Copyright © 2010 Tech Science Press CMES, vol.69, no.1, pp.1-17, 2010

Self-Adaptive Differential Evolution Based on the Concept of Population Diversity Applied to Simultaneous Estimation of Anisotropic Scattering Phase Function, Albedo and Optical Thickness

F. S. Lobato[1], **V. Steffen Jr**[2] **and A. J. Silva Neto**[3]

Abstract: Differential Evolution Algorithm (DE) has shown to be a powerful evolutionary algorithm for global optimization in a variety of real world problems. DE differs from other evolutionary algorithms in the mutation and recombination phases. Unlike some other meta-heuristic techniques such as genetic algorithms and evolutionary strategies, where perturbation occurs in accordance with a random quantity, DE uses weighted differences between solution vectors to perturb the population. Although the efficiency of DE algorithm has been proven in the literature, studies indicate that the efficiency of the DE methods is sensitive to its control parameters (perturbation rate and crossover rate) and there is not any guarantee that premature convergence will be avoided. To overcome this problem, the present work proposes an Self-Adaptive Differential Evolution (SADE) as based on the concept of population diversity aiming at dynamically updating the control parameters. The methodology proposed is applied to the simultaneous estimation of the radiation phase function of anisotropic scattering, albedo and optical thickness in an inverse radiative transfer problem. The results show that the procedure represents a promising alternative for the type of problem presented above.

Keywords: Adaptive Differential Evolution, Population Diversity, Inverse Problems, Radiative Transfer.

1 Introduction

Nowadays, the application of numerical optimization techniques for parameter identification has increased significantly due to the difficulty in building theoretical

[1] School of Chemical Engineering, Universidade Federal de Uberlândia (fslobato@ufu.br)
[2] School of Mechanical Engineering, Universidade Federal de Uberlândia (vsteffen@mecanica.ufu.br)
[3] Department of Mechanical Engineering and Energy, Universidade do Estado do Rio de Janeiro, (ajsneto@iprj.uerj.br)

models that are able to represent physical phenomena under real operating conditions. Basically, the estimation problem, also known as inverse problem, consists of minimizing the difference between experimental and calculated values. Traditionally, three research lines are proposed for the solution of parameter identification problems by using optimization techniques: the Deterministic, the Non-Deterministic and the Hybrid Approach [Wang, Su and Jang (2001); Silva Neto and Soeiro (2002); Silva Neto and Soeiro (2003a); Silva Neto and Silva Neto (2003b); Chang, Liu and Chang (2005); Liu (2006); Chalhoub, Campos Velho and Silva Neto (2007), Harris, Mustata, Elliott, Ingham and Lesnic (2008); Yeih and Liu (2009); Lobato, Steffen Jr and Silva Neto (2010)].

In this context, the inverse analysis of radiative transfer in participating media has numerous practical applications, such as the one-dimensional plane-parallel [Silva Neto and Özişik (1995); Acevedo, Roberty and Silva Neto (2004); Lobato, Steffen Jr and Silva Neto (2008); Lobato, Steffen Jr and Silva Neto (2009)] and two-dimensional media [Carita Montero, Roberty and Silva Neto (2001); Carita Montero, Roberty and Silva Neto (2004)], and radiative transfer in composite layer media [Siegel and Spuckler (1993); Wang, Cheng and Tan (2001)], which are devoted to applications in scientific and technological areas that are related to environmental sciences [Hanan (2001)], parameter estimation [Sousa, Soeiro, Silva Neto and Ramos (2007)], and tomography [Kim and Charette (2007)].

The main difficulty found in the so-called non-deterministic approach is the high number of objective function evaluations needed to solve optimization problems. Besides, in spite of the performance and the number of applications encompassed when fixed parameters are used by the algorithm, there is no guarantee that premature convergence will be avoided [Coelho and Mariani (2006)]. In addition, the DE algorithm is sensitive to control parameters [Storn, Price and Lampinen (2005); Gämperle, Müller and Koumoutsakos (2002)] and it is highly problem dependent [Zaharie (2002); Qin and Suganthan (2005); Brest et al (2006)], thus claiming for ad-hoc configurations. According to Nobakhti and Wang (2006), because of the special mutation mechanism used in DE, if for any reason (such as an incorrect choice of the perturbation rate - F) the DE population looses diversity, then the search will completely stop as mutation becomes zero. To overcome this difficulty, various methodologies have been proposed. Zaharie (2003) proposes a feedback update rule for F that is designed to maintain the diversity of the population at a given level. According the authors, this procedure is able to avoid premature convergence. Recently, chaotic search models have been used for the adaptation of parameters in non-deterministic approach due to its ability in escaping premature convergence [Coelho and Mariani (2006)]. In Tavazoei and Haeri (2007), a study about the performance of different chaotic search models when they are incorporated to classic optimization is addressed. Coelho and Mariani (2007) have used the Ant Colony algorithm with logistic maps in engineering problems. Another research line to make the number of objective function evaluations to decrease is using special strategies to update the population size. In this context, Lobato and Steffen Jr (2009) defined the convergence rate concept based on the homogeneity of the population and applied this concept to solve optimal control problems. In the present contribution the Self-Adaptive Differential Evolution algorithm (SADE) is used for the solution of the inverse radiative transfer problem related to the simultaneous estimation of the optical thickness, single scattering albedo, diffuse reflectivies and anisotropic scattering phase function of a one-dimensional homogeneous participating media. The results obtained with this methodology are compared with the standard Differential Evolution (DE) algorithm with fixed parameters. This work is organized as follows. The mathematical formulation of the direct and inverse problems is presented in Sections 2 and 3, respectively. A review of the Differential Evolution method and the strategy for the dynamic adapting of parameters is presented in Section 4. The results and discussion are described in Section 5. Finally, the conclusions and suggestions for future work conclude the paper.

