers the hydrodynamic stability of round jet flow in which there are no solid boundaries in the field. To construct a basis function set for unbounded domains, it is necessary to assume the asymptotic behavior of the approximated functions for large *r*. if the approximated functions decay exponentially as *r* trends to infinite, there are many options for the basis functions (Matsushima & Marcus. 1997). Xie *et al.* (2009a, 2009d) have proposed a exponential mapping for the computation of stability of circular jet, and the results are in agreement with previous work very well. But the coherent structure for super critical condition of jet flow cannot be obtained directly, and it needs the inverse coordinate transformation to draw the streamline. One way to treat this problem efficiently is the domain truncation method which imposes artificial boundary conditions at sufficiently large radius.

In the present work, a spectral Petrov-Galerkin scheme for the numerical approximation of flow in a circular jet is presented; rational basis functions in an unbounded domain are used for expansion in the radial direction of polar coordinates. They satisfy the pole condition exactly at the coordinate singularity. Solenoidal vector fields are treated efficiently by the toroidal and poloidal decomposition which reduces the number of dependent variables from 3 to 2. It can be used in analytical studies and it is particularly useful in numerical solutions.

2   The mathematical formulation

The impressible dimensionless Navier-Stokes equations in cylindrical polar coordinates, these equations become:

$$\frac{\partial u_r}{\partial r} + \frac{u_r}{r} + \frac{1}{r}\frac{\partial u_\theta}{\partial \theta} + \frac{\partial u_z}{\partial z} = 0 \tag{1}$$

$$\frac{Du_r}{Dt} - \frac{u_\theta^2}{r} = -\frac{1}{\rho}\frac{\partial p}{\partial r} + \frac{1}{Re}\left(\Delta u_r - \frac{u_r}{r^2} - \frac{2}{r^2}\frac{\partial u_\theta}{\partial \theta}\right) \tag{2}$$

$$\frac{Du_\theta}{Dt} + \frac{u_r u_\theta}{r} = -\frac{1}{\rho r}\frac{\partial p}{\partial \theta} + \frac{1}{Re}\left(\Delta u_\theta - \frac{u_\theta}{r^2} + \frac{2}{r^2}\frac{\partial u_r}{\partial \theta}\right) \tag{3}$$

$$\frac{Du_z}{Dt} = -\frac{1}{\rho}\frac{\partial p}{\partial z} + \frac{1}{Re}\Delta u_z \tag{4}$$

where

$$\frac{D}{Dt} \equiv \frac{\partial}{\partial t} + u_r\frac{\partial}{\partial r} + \frac{u_\theta}{r}\frac{\partial}{\partial \theta} + u_z\frac{\partial}{\partial z} \tag{5}$$

$$\Delta \equiv \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2} + \frac{\partial^2}{\partial z^2} \tag{6}$$

These equations are non-dimensionalised with respect to a length scale *Lc*, a velocity scale *Uc*, and the Reynolds number is *Re=LcUc/ν*. The length scale and velocity scale is usually based on the jet core velocity and momentum thickness. It is used to characterize jet velocity profiles for different axial locations. The boundary condition now takes the form:

$$u_r = u_\theta = u_z = 0 \text{ for } r = \infty \tag{7}$$

It is readily seen that independently of Reynolds number, there is a steady solution to Eqs.(1)-(4), the laminar parallel flow, consisting of pure axisymmetric axial motion (Schmid & Henningson, 2000; Drazin & Reid, 2004):

$$U_r = 0; \quad U_\theta = 0; \quad U_z = U(r) = 1/(1+r^2)^2; P = P(r) \tag{8}$$

Our concern in this paper is the linearized problem in which only infinitesimal perturbations from the laminar flow are considered, Let

$$u_r = U_r + u'_r; \quad u_\theta = U_\theta + u'_\theta; \quad u_z = U_z + u'_z; p = P + p' \tag{9}$$

and the perturbation can be expressed as superpositions of complex Fourier modes of the form:

$$\frac{u'_r}{u_r(r)} = \frac{u'_\theta}{u_\theta(r)} = \frac{u'_z}{u_z(r)} = \frac{p'}{\rho p(r)} = e^{i(n\theta+kz-\beta t)} = e^{i(n\theta+kz-kct)} \tag{10}$$

