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Fluid Flow Simulation Using Particle Method and Its Physics-based Computer Graphics

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Abstract: The application of a particle method to incompressible viscous fluid flow problem and its physics-based computer graphics are presented. The method is based on the MPS (Moving Particle Semi-implicit) scheme using logarithmic weighting function to stabilize the spurious oscillatory solutions for the pressure fields which are governed by Poisson equation. The physics-based computer graphics consist of the POV-Ray (Persistence of Vision Raytracer) rendering using marching cubes algorithm as polygonization. The standard MPS scheme is widely utilized as a particle strategy for the free surface flow, the problem of moving boundary, multi-physics/multi-scale ones, and so forth. Numerical results demonstrate the workability and the validity of the present approach through dam-breaking flow problem.

Keywords: particle method, MPS, logarithmic weighting function, dam-break problem, physics-based CG.

1 Introduction

From a practical point of view, the numerical simulations of moving boundary, multi-physics and multi-scale problems are indispensable in the wide fields of science and engineering. It is not easy to simulate such problems by using the grid/element-based schemes, namely finite difference method and finite element method. There are various gridless/meshless-based methodologies, such as SPH (Smoothed Particle Hydrodynamics) method [Lucy (1977);Gingold and Monaghan (1977)], MPS (Moving Particle Semi-implicit) method [Koshizuka and Oka (1996)], EFG (Element Free Galerkin) method [Belytschko, Lu and Gu (1994)], MLPG (Meshless Local Petrov-Galerkin) method [Atluri and Zhu (1998)], and MP (Meshfree Particle) one [Li and Liu (2002)], to simulate effectively such complex problems.

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The SPH methods for solving compressible fluid flows with gravity have been firstly developed in the field of astrophysics [Lucy (1977);Gingold and Monaghan (1977)], and applied successfully to a wide variety of complicated physical problems, including free surface incompressible flows [Monaghan (1994);Sakai, Yang and Jung (2004)] involving breaking dam, wave propagation, and so forth, thermal conduction with heat flux across discontinuities in material properties [Cleary and Monaghan (1999)], impact fracture in solids [Swegle, Hicks and Attaway (1995);Hoover (2006)], and the behaviors of arctic sea ice in oceanography [Lindsay and Stern (2004)]. The MPS method [Koshizuka and Oka (1996)] as an incompressible fluid flow solver has been widely applied to the problem of breaking wave with large deformation [Koshizuka, Nobe and Oka (1998)], the fluid-structure interaction problem [Chikazawa, Koshizuka and Oka (2001)], and the micro multiphase flow one [Harada, Suzuki, Koshizuka, Arakawa and Shoji (2006)]. However, the standard/original MPS method leads to the unphysical numerical oscillation of pressure fields which are described by the discretized Poisson equation. To improve some shortcomings of the standard MPS method, Khayyer and Gotoh have proposed the modified MPS method for the prediction of wave impact pressure on a coastal structure to ensure more exact momentum conservation [Khayyer and Gotoh (2009)]. The improvement of stability in the original MPS method [Koshizuka and Oka (1996)] has been more recently achieved by adding some source terms into Poisson pressure equation [Kondo and Koshizuka (2011)]. Belytschko et al. [Belytschko, Lu and Gu (1994)] have proposed significantly the element-free Galerkin method which was based upon the moving least-squares interpolants [Lancaster and Salkauskas (1981)] and Galerkin weak formulation. They applied the EFG method to two-dimensional problems of elasticity and heat conduction and obtained excellent results. Atluri and Zhu [Atluri and Zhu (1998)] have developed the MLPG approach based on the local symmetric weak form and the moving least squares for solving accurately potential problems, and the approach was extended to deal with the problems for convection-diffusion equation [Lin and Atluri (2000)] and incompressible Navier-Stokes equations [Lin and Atluri (2001)] in fluid dynamics. Some reviews of meshfree and particle methods have been excellently presented by Li and Liu [Li and Liu (2002)].

The purpose of this paper is to present application of the MPS method using logarithmic weighting function to incompressible viscous fluid flow problem, namely flow in dam-break problem [Martin and Moyce (1952); Hirt and Nichols (1981); Ramaswamy and Kawahara (1987)], which is one of the well-known typical ones in the framework of incompressible fluid flow. To overcome such spurious oscillations in the standard MPS method, we propose to utilize the logarithmic weighting function and also take into consideration the reduction of *ad hoc* influence radius for solving an auxiliary Poisson equation for the pressure field. As a reason for the consideration, the numerical treatment of the incompressibility constraint is one of more difficult aspects of incompressible Navier-Stokes equations, and the pressure field is determined such that the continuity equation is satisfied on the average in each grid/element. In the finite element framework, the approach is successful only if the pressure field is interpolated with functions at least one order lower than those the velocity vector field [Brooks and Hughes (1982)]. As the physics-based computer graphics, the polygonization of numerical data is constructed by using the well-known marching cubes technique [Lorensen and Cline (1987)] as the most popular isosurfacing extraction algorithm, and also the rendering is illustrated in using the generated polygons and POV-Ray [POV-Ray (1989)]. The workability and validity of the present approach are demonstrated through the dam-breaking flow problem, and compared with experimental data and other numerical ones.

