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Application of Symmetric Hyperbolic Systems for the Time-Dependent Maxwell's Equations in Bi-Anisotropic Media

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Abstract: The time-dependent Maxwell's equations in non-dispersive homogeneous bi-anisotropic materials are considered in the paper. These equations are written as a symmetric hyperbolic system. A new method of the computation of the electric and magnetic fields arising from electric current is suggested in the paper. This method consists of the following. The Maxwell's equations are written in terms of the Fourier transform with respect to the space variables. The Fourier image of the obtained system is a system of ordinary differential equations whose coefficients depend on the 3D Fourier parameter. The formula for the solution of the obtained system is derived by the matrix transformations. Finally, the electric and magnetic fields are computed by the inverse Fourier transform. Using this formula the computation of the electric and magnetic fields has been made for the case when the current is a polarized dipole.

Keywords: Maxwell's equations, bi-anisotropic material, symmetric hyperbolic system, analytical method, simulation.

1 Introduction

The bi-anisotropic materials have been predicted by Landau and Lifshitz (1960) and Dzyalovskii (1960) in their theoretical study. Astrov (1960) was the first who observed the magnetoelectric effect in chromium oxide. In recent years, many studies of bi-anisotropic materials have been carried out with the objective to find materials that show magnetic and electric coupled properties for applications in the magnetoelectronics technology such as non-volatile memories [Kimuraet et al. (2003)]. These materials could potentially be used for fabricating devices that include sensors, actuators and data storage. Now, scientists have identified a potential magneto-electric material based on a bismuth based oxide (Bi2Fe4O9) with

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favorable properties for magnetoelectric applications [Singhand et al. (2008)]. A simple classification of the ways magnetoelectric effects may be used in devices is described in [Wood and Austin (1974)]. It is mentioned there that there exists the following areas of possible applications for magnetoelectric materials: (1) magnetic field switching or modulation of electric polarization; (2) exploitation of high dielectric constant in high-frequency low-loss microwave Faraday rotators; (3) efficient generation, modulation, or modification of spin waves or hybrid spin-electromagnetic waves; (4) use of irreversible light propagation in a sensitive interference sensor. The concept of magnetoelectric composites and their applications have been discussed in [Grossinger et al. (2007)]. Although the comprehension of the mechanisms that favor the interaction between electric and magnetic fields is not yet absolutely established, there is a great interest in the study of the electromagnetic wave propagations in bi-anisotropic materials. In the past decade the significant research progress has been achieved in potential applications of bi-anisotropic materials. Thus for example, the dyadic Green's functions for source radiation and wave propagation in homogeneous bi-anisotropic materials have been studied in [Li and Lim (2003)], [Dmitriev (2004)]. Plane wave propagations in homogeneous bi-anisotropic materials have been treated in [Sedyukov et al. (2001)]. The electromagnetic scattering of various homogeneous bi-anisotropic objects has been examined in [Sedyukov et al. (2001)], [Zhang, Yeo and Leong (2003)]. However we need to note that the study of electromagnetic fields has been made only for particular cases of bi-anisotropic materials. The theoretical study of electromagnetic bi-anisotropic materials by the time-dependent Maxwell's equations has not been done so far. The complexity of the Maxwell's equations in bi-anisotropic materials with general structures of anisotropy depends on the complexity of the constitutive relations for electric and magnetic fluxes. The electric and magnetic fluxes in bi-anisotropic materials are governed by constitutive relations containing matrices of permittivity, permeability and magnetoelectric characteristics whose elements are functions of the position in the three dimensional space. These constitutive relations are more complicated in comparison with constitutive relations of ordinary isotropic or anisotropic materials.

The motivation of our study is to develop a new method for solving the problems of electromagnetic radiation in a class of bi-anisotropic materials with the general structure of anisotropy.

In the present paper the time dependent Maxwell's equations for homogeneous non-dispersive bi-anisotropic materials, characterized by arbitrary symmetric positive definite electric permittivity and magnetic permeability tensors and symmetric magneto-elastic tensors, are considered. These Maxwell's equations are written as a symmetric hyperbolic system. A new method of the computation of electric and magnetic fields arising from electric current is suggested in the paper. This method consists of the following. The Maxwell's equations as a symmetric hyperbolic system are written in terms of the Fourier transform with respect to the space variables. The Fourier image of the obtained system is a system of ordinary differential equations whose coefficients depend on the 3D Fourier parameter. The formula for the solution of the obtained ordinary differential system is derived by the matrix transformations. Finally, the electric and magnetic fields are calculated by the inverse Fourier transform. Using this formula the computation and simulation of the electric and magnetic fields has been obtained for the case when the current is a polarized dipole. These computational examples confirm the robustness of the suggested method.

2 The time-dependent Maxwell's equations in bi-anisotropic materials

The electromagnetic wave propagation in bi-anisotropic materials is governed by Maxwell's equations [Kong (1990)]:

$$curl_x\vec{H} = \frac{1}{c}\frac{\partial\vec{D}}{\partial t} + \frac{4\pi}{c}\vec{J}, \quad curl_x\vec{E} = -\frac{1}{c}\frac{\partial\vec{B}}{\partial t}, \tag{1}$$

$$div_x(\vec{D}) = \rho, \quad div_x(\vec{B}) = 0, \tag{2}$$

where $\vec{E} = (E_1, E_2, E_3)$, $\vec{H} = (H_1, H_2, H_3)$ are electric and magnetic fields, $\vec{D} = (D_1, D_2, D_3)$ is the electric displacement, $\vec{B} = (B_1, B_2, B_3)$ is the magnetic flux density, $\vec{J} = (J_1, J_2, J_3)$ is the electric current density, ρ is the density of electric charge, c is the speed of light in vacuum. The components of the vector functions $\vec{E}$, $\vec{H}$, $\vec{D}$, $\vec{B}$, $\vec{J}$ and ρ are real valued functions of the position $x = (x_1, x_2, x_3)$ from $\mathbf{R}^3$ and the time t from $\mathbf{R}$. Taking the divergence of (1) and using (2), we find the conservation law for electric charge and current density:

$$\frac{\partial\rho}{\partial t} + div_x\vec{J} = 0. \tag{3}$$

Remark 1. Equality (3) is a necessary condition that the pair of vector functions $\vec{E}$, $\vec{H}$ is a solution of Maxwell's equations (1), (2). We assume that $\vec{J}$ and ρ satisfy this conservation law.

