Geometrically Nonlinear Analysis of Anisotropic Composite Plates Resting On Nonlinear Elastic Foundations

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Abstract: Geometrically nonlinear static analysis of an anisotropic thick plate resting on nonlinear two-parameter elastic foundations has been studied. The plate formulation is based on first-order shear deformation theory (FSDT). The governing equation of bending for rectangular orthotropic thick plate is derived by using von Karman equation. The nonlinear static deflections of orthotropic plates on elastic foundation are investigated using the discrete singular convolution method. The effects of foundation, material and geometric parameters of orthotropic plates on nonlinear deflections are investigated.

Keywords: Orthotropic plate, nonlinear analysis, static deflection, discrete singular convolution.

Introduction

In recent years, a great deal of studies has been devoted to linear and nonlinear analyses of plates and shells. The linear and nonlinear static, dynamic and buckling analysis of plates may be either analytical or numerical. The analytical or rigorous approach consists of methods for seeking direct solutions to the governing differential equations of plates. It is well known that the analytical solution of problems can be obtained for only a certain simple cases. Generally, analytical solution cannot be found. Consequently, approximate numerical methods are the only alternative that can be employed (Atluri et al., 2000; Zhu et al., 1998; Zhu et al. 1999; Sladek et al., 2008a; Reutskiy, 2005; Atluri and Zhu, 1998).

For this reason, numerical approaches are widely used in various engineering and sciences problems. Therefore, various methods have been used for numerical solution of mathematical physics and engineering problems (Atluri and Shen, 2005;

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Sladek et al., 2005; Sladek et al. 2006; Atluri et al., 2006; Atluri et al., 2006a). Differential quadrature, meshless methods and discrete singular convolution methods are recently proposed numerical solution technique in engineering and science problems (Sladek et al., 2006a; Xiang et al., 2002; Sladek et al., 2007; Atluri and Zhu, 2000; Atluri and Shen, 2002; Civalek 2005; Atluri et al., 1999; Atluri et al., 2002).

It is generally known that, beam and plates are generally placed on an elastic foundation in many practical applications of civil, mechanical, aerospace, and railroad engineering (Nath et Shukla et al. 2005; al. 1986; Wang et al. 1992; Nath 1982; Dumir 1988; Nath and Kumar 1995;). So, the effect of foundation on the linear and nonlinear static and dynamic response of plates is important (Shen 1999; 2000; 2000a; 200b; Reutskiy, 2007; Shukla and Nath 2000; Sladek and Atluri, 2004; Sladek et al., 2002; Civalek 2004; Sladek et al., 2003; Sori'c et al., 2004; Civalek 2006; Nath and Kumar 1995a; Civalek 2007). In recent years, linear and nonlinear analysis of laminated composite plate and shell structures have found increased applications (Kant et al., 1988; Nath and Shukla, 2001; Kant et al. 1989; Kant et al. 1990; Nath et al. 2006; Nath et al., 1985; Kant et al., 1992; Nath and Alwar 1980). However, relatively few studies can be found in the literature about the nonlinear analysis of anisotropic plates on nonlinear elastic foundation. The method of discrete singular convolution (DSC) is an effective and simple approach for the numerical verification of singular convolutions, which occur commonly in mathematical physics and engineering. The discrete singular convolution method has been extensively used in scientific computations in past ten years (Lai and Xiang, 2009; Wei 2000; Civalek 2008). For more details of the mathematical background and application of the DSC method in solving problems in engineering, the readers may refer to some recently published reference (Wei et al. 2001; Wei et al. 2002).

In this paper, the method of discrete singular convolution is extended to the solution of nonlinear boundary value problems in solid mechanics such as the geometrically nonlinear analysis of anisotropic pates resting on nonlinear elastic foundations.

1 Governing equations

A laminated rectangular plate of dimensions a, b, and h resting on nonlinear elastic foundations, shown in Fig.1 is considered. A Cartesian coordinate Oxy is located in the middle surface of the plate. The displacement field at a point in the plate using the first-order shear deformation theory is given as

$$u(x,y,z) = u_0(x,y) + z\psi_x(x,y),$$

$$v(x,y,z) = v_0(x,y) + z\psi_y(x,y),$$

$$w(x,y,z) = w_0(x,y).$$ (1)

where u, v and w are the displacement components of point (x, y, z), u_0, v_0 and w_0 are the displacement components at a point on the mid-plane of the plate in x, y and z directions respectively, ψ_x and ψ_y are the rotations of the transverse normal about yz- and xz- planes of the plate, respectively.

Based on the von Karman nonlinear theory, which takes into account moderately large deflections and small strains, the strain-displacement relations are written as (Nath et al. 2006)

$$
\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \\ \gamma_{xz} \\ \gamma_{yz} \end{Bmatrix} = \begin{Bmatrix} \frac{\partial u_0}{\partial x} + \frac{1}{2}\left(\frac{\partial w_0}{\partial x}\right)^2 \\ \frac{\partial v_0}{\partial y} + \frac{1}{2}\left(\frac{\partial w_0}{\partial y}\right)^2 \\ \frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x} + \frac{\partial w_0}{\partial x}\frac{\partial w_0}{\partial y} \\ \frac{\partial w_0}{\partial x} + \psi_x \\ \frac{\partial w_0}{\partial y} + \psi_y \end{Bmatrix} + z \begin{Bmatrix} \frac{\partial \psi_x}{\partial x} \\ \frac{\partial \psi_y}{\partial y} \\ \frac{\partial \psi_x}{\partial x} + \frac{\partial \psi_y}{\partial y} \\ 0 \\ 0 \end{Bmatrix}
$$ (2)