Then the linearized Navier-Stokes equations become:

$$-ikcu_r = -Dp + \frac{1}{Re}\left(D^2u_r + \frac{1}{r}u_r - \frac{n^2+1}{r^2}u_r - k^2u_r - \frac{2}{r^2}inu_\theta\right) - ikU_zu_r \tag{11}$$

$$-ikcu_\theta = -\frac{in}{r}p + \frac{1}{Re}\left(D^2u_\theta + \frac{1}{r}u_\theta - \frac{n^2+1}{r^2}u_\theta - k^2u_\theta + \frac{2}{r^2}inu_r\right) - ikU_zu_\theta \tag{12}$$

$$-ikcu_z = -ikp + \frac{1}{Re}\left(D^2u_z + \frac{1}{r}u_z - \frac{n^2}{r^2}u_z - k^2u_z\right) - u_rDU_z - ikU_zu_z \tag{13}$$

$$\left(D + \frac{1}{r}\right)u_r + i\left(\frac{1}{r}nu_\theta + ku_z\right) = 0 \tag{14}$$

The boundary conditions are set for different azimuthal modes. The following boundary conditions of disturbance were first derived by Batchelor and Gill (1962), and then adopted by many researchers (Lessen and Singh, 1973; Morris, 1976). The boundary conditions take the form:

Case $n = 0$:

$$u_r(0) = u_\theta(0) = Du_z(0) = Dp(0) = 0; u_r(\infty) = u_\theta(\infty) = u_z(\infty) = p(\infty) = 0 \tag{15}$$

Case $n = 1$

$$u_r(0) + u_\theta(0) = Du_z(0) = Dp(0) = 0; u_r(\infty) = u_\theta(\infty) = u_z(\infty) = p(\infty) = 0 \tag{16}$$

Although the outer boundary of the jet is located at infinity, it should be truncated at some finite values in the present numerical simulation. This finite domain size is denoted by R.

3 Solenoidal Petrov-Galerkin discretisation

The solenoidal condition (Eq.(14)) introduces a linear dependence between the components, u_r, $u_{\theta,}$, u_z, therefore, there are only two degrees of freedom. There are many different ways of obtaining divergence free fields in polar coordinate; we proceed in the way similar to that of Meseguer and Trefethen (2003), the solenoidal basis for the approximation of the perturbation vector field takes the form:

$$\mathbf{u} = e^{i(kz+n\theta-kct)}\sum_{m=0}^{M} a_m^{(1)}\mathbf{w}_m^{(1)}(r/R)\mathbf{u}_m^{(1)}(r/R) + a_m^{(2)}\mathbf{w}_m^{(2)}(r/R)\mathbf{u}_m^{(2)}(r/R) \tag{17}$$

where $\mathbf{u}_m$ belongs to the physical or trial space and $\mathbf{w}_m$ is a solenoidal vector field belongs to the test or projection space. Then the physical or trial basis is:

$$\mathbf{u}_m^{(1)} = \begin{pmatrix} -inr^{n-1}g_m(r/R) \\ D_*[r^ng_m(r/R)] \\ 0 \end{pmatrix}, \mathbf{u}_m^{(2)} = \begin{pmatrix} 0 \\ -ikr^nh_m(r/R) \\ inr^{n-1}h_m(r/R) \end{pmatrix} \tag{18}$$

where $D_* = D + 1/r,$; and $h_m(r/R) = (R^2 - r^2)T_{2m}(r/R)/R^2$, $g_m(r/R) = (R^2 - r^2)h_m(r/R)/R^2$, in which $T_{2m}(r/R)$ stands for the Chebyshev polynomial of order $2m$, the factor $(R^2 - r^2)/R^2$ is added to make the vector field vanish over the

boundary. In order to take the advantage of orthogonality properties of Chebyshev polynomials, the test functions should be built up suitably. In essence, the projection fields are going to have the same structure as the trial fields but the functions will be modified by the Chebyshev weight $(1-x^2)^{-1/2}$.