Throughout this paper, the summation convention on repeated indices is employed. A comma following a variable is used to denote partial differentiation with respect to the spatial variable.

2 Statement of the problem

Let Ω be a bounded domain in Euclidean space with a piecewise smooth boundary Γ. The unit outward normal vector to Γ is denoted by $\boldsymbol{n}$. Also, $\Im$ denotes a closed time interval.

The motion of an incompressible viscous fluid flow is governed by the following Navier-Stokes equations :

$$\frac{Du_i}{Dt} = -\frac{1}{\rho}p_{,i} + \nu u_{i,jj} + f_i \qquad in\ \Im \times \Omega \tag{1}$$

$$\frac{D\rho}{Dt} = 0 \qquad in\ \Im \times \Omega \tag{2}$$

where u_i is the velocity vector component, ρ is the density, p is the pressure, f_i is the external force, ν is the kinematic viscosity, and D/Dt denotes the Lagrangian differentiation.

In addition to Eq. 1 and Eq. 2, we prescribe the initial condition $u_i(\boldsymbol{x},0) = u_i^0$, where u_i^0 denotes the given initial velocity, and the Dirichlet and Neumann boundary conditions.

3 Standard MPS formulation

Let us briefly describe the MPS as one of the particle methods [Koshizuka and Oka (1996)]. The particle interaction models as illustrated in Fig. 1(a) are prepared with respect to differential operators, namely, gradient, divergence and Laplacian. The incompressible viscous fluid flow is calculated by a semi-implicit algorithm, such as SMAC (Simplified MAC) scheme [Amsden and Harlow (1970)].

The particle number density n at particle i with the neighboring particles j is defined as

$$n_i = \sum_{j \neq i} w(|\boldsymbol{r}_j - \boldsymbol{r}_i|) \tag{3}$$

in which the weighting function $w(r)$ is

$$w(r) = \begin{cases} \dfrac{r_e}{r} - 1 & (r < r_e) \\ 0 & (r \geq r_e) \end{cases} \tag{4}$$

where r_e is the radius of the interaction area as shown in Fig. 1(a).

The model of the gradient vectors at particle i between particles i and j are weighted with the kernel function and averaged as follows :

$$< \boldsymbol{\nabla}\phi >_i = \frac{d}{n^0} \sum_{j \neq i} [\frac{\phi_j - \phi_i}{|\boldsymbol{r}_j - \boldsymbol{r}_i|^2} (\boldsymbol{r}_j - \boldsymbol{r}_i) w(|\boldsymbol{r}_j - \boldsymbol{r}_i|)] \tag{5}$$

where d is the number of spatial dimensions, ϕ_i and ϕ_j denote the scalar quantities at coordinates $\boldsymbol{r}_i$ and $\boldsymbol{r}_j$, respectively, and n^0 is the constant value of the particle number density.

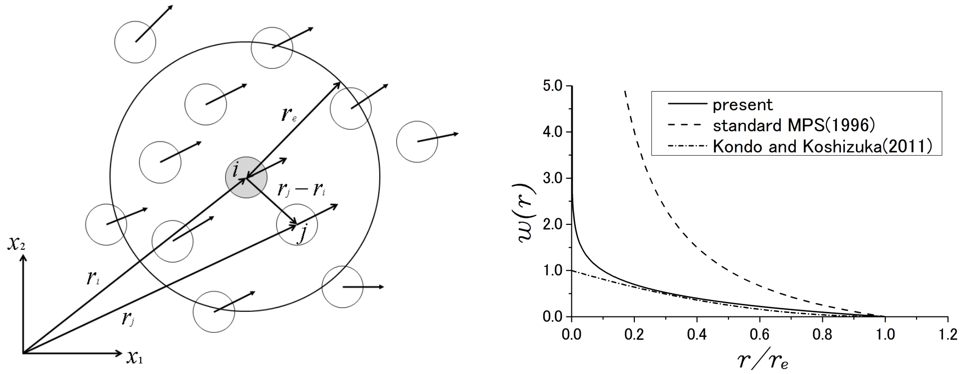
The Laplacian model at particle i is also given by

$$< \boldsymbol{\nabla}^2\phi >_i = \frac{2d}{n^0 \lambda} \sum_{j \neq i} (\phi_j - \phi_i) w(|\boldsymbol{r}_j - \boldsymbol{r}_i|) \tag{6}$$

where λ is an *ad hoc* coefficient.