For laminated composite plate, the constitutive relationship, the in-plane force and moment resultants can be given by

$$
\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} & B_{11} & B_{12} & B_{16} \\ A_{12} & A_{22} & A_{26} & B_{12} & B_{22} & B_{26} \\ A_{16} & A_{26} & A_{66} & B_{16} & B_{26} & B_{66} \\ B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & D_{16} \\ B_{12} & B_{22} & B_{26} & D_{12} & D_{22} & D_{26} \\ B_{16} & B_{26} & B_{66} & D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{Bmatrix} \frac{\partial u_0}{\partial x} + \frac{1}{2}\left(\frac{\partial w_0}{\partial x}\right)^2 \\ \frac{\partial v_0}{\partial y} + \frac{1}{2}\left(\frac{\partial w_0}{\partial y}\right)^2 \\ \frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x} + \frac{\partial w_0}{\partial x}\frac{\partial w_0}{\partial y} \\ \frac{\partial \psi_x}{\partial x} \\ \frac{\partial \psi_y}{\partial x} \\ \frac{\partial \psi_x}{\partial y} + \frac{\partial \psi_y}{\partial x} \end{Bmatrix}
$$ (3)

The transverse shear force can be expressed as

$$
\begin{Bmatrix} Q_y \\ Q_x \end{Bmatrix} = \begin{bmatrix} A_{44} & A_{45} \\ A_{45} & A_{55} \end{bmatrix} \begin{Bmatrix} \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix}
$$ (4)

where the laminate stiffness coefficients (A_{ij}, B_{ij}, D_{ij}) are in-plane, bending stretching coupling, bending, and thickness shear stiffness, respectively, defined in terms of the reduced stiffness coefficients for the layers $k=1,2,\ldots,n$. These are given as

$$(A_{ij}, B_{ij}, D_{ij}) = \sum_{k=1}^{n} \int_{z_{k-1}}^{z_k} (1, z, z^2)(\bar{Q}_{ij})_k dz \quad (i, j = 1, 2, 6),$$ (5)

$$(A_{ij}) = \sum_{k=1}^{n} k_i k_j \int_{z_{k-1}}^{z_k} (\bar{Q}_{ij})_k dz \quad (i, j = 4, 5).$$ (6)
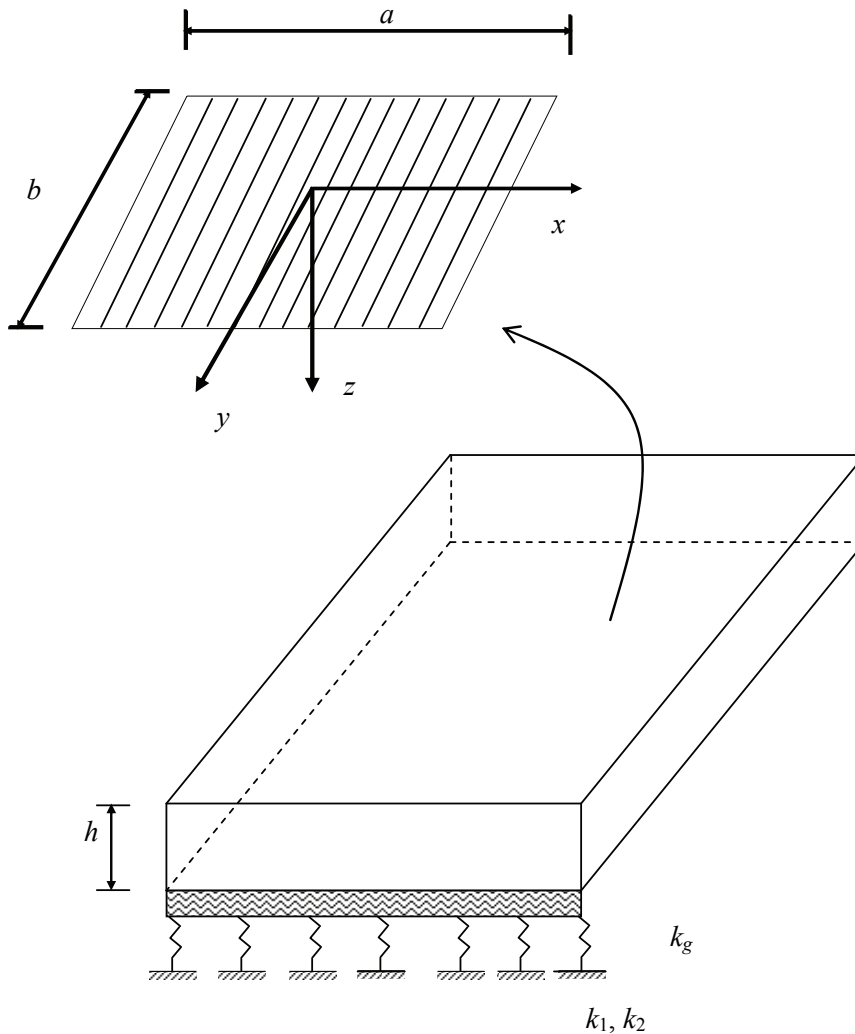
Figure 1: Geometry and coordinate system of the anisotropic rectangular plate on elastic foundation

where $k_i^2 = 5/6$ (i=4,5) are the shear correction factors. The equations of motion for geometrically nonlinear analysis of laminated plates resting on nonlinear elastic foundation are expressed as (Shukla and Nath, 2000)

$$\frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} - P\frac{\partial^2 u}{\partial t^2} - R\frac{\partial^2 \psi_x}{\partial t^2} = 0, \tag{7}$$

$$\frac{\partial N_{xy}}{\partial x} + \frac{\partial N_y}{\partial y} - P\frac{\partial^2 v}{\partial t^2} - R\frac{\partial^2 \psi_y}{\partial t^2} = 0, \tag{8}$$

$$\frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y} + \frac{\partial N_x}{\partial x}\frac{\partial w}{\partial x} + \frac{\partial^2 w}{\partial x^2}N_x + \frac{\partial N_{xy}}{\partial y}\frac{\partial w}{\partial x} + \frac{\partial^2 w}{\partial x\partial y}N_{xy}$$