In the present study, the temporal instability of round jet is considered. Hence, k and n is real quantity while $c = c_r + ic_i$ is generally complex. The disturbances will grow with time if $c_i > 0$ and will decay $c_i < 0$ in Eq.(10). The neutral disturbances are then characterized by $c_i = 0$. The Petrov-Galerkin projection scheme is carried out by substituting the spectral approximation in equations and projecting over the dual space. This procedure leads to a discretized generalized eigenvalues problem, and the coefficient $a_m^{(1,2)}$ govern the temporal behavior of the perturbation.

$$\mathbf{A}\mathbf{X} = \lambda \mathbf{B}\mathbf{X} \tag{19}$$

where $\lambda = -ikc$, the matrixes, $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{X}$ represent as follows respectively.

$$\mathbf{A} = \begin{bmatrix} \left(\mathbf{w}_m^{(1)} \cdot \ell[\mathbf{u}_m^{(1)}]\right) & \left(\mathbf{w}_m^{(1)} \cdot \ell[\mathbf{u}_m^{(2)}]\right) \\ \left(\mathbf{w}_m^{(2)} \cdot \ell[\mathbf{u}_m^{(1)}]\right) & \left(\mathbf{w}_m^{(2)} \cdot \ell[\mathbf{u}_m^{(2)}]\right) \end{bmatrix}$$

$$\mathbf{B} = \begin{bmatrix} \left(\mathbf{w}_m^{(1)} \cdot \mathbf{u}_m^{(1)}\right) & \left(\mathbf{w}_m^{(1)} \cdot \mathbf{u}_m^{(2)}\right) \\ \left(\mathbf{w}_m^{(2)} \cdot \mathbf{u}_m^{(1)}\right) & \left(\mathbf{w}_m^{(2)} \cdot \mathbf{u}_m^{(2)}\right) \end{bmatrix}$$

$$\mathbf{X} = \begin{bmatrix} a_m^{(1)} \\ a_m^{(2)} \end{bmatrix}$$

where ℓ stands for the linear operator of linear stability equations

$$\ell[\cdot] = \frac{1}{Re}\Delta[\cdot] - \mathbf{u}_B \cdot \nabla[\cdot] - [\cdot] \cdot \nabla \mathbf{u}_B \tag{20}$$

where $\mathbf{u}_B$ are the basic flow velocity vector $(0, 0, U_z)$. The pressure term should be formally included in the operator ℓ, but it is cancelled when projecting it over $\mathbf{w}$, that is $(\mathbf{w}, p) = 0$.

The solenoidal perturbation vector fields defined in Eq.(14) are invariant under spiral transformations of the form $dz/d\theta = -n/k$. Making use of the spiral coordinate $\zeta = n\theta + kz$, the Solenoidal condition Eq.(14) can be expressed in this new variable as

$$\frac{\partial r u_r}{\partial r} + \frac{\partial}{\partial \zeta}(nu_\theta + rku_z) = 0 \tag{21}$$

which implies the existence of a first integral $\chi(r, \zeta)$ of perturbed field satisfying

$$\frac{\partial \chi}{\partial r} = nu_\theta + rku_z; \frac{\partial \chi}{\partial \zeta} = -ru_r \tag{22}$$

the function $\chi(r, \zeta)$ plays the role of a stream function which is tangent to the perturbation vector field **u**. Then we can obtain $\chi(r, \zeta)$ for the vector field for $n>0$:

$$\chi = 2nr^n Re \left\{ e^{i(kz+n\theta-\beta t)} \sum_{m=0}^{M} a_m^{(1)} g_m(r) \right\} + C \tag{23}$$

where C is an arbitrary constant of integration.

4 Results and discussion

In stability analysis the most important eigenvalue is the one that is the most unstable or least stable. For the present framework, this corresponds to the eigenvalue with the least imaginary part. In particular, the flow will be temporal unstable if the imaginary part of the complex amplification is positive. The results in the present section have been obtained by parametrically varying the Reynolds number and frequency for an azimuthal wave number n of 1. The mode of $n=0$ is not considered in present study in that the linear stability problem of round jet reduces to the ordinary Orr-Sommerfeld equation.