2 Mathematical Formulation of the Direct Problem

A plane-parallel, gray, anisotropically scattering slab of optical thickness τ_o, with diffusely reflecting boundaries is subjected to external isotropic irradiation at both boundaries, τ=0 and τ=τ_o as shown in Fig. 1.

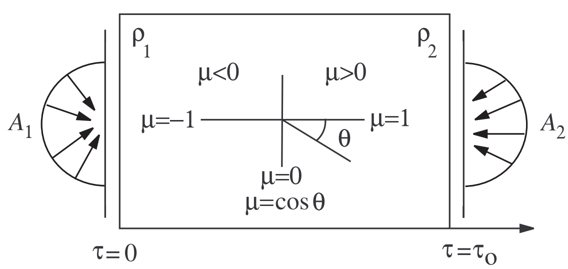


Figure 1: Schematic representation of the one-dimensional participating medium.

It is assumed that the emission of radiation by the medium due to its temperature is negligible in comparison to the intensity of the external incoming radiation. Also the effects of possible differences on the refractive indices of the participating medium and surrounding environment are not taken into account. The mathematical formulation of the direct radiative transfer problem is given by [Özişik (1973)];

$$\mu \frac{\partial I(\tau,\mu)}{\partial \tau} + I(\tau,\mu) = \frac{\omega}{2}\int_{-1}^{1} p(\mu,\mu') I(\tau,\mu')\, d\mu', 0 < \tau < \tau_o,\ -1 \le \mu \le 1 \quad (1)$$

$$I(0,\mu) = A_1 + 2\rho_1 \int_{0}^{1} I(0,-\mu')\,\mu' d\mu',\ \mu > 0 \quad (2)$$

$$I(\tau_o,\mu) = A_2 + 2\rho_2 \int_{0}^{1} I(\tau_o,\mu')\,\mu' d\mu',\ \mu < 0 \quad (3)$$

where $I(\tau,\mu)$ is the dimensionless radiation intensity, τ is the optical variable, μ is the direction cosine of the radiation beam with the positive τ axis, ω is the single scattering albedo, ρ_1 and ρ_2 are the diffuse reflectivities at boundaries $\tau = 0$ and $\tau = \tau_o$, respectively, A_1 and A_2 are the strength of the external irradiation at these boundaries, and $p(\mu,\mu')$ is the phase function of anisotropic scattering which is represented in terms of a series of Legendre polynomials as

$$p(\mu,\mu') = \sum_{m=0}^{M} (2m+1) f_m P_m(\mu) P_m(\mu') = \sum_{m=0}^{M} b_m P_m(\mu) P_m(\mu') \quad (4)$$

with $b_o = 1$, where the coefficients b_m, $m = 1,2, \ldots, M$ are tabulated [Chu, Clark, and Churchill (1957)].

In the direct problem defined by Eqs. (1) to (3) the radiative properties and boundary conditions are considered known, then the problem becomes the one of determining the radiation intensity $I(\tau,\mu)$. For that purpose a Collocation Method [Villadsen and Michelsen (1978); Wylie and Barrett (1985)] was used together with a Gauss-Legendre quadrature for the terms given on the right hand sides of Eqs. (1) to (3).

3 Mathematical Formulation of the Inverse Problem

In the inverse problem considered here the optical thickness τ_o, the single scattering albedo ω, the diffuse reflectivities ρ_1 and ρ_2, and the coefficients of phase function of anisotropic scattering, b_m, $m = 1,2, \ldots, M$ are considered unknown. Note that M is also unknown and from now on it will be referred to as M^*. Measured exit intensities at both surfaces of the plate $\{Yi\}$, at different polar angles corresponding to $i = 1,2, \ldots, K$, are considered available, where K is the total number of measured data. Therefore, the inverse problem can be stated as: utilizing the measured data

$\{Yi\}$, $i = 1, 2, \ldots, K$, determine the M^*+4 elements of the vector of unknowns Z defined as:

$$\vec{Z} = \{\tau_o, \omega, \rho_1, \rho_2, b_1, b_2, ..., b_{M^*}\}^T \tag{5}$$