$$+\frac{\partial N_{xy}}{\partial x}\frac{\partial w}{\partial y} + \frac{\partial^2 w}{\partial x\partial y}N_{xy} + \frac{\partial N_y}{\partial y}\frac{\partial w}{\partial y} + \frac{\partial^2 w}{\partial y^2}N_y + q(x,y,t) - P\frac{\partial^2 w}{\partial t^2} = 0, \tag{9}$$

$$\frac{\partial M_x}{\partial x} + \frac{\partial M_{xy}}{\partial y} - Q_x - R\frac{\partial^2 u}{\partial t^2} - I\frac{\partial^2 \psi_x}{\partial t^2} = 0, \tag{10}$$

$$\frac{\partial M_y}{\partial y} + \frac{\partial M_{xy}}{\partial x} - Q_y - R\frac{\partial^2 v}{\partial t^2} - I\frac{\partial^2 \psi_y}{\partial t^2} = 0, \tag{11}$$

where the coefficients R, P and I are the coupled normal rotary, normal rotary, and rotary inertia, respectively, and defined as

$$(P,R,I) = \int_{-h/2}^{h/2} (1,z,z^2)\rho \, dz, \tag{12}$$

Substituting the Eqs. (2-6) into Eqs. (7-11) and ignoring the time derivation for static analysis, the governing equations of deflection can be given in the non-dimensional form (Civalek, 2004)

$$L_{11}(U) + L_{12}(V) + L_{15}(W) = 0 \tag{13}$$

$$L_{21}(U) + L_{22}(V) + L_{25}(W) = 0 \tag{14}$$

$$L_{31}(U) + L_{32}(V) + L_{35}(W) = 0 \tag{15}$$

$$L_{43}(\Psi_X) + L_{44}(\Psi_Y) + L_{45}(W) = 0 \tag{16}$$

$$L_{52}(V) + L_{53}(\Psi_X) + L_{54}(\Psi_Y) + L_{55}(W) = 0 \tag{17}$$

The differential operators given in the governing equations (13a-13e) are given as (Civalek, 2004)

$$L_{11} = A_{11}\frac{\partial^2}{\partial X^2} + \lambda^2 A_{66}\frac{\partial^2}{\partial Y^2} + 2\lambda A_{16}\frac{\partial^2}{\partial X\partial Y} \tag{18}$$

$$L_{12} = A_{16}\frac{\partial^2}{\partial X^2} + \lambda^2 A_{26}\frac{\partial^2}{\partial Y^2} + \lambda(A_{12}+A_{66})\frac{\partial^2}{\partial X\partial Y} \tag{19}$$

$$L_{15} = 2\alpha_1 \left[A_{11}\frac{\partial^3}{\partial X^3} + \lambda^2 A_{66}\frac{\partial^3}{\partial Y^2\partial X} + 2\lambda A_{16}\frac{\partial^3}{\partial X^2\partial Y} \right]$$

$$+2\alpha_2 \left[A_{16}\frac{\partial^3}{\partial X\partial Y^2} + \lambda^2 A_{26}\frac{\partial^3}{\partial Y^3} + \lambda(A_{12}+A_{66})\frac{\partial^3}{\partial X\partial Y^2} \right] \tag{20}$$

$$L_{21} = A_{16}\frac{\partial^2}{\partial X^2} + \lambda^2 A_{26}\frac{\partial^2}{\partial Y^2} + \lambda(A_{12}+A_{66})\frac{\partial^2}{\partial X\partial Y} \tag{21}$$

$$L_{22} = A_{66}\frac{\partial^2}{\partial X^2} + \lambda^2 A_{22}\frac{\partial^2}{\partial Y^2} + 2\lambda A_{26}\frac{\partial^2}{\partial X\partial Y} \tag{22}$$

$$L_{25} = 2\alpha_1 \left[A_{16}\frac{\partial^3}{\partial X^3} + \lambda^2 A_{26}\frac{\partial^3}{\partial Y^2\partial X} + \lambda(A_{12}+A_{66})\frac{\partial^3}{\partial X^2\partial Y} \right]$$

$$+2\alpha_2 \left[A_{66}\frac{\partial^3}{\partial X^2\partial Y} + \lambda^2 A_{22}\frac{\partial^3}{\partial Y^3} + 2\lambda A_{26}\frac{\partial^3}{\partial X\partial Y^2} \right] \tag{23}$$

$$L_{31} = \frac{\partial}{\partial X}\left[L_{31}^1 \right] + \frac{\partial}{\partial Y}\left[L_{31}^2 \right] \tag{24}$$

$$L_{31}^1 = \frac{2}{\beta}A_{11}\frac{\partial^2 W}{\partial X^2} + 2A_{12}\frac{\lambda^2}{\beta}\frac{\partial^2 W}{\partial Y^2} + 4A_{16}\frac{\lambda}{\beta}\frac{\partial^2 W}{\partial X\partial Y} \tag{25}$$

$$L_{31}^2 = \left[\lambda A_{16}\frac{\partial^2 W}{\partial X^2} + 4A_{66}\frac{\lambda^2}{\beta}\frac{\partial^2 W}{\partial X\partial Y} \right] \tag{26}$$

$$L_{32} = \frac{\partial}{\partial x}\left[L_{32}^1 \right] + \frac{\partial}{\partial y}\left[L_{32}^2 \right] \tag{27}$$

$$L_{32}^1 = A_{16}\frac{\partial^2 W}{\partial X^2} + A_{26}\frac{\partial^2 W}{\partial Y^2} + A_{66}\frac{\partial^2 W}{\partial X\partial Y} \tag{28}$$

$$L_{32}^2 = \lambda A_{12}\frac{\partial^2 W}{\partial X^2} + \lambda A_{22}\frac{\partial^2 W}{\partial Y^2} + \lambda A_{26}\frac{\partial^2 W}{\partial X\partial Y} \tag{29}$$

$$L_{35} = k^2 A_{55}\frac{\partial^2}{\partial X^2} + \lambda^2 k^2 A_{44}\frac{\partial^2}{\partial Y^2} + 2\lambda k^2 A_{45}\frac{\partial^2}{\partial X\partial Y}$$