The properties of bi-anisotropic materials, in which we study electric and magnetic fields, are constituted by the constitutive relations. These constitutive relations are given in the following form (see, for example, [Kong (1990)])

$$\vec{D} = \bar{\bar{\varepsilon}}\vec{E} + \bar{\bar{\xi}}\vec{H}, \quad \vec{B} = \bar{\bar{\eta}}\vec{E} + \bar{\bar{\mu}}\vec{H}, \tag{4}$$

where $\bar{\bar{\varepsilon}} = (\varepsilon_{km})_{3\times 3}$, $\bar{\bar{\mu}} = (\mu_{km})_{3\times 3}$, $\bar{\bar{\xi}} = (\xi_{km})_{3\times 3}$, $\bar{\bar{\eta}} = (\eta_{km})_{3\times 3}$ are matrices of electric permittivity, magnetic permeability and magnetoelectric tensors, respectively. Here $\bar{\bar{\xi}}$ and $\bar{\bar{\eta}}$ are not zero if the medium is bi-anisotropic. If there is no coupling between electric and magnetic fields, i.e. when $\bar{\bar{\xi}} = \bar{\bar{\eta}} = 0$, the medium is anisotropic. If $\bar{\bar{\xi}} = \bar{\bar{\eta}} = 0$ and $\bar{\bar{\varepsilon}} = \mathbf{I}$ and $\bar{\bar{\mu}} = \mathbf{I}$, where $\mathbf{I}$ is the identity 3×3 matrix, the medium is isotropic.

According to the theory of continuous groups of symmetry (CGS) developed by Dmitriev (2000), the number of independent elements in $\bar{\bar{\varepsilon}}$, $\bar{\bar{\mu}}$, $\bar{\bar{\xi}}$, $\bar{\bar{\eta}}$ for a given bi-anisotropic medium is completely governed by certain magnetic group symmetry. From CGS theory we can find a classification of the constitutive tensors $\bar{\bar{\varepsilon}}$, $\bar{\bar{\mu}}$, $\bar{\bar{\xi}}$, $\bar{\bar{\eta}}$ for 23 linear complex media [Li and Yin (2005)]. In the present paper we assume that the following 6×6 matrix

$$\bar{\bar{A}}_0 = \begin{pmatrix} \bar{\bar{\varepsilon}} & \bar{\bar{\xi}} \\ \bar{\bar{\eta}} & \bar{\bar{\mu}} \end{pmatrix} \tag{5}$$

is symmetric and positive definite with constant elements. We suppose also that

$$\vec{E} = 0, \quad \vec{H} = 0, \quad \vec{J} = 0, \quad \rho = 0 \quad \text{for} \quad t < 0. \tag{6}$$

Remark 2. We note that equality (1) under conditions (3), (6) implies (2). This means that for given $\bar{\bar{\varepsilon}} = (\varepsilon_{km})_{3\times 3}$, $\bar{\bar{\mu}} = (\mu_{km})_{3\times 3}$, $\bar{\bar{\xi}} = (\xi_{km})_{3\times 3}$, $\bar{\bar{\eta}} = (\eta_{km})_{3\times 3}$, $\vec{J}$ and ρ the vector functions $\vec{E}$, $\vec{H}$ satisfying (1), (6) are a solution of the Maxwell's equations under assumptions (3), (4).

The main object of the paper is the initial value problem (IVP) of finding $\vec{E}$, $\vec{H}$ satisfying (1), (6) for given $\bar{\bar{\varepsilon}} = (\varepsilon_{km})_{3\times 3}$, $\bar{\bar{\mu}} = (\mu_{km})_{3\times 3}$, $\bar{\bar{\xi}} = (\xi_{km})_{3\times 3}$, $\bar{\bar{\eta}} = (\eta_{km})_{3\times 3}$, $\vec{J}$.

3 Representation of (1), (6) as the initial value problem for a symmetric hyperbolic system

Equations (1), (6) can be written in terms of $\vec{E}(x,t)$ and $\vec{H}(x,t)$ as follows

$$\frac{1}{c}\bar{\bar{\varepsilon}}\frac{\partial \vec{E}(x,t)}{\partial t} + \frac{1}{c}\bar{\bar{\xi}}\frac{\partial \vec{H}(x,t)}{\partial t} = curl_x \vec{H}(x,t) - \frac{4\pi}{c}\vec{J}, \tag{7}$$

$$\frac{1}{c}\bar{\bar{\eta}}\frac{\partial \vec{E}(x,t)}{\partial t} + \frac{1}{c}\bar{\bar{\mu}}\frac{\partial \vec{H}(x,t)}{\partial t} = -curl_x \vec{E}(x,t), \tag{8}$$

with the following conditions

$$\vec{E}(x,t)\Big|_{t\leq 0} = 0, \quad \vec{H}(x,t)\Big|_{t\leq 0} = 0. \tag{9}$$

Equations (7)–(8) with conditions (9) can be presented in the form of an initial value problem for the following symmetric hyperbolic system:

$$\frac{1}{c}\bar{\bar{A}}_0\frac{\partial \vec{U}(x,t)}{\partial t} = \sum_{j=1}^{3}\bar{\bar{A}}_j\frac{\partial \vec{U}(x,t)}{\partial x_j} + \frac{1}{c}\vec{f}(x,t), \quad \vec{U}(x,t)\Big|_{\leq 0} = 0, \tag{10}$$

where $\vec{U}(x,t)$ and $\vec{f}(x,t)$ are vector columns with six components defined by $\vec{U}(x,t) = (\vec{E}(x,t),\vec{H}(x,t))^T$ and $\vec{f} = -4\pi(\vec{J},0,0,0)^T$ (the upper index T means the transpose of the row vectors to the column vectors); $\bar{\bar{A}}_0$ is the 6×6 matrix defined by (5) and $\bar{\bar{A}}_j$, $j = 1,2,3$ are matrices of the order 6×6 determined as

$$\bar{\bar{A}}_j = \begin{pmatrix} 0_{3\times 3} & \bar{\bar{B}}_j \\ (\bar{\bar{B}}_j)^T & 0_{3\times 3} \end{pmatrix}, \tag{11}$$

where $0_{3\times 3}$ is the 3×3 zero matrix, $(\bar{\bar{B}}_j)^T$ is the transpose matrix of $\bar{\bar{B}}_j$;

$$\bar{\bar{B}}_1 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}, \ \bar{\bar{B}}_2 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{pmatrix}, \ \bar{\bar{B}}_3 = \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