4 Logarithmic-type weighting function

For the standard MPS formulation mentioned above, the weighting function of Eq. 4 is a key factor in the particle-based framework. If the distance r between the coordinates $\boldsymbol{r}_i$ and $\boldsymbol{r}_j$ is close, then there is a possibility that the computation fails suddenly with unphysical numerical oscillations. Therefore, in order to stabilize such spurious oscillations generated by the standard MPS strategy, we adopt the following logarithmic-type weighting function as shown in Fig. 1(b), and also consider the reduction of *ad hoc* influence radius, r_e, for solving the pressure fields (see Fig. 2). The logarithmic-type weighting function is generally similar to the

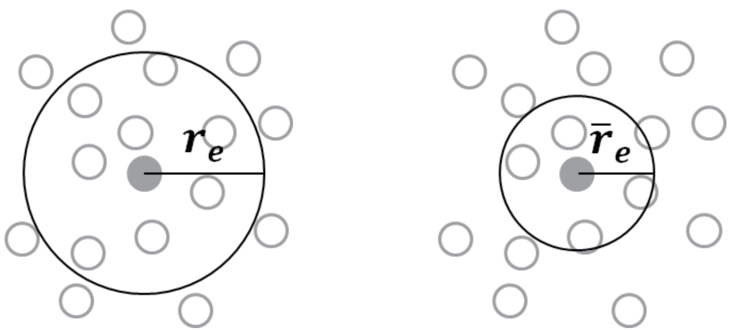


(a) Particle interaction models (b) Profiles of weighting functions

Figure 1: Particle interaction models and weighting functions

profile of the weighting function proposed by Kondo and Koshizuka to stabilize the pressure calculations [Kondo and Koshizuka (2011)](see Fig. 1(b)).

$$w(r) = \begin{cases} log(\frac{r_e}{r}) & (r < r_e) \\ 0 & (r \geq r_e) \end{cases} \tag{7}$$



(a) Velocity field (b) Pressure field

Figure 2: Basic idea for taking the influence radius r_e

5 Physics-based computer graphics

It is also important to visualize realistically the obtained numerical scalar data sets. The marching cubes algorithm proposed by Lorensen and Cline [Lorensen and

Cline (1987)] is the most popular isosurface extraction in the computer-graphics framework, namely scientific visualization. The algorithm has been widely applied to various fields, including biochemistry [Heiden, Goetze and Brickmann (1993)], computational fluid dynamics[Müller, Charypar and Gross (2003)], biomechanics and/or biomedicine[Yim, Vasbinder, Ho and Choyke (2003);Peiró, Formaggia, Gazzola, Radaelli and Rigamonti (2007)], and so forth. Fig. 3 shows the 15 unique intersection topologies by contracting the patterns of reflective and rotational symmetries as illustrated in Fig. 4. A survey of the literature involving the marching cubes algorithm has been presented in detail by Newman and Yi [Newman and Yi (2006)]. The generated polygon data are rendered rapidly by using POV-Ray which is the well-known ray-tracing software [POV-Ray (1989)].