$$+\frac{\partial^2}{\partial X^2}\left[\frac{2}{\beta^2}A_{11}(\frac{\partial}{\partial X})^2 + \frac{2\lambda^2}{\beta^2}A_{12}(\frac{\partial}{\partial Y})^2 \right]$$

$$+\frac{\partial^2}{\partial Y^2}\left[\lambda^2 A_{12}(\frac{\partial}{\partial X})^2 + \lambda^2 A_{22}(\frac{\partial}{\partial Y})^2 \right]$$

$$+\frac{\partial^2}{\partial X\partial Y}\left[2\lambda A_{26}(\frac{\partial}{\partial X})^2 + 2\lambda A_{26}(\frac{\partial}{\partial Y})^2 + \frac{4\lambda}{\beta^2}A_{16}\frac{\partial^2}{\partial X^2} + \lambda^2 A_{26}\frac{\partial^2}{\partial Y^2} + 2\lambda A_{66}\frac{\partial^2}{\partial X\partial Y} \right]$$

$$+qa^2/4h - A_{22}\left[K_1 W - K_2 W^3 + G(\frac{\partial^2}{\partial X^2} + \frac{\partial^2}{\partial Y^2}) \right] \tag{30}$$

$$L_{43} = \frac{\partial^2}{\partial X^2} + \lambda^2 D_{66}\frac{\partial^2}{\partial Y^2} + \frac{(\beta kh)^2}{4}A_{55}\Psi_x \tag{31}$$

$$L_{44} = \lambda\left(D_{12} + D_{66}\right)\frac{\partial^2}{\partial X \partial Y} + \frac{(\beta kh)^2}{4}A_{45}\Psi_y \tag{32}$$

$$L_{45} = \frac{(\beta kh)^2}{2}A_{55}\frac{\partial}{\partial X} + \frac{\lambda \beta k^2 h^2}{2}A_{45}\frac{\partial}{\partial Y} \tag{33}$$

$$L_{52} = D_{66}h\frac{\partial^2}{\partial X^2} \tag{34}$$

$$L_{53} = \lambda\left(D_{12} + D_{66}\right)\frac{\partial^2}{\partial X \partial Y} + \frac{(\beta kh)^2}{4}A_{45}\Psi_x \tag{35}$$

$$L_{54} = D_{66}\frac{\partial^2}{\partial X^2} + \lambda^2\frac{\partial^2}{\partial Y^2} + \frac{(\beta kh)^2}{2}A_{44}\Psi_y \tag{36}$$

$$L_{55} = -\frac{\beta(kh)^2}{2}A_{25}\frac{\partial}{\partial X} - \frac{\lambda \beta(kh)^2}{2}D_{22}\frac{\partial}{\partial Y} \tag{37}$$

The following non-dimensional quantities are used.

$$U = u/h, \quad V = v/h, \quad W = w/h, \quad \Psi_x = \psi_x, \quad \Psi_y = \psi_y,$$

$$X = 2x/a, \quad Y = 2y/b, \quad \lambda = a/b, \quad \beta = a/h, \quad \alpha_1 = h/a,$$

$$\alpha_2 = h/b, \quad K_1 = k_1 a^4/D_{11}, \quad K_2 = k_2 a^4 h^2/D_{11}, \quad G = k_p a^2/D_{11} \tag{38}$$

2 Discrete singular convolution (DSC)

Discrete singular convolution (DSC) method is a relatively new numerical technique in applied mechanics. The method of discrete singular convolution (DSC) was proposed to solve linear and nonlinear differential equations by Wei (1999), Wei et al. (1998) and later it was introduced to solid and fluid mechanics by Wei (2001a), Wei et al. (2002a), Zhao et al.(2002; 2002a; 2005), Xiang et al. (2005), Seçkin and Sarıgül (2009, 2009a) and Civalek (2008, 2008a, 2009).

In the context of distribution theory, a singular convolution can be defined by (Wei et al., 2002)

$$F(t) = (T * \eta)(t) = \int\limits_{-\infty}^{\infty} T(t - x)\eta(x)dx \tag{39}$$

Where T is a kind of singular kernel such as Hilbert, Abel and delta type, and $\eta(t)$ is an element of the space of the given test functions. In the present approach, only singular kernels of delta type are chosen. This type of kernel is defined by (Zhao and Wei, 2002)

$$T(x) = \delta^{(r)}(x); \quad (r = 0, 1, 2, ...,). \tag{40}$$

where subscript r denotes the rth-order derivative of distribution with respect to parameter x.

In order to illustrate the DSC approximation, consider a function $F(x)$. In the method of DSC, numerical approximations of a function and its derivatives can be treated as convolutions with some kernels. According to DSC method, the rth derivative of a function $F(x)$ can be approximated as (Zhao and Wei, 2003)

$$F^{(r)}(x) \approx \sum_{k=-M}^{M} \delta_{\Delta,\sigma}^{(r)}(x_i - x_k) f(x_k); \quad (r = 0, 1, 2, ...,). \tag{41}$$

where Δ is the grid spacing, x_k are the set of discrete grid points which are centered around x, and $2M + 1$ is the effective kernel, or computational bandwidth. It is also known, the regularized Shannon kernel (RSK) delivers very small truncation errors when it use the above convolution algorithm. The regularized Shannon kernel (RSK) is given by (Wei et al., 2001)

$$\delta_{\Delta,\sigma}(x - x_k) = \frac{\sin[(\pi/\Delta)(x - x_k)]}{(\pi/\Delta)(x - x_k)} \exp\left[-\frac{(x - x_k)^2}{2\sigma^2}\right]; \quad \sigma > 0 \tag{42}$$

The researchers is generally used the regularized delta Shannon kernel by this time. The required derivatives of the DSC kernels can be easily obtained using the below formulation (Wei, 2001)

$$\delta_{\Delta,\sigma}^{(r)}(x - x_j) = \frac{d^r}{dx^r} \left[\delta_{\Delta,\sigma}(x - x_j)\right] \Big|_{x=x_i}, \tag{43}$$