4.1 The effect of finite domain size R

The wave amplifications (c_i) of round jet for R ranging from 5 to 10 are shown in Fig.1. It shows that the wave amplification of jet at $R=7.5$ is almost the same trend as that of $R=10$. So much it can be concluded that $R=10$ is far away enough for outer boundary of an axisymmetric jet flow. Hence, the outer boundary is defined at $R=10$ in the present study. And the normalized jet velocity and its derivation profile is shown in Fig.2. There is a inflexion at the point $r=(1/5)^{1/2}$.

4.2 Critical Reynolds number

To obtain the critical Reynolds number, the amplification factor for some values of Re close to the critical Reynolds number is plot in Fig.3. The critical Reynolds number is the point where the curve $c_i(k)$ becomes tangent to the $c_i=0$ line. And in k-Re plane the neutral curve ($c_i=0$ line) separates the space into two zones: one is stable and the other is unstable, which is shown in Fig.4.

From the graph the critical Reynolds number is found to be 37.89 and the corresponding wavenumber is 0.45 for $n =1$ mode, under the conditions, the amplification is -0.102758529595647-0.000000635742628i. The distribution of the eigenvalues is plotted in Fig.5. The present result is also compared with some of the other values reported by researchers in Table 1.

4.3   Stream function

From the previous expression, the eigen-stream functions have been computed associated with the least stable eigenvalues of the spectra of the operator. The least stable eigenmode of various Reynolds number under the conditions $k =0.45$ is plotted in Fig.8. The dynamics is localized around the axis of the cylinder. The lower Reynolds number, the more smoothness of the distribution of stream functions; the larger Reynolds number, the farther downstream the locations, the more concentrated of stream function on the axis. The similar things are founded in the effect of wave number.
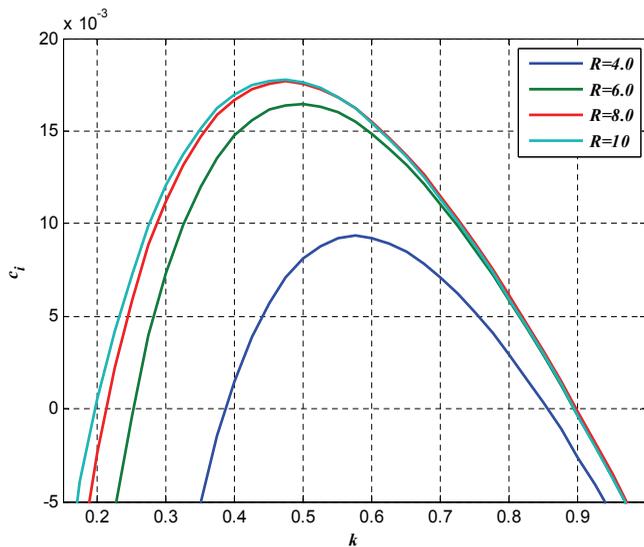
Figure 1: The effect of domain size on wave amplification.