Considering that the number of measured data, K, is larger than the number of parameters to be estimated, M^*+4, an implicit formulation based on a optimization problem is used for the inverse radiation problem at hand, in which it is required the minimization of the least squares norm as given below:

$$Q\left(\vec{Z}\right) = \sum_{i=1}^{K} \left[I_i\left(\tau_o, \omega, \rho_1, \rho_2, b_1, b_2, ..., b_{M^*}\right) - Y_i\right]^2 = \vec{G}^T \vec{G} \tag{6}$$

where I_i and Y_i are computed and measured exit intensities, respectively, and the elements of the vector of residues are

$$G_i = I_i\left(\tau_o, \omega, \rho_1, \rho_2, b_1, b_2, ..., b_{M^*}\right) - Y_i, \;\; i = 1,\, 2,\, \ldots,\, K \tag{7}$$

As real experimental data are not available, the measured exit intensities, Y_i, were obtained from simulation. For this aim, random error E (with normal distribution and unitary standard deviation) was added to the exact intensities, I_{exact}, obtained from the solution of the direct problem,

$$Y_i = I_{exact_i} + \sigma E_i, \;\; i = 1,\, 2,\, \ldots,\, K \tag{8}$$

and σ represents the standard deviation of measurement errors.

4 Solution of the Inverse Problem

4.1 Differential Evolution Algorithm

Differential Evolution (DE) is a recent optimization technique in the family of evolutionary computation proposed by Storn and Price (1995), which differs from other evolutionary algorithms in the mutation and recombination phases. According to several authors, DE has as main advantages the conceptual simplicity and faster convergence. However, the main difficulty with the technique appears to be in the slowing down of convergence as the region of global minimum is approached and stagnation of the population [Lampinen and Zelinka (2000)].

This methodology consists in generating trial parameter vectors by adding the weighted difference between two population vectors to a third vector. The control parameters in DE are: NP, the population size, CR, the crossover rate, and, F, the weight applied to random differential (perturbation rate). According to Storn, Price and Lampinen (2005), *NP* should be about 5 to 10 times the problem dimension (number of parameters in a vector), *F* should be in the range 0.1 to 2.0 and *CR* in the range 0.01 to 1.0.

The iterative procedure of the canonical DE is shown in Fig. 2 and summarized as follows [Storn, Price and Lampinen (2005)]:

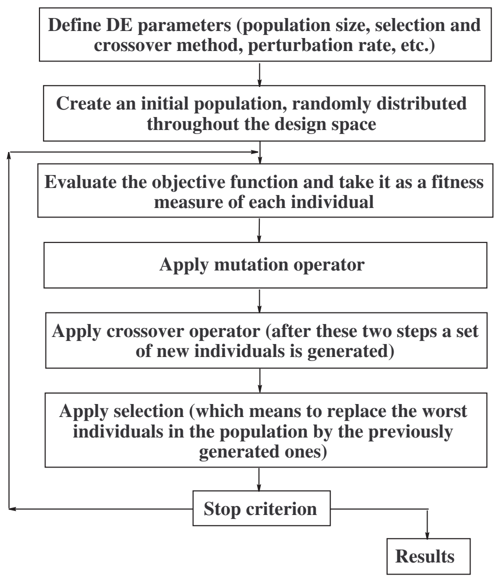


Figure 2: Differential Evolution Structure.

- Step 1: Randomly initialize the population of individuals for DE, where each individual contains *n* variables;
- Step 2: Evaluate the objective values of all individuals, and determine the individual that has the best objective value;
- Step 3: Perform mutation operation for each individual to obtain each individual's mutant counterpart;
- Step 4: Perform crossover operation between each individual and its corresponding mutant counterpart to obtain each individual's trial individual;
- Step 5: Evaluate the objective function values of the trial individuals;

- Step 6: Perform selection operation between each individual and its corresponding trial counterpart to generate the new individual for the next generation;
- Step 7: Determine the best individual of the current new population with the best objective value. If the objective function value is better than the objective function value of X_{best}, then update X_{best} and its objective function value with the value of the current best individual;
- Step 8: If a stopping criterion is met, then output X_{best} and its objective function value; otherwise go back to Step 3.

Storn and Price (1995) proposed various mutation schemes for the generation of new vectors (candidate solutions) by the combination of the vectors that are randomly chosen from the current population as shown in Tab. 1.

Table 1: Mutation Schemes used in the Differential Evolution Algorithm ($r_1, \ldots, r_5$ are random integer indexes and mutually different, x_o is the trial vector (individual), X_{best} is the best individual of the current population and x is the current individual).