4 Derivation of the formula for the solution of (10)

4.1 Equation for the Fourier image of $\vec{U}(x,t)$

Let $\tilde{\mathbf{U}}(\nu,t)$ be the Fourier transform image of the $\vec{U}(x,t)$ with respect to $x = (x_1,x_2,x_3) \in R^3$, i.e. $\tilde{\mathbf{U}}(\nu,t) = (\tilde{U}_1(\nu,t),\tilde{U}_2(\nu,t),\tilde{U}_3(\nu,t),\tilde{U}_4(\nu,t),\tilde{U}_5(\nu,t),\tilde{U}_6(\nu,t))$, where $\tilde{U}_j(\nu,t) = \mathscr{F}_x[E_j](\nu,t)$ and $\tilde{U_{j+3}}(\nu,t) = \mathscr{F}_x[H_j](\nu,t)$ $j = 1,2,3$, and the Fourier operator $\mathscr{F}_x$ is given by (see, for example, Vladimirov (1971))

$$\mathscr{F}_x[E_j](\nu,t) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} E_j(x,t)e^{i\nu\cdot x}dx_1dx_2dx_3,$$

$$\nu = (\nu_1,\nu_2,\nu_3) \in R^3; \quad \nu\cdot x = x_1\nu_1 + x_2\nu_2 + x_3\nu_3, \quad i^2 = -1.$$

Equations (10) can be written in the form of their Fourier images as follows

$$\bar{\bar{A}}_0\frac{\partial \tilde{\mathbf{U}}(\nu,t)}{\partial t} + ic\mathscr{A}(\nu)\tilde{\mathbf{U}}(\nu,t) = \tilde{\mathbf{F}}(\nu,t), \quad \tilde{\mathbf{U}}(\nu,t)\big|_{t<0} = 0. \tag{12}$$

where $\mathscr{A}(\nu) = \sum_{j=1}^{3}\nu_j\bar{\bar{A}}_j$ is the symmetric 6×6 matrix, $\tilde{\mathbf{F}}(\nu,t) = \mathscr{F}_x[\vec{f}](\nu,t)$.

4.2 Construction of the formula for the solution of (12)

The construction of the exact solution of (12) consists of several steps. In the first step, using the matrix formalism for given symmetric matrix $\mathscr{A}(\nu)$ and the symmetric positive definite matrix $\bar{\bar{A}}_0$ we construct a non-singular matrix $\bar{\bar{T}}$ and a diagonal matrix $\bar{\bar{D}}(\nu) = diag(d_1(\nu), d_2(\nu), ..., d_6(\nu))$ with real valued elements such that

$$\bar{\bar{T}}^T(\nu)\bar{\bar{A}}_0\bar{\bar{T}}(\nu) \;=\; \bar{\bar{I}}, \qquad \bar{\bar{T}}^T(\nu)\mathscr{A}(\nu)\bar{\bar{T}}(\nu) = \bar{\bar{D}}(\nu), \tag{13}$$

where $\bar{\bar{I}}$ is the identity matrix, $\bar{\bar{T}}^T(\nu)$ is the transposed matrix to $\bar{\bar{T}}(\nu)$.

Remark 3. Computing $\bar{\bar{D}}(\nu)$ and $\bar{\bar{T}}(\nu)$ explicitly can be made successively: first, find $\bar{\bar{A}}_0^{-1/2}$ and then construct $\bar{\bar{D}}(\nu)$ and $\bar{\bar{T}}(\nu)$. We note that for a given diagonal matrix $\bar{\bar{A}}_0 = diag(a_{jj}, \quad j = 1,2,...6)$ with positive elements on the diagonal the matrix $\bar{\bar{A}}_0^{-1/2}$ is given by

$$\bar{\bar{A}}_0^{-1/2} = diag\left(\frac{1}{\sqrt{a_{jj}}}, \quad j = 1,2,...6\right).$$

For the given positive definite non-diagonal matrix $\bar{\bar{A}}_0$ we construct an orthogonal matrix $\bar{\bar{R}}$ by eigenvectors of $\bar{\bar{A}}_0$ such that

$$\bar{\bar{R}}^T\bar{\bar{A}}_0\bar{\bar{R}} = \bar{\bar{L}} \equiv diag(\lambda_j, \quad j = 1,2,...6),$$

where $\bar{\bar{R}}^T$ is the transpose matrix of $\bar{\bar{R}}$ and $\lambda_k > 0$, $k = 1,2,...6$ are eigenvalues of $\bar{\bar{A}}_0$. Then $\bar{\bar{A}}_0^{1/2}$ is defined by $\bar{\bar{A}}_0^{1/2} = \bar{\bar{R}}\bar{\bar{L}}^{1/2}\bar{\bar{R}}^T$, where $\bar{\bar{L}}^{1/2} = diag(\sqrt{\lambda_j}, \quad j = 1,2,...6)$. The matrix $\bar{\bar{A}}_0^{-1/2}$ is the inverse to $\bar{\bar{A}}_0^{1/2}$. Let us take the given symmetric matrix $\mathscr{A}(\nu)$ and the matrix $\bar{\bar{A}}_0^{-1/2}$ which assumed to be found. Let us consider the matrix $\bar{\bar{A}}_0^{-1/2}\mathscr{A}(\nu)\bar{\bar{A}}_0^{-1/2}$ which is symmetric. The diagonal matrix $\bar{\bar{D}}(\nu) = diag(d_1(\nu), d_2(\nu), ..., d_6(\nu))$ is constructed by eigenvalues of $\bar{\bar{A}}_0^{-1/2}\mathscr{A}(\nu)\bar{\bar{A}}_0^{-1/2}$. The columns of the orthogonal matrix $\bar{\bar{Q}}(\nu)$ are formed by normalized orthogonal eigenvectors of $\bar{\bar{A}}_0^{-1/2}\bar{\bar{A}}(\nu)\bar{\bar{A}}_0^{-1/2}$ corresponding to eigenvalues $d_k(\nu)$, $k = 1,2,...,6$. The matrix $\bar{\bar{T}}(\nu)$ is defined by the formula $\bar{\bar{T}}(\nu) = \bar{\bar{A}}_0^{-1/2}\bar{\bar{Q}}(\nu)$. We note that computing $\bar{\bar{D}}(\nu)$, $\bar{\bar{T}}(\nu)$ and $\bar{\bar{T}}^T(\nu)$ for the diagonalization of matrices $\mathscr{A}(\nu)$ and $\bar{\bar{A}}_0$ is similar to the procedure from the paper [Yakhno (2011)].