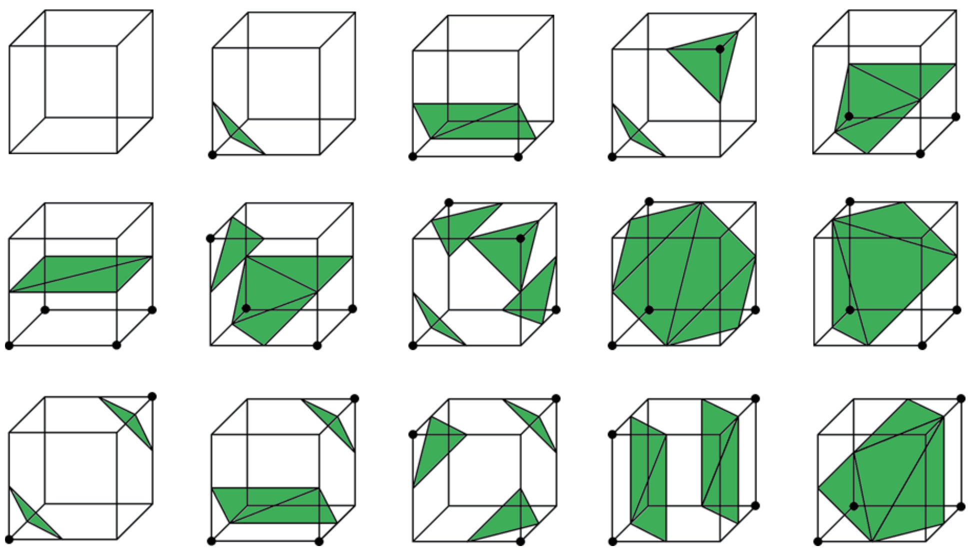


Figure 3: Fifteen basic intersection patterns

6 Numerical example

In this section we present numerical results obtained from applications of the above-mentioned numerical method to dam-breaking incompressible flow problem involving free surface and gravity. The dam-breaking flow problem has been used widely to verify the applicability and validity of the numerical methods [Hirt and Nichols (1981); Ramaswamy and Kawahara (1987); Koshizuka and Oka (1996); Khayyer and Gotoh (2009); Kondo and Koshizuka (2011)]. The initial velocities are assumed to be zero everywhere in the interior domain.

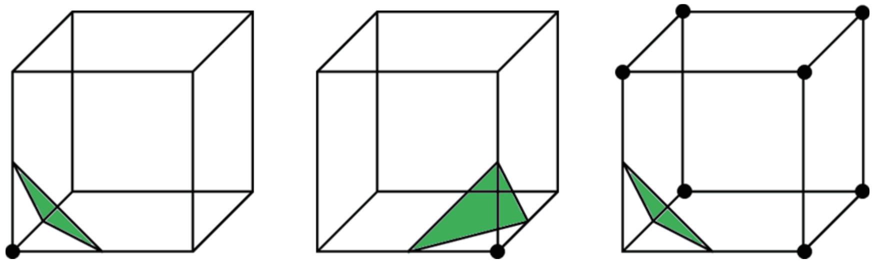
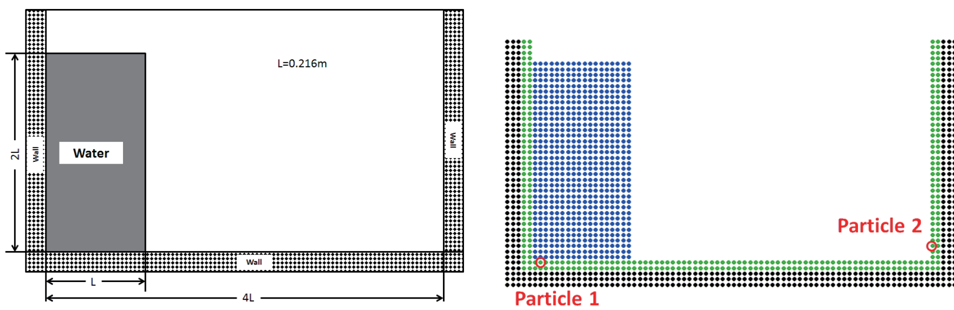


Figure 4: Examples of reflective and rotational symmetries

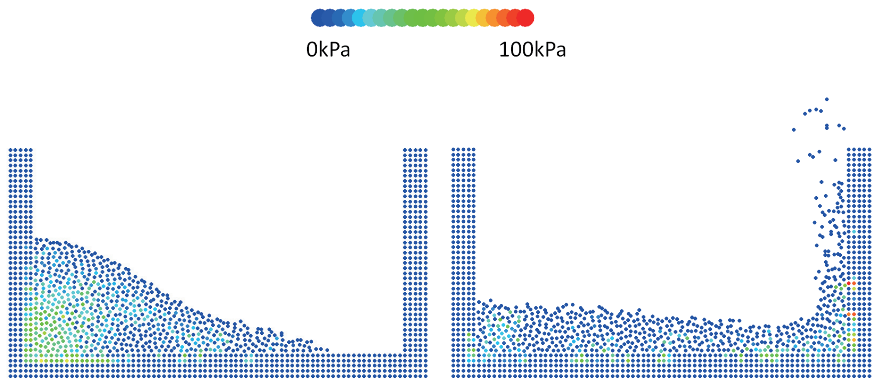


(a) Geometry (b) Initial state of particles

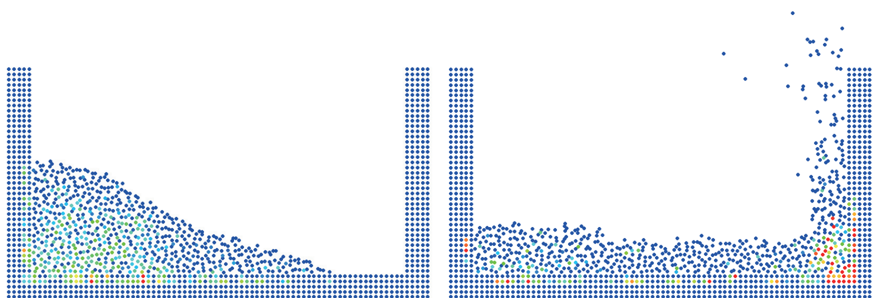
Figure 5: Geometrical configuration and initial state of particles