The second-order derivative at $x \neq x_k$, for example can be given as follows (Wei, 2001)

$$
\begin{aligned}
\delta_{\sigma,\Delta}^{(2)}(x - x_k) = & -\frac{(\pi/\Delta)\sin(\pi/\Delta)(x - x_k)}{(x - x_k)} \exp[-(x - x_k)^2/2\sigma^2] \\
& -2\frac{\cos(\pi/\Delta)(x - x_k)}{(x - x_k)^2} \exp[-(x - x_k)^2/2\sigma^2)] \\
& -2\frac{\cos(\pi/\Delta)(x - x_k)}{\sigma^2} \exp[-(x - x_k)^2/2\sigma^2] \\
& +2\frac{\sin(\pi/\Delta)(x - x_k)}{\pi(x - x_k)^3/\Delta} \exp[-(x - x_k)^2/2\sigma^2)] \\
& +\frac{\sin(\pi/\Delta)(x - x_k)}{\pi(x - x_k)\sigma^2/\Delta} \exp[-(x - x_k)^2/2\sigma^2] \\
& +\frac{\sin(\pi/\Delta)(x - x_k)}{\pi\sigma^4/\Delta}(x - x_k) \exp[-(x - x_k)^2/2\sigma^2)]
\end{aligned}
\tag{44}
$$

For $x = x_k$, this derivative is given by

$$\delta_{\sigma,\Delta}^{(2)}(0) = -\frac{3 + (\pi^2/\Delta^2)\sigma^2}{3\sigma^2} = -\frac{1}{\sigma^2} - \frac{\pi^2}{3\Delta^2} \tag{45}$$

3 Method of Solution

Using DSC method to discretize the spatial derivatives in Eqs. (13), the derivatives of the displacement components can be given by (Baltacıo?lu, 2010)

$$D_{11}(U) + D_{12}(V) + D_{15}(W) = 0 \tag{46}$$

$$D_{21}(U) + D_{22}(V) + D_{25}(W) = 0 \tag{47}$$

$$D_{31}(U) + D_{32}(V) + D_{35}(W) = 0 \tag{48}$$

$$D_{41}(U) + D_{42}(V) + D_{45}(W) = 0 \tag{49}$$

$$D_{52}(V) + D_{53}(\Psi_X) + D_{54}(\Psi_Y) + D_{55}(W) = 0 \tag{50}$$

The differential operators given in the governing equations (42) are given as (Baltacıoğlu, 2010)

$$D_{11} = A_{11}\Re_X^2 + \lambda^2 A_{66}\Re_Y^2 + 2\lambda A_{16}\Re_{XY}^2 \tag{51}$$

$$D_{12} = A_{16}\Re_X^2 + \lambda^2 A_{26}\Re_Y^2 + \lambda(A_{12} + A_{66})\Re_{XY}^2 \tag{52}$$

$$D_{15} = 2\alpha_1 \left[A_{11}\Re_X^3 + \lambda^2 A_{66}\Re_Y^2\Re_X^1 + 2\lambda A_{16}\Re_X^2\Re_Y^1 \right]$$
$$+ 2\alpha_2 \left[A_{16}\Re_X^1\Re_Y^2 + \lambda^2 A_{66}\Re_Y^3 + \lambda(A_{12} + A_{66})\Re_X^1\Re_Y^2 \right] \tag{53}$$

$$D_{21} = A_{16}\Re_X^2 + \lambda^2 A_{26}\Re_Y^2 + \lambda(A_{12} + A_{66})\Re_{XY}^2 \tag{54}$$

$$D_{22} = A_{66}\Re_X^2 + \lambda^2 A_{22}\Re_Y^2 + 2\lambda A_{26}\Re_{XY}^2 \tag{55}$$

$$D_{25} = 2\alpha_1 \left[A_{16}\Re_X^3 + \lambda^2 A_{26}\Re_Y^2\Re_X^1 + \lambda(A_{12} + A_{66})\Re_X^2\Re_Y^1 \right]$$
$$+ 2\alpha_2 \left[A_{66}\Re_X^2\Re_Y^1 + \lambda^2 A_{22}\Re_Y^3 + 2\lambda A_{26}\Re_X^1\Re_Y^2 \right] \tag{56}$$

$$D_{31} = \frac{\partial}{\partial X}\left[D_{31}^1 \right] + \frac{\partial}{\partial Y}\left[D_{31}^2 \right] \tag{57}$$

$$D_{31}^1 = \frac{2}{\beta}A_{11}\Re_X^2 + 2A_{12}\frac{\lambda^2}{\beta}\Re_Y^2 + 4\frac{\lambda}{\beta}A_{16}\Re_{XY}^2 \tag{58}$$

$$D_{31}^2 = \lambda A_{16}\Re_X^2 + 4A_{66}\frac{\lambda^2}{\beta}\Re_{XY}^2 \tag{59}$$

$$D_{32} = \frac{\partial}{\partial x}\left[D_{32}^1\right] + \frac{\partial}{\partial y}\left[D_{32}^2\right] \tag{60}$$

$$D_{32}^1 = A_{16}\mathfrak{R}_X^2 + A_{26}\mathfrak{R}_Y^2 + A_{66}\mathfrak{R}_{XY}^2 \tag{61}$$

$$D_{32}^1 = \lambda A_{12}\mathfrak{R}_X^2 + \lambda A_{22}\mathfrak{R}_Y^2 + \lambda A_{26}\mathfrak{R}_{XY}^2 \tag{62}$$