In the second step, we are looking for the solution of (12) in the form $\tilde{\mathbf{U}}(\nu,t) = \bar{\bar{T}}(\nu)\mathbf{V}(\nu,t)$, where the matrix $\bar{\bar{T}}(\nu)$ is constructed in the first step and a vector function $\mathbf{V}(\nu,t)$ is unknown. Substituting $\tilde{\mathbf{U}}(\nu,t) = \bar{\bar{T}}(\nu)\mathbf{V}(\nu,t)$ into (12) and then multiplying the obtained differential equation by $\bar{\bar{T}}^T(\nu)$ and using (13) we find for $\nu \in R^3, t \in R$

$$\frac{d\mathbf{V}(\nu,t)}{dt} + ic\bar{\bar{D}}(\nu)\mathbf{V}(\nu,t) = \bar{\bar{T}}^T(\nu)\tilde{\mathbf{F}}(\nu,t), \quad \mathbf{V}(\nu,t)|_{t\leq 0} = 0. \tag{14}$$

In the third step of the method, using the ordinary differential equations technique (see, for example Boyce and DiPrima (1992)), the solution of the initial value problem (14) is obtained in the following form

$$\mathbf{V}(\nu,t) = \theta(t)\int_{-\infty}^{t}\Big[\cos\Big(c\bar{\bar{D}}(\nu)(t-\tau)\Big) - i\sin\Big(c\bar{\bar{D}}(\nu)(t-\tau)\Big)\Big]\bar{\bar{T}}^T(\nu)\tilde{\mathbf{F}}(\nu,\tau)d\tau, \tag{15}$$

where $\nu \in R^3, t \in R$; $\theta(t) = 1$ for $t \geq 0$ and $\theta(t) = 0$ for $t < 0$; $\cos\Big(\bar{\bar{D}}(\nu)t\Big)$ and $\sin\Big(\bar{\bar{D}}(\nu)t\Big)$ are the diagonal matrices whose diagonal elements are $\cos(d_n(\nu)t)$ and $\sin(d_n(\nu)t)$, respectively, $n = 1,2,...,6$.

In the last step, using (15) and the equality $\tilde{\mathbf{U}}(\nu,t) = \bar{\bar{T}}(\nu)\mathbf{V}(\nu,t)$, we find the exact solution of (12) by the formula

$$\tilde{\mathbf{U}}(\nu,t) = \theta(t)\bar{\bar{T}}(\nu)\int_{-\infty}^{t}\Big[\cos\Big(c\bar{\bar{D}}(\nu)(t-\tau)\Big) - i\sin\Big(c\bar{\bar{D}}(\nu)(t-\tau)\Big)\Big]\bar{\bar{T}}^T(\nu)\tilde{\mathbf{F}}(\nu,\tau)d\tau. \tag{16}$$

Here $\bar{\bar{T}}(\nu)$, $\bar{\bar{D}}(\nu)$, $\bar{\bar{T}}^T(\nu)$ are matrices obtained from $\bar{\bar{\varepsilon}} = (\varepsilon_{km})_{3\times 3}$, $\bar{\bar{\mu}} = (\mu_{km})_{3\times 3}$, $\bar{\bar{\xi}} = (\xi_{km})_{3\times 3}$, $\bar{\bar{\eta}} = (\eta_{km})_{3\times 3}$ and the procedure, described in the Remark 3.

4.3 The formula for the solution of (10)

Applying the inverse Fourier transform with respect to $\nu = (\nu_1,\nu_2,\nu_3) \in R^3$ to the formula (16) the real valued solution of (10) is obtained in the following form

$$\vec{U}(x,t) = \frac{\theta(t)}{(2\pi)^3}\int_{-\infty}^{t}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\bar{\bar{T}}(\nu)\Big[\cos\Big(c\bar{\bar{D}}(\nu)(t-\tau)\Big)\cos(x\cdot\nu) - \sin\Big(c\bar{\bar{D}}(\nu)(t-\tau)\Big)\sin(x\cdot\nu)\Big]\bar{\bar{T}}^T(\nu)\tilde{\mathbf{F}}(\nu,\tau)d\nu_1 d\nu_2 d\nu_3 d\tau, \tag{17}$$

where $\nu = (\nu_1,\nu_2,\nu_3) \in R^3$; $\quad x\cdot\nu = x_1\nu_1 + x_2\nu_2 + x_3\nu_3$.

5 Computation of the electromagnetic radiation in bi-anisotropic materials arising from a polarized dipole

The formula (17) has been used for the computation of the electric and magnetic fields arising from the current $\vec{J} = \vec{e}\delta(x_1)\delta(x_2)\delta(x_3)\delta(t)$, where $\vec{e}$ is a given vector from R^3 and $\delta(\cdot)$ is the Dirac delta function. We note that for this case the formula (17) has the form

$$\vec{U}(x,t) = \frac{\theta(t)}{(2\pi)^3} \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \bar{\bar{T}}(\nu)\left[\cos\left(c\bar{\bar{D}}(\nu)t\right)\cos(x\cdot\nu) - \sin\left(c\bar{\bar{D}}(\nu)t\right)\sin(x\cdot\nu)\right]\bar{\bar{T}}^T(\nu)\vec{e}d\nu_1 d\nu_2 d\nu_3. \tag{18}$$

The values of matrices $\mathbf{D}(\nu)$, $\mathbf{T}(\nu)$ appearing in (18) have been computed for the given symmetric positive definite matrix $\bar{\bar{A}}_0$ and symmetric matrices $\bar{\bar{A}}_j$, $j = 1,2,3$ using the following MATLAB codes:

$\texttt{Input:}\ A_0$
$[EigVecA0, EigValA0] = eig(A_0);$
$\mathscr{R} = EigVecA0;$
$PT = \mathscr{R}';$
$\mathscr{L} = EigValA0;$
$Mh = sqrt(\mathscr{L});$
$SqrA0 = P * Mh * PT;$
$InvSqrA0 = inv(SqrA0);$
$\texttt{Output:}\ InvSqrA0 = inv(SqrA0)$

and

$\texttt{Input:}\ \nu_1,\ \nu_2,\ \nu_3,\ A_1,\ A_2,\ A_3,\ InvSqrA0$
$[EigVecA0, EigValA0] = eig(A_0);$
$B = \nu_1 A_1 + \nu_2 A_2 + \nu_3 A_3;$
$H = InvSqrA0 * B * InvSqrA0;$
$[EigVecH, EigValH] = eig(H);$
$\mathbf{De}(\nu) = simplify(EigValH);$
$\mathbf{Q}(\nu) = simplify(EigVecH);$
$T(\nu) = simplify(InvSqrA0 * \mathbf{Q}(\nu));$
$\mathbf{D}(\nu) = diag(\mathbf{De}(\nu))$
$\texttt{Output:}\ T(\nu), \mathbf{D}(\nu)$

In this section we consider three homogenous bi-anisotropic materials. The first one is characterized by the permittivity $\overline{\overline{\varepsilon}} = diag(2.25, 1, 0.25)$, permeability $\overline{\overline{\mu}} = diag(0.25, 1, 2.25)$ and magnetoelectric tensors

$$\overline{\overline{\eta}} = \overline{\overline{\xi}} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0.1 & 0 \\ 0 & 0 & 0.01 \end{bmatrix}.$$

The second bi-anisotropic material is characterized by the permittivity $\overline{\overline{\varepsilon}} = diag(2.25, 1, 0.25)$, permeability $\overline{\overline{\mu}} = diag(0.25, 1, 2.25)$ and magnetoelectric tensors

$$\overline{\overline{\eta}} = \overline{\overline{\xi}} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

The third material has the permittivity, permeability and magnetoelectric tensors given by

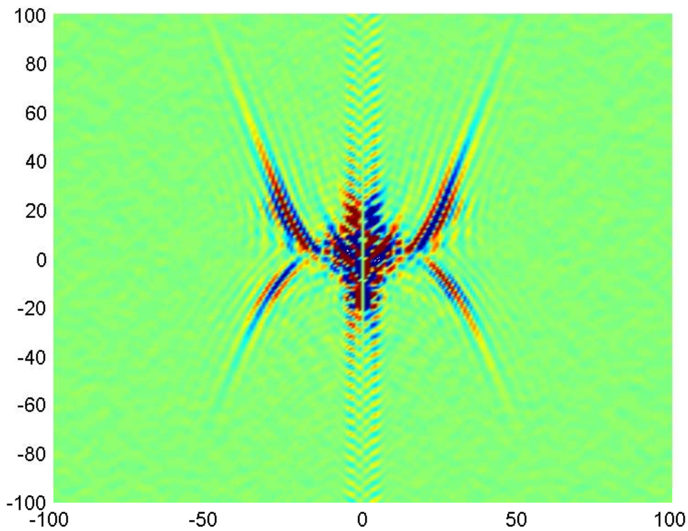
$$\overline{\overline{\varepsilon}} = \begin{bmatrix} 30.7929 & -12.7337 & -14.3432 \\ -12.7337 & 5.51479 & 5.86982 \\ -14.3432 & 5.86982 & 6.74556 \end{bmatrix}, \ \overline{\overline{\mu}} = \mathbf{I},$$

$$\overline{\overline{\eta}} = \overline{\overline{\xi}} = \begin{bmatrix} 0 & 0.25 & 0 \\ 0.25 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

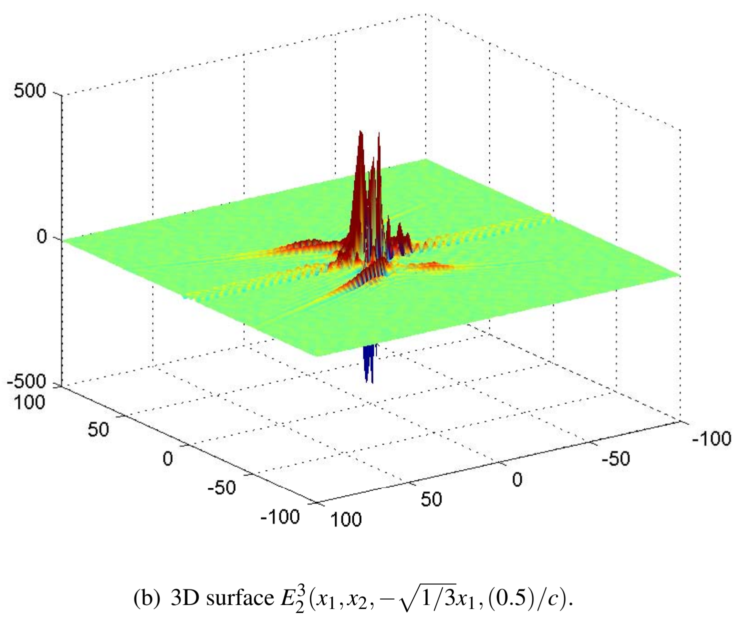
Applying the method developed in the present paper, we have computed the matrices $\mathbf{T}^*(\nu), \mathbf{T}(\nu), \mathbf{D}(\nu)$. After that, using formula (18) we have calculated the electric and magnetic fields.

Denoting the solution $\vec{E}^k(x,t)$, $\vec{H}^k(x,t))$ of (7)–(9) for $\vec{J} = \mathbf{e}^k\delta(x_1)\delta(x_2)\delta(x_3)\delta(t)$, the electric and magnetic fields $\vec{E}^k(x,t)$, $\vec{H}^k(x,t))$, arising from the dipole $\vec{J} = \mathbf{e}^k\delta(x_1)\delta(x_2)\delta(x_3)\delta(t)$, $k = 1,2,3$, in non-dispersive bi-anisotropic materials, have been computed. Here $\mathbf{e}^1 = (1,0,0)$, $\mathbf{e}^2 = (0,1,0)$, $\mathbf{e}^3 = (0,0,1)$. The results of the computation of (E_1^3, E_2^3, E_3^3), (H_1^3, H_2^3, H_3^3) in above mentioned bi-anisotropic materials are presented in Figs. 1-6.