Strategy	Updating Equation
rand/1	$x' = x_{r1} + F\left(x_{r2} - x_{r3}\right)$
rand/2	$x' = x_{r1} + F\left(x_{r2} - x_{r3} + x_{r4} - x_{r5}\right)$
best/1	$x' = X_{best} + F\left(x_{r2} - x_{r3}\right)$
best/2	$x' = X_{best} + F\left(x_{r2} - x_{r3} + x_{r4} - x_{r5}\right)$
rand/best/1	$x' = x_{r1} + F\left(X_{best} - x_{r1} + x_{r1} - x_{r2}\right)$
rand/best/2	$x' = x_{r1} + F\left(X_{best} - x_{r1}\right) + F\left(x_{r1} - x_{r2} + x_{r3} - x_{r4}\right)$

Applications of the above technique are found in various fields of science and engineering, such as: digital filter design [Storn (1999)], batch fermentation process [Chiou and Wang (1999)], parameter estimation in fed-batch fermentation process [Wang, Su and Jang (2001)], parameter estimation in biofilter modeling [Bhat, Venkataramani, Ravic and Murty (2006)], economic load dispatch problem [Coelho and Mariani (2007)], engineering system design applied to a multi-objective context [Lobato and Steffen Jr (2007)], apparent thermal diffusivity estimation of fruits drying [Mariani, Lima and Coelho (2008)], solution of inverse radiative transfer problems in two-layer participating media [Lobato, Steffen Jr and Silva Neto (2008)], optimal control problems [Lobato, Murata, Oliveira-Lopes and Steffen Jr (2008)], multi-objective optimization [Lobato, Steffen Jr and Silva Neto (2009)], estimation of radiative properties [Lobato, Steffen Jr and Silva Neto (2010)], and other applications [Storn, Price and Lampinen (2005)].

4.2 Self-Adaptive Differential Evolution Algorithm

In this work, parameter updating is performed according to the previous work from Zaharie (2003). This methodology is based on the evolution of the population variance (viewed as a measure of the diversity population) given by:

$$Var(x) = x^2 - \left(\sum_{i=1}^{NP} \frac{x_i}{NP}\right)^2 \tag{9}$$

According to Zaharie (2002, 2003) the expected value of the variance of population obtained after recombination, if the best element of the population is not taken into consideration, is:

$$E(Var(x)) = \left(1 + 2F^2CR - \frac{2CR}{NP} + \frac{CR^2}{NP}\right) Var(x) \tag{10}$$

Consider that $x(g)$ is the population obtained at generation g-1 (previous population). During the g-th generation the vector x is transformed into x' (recombination); then in x" (selection). x" will represent the starting population for the next generation, $x(g+1)$. Defining γ as

$$\gamma \equiv \frac{Var(x(g+1))}{Var(x(g))} \tag{11}$$

information about the variance tendency is provided: if $\gamma < 1$ we can compensate an increase of the variance, thus we could accelerate the convergence but with the risk of inducing premature convergence and if $\gamma > 1$ we can compensate a decrease of the variance, thus we can avoid premature convergence situations. The controlling idea is to choose the parameter F such that the recombination applied in generation g compensates the effect of the previous application of recombination and selection. The idea of the parameter adaptation is to solve, with respect to F:

$$1 + 2F^2CR - \frac{2CR}{NP} + \frac{CR^2}{NP} = c \tag{12}$$

where $c \equiv \gamma \dfrac{Var(x(g+1))}{Var(x(g))}$.

Equation (12) can be solved with respect to F:

$$F = \begin{cases} \sqrt{\frac{1}{NP}}\sqrt{\frac{\eta}{2CR}} & \text{if } \eta \geq 0 \\ F_{\min} & \text{otherwise} \end{cases} \tag{13}$$

where $\eta \equiv NP(c-1)+CR(2-CR)$ and $F_{\min}$ is the minimal value for F. A sufficient condition for increasing the population variance by recombination is that $F > \sqrt{1/NP}$, thus $F_{min} = \sqrt{1/NP}$ should be used. An upper bound for F can also be imposed as suggested by Storn, Price and Lampinen (2005) ($F_{\max} = 2$). By solving Eq. (12) with respect to CR one obtains the following adaptation rule for CR:

$$CR = \begin{cases} -\left(NP \times F^2 - 1\right) + \sqrt{\left(NP \times F^2 - 1\right)^2 - NP(1-c)} & \text{if } c \geq 1 \\ CR_{\min} & \text{otherwise.} \end{cases} \tag{14}$$

with $CR_{min} = 0.01 < CR < 1$.

5 Results and Discussion

In the inverse radiative transfer problem described before, the goal is the simultaneous estimation of the optical thickness, τ_o, the single scattering albedo ω, the diffuse reflectives ρ_1 and ρ_2, and the coefficients of the anisotropic scattering phase function b_m, $m = 1, 2, \ldots, M^*$. Due to space limitation, the present contribution will focus on the estimation of the phase functions represented by PF-1 and PF-2, whose coefficients are listed in Tab. 2. Therefore, even though the other four parameters are also estimated, only test cases with one set of exact values τ_o=1.0, ω=0.5 and ρ_1=ρ_2=0.2 were considered.

Table 2: Phase function expansion coefficients [Chu, Clark and Churchill (1957)] (m is the index of refraction of the particle relative to the surrounding media and α is $\pi D/\lambda$, D is the particle diameter and λ is the wavelength of incident radiation).