$$
\begin{aligned}
D_{35} =& k^2 A_{55}\mathfrak{R}_X^2 + \lambda^2 k^2 A_{44}\mathfrak{R}_Y^2 + 2\lambda k^2 A_{45}\mathfrak{R}_{XY}^2 \\
&+ \frac{\partial^2}{\partial X^2}\left[\frac{2}{\beta^2}A_{11}(\mathfrak{R}_X)^2 + \frac{2}{\beta^2}\lambda^2 A_{12}(\mathfrak{R}_Y)^2\right] \\
&+ \frac{\partial^2}{\partial Y^2}\left[\lambda^2 A_{12}(\mathfrak{R}_X)^2 + \lambda^2 A_{22}(\mathfrak{R}_Y)^2\right] \\
&+ \frac{\partial^2}{\partial X \partial Y}\left[2\lambda A_{26}(\mathfrak{R}_X)^2 + 2\lambda A_{26}(\mathfrak{R}_Y)^2 + \frac{4\lambda}{\beta^2}A_{16}\mathfrak{R}_X^2 + \lambda^2 A_{26}\mathfrak{R}_Y^2 + 2\lambda A_{66}\mathfrak{R}_{XY}^2\right] \\
&+ qa^2/4h - A_{22}\left[K_1 W - K_2 W^3 + G(\mathfrak{R}_X^2 + \mathfrak{R}_Y^2)\right]
\end{aligned}
\tag{63}
$$

$$D_{43} = \mathfrak{R}_X^2 + \lambda^2 D_{66}\mathfrak{R}_Y^2 + \frac{(\beta kh)^2}{4}A_{55}\Psi_X \tag{64}$$

$$D_{44} = \lambda(D_{12} + D_{66})\mathfrak{R}_{XY}^2 + \frac{(\beta kh)^2}{4}A_{45}\Psi_Y \tag{65}$$

$$D_{45} = \frac{(\beta kh)^2}{4}A_{55}\mathfrak{R}_X^1 + \frac{\lambda\beta(kh)^2}{2}A_{45}\mathfrak{R}_Y^1 \tag{66}$$

$$D_{52} = D_{66}h\mathfrak{R}_X^2 \tag{67}$$

$$D_{53} = \frac{(\beta kh)^2}{4}A_{45}\Psi_X \tag{68}$$

$$D_{54} = D_{66}\mathfrak{R}_X^2 + \lambda^2\mathfrak{R}_Y^2 + \frac{(\beta kh)^2}{2}A_{44}\Psi_Y \tag{69}$$

$$D_{55} = -\frac{\beta(kh)^2}{2}A_{25}\mathfrak{R}_X^1 - \frac{\lambda\beta(kh)^2}{2}D_{22}\mathfrak{R}_Y^1 \tag{70}$$

The resulting equation can be given as

$$\left\{\left[{}^L K\right] + \left[{}^{NL} K\right]\right\}\{X\} = \{F\} \tag{71}$$

Where the ^{L}K and ^{NL}K are the linear and nonlinear operators, respectively and are defined as

$$^{L}K = \begin{bmatrix} K_{11} & K_{12} & K_{13} & K_{14} & K_{15} \\ K_{21} & K_{22} & K_{23} & K_{24} & K_{25} \\ 0 & 0 & 0 & 0 & 0 \\ K_{41} & K_{42} & K_{43} & K_{44} & K_{45} \\ K_{51} & K_{52} & K_{53} & K_{54} & K_{55} \end{bmatrix} \tag{72}$$

$$^{NL}K = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ K_{31} & K_{32} & K_{33} & K_{34} & K_{35} \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{73}$$

The displacement and load vector are given by

$$\{X\} = \{U \quad V \quad \Psi_X \quad \Psi_Y \quad W\}^{T} \tag{74}$$

$$\{F\} = \{0 \quad 0 \quad Q \quad 0 \quad 0\}^{T} \tag{75}$$

and the elements of the DSC operators are defined below

$$\Re_X^n(\ ) = \frac{\partial^{(n)}(\ )}{\partial X^{(n)}} = \sum_{k=-M}^{M} \delta_{\Delta,\sigma}^{(n)}(k\Delta X)(\ )_{i+k,j} \tag{76}$$

$$\Re_Y^n(\ ) = \frac{\partial^{(n)}(\ )}{\partial Y^{(n)}} = \sum_{k=-M}^{M} \delta_{\Delta,\sigma}^{(n)}(k\Delta Y)(\ )_{i,j+k} \tag{77}$$

$$\Re_X^1 \Re_Y^{(n-1)}(\ ) = \frac{\partial^{(n)}(\ )}{\partial X \partial Y^{(n-1)}} = \sum_{k=-M}^{M} \delta_{\Delta,\sigma}^{(1)}(k\Delta X)(\ )_{i+k,j} \sum_{k=-M}^{M} \delta_{\Delta,\sigma}^{(n-1)}(k\Delta Y)(\ )_{i,k+j} \tag{78}$$

$$\Re_X^{(n-1)} \Re_Y^1(\ ) = \frac{\partial^{(n)}(\ )}{\partial X^{(n-1)} \partial Y} = \sum_{k=-M}^{M} \delta_{\Delta,\sigma}^{(n-1)}(k\Delta X)(\ )_{i+k,j} \sum_{k=-M}^{M} \delta_{\Delta,\sigma}^{(1)}(k\Delta Y)(\ )_{i,k+j} \tag{79}$$

The resulting nonlinear equation has been solved using the Newton-Raphson method (Civalek, 2004; Civalek, 2006; Baltacıoğlu, 2010). The procedure is based on an incremental iterative method.