The 3D surfaces of $z = E_2^3(x_1, x_2, -\sqrt{1/3}x_1, t)$ and $z = H_2^3(x_1, x_2, -\sqrt{1/3}x_1, t)$ for the first material at the time $t = (0.5)/c)$ and points x belonging to the plane $x \cdot \overrightarrow{n} = 0$, where $\overrightarrow{n} = (\frac{\sqrt{3}}{2}, 0, \frac{1}{2})$, are presented in Figs.1, 2. In Fig.1(b), the vertical axis is the magnitude of $z = E_2^3(x_1, x_2, -\sqrt{1/3}x_1, t)$, $t = (0.5)/c$, and the horizontal ones are x_1 and x_2. In Fig.2(b) the vertical axis is the magnitude of $z =$

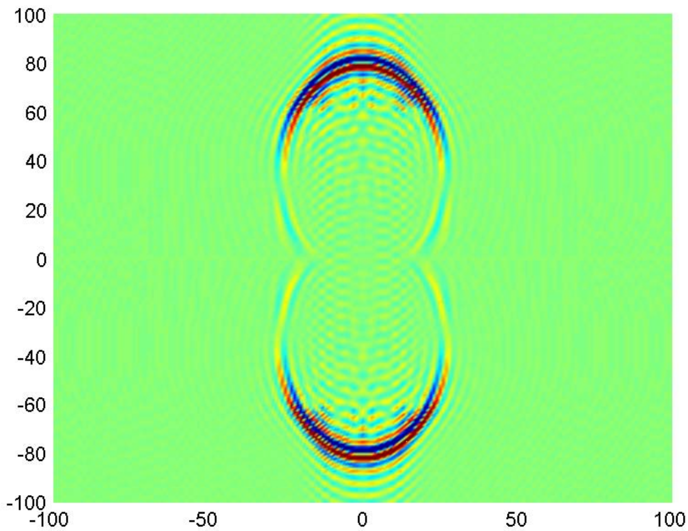


(a) The map of the surface of $E_2^3(x_1, x_2, -\sqrt{1/3}x_1, (0.5)/c)$.

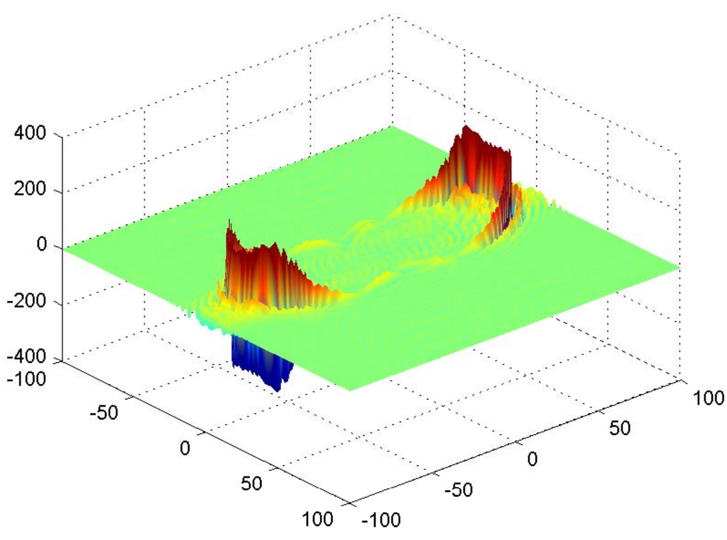


(b) 3D surface $E_2^3(x_1, x_2, -\sqrt{1/3}x_1, (0.5)/c)$.

Figure 1: Bi-anisotropic material 1. Graphs of the second component of the electric field $\vec{E}^3(x,t)$ for points of the plane $x_1 + \sqrt{3}x_3 = 0$ at $t = 1/(2c)$.

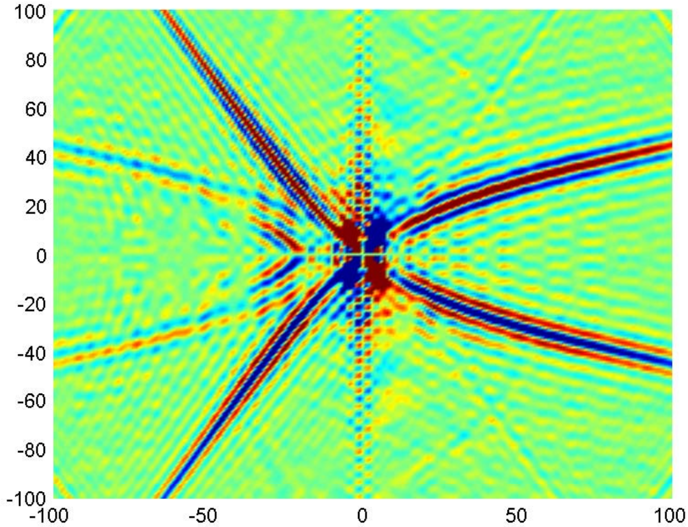


(a) The map of the surface of $H_2^3(x_1, x_2, -\sqrt{1/3}x_1, (0.5)/c)$.

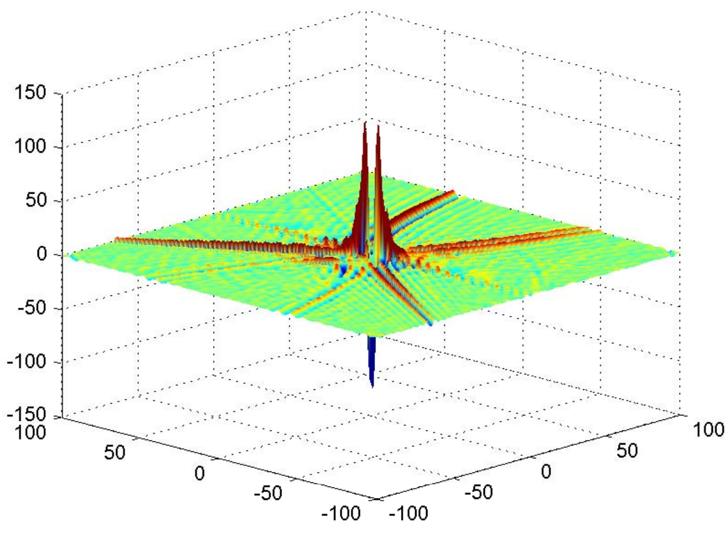


(b) 3D surface $H_2^3(x_1, x_2, -\sqrt{1/3}x_1, (0.5)/c)$.