Coefficient	Forward Scattering PF-1 (M=4) m=1.4, α=1	Backward Scattering PF-2 (M=5) m=∞, α=1
b_1	0.57024	-0.56524
b_2	0.56134	0.29783
b_3	0.11297	0.08571
b_4	0.01002	0.01003
b_5	0.00000	0.00063
b_6	-	0.00000

For evaluating the methodology proposed in this work, some practical points should be emphasized:

- To compare the results obtained by the proposed methodology, the canonical DE algorithm is used with the following parameters: DE-1 $\{F$=0.5 and $CR = 0.5\}$, DE-2 $\{F$=0.5 and $CR = 0.8\}$ and DE-3 $\{F$=0.8 and $CR = 0.5\}$. Parameters used in all algorithms tested: 10 individuals, 1000 generations and DE/rand/1/bin strategy for the generation of potential candidates;

- To perform the SADE algorithm γ equal to 1 was adopted;

- In all test cases it is also considered the following external illumination: A_1=1 and A_2=0;

- Finally, the stopping criterion for all the algorithms is associated to the difference between the best and the worst values of the objective function; this difference should be smaller than 10^{-9}. All the algorithms were executed 10 times to obtain the average values presented.

In order to examine the accuracy of the inverse methodology of analysis considered, test cases with noise (σ=0.02 in Eq. (8), i. e., corresponding to 5% error) or without noise (σ=0) have been studied.

Table 3 presents the results obtained by DE and SADE algorithms considering the Phase Functions PF-1 and PF-2. In this table it is important to observe that, considering noiseless data, both DE and SADE were able to estimate the parameters satisfactorily as shown by the values obtained for the objective function. However, the SADE algorithm leads to a smaller number of objective function evaluations as compared with the original DE algorithm (a reduction of 32%, 34% and 22% in the number of objective function evaluations with respect to DE-1, DE-2 and DE-3, respectively). Good estimates are obtained even when noise is taken into account.

Figure 3 presents the exit radiation intensities profiles at boundaries τ=0 and $\tau=\tau_o$ calculated with the radiative parameters estimated using the SADE algorithm without noise (σ=0), and with the exact radiative parameters shown in Tab. 3.

In theses figures it is possible to observe that the SADE algorithm is able to perform satisfactorily at the boundaries τ=0 and $\tau=\tau_o$ for all cases considered without noise. It is important to emphasize that similar behavior can also be observed when noise is considered, however larger deviations with respect to the profiles obtained by using the exact values of the parameters appear.

Table 4 presents the average errors obtained by DE and SADE algorithms considering the Phase Functions PF-1 and PF-2.

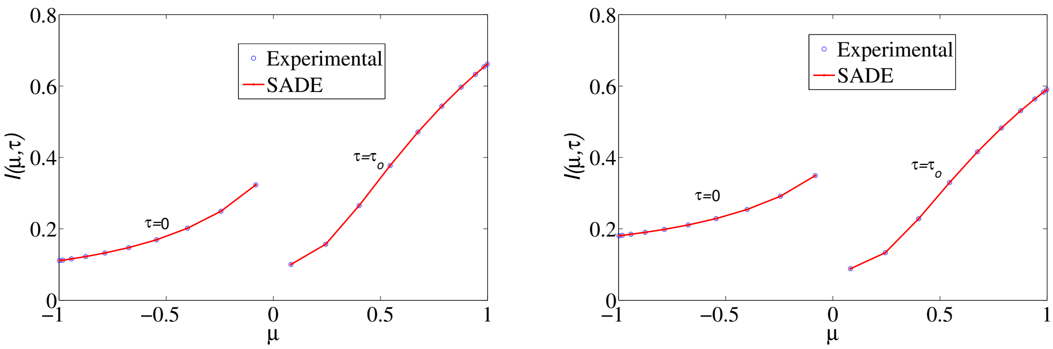
In this table is possible to observe that both the algorithms are able to estimate satisfactorily the anisotropic scattering radiation phase function, albedo and optical thickness. However, it is important to emphasize that the SADE algorithm leads to

Table 3: Results obtained using the DE and SADE methods, where OF is the objective function value, Eq. (6), and N is the number of function evaluations.