5   Conclusion

The incompressible linear stability equation of round jet in cylindrical polar coordinates with Petrov Galerkin method is presented. To construct a basis function set
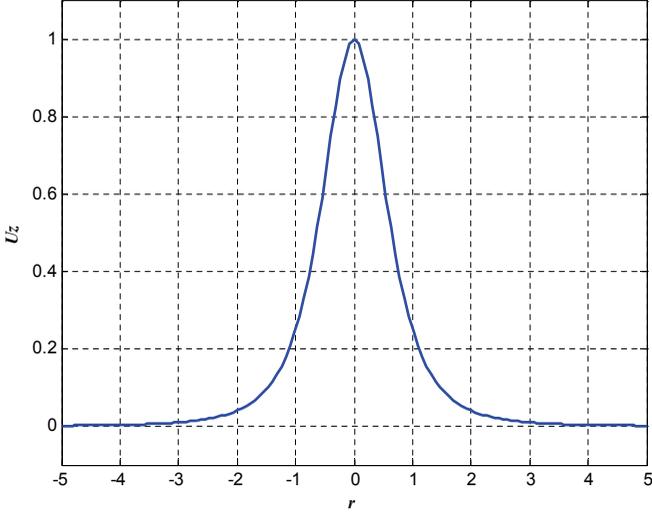
(a)
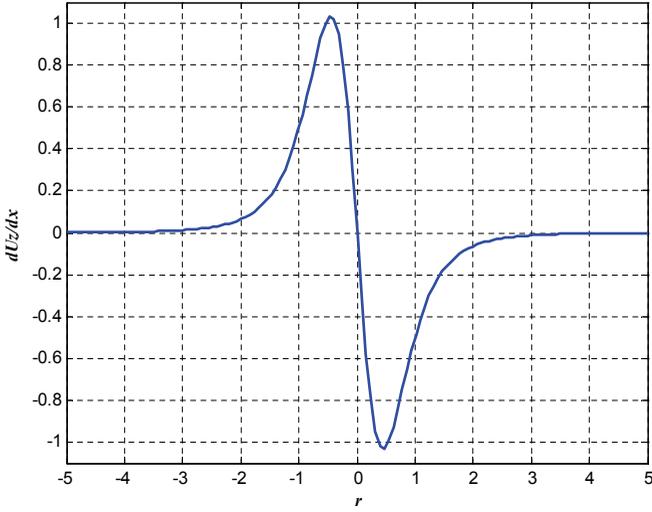
(b)

Figure 2: The velocity and its derivation distribution in the direction of axis.
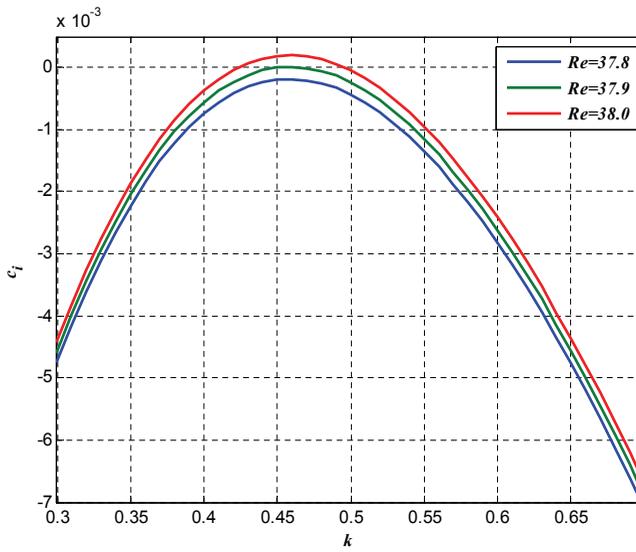
Figure 3: Amplification factor as a function of *Re* and *k*.
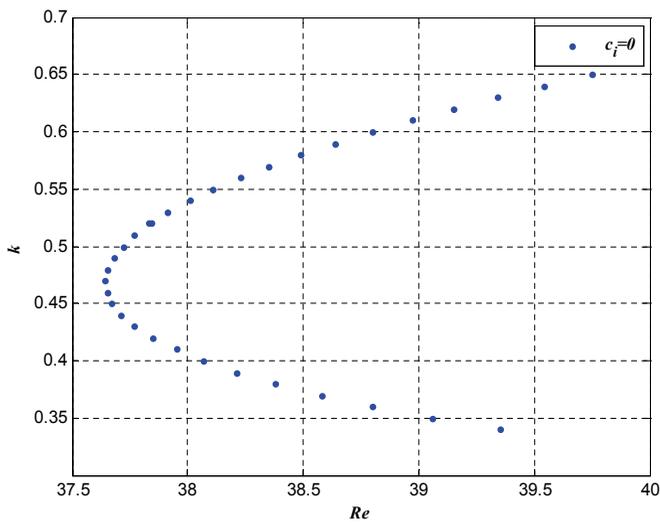
Figure 4: The neutral curve in *k-Re* plane based on numerically computed critical c_i.
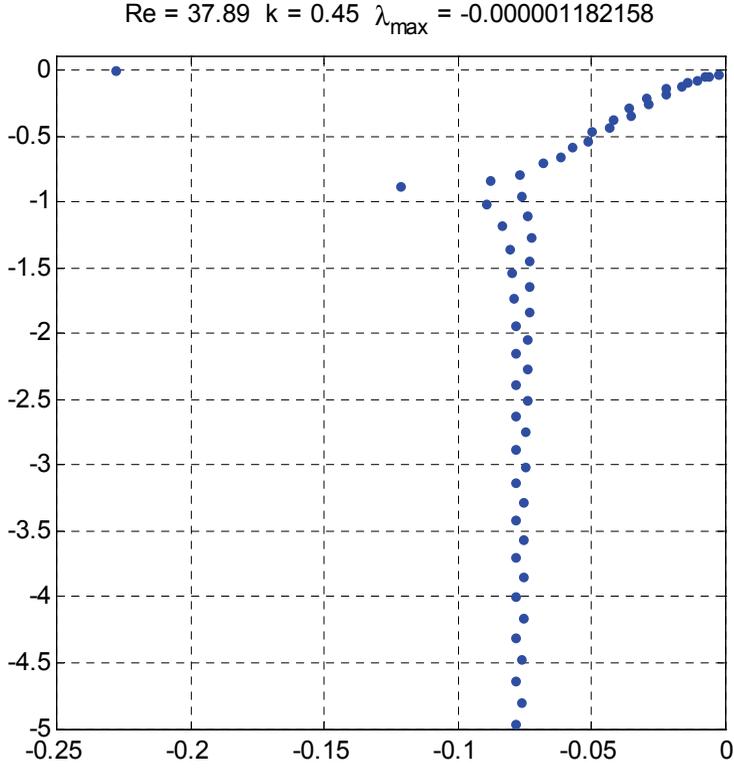
Figure 5: The distribution of eigenvalues for $n = 1$ mode.

