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Turbulentlike Quantitative Analysis on Energy Dissipation in Vibrated Granular Media

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Abstract: A quantitative rule of the vibrated granular media's energy dissipation is obtained by adopting the turbulence theory in this letter. Our results show that, similar to the power spectrum in fully developed fluid turbulence as described in Kolmogorov's theory, the power spectrum of vibrated granular media also exhibits a $k^{-5/3}$(k is the wave number) power which characterizes the local isotropic flow. What's more, the mean energy dissipation rate in vibrated granular media rises with the increase of particle size and volume ratio. The theoretical results in this letter can be verified by the previous experimental results as well.

Keywords: Vibrated granular media, Turbulence, Kolmogorov's hypothesis, Energy spectral density.

1 Introduction

Granular materials have attracted intense interests in recent years. This letter is to obtain a quantitative rule of energy dissipation in a vibrated container filled with granular media by adopting the turbulence theory. The velocity fluctuations in granular media have been studied by several authors [Gioia, Ott-Monsivais and Hill (2006); Drozd and Denniston (2008)],and the fluctuating properties of homogeneous quasistatic sheared granular media has been found to be similar to turbulence, although their origins of the fluctuations are different [Radjai and Roux (2002)]. The energy flow in vibrated granular media has been studied by considering the instaneous collisions with neglecting the sliding process among particles [McNamara and Luding (1998)].

2 Statical analysis

In general fluids, the turbulence would appear when the Reynolds number is beyond the critical value, and then the fluctuating velocity, which is caused by the nonlinear

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term in the Reynolds equation, brings about energy transport from large eddies to small ones. The shaking strength, a dimensionless parameter defined in refs. [Eshuis, van der Meer, Alam, van Gerner, van der Weele and Lohse (2010); Taguchi (1992)], plays the same role in vibrated granular media as the Reynolds number in fluids. At low strength, the particles move together in a layer, whose structure was found to be similar to the structure of a laminar liquid flow [Hill, Gioia and Tota (2003)]. While the structure of the granular media is changed with the increase of the shaking strength, the fluidization in granular media starts. At high strength, the fluidization is dominated by particle-particle collisions [Hui, Haff, Ungar and Jackson (1984)], which would destroy the laminar-like structure and bring about the energy transport in the granular media, and the convection flow with eddy-like turbulence would appear when the strength is beyond the critical value [Eshuis, van der Meer, Alam, van Gerner, van der Weele and Lohse (2010); Taguchi (1993a)]. In fact, the convection could be regarded as the local regular periodic relative shearing motion (simple shear) between two adjacent layers with different motion directions if there is a steady energy input [Hill, Gioia and Tota (2003)]. Similar to the fluid turbulence in which the energy transport and cascade decay are driven by the nonlinear term in the Reynolds equation, the transition from laminar structure to turbulent in vibrated granular media is also dominated by the nonlinear term in the Reynolds equation of granular media due to the particle collisions [Khosropour, Zirinsky, Pak and Behringer (1997)]. The powder turbulent-flow with $k^{-5/3}$ power spectrum in a vibrated bed filled with granular media was numerically found by Taguchi, who adopted the powder turbulence model to study fluid turbulence [Taguchi (1993a); Taguchi (1993b)]. It showed that, the convection appeared and looked like fully developed fluid turbulence whose power spectrum exhibits a $k^{-5/3}$ power which characterizes the local isotropic flow in turbulence as described in Kolmogorov's theory [Frisch (1987)], after the fluidization of the vibrated granular media at high shaking strength. In this latter, a quantitative analysis on energy dissipation in vibrated granular media is obtained by adopting the turbulence theory and a local isotropic model.

We assume that structure of the vibrated container is large enough in comparison with the size of rigid particles, then the motion of particle is least influenced by the side walls of the container [Taguchi (1992)]. Thus, in the case of large aspect ratio, the energy dissipation is mainly caused by the turbulent-flow in the granular media, not by the particle-wall interactions. The velocity fields of granular flow are expressed as follows:$U_i = \bar{U}_i + u_i$, where $\bar{U}_i$ is the mean velocity ($\overline{(\cdot)}$ denotes time average) and u_i is the fluctuation velocity. The simplified general constitutive equation of the simple shear granular flow, which is appropriate for both slow and rapid granular flow ($U_1 = U(\mathrm{y})$, $U_2 = U_3 = 0$), was given in the ref. [Wang, Xiong and Fang (1998)] as follows:

$$\begin{cases} T = T_{xy} = T_{yx} = p\sin\phi_0 + k_{\tau 1}\rho_s d^{3/2} g^{1/2} \frac{du}{dy} + k_{\tau 2}\rho_s d^2 \left(\frac{du}{dy}\right)^2, \\ P = T_{xx} = T_{yy} = -p + k_{p1}\rho_s d^{3/2} g^{1/2} \frac{du}{dy} + k_{p2}\rho_s d^2 \left(\frac{du}{dy}\right)^2 \end{cases} \tag{1}$$

where p is the static pressure, Φ_0 is the static internal frictional angle, ρ_s is the density of granular material, d is the particle's diameter and g is the gravity acceleration. k_τ and k_p are the stress factors relevant to the granular matter volume ratio and granular restitution coefficient. Owing to the assumption of rigid particles, there is no plastic deformation and fragmentation of particles during the energy dissipation process in this letter. So the energy dissipation in the vibrated granular matter can be separated into two parts: a) the energy transferred among particles; b) the energy dissipated by viscous property among particles. We adopt two parameters, ε (mean energy dissipation rate per unit mass) and ν (equivalent kinematical viscosity coefficient) to describe the energy dissipation mechanism of the vibrated granular flow. The "viscosity" is caused by the friction motions among particles, which appears as the internal friction of the granular media. We obtain the ν by comparing the simplified general constitutive equation of the granular flow with the stress equation of liquid as follows: ν=$d^{3/2}g^{1/2}$. According to the similarity hypothesis, we apply a non-dimensional analysis to obtain the energy spectrum of the velocity fluctuations at high shaking strength is obtained as follows:

$$E(k) \sim \varepsilon^{2/3} k^{-5/3} \tag{2}$$

where k is the wave number. This theoretical result is verified by the simulating results in refs. [Taguchi (1993a); Taguchi (1993b)].

As was found that, the convection in strongly shaken granular matter can be described by a linear continuum model [Eshuis, van der Meer, Alam, van Gerner, van der Weele and Lohse (2010)]. The generalized Navier-Stokes equation for the granular media (N-S-G) can be obtained by substituting the constitutive equation of the granular media into the N-S equation, then the fluctuating energy conservation equation is obtained as the product of the fluctuating motion equation which is the N-S-G equation minus the averaged N-S-G equation, and the fluctuating velocity as follows:

$$\begin{aligned} &\frac{\partial}{\partial t}\left(\frac{\overline{u_i^2}}{2}\right) + \overline{U}_j \overline{u_i \frac{\partial u_i}{\partial x_j}} + \overline{u_i u_j}\frac{\partial \overline{U}_i}{\partial x_j} + \overline{u_j \frac{\partial}{\partial x_j}\left(\frac{u_i^2}{2}\right)} \\ &\qquad = \frac{1}{2}\overline{A u_i \nabla^2 u_i} + \overline{B u_i \left[\left(D_{ij} + D'_{ij}\right)\nabla^2 u_i + D'_{ij}\nabla^2 \overline{U}_i\right]} \end{aligned} \tag{3}$$

where

$$A=\frac{p\sin\phi_0}{\rho_s\sqrt{J_2'}}+2\tan\phi_1 k_{p1}d^{3/2}g^{1/2}+4\tan\phi_2 k_{p2}d^2\sqrt{J_2'}\ ,$$

$$B=4k_{p1}d^2g^2\frac{1}{\sqrt{J_2'}}+2k_{p2}d^2,$$

$$D_{ij}'=\frac{1}{2}\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right)$$

Considering the continuity equation: u_j'=0, the first term of the right side of eq. (3) can be written as follows:

$$\overline{Au_i\nabla^2 u_i}=\underbrace{A\frac{\partial}{\partial x_j}\left[\overline{\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right)u_i}\right]}_{\mathrm{I}}\underbrace{-A\overline{\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right)\frac{\partial u_i}{\partial x_j}}}_{\mathrm{II}}\ , \tag{4}$$

where I stands for the work done by the viscous shearing force among layers which causes the momentum transport, and II is the term of energy dissipation during the fluctuating process, which is just defined as the ε (mean energy dissipation rate per unit mass) above.

3 Numerical results and analysis

As we mentioned, in the local regions the granular motion can be regarded as a simple shear ($U_1=U$(y), $U_2=U_3=$0). The mean energy dissipation rate ε can be expressed as follows:

$$\varepsilon=(1+\tan\phi_1)k_{p1}d^{32}g^{1/2}\overline{\left(\frac{du}{dy}\right)^2} \tag{5}$$

The stress factor can be obtained from the empirical correlation: k_{p1}=$\alpha_1\psi_{vr}^{n1}$, where ψ_{vr} is the relative volume ratio which can be expressed as follows: ψ_{vr}=ψ_{vr}/(ψ_{vm}-ψ_v), ψ_{vm} is the critical volume ratio of shear flow, and its relation to maximum volume ratio of loose piled particles (ψ_{vm0}) is given by the relation: $\psi_{vm}\approx 0.92\psi_{vm0}$. ψ_v=V_p/V_c is the volume ratio, where V_p is the volume of particles and V_c is the volume of the container. The approximate values of other parameters can be obtained by experiments in ref. [Wang, Xiong and Fang (1998)] as follows:n_1=1, α_1=0.7, tanΦ_1=0.5, where α_1 presents some properties of the granular material.