			M=4 (m=1.4, α=1)					M=5 (m=∞, α=1)			
	Error (%)	Exact	DE-1	DE-2	DE-3	SADE	Exact	DE-1	DE-2	DE-3	SADE
τ_o	0	1.0	0.999	0.998	0.999	1.000	1.0	0.999	0.999	0.999	0.999
	5		0.978	1.019	0.987	0.986		1.000	0.994	0.994	0.992
ω	0	0.5	0.499	0.4999	0.499	0.500	0.5	0.499	0.499	0.499	0.499
	5		0.491	0.510	0.495	0.495		0.491	0.488	0.488	0.483
ρ_1	0	0.2	0.199	0.200	0.200	0.199	0.2	0.200	0.200	0.199	0.200
	5		0.199	0.199	0.200	0.200		0.204	0.204	0.203	0.203
ρ_2	0	0.2	0.195	0.199	0.210	0.199	0.2	0.198	0.198	0.199	0.199
	5		0.224	0.230	0.182	0.198		0.220	0.214	0.201	0.194
b_1	0	0.570	0.570	0.560	0.570	0.570	-0.565	-0.570	-0.570	-0.570	-0.570
	5		0.496	0.492	0.484	0.493		-0.621	-0.666	-0.666	-0.666
b_2	0	0.561	0.561	0.558	0.561	0.561	0.297	0.294	0.295	0.294	0.296
	5		0.510	0.532	0.511	0.536		0.268	0.265	0.265	0.265
b_3	0	0.112	0.111	0.110	0.111	0.113	0.085	0.084	0.084	0.084	0.084
	5		0.093	0.099	0.102	0.104		0.077	0.088	0.093	0.079
b_4	0	0.010	0.009	0.010	0.010	0.009	0.010	0.009	0.009	0.009	0.009
	5		0.008	0.012	0.010	0.009		0.011	0.011	0.011	0.011
b_5	0	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	5		0.001	0.001	0.001	0.001		0.000	0.001	0.001	0.001
b_6	0	-	-	-	-	-	0.000	0.000	0.000	0.000	0.000
	5		-	-	-	-		0.000	0.001	0.001	0.001
OF	0	-	6E-10	2E-10	8E-10	4E-10	-	4E-10	5E-10	4E-10	3E-10
	5		1E-02	1E-02	1E-02	1E-02		1E-02	1E-02	1E-02	1E-02
N	0	-	2960	3050	2580	2010	-	2420	2880	2380	1920
	5		1220	1660	1480	1060		1200	1680	1540	1100

Table 4: Average errors obtained using the DE and the SADE methods.

	M=4 (m=1.4, α=1)				M=5 (m=∞, α=1)			
Error (%)	DE-1 (%)	DE-2 (%)	DE-3 (%)	SADE (%)	DE-1 (%)	DE-2 (%)	DE-3 (%)	SADE (%)
0	0.0743	0.1981	0.1353	0.0067	0.1267	0.1108	0.1081	0.0783
5	2.2508	2.0608	2.0407	1.4522	1.7072	1.7501	1.7657	1.6689



(a) Phase Function 1. (b) Phase Function 2.

Figure 3: Radiation intensities profiles at boundaries τ=0 and τ=τ_o.

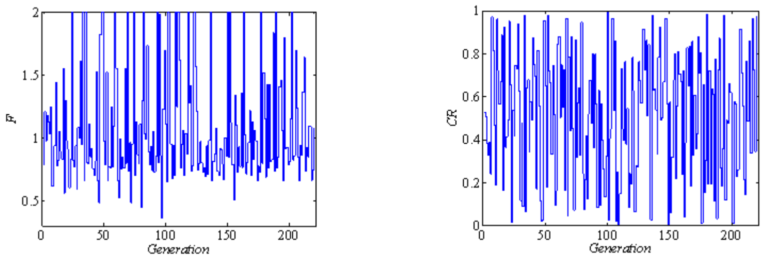
a smaller error in all cases studied as compared with the canonical DE algorithm (DE-1, DE-2 and DE-3).

Figure 4 shows the perturbation rate and crossover rate profiles obtained by using the SADE algorithm for noiseless data.

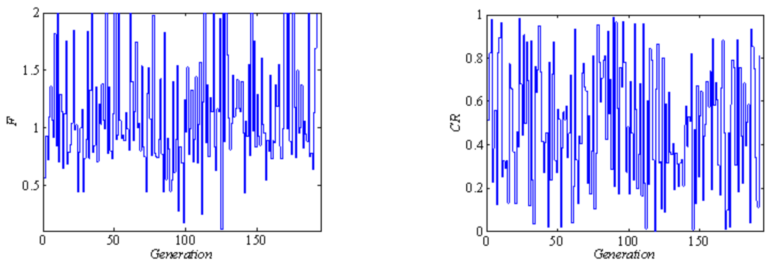
6 Conclusions

In this work, the Self-Adaptive Differential Evolution (SADE) algorithm, which is based on the concept of population diversity to dynamically updating the control parameters, was applied to the simultaneous estimation of the anisotropic scattering radiation phase function, single scattering albedo and optical thickness of an inverse radiative transfer problem. The SADE has been found to be beneficial for adjusting control parameters during the evolutionary process, specially when compared with the standard Differential Evolution algorithm with fixed parameters. The main characteristics of the proposed methodology are: dynamic updating of control parameters based on the diversity of the population and the easiness of incorporating this strategy to other evolutionary strategies. This first characteristic avoided the necessity of choosing these parameters and, as a consequence, the premature convergence of the evolutionary process was not necessary. Finally, the results showed that the methodology conveyed represents a promising alternative for dealing with optimization problems. Further research work will be focused on the influence of the parameter values required by SADE on the solution of others case studies.

Acknowledgement: Dr. Lobato acknowledges the financial support provided by FAPEMIG Proc. No. 00233/08. Dr. Steffen Jr acknowledges the Brazilian Re-



Best result obtained for Phase Function 1.