Fig. 5 shows the geometry and the initial state of particles for flow in the dam-breaking problem. In this two-dimensional simulation, we set $1,458$ particles in the initial configuration, and the CFL condition $u_{max}\Delta t/l_{min} \leq C$, where C is the Courant number $(= 0.025)$. The kernel sizes for the particle number density and the gradient/Laplacian models are $r_e = 4.0l_0$ and $\bar{r}_e = 2.0l_0$ for velocity and pressure calculations, respectively, in which l_0 is the distance between two neighboring particles in the initial state. In this case, we set $l_0 = 0.012m$.

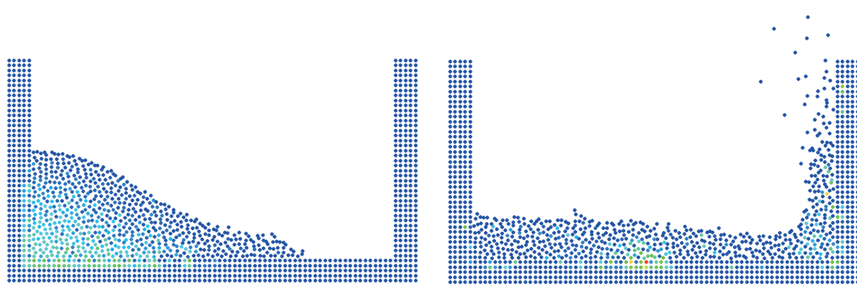
Fig. 6 shows the instantaneous particle/pressure behaviors without the influence radius reduction for different weighting functions. The standard MPS method leads to irregular pressure distributions at early times, while the present distributions are slightly improved as well as the results of Fig. 6(c). Fig. 7 shows the time histories of the pressure at particles 1 and 2 as shown in Fig. 5(b). We see also from Fig. 7 that the pressure-peak values at particle 1 in Fig. 7(b) and (c) are smoother than the standard MPS calculations of Fig. 7(a). The instantaneous particle/pressure behaviors with the influence radius reduction for different weighting functions are