Figure 2: Bi-anisotropic material 1. Graphs of the second component of the magnetic field $\vec{H}^3(x,t)$ for points of the plane $x_1 + \sqrt{3}x_3 = 0$ at $t = 1/(2c)$.

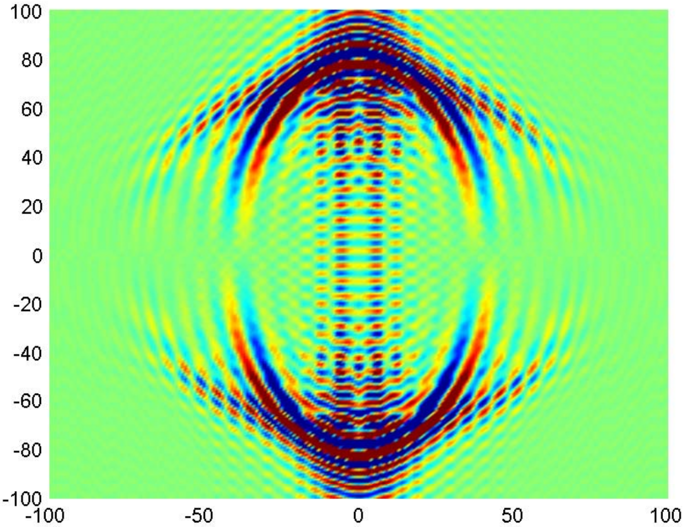


(a) The map of the surface of $E_2^3(x_1, x_2, -\sqrt{1/3}x_1, (0.5)/c)$.

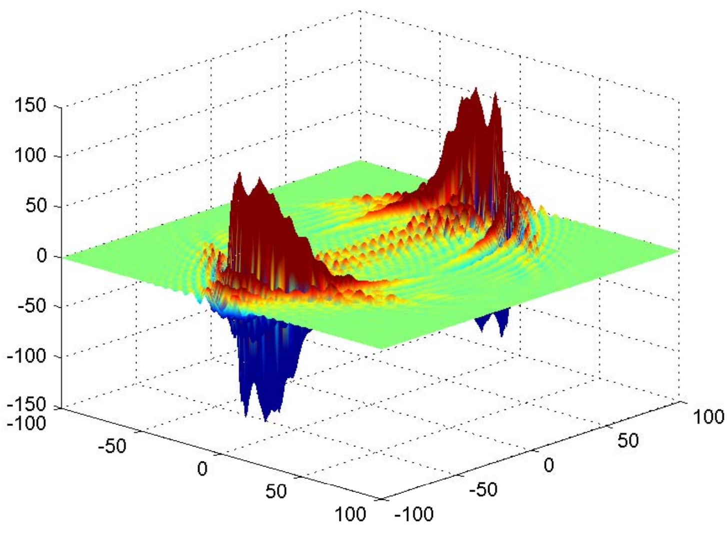


(b) 3D surface $E_2^3(x_1, x_2, -\sqrt{1/3}x_1, (0.5)/c)$.

Figure 3: Bi-anisotropic material 2. Graphs of the second component of the electric field $\vec{E}^3(x,t)$ for points of the plane $x_1 + \sqrt{3}x_3 = 0$ at $t = 1/(2c)$.

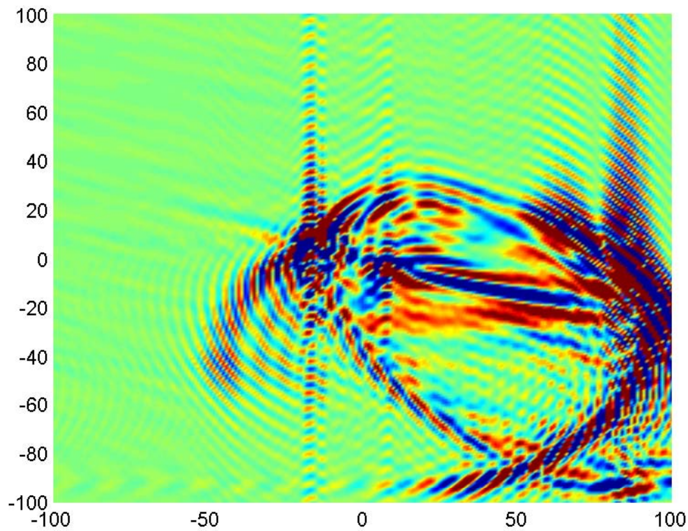


(a) The map of the surface of $H_2^3(x_1, x_2, -\sqrt{1/3}x_1, (0.5)/c)$.

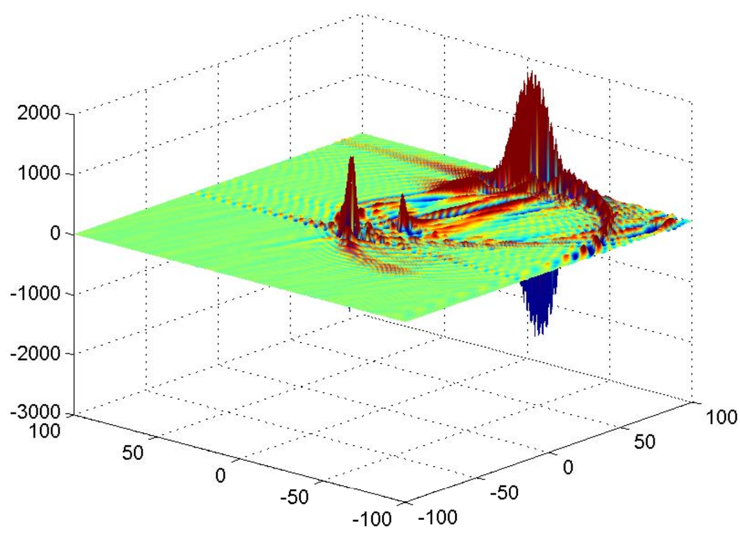


(b) 3D surface $H_2^3(x_1, x_2, -\sqrt{1/3}x_1, (0.5)/c)$.

Figure 4: Bi-anisotropic material 2. Graphs of the second component of the magnetic field $\vec{H}^3(x,t)$ for points of the plane $x_1 + \sqrt{3}x_3 = 0$ at $t = 1/(2c)$.