Best result obtained for Phase Function 2.

Figure 4: Perturbation rate (F) and crossover rate (CR) for the case without noise using the SADE algorithm.

search Agencies CNPq and FAPEMIG for the partial funding of this research work through the INCT-EIE. Dr. Silva Neto acknowledges the financial support provided by CNPq and FAPERJ.

References

Acevedo, N. I. A.; Roberty, N. C.; Silva Neto, A. J. (2004): A one-dimensional inverse radiative transfer problem with time-varying boundary conditions. *Inverse Problems in Engineering*, 12, 123-140.

Bhat, T. R.; Venkataramani, D.; Ravic, V.; Murty, C. V. S. (2006): An improved differential evolution method for efficient parameter estimation in biofilter modeling, *Biochemical Engineering Journal*, 28, 167-176.

Brest, J.; Greimer, S.; Boskovi, B.; Mernik, M.; Zumer, V. (2006): Self-adapting control parameters in differential evolution: a comparative study on numerical benchmark problems, *IEEE Transactions on Evolutionary Computation*, 82-102.

Carita Montero, R. F.; Roberty, N. C.; Silva Neto, A. J. (2001): Absorption coefficient estimation in heterogeneous media using a domain partition consistent with divergent beams. *Inverse Problems in Engineering*, 9, 587-617.

Carita Montero, R. F.; Roberty, N. C.; Silva Neto, A. J. (2004): Reconstruction of a combination of the absorption and scattering coefficients with a discrete ordinates method consistent with the source-detector system. *Inverse Problems in Engineering*, 12 (1), 81-101.

Chalhoub, E. S.; Campos Velho, H. F.; Silva Neto, A. J. (2007): A comparison of the one-dimensional radiative transfer problem solutions obtained with the Monte Carlo method and three variations of the discrete ordinates method. *In Proceedings in 19th International Congress of Mechanical Engineering*.

Chang, C. W.; Liu, C-S.; Chang, J. R. (2005): A group preserving scheme for inverse heat conduction problems. *CMES: Computer Modeling in Engineering & Sciences*, vol. 10, pp. 13-38.

Chiou, J. P.; Wang, F. S. (1999): Hybrid method of evolutionary algorithms for static and dynamic optimization problems with application to a fed-batch fermentation process, *Computers & Chemical Engineering*, 23, 1277-1291.

Chu, C. M.; Clark, G. C.; Churchill, S. W. (1957): *Tables of angular distribution coefficients for light scattering by spheres*, University of Michigan Press, Ann Arbor, MI.

Coelho, L. S.; Mariani, V. C. (2006): Combining of chaotic differential evolution and quadratic programming for economic dispatch optimization with valve-point effect, *IEEE Transactions on Power Systems*, 21, 2, 989-996.

Coelho, L. S.; Mariani, V. C. (2007): Improved differential evolution algorithms for handling economic dispatch optimization with generator constraints, *Energy Conversion and Management*, 48, 1631-1639.

Gämperle, R.; Müller, S. D.; Koumoutsakos, P. (2002): A parameter study for differential evolution, *Advances in Intelligent Systems, Fuzzy Systems, Evolutionary Computation*, 10, 293-298.

Hanan, N. P. (2001): Enhanced two-layer radiative transfer scheme for a land surface model with a discontinuous upper canopy. *Agricultural and Forest Meteorology*, 109, 265-281.

Harris, S. D.; Mustata, R.; Elliott, L.; Ingham, D. B.; Lesnic, D. (2008): Numerical identification of the hydraulic conductivity of composite anisotropic materials. *Computer Modeling in Engineering & Sciences*, vol. 25, pp. 69-79.

Kim, H. K.; Charette, A. (2007): A sensitivity function based conjugate gradient method for optical tomography with the frequency domain equation of radiative transfer. *Journal of Quantitative Spectroscopy and Radiative Transfer*, vol. 104, pp. 24-39.

Lampinen, J.; Zelinka. I. (2000): On stagnation of the differential evolution algorithm, *In Proceedings of MENDEL.*

Liu, C-S. (2006): An efficient simultaneous estimation of temperature-dependent thermophysical properties. *CMES: Computer Modeling in Engineering & Sciences*, vol. 14, pp. 77-90.

Lobato, F. S.; Steffen Jr., V. (2007): Engineering system design with multi-objective differential evolution, *In Proceedings in 19th International Congress of Mechanical Engineering*, Brasília, Brazil.

Lobato, F. S.; Steffen Jr., V.; Silva Neto, A. J. (2008): Solution of inverse radiative transfer problems in two-layer participating media with differential evolution, *International Conference on Engineering Optimization*, 2008, Rio de Janeiro, Brazil.

Lobato, F. S.; Steffen Jr., V. (2009): Adaptive differential evolution algorithm and differential index reduction strategy applied to solution of optimal control problems, *In Proceedings in 8th Brazilian Conference on Dynamics, Control and Applications*, Bauru, Brazil.