(a) Standard MPS method with Eq. 4

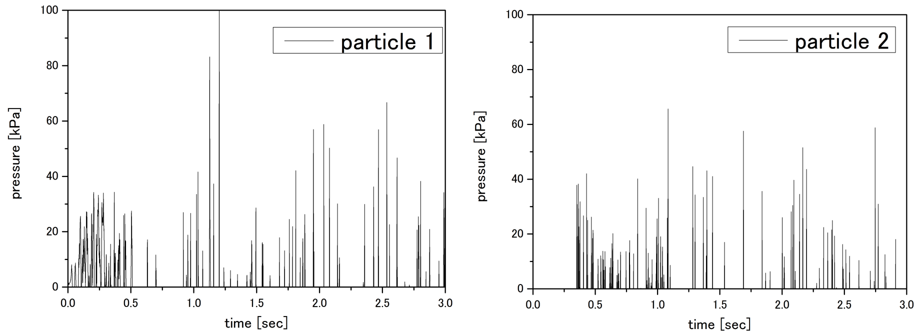


(b) Present; logarithmic-type weighting function of Eq. 7

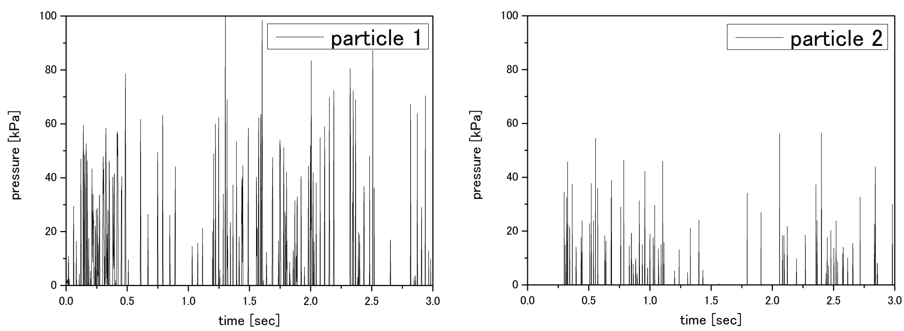


(c) Weighting function proposed by Kondo and Koshizuka

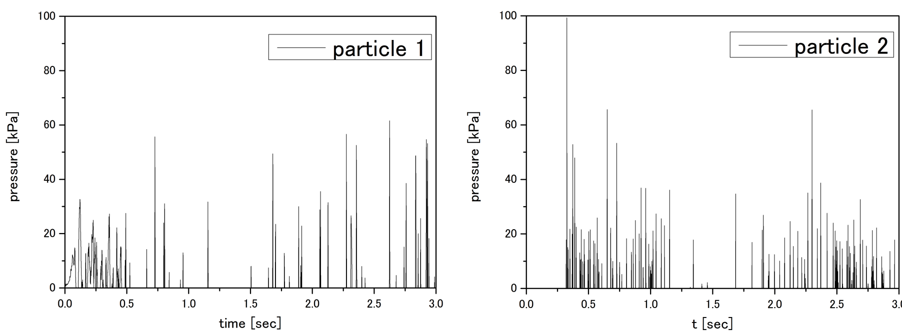
Figure 6: Particle and pressure behaviors at times 0.25 (left) and 0.5 (right) without the influence radius reduction



(a) Standard MPS method with Eq. 4

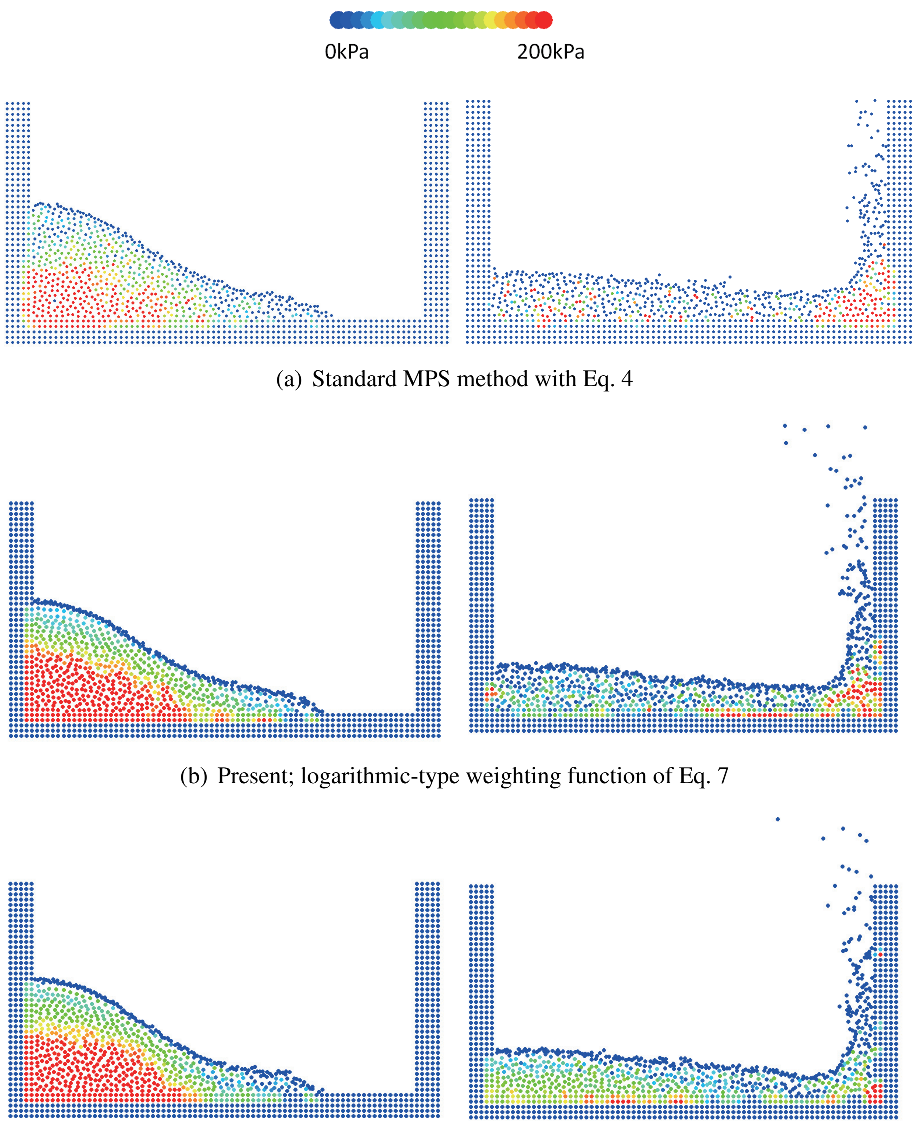


(b) Present; logarithmic-type weighting function of Eq. 7



(c) Weighting function proposed by Kondo and Koshizuka

Figure 7: Time histories of the pressure at particles 1 (left) and 2 (right) without the influence radius reduction