4   Numerical Results and Discussions

In this section, several examples of the linear and nonlinear analyses of anisotropic plates are provided to demonstrate the validity and accuracy of the proposed method. In the following solutions, four sets of orthotropic materials, unless mentioned otherwise, are considered:

Material-I (Glass-Epoxy)

$$E_y/E_x = 3, \quad G_{xy}/E_x = 0.5, \quad \upsilon_y = 0.25$$

Material-II (Boron-Epoxy)

$$E_y/E_x = 10, \quad G_{xy}/E_x = 0.333, \quad \upsilon_y = 0.22$$

Material-III (Graphite- Epoxy)

$$E_y/E_x = 40, \quad G_{xy}/E_x = 0.60, \quad \upsilon_y = 0.25$$

Material-IV (Isotropic)

$$E_y/E_x = 1, \quad G_{xy}/E_x = 0.385, \quad \upsilon_y = 0.3$$

To test the accuracy of the proposed DSC approach, some convergence studies have been made. Table 1 presents the numerical results obtained by DSC method using different grid points for SSSS orthotropic plates. The results are compared with those exact (Timoshenko, Woinowsky-Krieger, 1959) values and results given by Tsay and Reddy (1978). It is observed from the comparison that for all the results presented in Table 1, the error is within 0.5% using 13 grid points. Further comparison has been made and presented in Table 2 for isotropic plate on two-parameter elastic foundations subjected to a uniformly distributed load. Excellent agreement is achieved between the present results and the literature results obtained by boundary element method (Wang et al., 1992) and differential cubature method (Teo and Liew, 2002). The last comparison study is related to central deflection of CCCC anisotropic plate on two-parameter elastic foundations for two different materials (Table 3). The present DSC solutions are compared with the solutions of the Kirchhoff thin plate theory given by Dumir and Bhaskar (1988). It can be seen that the results are reasonable. It is also noted that the results given by Dumir and Bhaskar (1988) is based on the thin plate theory. Thus, 15 grid points are used in the following computations.

Nonlinear central deflection of SSSS anisotropic plate on two-parameter elastic foundations is listed in Table 4 for different material and foundation parameters.

Table 1: Comparisons of linear deflection $W(H/QL^4)$ of SSSS orthotropic plate subjected to a uniformly distributed load

Material	Tsay and Reddy (1978)	Timoshenko, Woinowsky-Krieger (1959)	Present DSC results			
			N=11	N=13	N=15	N=17
I	0.003089	0.003087	0.003095	0.003094	0.003093	0.003093
III	0.000920	0.000920	0.000932	0.000930	0.000928	0.000928

Table 2: Comparisons of linear deflection $W(D/qa^4)$ of SSSS isotropic plate on two-parameter elastic foundations subjected to a uniformly distributed load(G=20; K1=200)

h/a	BEM [48]	DQ [49]	Present DSC results			
			N=11	N=13	N=15	N=17
0.1	0.001582	0.001587	0.001632	0.001632	0.001625	0.001625
0.2	0.001637	0.001642	0.001650	0.001649	0.001647	0.001647

The effect of the increase in Winkler and Pasternak foundation parameters is to decrease the nonlinear deflections of plate. Nonlinear central deflection of Glass-Epoxy and Boron-Epoxy square plates under uniformly distributed load are calculated and listed in Tables 5-6. Two different boundary conditions are considered. Results are presented for different Winkler, Pasternak and nonlinear foundation parameters.

The effects of aspect ratio of anisotropic plate on the nonlinear deflections are studied and results are depicted in Figs. 2-3.
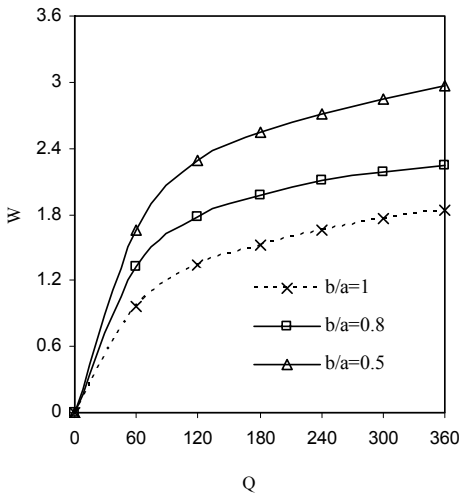

Figure 2: Variation of non-linear central deflection of SSSS Graphite-Epoxy plate under uniformly distributed load (a/h=100)
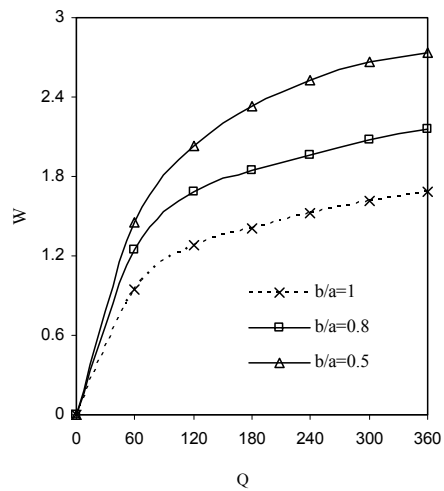
Figure 3: Variation of non-linear central deflection of SSSS Graphite-Epoxy plate under sinusoidal load (a/h=100)

Nonlinear central deflection of SSSS anisotropic plate under uniformly distributed and sinusoidal load for different aspect ratio (a/h=100; Graphite-Epoxy) are given in these figures. It can be seen from these figures that deflections increase with increasing value of aspect ratio. It is concluded from these figures that aspect ratio of plate exhibit a significant effect on the nonlinear static response of plate. Variations of central deflection with three different foundation parameters for different material properties are investigated. The results are presented in Figs. 4-6 for two different boundary conditions. Variation of central deflection with Winkler parameters for different material properties (SCSC; a/h=100; G=0; K2=0) is depicted in Fig. 4. Similarly, variation of central deflection with Pasternak parameters for different material properties (SCSC; a/h=100; G=0; K2=0) is depicted in Fig. 5. The

Table 3: Comparisons of central deflection $W(E_x h^4 / qa^4)$ of CCCC anisotropic plate on two-parameter elastic foundations (K1=50; G=50)

Material	Dumir and Bhaskar* [14]	Present DSC results				
		N=11	N=13	N=15	N=17	
Glass-Epoxy	0.03689	0.03703	0.03703	0.03702	0.03702	
Boron-Epoxy	0.02597	0.02715	0.02714	0.02711	0.02711	

* Thin plate

Table 4: Nonlinear central deflection $W(E_x h^4 / qa^4)$ of SSSS anisotropic plate on two-parameter elastic foundations