Table 1: Comparison of critical Reynolds number for $n = 1$ mode

Reference	*Re*	*k*	λ_r
Morris (1976)	37.64	0.44	0.1
Lessen & Singh (1973)	37.9	0.3989	0.08
Salgado & Sandham (2007)	37.8	0.417	0.09
Kulkarni & Agarwal (2007)	37.68	0.4505	0.104
Xie & Lin (2009a)	37.6829	0.459	0.103
Xie et al. (2009d)	37.64	0.469	0.119
present	37.89	0.45	0.1028
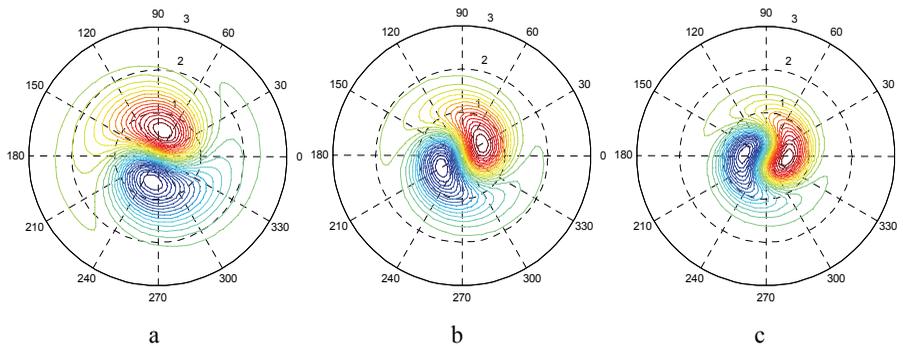
Figure 6: Stream functions for various Reynolds number of the $n = 1, k = 0.45$ mode. The angular orientation is arbitrary. a) $Re=50$; b) $Re=100$; c) $Re=200$.

for unbounded domains, it is necessary to assume the asymptotic behavior of the approximated functions for large r, and the outer boundary condition is truncated at a large enough value in the present study. The numerical simulation was performed by a Fourier-Chebyshev Petrov-Galerkin spectral. The critical Reynolds number is also computed and shown to be in good agreement with those reported in the literature. The method is validated against the results with previous work. And the coherent structure of round jet is also presented.

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