Lobato, F. S.; Steffen Jr. V.; Silva Neto, A. J. (2009), Solution of the coupled inverse conduction-radiation problem using multi-objective optimization differential evolution, *8th World Congress on Structural and Multidisciplinary Optimization*, Lisboa, Portugal.

Lobato, F. S.; Steffen Jr. V.; Silva Neto, A. J. (2010), A comparative study of the application of Differential Evolution and Simulated Annealing in inverse radiative transfer problems, *Journal of the Brazilian Society of Mechanical Sciences*, accept for publication.

Mariani, V. C.; Lima, A. G. B.; Coelho, L. S. (2008): Apparent Thermal diffusivity estimation of the banana during drying using inverse method, *Journal of Food Engineering*, 85, 569-579.

Nobakhti, A.; Wang, H. (2006): A Self-adaptive differential evolution with application on the ALSTOM gasifer, *Proceedings of the 2006 American Control Conference Minneapolis*, Minnesota, USA, June 14-16.

Özişik, M. N. (1973): *Radiative transfer and interactions with conduction and convection*, John Wiley.

Qin, A. K.; Suganthan, P. N. (2005): Self-adaptive differential evolution algorithm for numerical optimization, *Proceedings of the 2005 Congress on Evolutionary Computation*, vol. 2, pp. 1785-1791.

Siegel, R.; Spuckler, C. M. (1993): Refractive index effects on radiation in an absorbing, emitting, and scattering laminated layer. *Journal of Heat Transfer*, 115, 194-200.

Silva Neto, A. J.; Özişik, M. N. (1995): An inverse problem of simultaneous estimation of radiation phase function, albedo and optical thickness. *Journal of Quantitative Spectroscopy and Radiative Transfer*, 53 (4), 397-409.

Silva Neto, A. J.; Soeiro, F. J. C. P. (2002): Estimation of the phase function of anisotropic scattering with a combination of gradient based and stochastic global optimization methods. *In Proceedings of 5th World Congress on Computational Mechanics*, Vienna, Austria, July, 7-12.

Silva Neto, C. A.; Silva Neto, A. J. (2003a): Estimation of optical thickness, single scattering albedo and diffuse reflectivities with a minimization algorithm based on an interior points method. *In Proceedings of 17th International Congress of Mechanical Engineering*, São Paulo, Brazil.

Silva Neto, A. J.; Soeiro, F. J. C. P. (2003b) Solution of implicitly formulated inverse heat transfer problems with hybrid methods. *In Mini-Symposium Inverse Problems from Thermal/Fluids and Solid Mechanics Applications*, 2nd MIT Conference on Computational Fluid and Solid Mechanics, Cambridge, USA.

Sousa, F. L.; Soeiro, F. J. C. P.; Silva Neto, A. J.; Ramos, F. M. (2007): Application of the generalized extremal optimization algorithm to an inverse radiative transfer problem, *Inverse Problems in Science and Engineering*, 15, 7, 699-714.

Storn, R. (1999): System design by constraint adaptation and differential evolution, *IEEE Transactions on Evolutionary Computation*, 3, 22-34.

Storn, R.; Price, K. (1995): Differential evolution: a simple and efficient adaptive scheme for global optimization over continuous spaces. *International Computer Science Institute*, vol. 12, pp. 1-16.

Storn, R.; Price, K.; Lampinen, J. A. (2005): *Differential evolution - a practical approach to global optimization*, Springer - Natural Computing Series.

Tavazoei, M. S.; Haeri, M. (2007): Comparison of different one-dimensional maps as chaotic search pattern in chaos optimization algorithms, *Applied Mathematics and Computation*, 187, 1076-1085.

Villadsen, J.; Michelsen, M. L. (1978): *Solution of differential equation models by polynomial approximation*. Prentice-Hall, Englenwood Cliffs.

Wang, P. Y.; Cheng, H. E.; Tan, H. P. (2001): Transient thermal analysis of semitransparent composite layer with an opaque boundary, *International Journal of Heat and Mass Transfer*, 45, 425-440.

Wang, F. S.; Su, T. L.; Jang, H. J. (2001): Hybrid differential evolution for problems of kinetic parameter estimation and dynamic optimization of an ethanol fermentation process. *Industry Engineering Chemical Research*, vol. 40, pp. 2876-2885.

Wylie, C. R.; Barrett, L. C. (1985): *Advanced engineering mathematics*. 5th Edition, McGraw-Hill, Singapura.

Zaharie, D. (2002): Critical values for the control parameters of differential evolution algorithms, *Proceedings of the 8th International Conference on Soft Computing*, pp. 62-67.

Zaharie, D. (2003): Control of population diversity and adaptation in differential evolution algorithms, R. Matouek, P. Omera (eds.), *Proc. of Mendel 2003, 9th Internat. Conference on Soft Computing*, 41-46.

Yeih, W.; Liu, C-S. (2009): A three-point BVP of time-dependent inverse heat source problems and solving by a TSLGSM, *CMES: Computer Modeling in Engineering & Sciences*, vol. 46, n. 2, pp. 107-127.