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Numerical Solutions of the Symmetric Regularized Long Wave Equation Using Radial Basis Functions

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Abstract: In this study, the nonlinear symmetric regularized long wave equation was solved numerically by using radial basis functions collocation method. The single solitary wave solution, the interaction of two positive solitary waves and the clash of two solitary waves were studied. Numerical results and simulations of the wave motions were presented. Validity and accuracy of the method was tested by compared with results in the literature.

Keywords: Radial basis function, collocation method, SRLW equation.

1 Introduction

The symmetric regularized long wave (SRLW) equation has the following form:

$$u_{tt}-u_{xx}+\frac{1}{2}(u^2)_{xt}-u_{xxtt}=0, \qquad x\in\mathbb{R}, \quad t>0. \tag{1}$$

This equation was introduced for the first time by Seyler and Fenstermacher and described derived from a weakly nonlinear ion acoustic and space charge waves [Seyler and Fenstermacher (1984)]. Also the SRLW equation is used in many areas of mathematical physics in [Bogolubsky (1977); Carlson (1989); Rosenau (1988); Shivamoggi (1986)].

The SRLW equation (1) is explicitly symmetric in the x and t derivatives and it is very similar to the regularized long wave (RLW) equation which has the following form

$$U_t+U_x+\varepsilon UU_x-\mu U_{xxt}=0. \tag{2}$$

RLW equation was proposed by Peregrine to describe the undular bore development [Peregrine (1966)] and later it was used to modelling of a class of physical phenomena by Benjamin [Benjamin, Bona, and Mahony (1972)] and to describe shallow

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water waves and plasma drift waves [Albert (1989); Amick, Bona, and Schonbek (1989)]. The RLW and SRLW equations have the solitary wave solutions. Solitary wave solution of the SRLW equation is defined as follows

$$u_c(x,t) = \frac{3(c^2-1)}{c}\operatorname{sech}^2\left(\sqrt{\frac{c^2-1}{4c^2}}(x-ct)\right) \tag{3}$$

where c is the velocity and $c^2 > 1$, [Seyler and Fenstermacher (1984)], therefore the SRLW equation has the bidirectional propagation as depends upon sign of the its velocity.

The SRLW equation (1) can be rewritten as an equivalent first order system form

$$\begin{aligned} u_{xxt} - u_t &= \rho_x + uu_x \\ \rho_t + u_x &= 0 \end{aligned} \tag{4}$$

where ρ and u are the dimensionless electron charge density and the fluid velocity, respectively.

Obviously, the equation (1) is obtained eliminating ρ from the system (4). Therefore derivative of u with respect to t is reduced to first order.

Boundary conditions for the system (4) are given as follows

$$u(a,t) = u(b,t) = 0, \quad \rho(a,t) = \rho(b,t) = 0, \quad x \in [a,b], \; t \in [0,T] \tag{5}$$

and initial conditions

$$u(x,0) = f(x), \quad \rho(x,0) = g(x). \tag{6}$$

In the literature there are some papers about the SRLW equation. Bogolubsky showed that interactions of solitary waves for the SRLW equation were inelastic, thus the solitary wave of the SRLW equation was not solution, [Bogolubsky (1977)]. The orbital stability and instability of solitary wave solutions of the generalized SRLW equations was studied in [Chen (1998)]. The existence, uniqueness and regularity of the periodic initial value problem for a class of the generalized SRLW equations was investigated in [Guo (1987)] and the error estimates of the spectral approximation were obtained. A Fourier pseudospectral method with a restraint operator for the SRLW equation was presented in [Zheng, Zhang, and Guo (1989)], its stability was proved and the optimum error estimates were obtained. The initial boundary value problem for symmetric regularized long wave equations with non homogenous boundary value was considered in [Miao (1994)].

Conservative schemes for the SRLW equation was presented in [Wang, Zhang, and Chen (2007)]. In the paper, numerical solutions were presented for the SRLW equation by using two-level and nonlinear implicit scheme, three-level and linear-implicit scheme and an uncoupled linear-implicit conservative scheme based on the finite difference methods.

The SRLW equation was solved by using the trigonometric integrator pseudospectral discretization method in [Dong (2011)] and presented some test problems and their numerical results.

The aim of this study to solve the SRLW equations system numerically by using radial basis functions collocation method and therefore solitary wave solutions will be obtained for the different test problems.

This method depends upon the meshless solution technique and so there is no need to an extra discretization. To apply the mentioned method firstly a linear form of the given system should be obtained and after a linear equations system will be obtained by using the needed approaches. Therefore, this system will be solved for the different test problems. The detailed instructions are presented at the next sections.