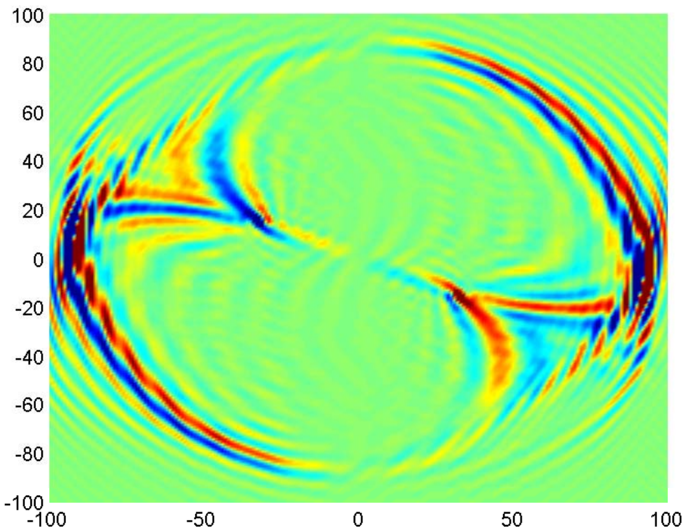


(a) The map of the surface of $E_2^3(x_1, x_2, -\sqrt{1/3}x_1, (0.15)/c)$.

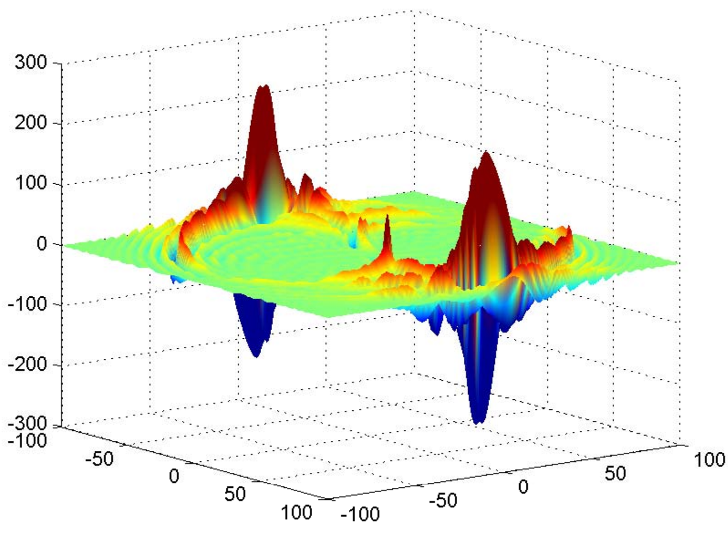


(b) 3D surface $E_2^3(x_1, x_2, -\sqrt{1/3}x_1, (0.15)/c)$.

Figure 5: Bi-anisotropic material 3. Graphs of the second component of the electric field $\vec{E}^3(x,t)$ for points of the plane $x_1 + \sqrt{3}x_3 = 0$ at $t = (0.15)/c$.



(a) The map of the surface of $H_2^3(x_1, x_2, -\sqrt{1/3}x_1, (0.5)/c)$.



(b) 3D surface $H_2^3(x_1, x_2, -\sqrt{1/3}x_1, (0.5)/c)$.

Figure 6: Bi-anisotropic material 3. Graphs of the second component of the magnetic field $\vec{H}^3(x,t)$ for points of the plane $x_1 + \sqrt{3}x_3 = 0$ at $t = 1/(2c)$.

$H_2^3(x_1, x_2, -\sqrt{1/3}x_1, (0.5)/c)$. The different colors correspond to different values of magnitude. Figs. 1(a), 2(a) contain the map of same surfaces, i.e. view of these surfaces from the top of z axis. Here x_1-axis is horizontal and x_2-axis is vertical. The 3D surface plots of $z = E_2^3(x_1, x_2, -\sqrt{1/3}x_1, t)$ and $z = H_2^3(x_1, x_2, -\sqrt{1/3}x_1, t)$ for the second material at the time $t = (0.5)/c)$ and points x belonging to the plane $x \cdot \overrightarrow{n} = 0$, where $\overrightarrow{n} = (\frac{\sqrt{3}}{2}, 0, \frac{1}{2})$, are presented in Figs.3, 4. In Fig.3(b), the vertical axis is the magnitude of $z = E_2^3(x_1, x_2, -\sqrt{1/3}x_1, (0.5)/c)$. In Fig.4(b) the vertical axis is the magnitude of $z = H_2^3(x_1, x_2, -\sqrt{1/3}x_1, (0.5)/c)$. Figs. 3(a), 4(a) contain the map of same surfaces.

The 3D surface plots of $z = E_2^3(x_1, x_2, -\sqrt{1/3}x_1, t)$ for the third material at the time $t = (0.15)/c)$ and points x belonging to the plane $x \cdot \overrightarrow{n} = 0$ ($\overrightarrow{n} = (\frac{\sqrt{3}}{2}, 0, \frac{1}{2})$) are presented in Figs.5(a), 5(b). In Fig.5(b), the vertical axis is the magnitude of $z = E_2^3(x_1, x_2, -\sqrt{1/3}x_1, (0.15)/c)$. Fig.5(a) contains the map of same surface $z = E_2^3(x_1, x_2, -\sqrt{1/3}x_1, t)$ at $t = (0.15)/c)$. In Fig.6(b) the vertical axis is the magnitude of $z = H_2^3(x_1, x_2, -\sqrt{1/3}x_1, t)$ ($t = (0.5)/c$). Fig.6(a) contains the map of the surface.

6 Summary and conclusion

Applying the approach, developed in this study, the Maxwell's equations for the bi-anisotropic materials have been written as a symmetric hyperbolic system. The Fourier image of this system, with respect to the space variables, is a system of the ordinary differential equations whose coefficients depend on the 3D Fourier parameter. The formula for the solution of the obtained system is derived by the matrix transformations. Finally, the electric and magnetic fields are computed by the inverse Fourier transform. Using this formula the calculation of the electric and magnetic fields has been made for the case when the current is a polarized dipole.

The computational experiments have confirmed the robustness of the suggested method. The visualization of electric and magnetic fields in homogeneous non-dispersive bi-anisotropic materials by modern computer tools allows engineers to observe and evaluate the dependence between the structure of the materials and the behavior of the electric and magnetic fields.

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