It is shown in fig. 1 that, the mean energy dissipation rate rises with the increase of particle size and the volume ratio. What's more, an increase of particle size causes more obvious increase of energy dissipation at higher volume ratio. This can be interpreted that, the increase of particle size will cause more friction opportunities in dense flow than in dilute flow. This result can be verified by the experimental discovery in ref. [Xu, Wang and Chen (2004)]. We notice from fig. 2 that higher volume ratio brings more energy dissipation when the particle size is the same. It can also be found in fig. 2 that the mean energy dissipation rate increases rapidly beyond a volume ratio of 60%, and tends to reach its maximum just before being fully filled. The same change rules have been presented in ref. [Wakasawa, Hashimoto and Marui (2004)], which showed that the vibration dissipation time becomes shorter with increase of the volume ratio and has a big drop at a volume ratio of 60%.

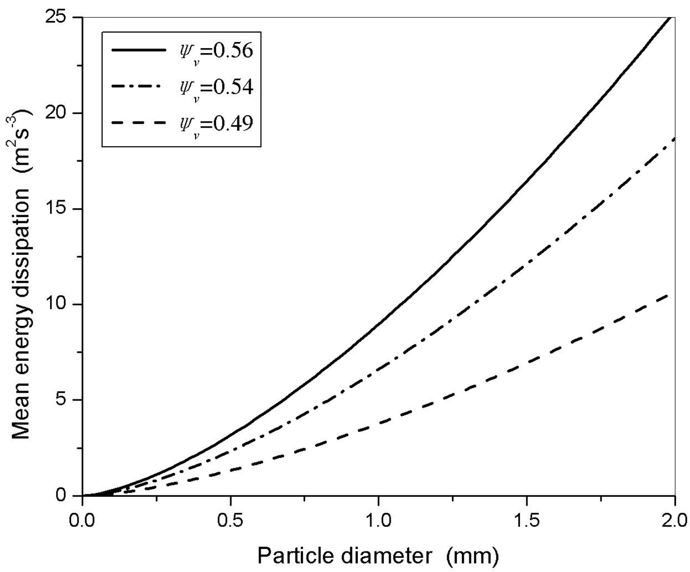

Figure 1: Mean energy dissipation (ε) curves for different values of particle diameter (d), when ψ_v =0.56, ψ_v =0.54, ψ_v =0.49, du/dy=100 s^{-1}

4 Conclusion

In conclusion, a quantitative analysis model is put forward for the vibrated granular matter's energy dissipation mechanism, and the satisfactory theoretical results are obtained, which show that the mean energy dissipation rate rises with the increase of particle size and volume ratio, and the energy spectrum has a clear power-law

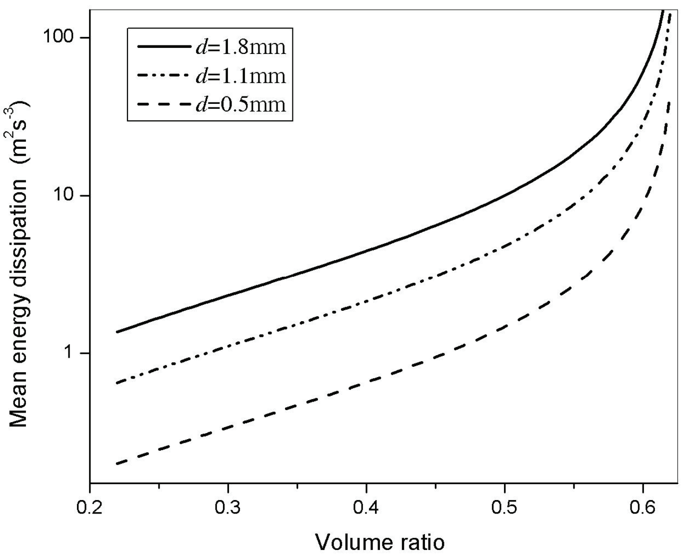

Figure 2: Mean energy dissipation (ε) curves (log-log) for different values of volume ratio (ψ_v), when d=1.8mm,d=1.1mm,d=0.5mm; du/dy=100 s^{-1}

shape $k^{-5/3}$. The results of the mean energy dissipation can be verified by the previous references as we mentioned above.

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