Material	Foundation parameters			
	K1=0; G=0	K1=40; G=0	K1=40; G=20	K1=40; G=40
Glass-Epoxy	0.4951	0.2475	0.0853	0.0621
Boron-Epoxy	0.2538	0.1838	0.0743	0.0593
Graphite-Epoxy	0.0693	0.0583	0.0425	0.0401

Table 5: Nonlinear central deflection $W(E_x h^4/qa^4)$ of Glass-Epoxy square plates under uniformly distributed load

Foundation parameters			SSSS	CCCC
K1	G	K2		
0	0	0	0.4933	0.1396
40	0	0	0.3028	0.1092
100	10	0	0.1275	0.0823
40	30	0	0.0893	0.0745
40	30	10	0.0603	0.0488
50	50	20	0.0397	0.0297

Table 6: Nonlinear central deflection $W(E_x h^4/qa^4)$ of Boron-Epoxy square plates under uniformly distributed load

Foundation parameters			SSSS	CCCC
K1	G	K2		
0	0	0	0.2468	0.0548
40	0	0	0.1681	0.0473
100	10	0	0.1026	0.0391
40	30	0	0.0855	0.0342
40	30	10	0.0546	0.0305
50	50	20	0.0387	0.0243

effect of the increase in Winkler and Pasternak foundation parameters is to decrease the nonlinear deflections of plate. Maximum response occurs for Glass-Epoxy plate and the minimum for Graphite-Epoxy plate.

The effect of nonlinear parameter of foundation on mode nonlinear deflections has also been investigated and results presented in Fig. 6. In general, the deflections are decreased with increasing value nonlinear parameter of foundation. Fig. 7 contains plots of nonlinear deflection versus applied load for different Pasternak foundation parameters of CCCC Glass-Epoxy plate under uniformly distributed load. It shown clearly shown that the nonlinear response of deflections increases with increase in load parameter.

5   Conclusions

A regularized Shannon delta kernel based discrete singular convolution method is adopted for the nonlinear analysis of anisotropic plate resting on nonlinear elastic
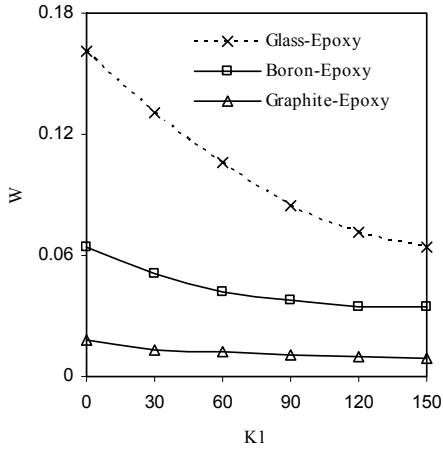
Figure 4: Variation of central deflection with Winkler parameters for different material properties (SCSC; a/h=100; G=0; K2=0)
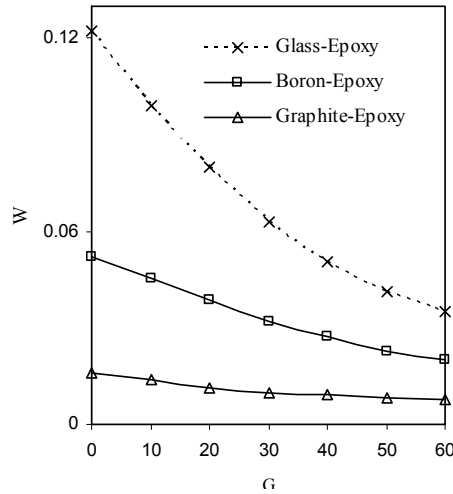

Figure 5: Variation of central deflection with Pasternak parameters for different material properties (SCSC; a/h=100; K1=40; K2=0)
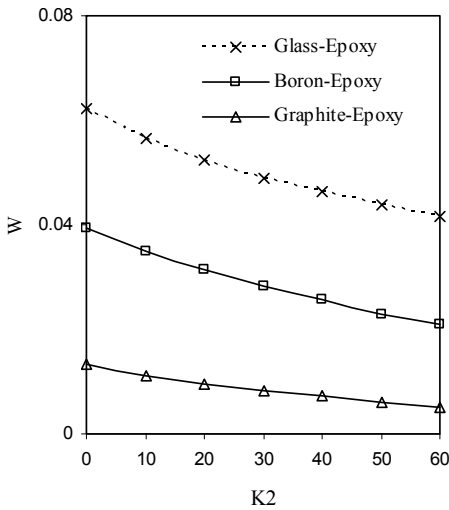

Figure 6: Variation of central deflection with nonlinear foundation parameters for different material properties (SSSS; a/h=100; K1=40; G=18)

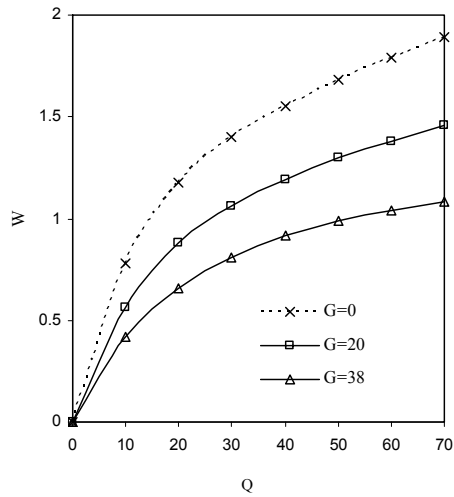Figure 7: Variation of non-linear central deflection of CCCC Glass-Epoxy plate under uniformly distributed load (a/h=100; K1=40)

foundations. The applicability of the DSC method is demonstrated from the linear and nonlinear solution of plates on elastic foundations. The effects of some geometric and material properties of plates and parameters of foundations on static response of plates are investigated. The investigation of the nonlinear dynamic analysis of anisotropic plates on nonlinear elastic foundations is deferred as future work by the method of discrete singular convolution and differential quadrature methods.

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