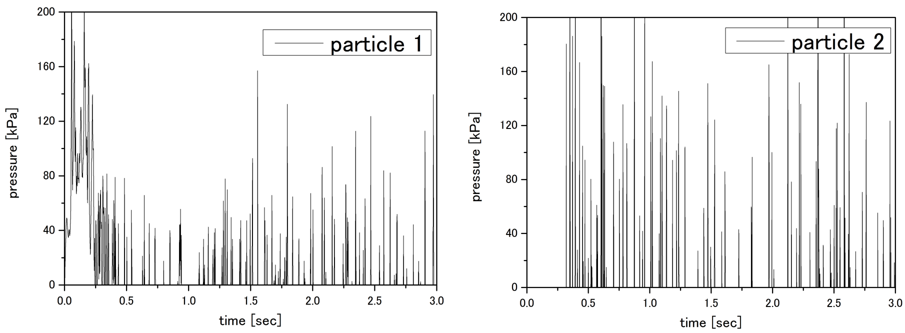


(a) Standard MPS method with Eq. 4

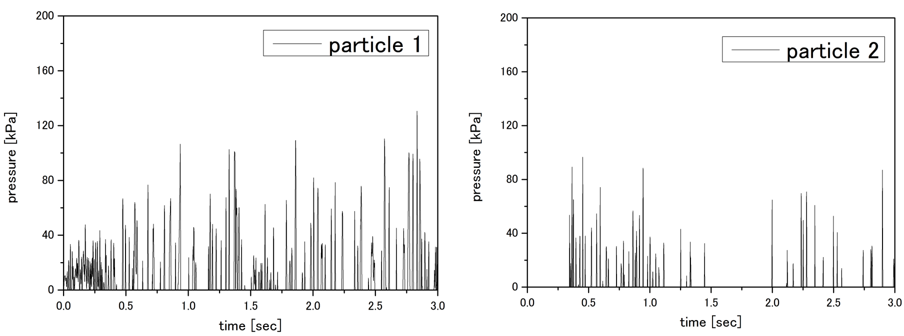
(b) Present; logarithmic-type weighting function of Eq. 7

(c) Weighting function proposed by Kondo and Koshizuka

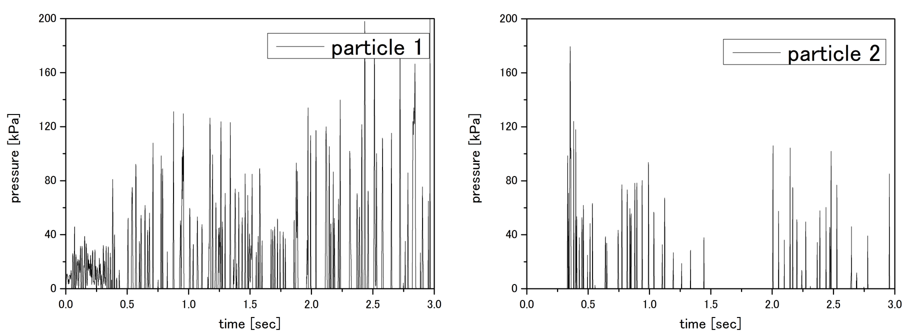
Figure 8: Particle and pressure behaviors at times 0.25 (left) and 0.5 (right) with the influence radius reduction



(a) Standard MPS method with Eq. 4



(b) Present; logarithmic-type weighting function of Eq. 7



(c) Weighting function proposed by Kondo and Koshizuka

Figure 9: Time histories of the pressure at particles 1 (left) and 2 (right) with the influence radius reduction

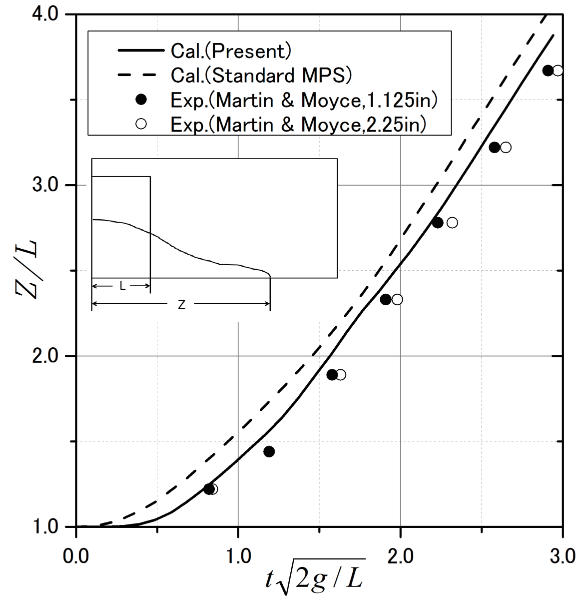
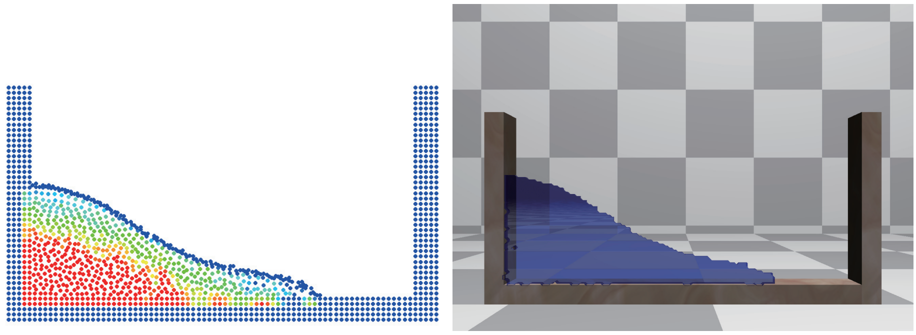


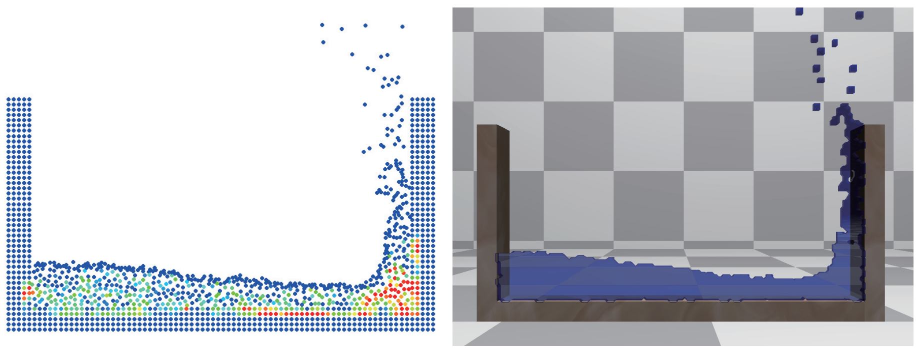
Figure 10: Comparison of present results with experimental data

shown in Fig. 8. The corresponding time histories of the pressure at particles 1 and 2 are also shown in Fig. 9. From both figures, the pressure distributions with the influence radius reduction are generally improved than those without the reduction shown in Fig. 6 and Fig. 7. In particular, the present results are qualitatively similar to the improved MPS simulation. Fig. 10 shows the time evolutions of the leading-edge of the water using the present approach and the standard MPS method through comparison with experimental data [Martin and Moyce (1952)]. The agreement between the present results, the standard MPS simulation and the experimental data appears satisfactory.

For the dam break problem, Fig. 11 shows the particle representations and the POV-Ray rendering using marching cubes algorithm. The convincing results are obtained when the iso-surface of the color field is visualized using the POV-Ray rendering with the marching cubes algorithm as illustrated in Fig. 11. Using the generated polygons and the POV-Ray, our computational results show satisfactory rendering effects.



(a) $t = 0.25$



(b) $t = 0.5$

Figure 11: Particle representations (left) and POV-Ray rendering (right) at different times

7 Conclusions

We have presented the MPS approach using logarithmic-type weighting function for solving numerically incompressible viscous fluid flow involving free surfaces and gravity. The standard MPS method has been widely utilized as a particle strategy for free surface flow, the problem of moving boundary, and multi-physics/multi-scale ones. To overcome spurious oscillations in the standard MPS method, we have proposed to utilize the logarithmic weighting function and also take into the influence radius reduction for solving an auxiliary Poisson equation for the pressure field. The polygonization of numerical data has been constructed by using the marching cubes algorithm, and also the rendering has been significantly illustrated in using the generated polygons and POV-Ray as the physics-based computer graphics. As the numerical example, the well-known dam-breaking flow problem has been carried out and compared with experimental data and other numerical ones. The numerical results obtained herein are summarized as follows:

(1) The standard MPS approach leads to irregular pressure distributions, while the present distributions using logarithmic weighting function are improved slightly.

(2) We can see that the pressure distributions with the influence radius reduction are generally improved than those without the reduction.

(3) The qualitative agreements between the present results and the experimental data appear satisfactory.

(4) The numerical results demonstrate that the approach is capable of solving qualitatively and in a stable manner the complicated flow phenomena involving free surfaces.

(5) The convincing images are visualized using the POV-Ray rendering and the marching cubes algorithm.

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