2 Discretization of the SRLW Equation

We discretize the SRLW equation system (4) as follows by using the forward difference approach for the time derivative and Crank-Nicolson formula for the space derivatives

$$
\begin{aligned}
&\frac{u_{xx}^{n+1}-u_{xx}^{n}}{\Delta t}-\frac{u^{n+1}-u^{n}}{\Delta t}=\frac{\rho_x^{n+1}+\rho_x^{n}}{2}+\frac{(uu_x)^{n+1}+(uu_x)^{n}}{2}\\
&\frac{\rho^{n+1}-\rho^{n}}{\Delta t}+\frac{u_x^{n+1}+u_x^{n}}{2}=0.
\end{aligned}
\tag{7}
$$

In this system the nonlinear term uu_x is linearized by using the following approach [Rubin and Graves (1975)]

$$
(uu_x)^{n+1}=u^{n+1}u_x^{n}+u^{n}u_x^{n+1}-u^{n}u_x^{n}. \tag{8}
$$

So the following linear system is obtained

$$
\begin{aligned}
u_{xx}^{n+1}-u_{xx}^{n}-u^{n+1}+u^{n}&=\frac{\Delta t}{2}(\rho_x^{n+1}+\rho_x^{n})+\frac{\Delta t}{2}(u^{n+1}u_x^{n}+u^{n}u_x^{n+1})\\
\rho^{n+1}-\rho^{n}+\frac{\Delta t}{2}(u_x^{n+1}+u_x^{n})&=0.
\end{aligned}
\tag{9}
$$

3 Numerical Method

In this study, to solve the SRLW equation numerically the meshless radial basis function (RBF) collocation method is used. The meshless RBF collocation method was used by Kansa for the first time in papers [Kansa (1990); Kansa (1990)]. In this method, the RBF is used as the kernel function. The general form of a RBF is defined as follows

$$\Phi(x) = \phi(r), \qquad r = \|x\|, \tag{10}$$

where $\|.\|$ is Euclidean norm between any point and reference point. In the literature there are several types of RBFs such as Gaussian, Multiquadric, Inverse multiquadric, Thin plate spline RBFs. Numerical results are calculated by using Gaussian RBF in our algorithms. The general form of the Gaussian RBF is $\phi(r) = e^{-c^2r^2}$. Where the parameter c is called as shape parameter since it dictates the flatness of RBF profile. Shape parameter c has very important on the convergence rate of the approximations and condition number of relevant matrices, in the literature there are some papers about this subject [Cheng, Golberg, Kansa, and Zammito (2003); Rippa (1999); Roque and Ferreira (2009); Schaback (1995); Wu (1992)]. The new strategies to determine the optimal value of shape parameter are still studied. In our algorithms to determine the optimal value of shape parameter is done experimentally choosing by scanning some real values of the interval chosen randomly. Effect of the shape parameter to the numerical error is presented at the test problem 1.

Now, we will show the implementation of the RBF collocation to the SRLW equation. The approximate values of functions $u(x,t)$ and $\rho(x,t)$ in the equations system (9) are approached as follows

$$u^n = \sum_{j=1}^{N} \lambda_j^n \phi(r_{ij}), \qquad \rho^n = \sum_{j=1}^{N} \xi_j^n \phi(r_{ij}) \tag{11}$$

where $\{\lambda_j\}_{j=1}^N$ and $\{\xi_j\}_{j=1}^N$ are unknown coefficients to be determined at each time iterations, and $\phi(r_{ij})$ is the RBFs. The first and second derivatives of the approximate solution (11) with respect to x are obtained as follows

$$u_x^n = \sum_{j=1}^{N} \lambda_j^n \phi^{'}(r_{ij}), \;\; \rho_x^n = \sum_{j=1}^{N} \xi_j^n \phi^{'}(r_{ij}), \;\; u_{xx}^n = \sum_{j=1}^{N} \lambda_j^n \phi^{''}(r_{ij}) \tag{12}$$

Inserting approaches (11) and (12) into equation system (9) at the collocation points

x_i, $i = 1, ..., N$

$$\sum_{j=1}^{N}\left[\phi^{''}(x_{ij})-\phi(x_{ij})-\frac{\Delta t}{2}\phi(x_{ij})\sum_{j=1}^{N}\lambda_j^n\phi^{'}(x_{ij})-\frac{\Delta t}{2}\phi^{'}(x_{ij})\sum_{j=1}^{N}\lambda_j^n\phi(x_{ij})\right]\lambda_j^{n+1}$$
$$+\sum_{j=1}^{N}\left[-\frac{\Delta t}{2}\phi^{'}(x_{ij})\right]\xi_j^{n+1}=\sum_{j=1}^{N}\lambda_j^n\phi^{''}(x_{ij})-\sum_{j=1}^{N}\lambda_j^n\phi(x_{ij})+\frac{\Delta t}{2}\sum_{j=1}^{N}\xi_j^n\phi^{'}(x_{ij})$$
$$\sum_{j=1}^{N}\left[\frac{\Delta t}{2}\phi^{'}(x_{ij})\right]\lambda_j^{n+1}+\sum_{j=1}^{N}\phi(x_{ij})\xi_j^{n+1}=-\frac{\Delta t}{2}\sum_{j=1}^{N}\lambda_j^n\phi^{'}(x_{ij})+\sum_{j=1}^{N}\xi_j^n\phi(x_{ij}) \tag{13}$$

a system of $2N \times 2N$ equations is obtained. This system (13) can be written in matrix form as follows:

$$\begin{pmatrix} A_1 & B_1 \\ A_2 & B_2 \end{pmatrix}\begin{pmatrix} \lambda_j^{n+1} \\ \xi_j^{n+1} \end{pmatrix}=\begin{pmatrix} F_1 \\ F_2 \end{pmatrix} \tag{14}$$

where entries of the matrix are defined as follows:

$$\begin{aligned}
A_1 &= \phi^{''}(x_{ij})-\phi(x_{ij})-\frac{\Delta t}{2}\phi(x_{ij})\sum_{j=1}^{N}\lambda_j^n\phi^{'}(x_{ij}) \\
&\quad -\frac{\Delta t}{2}\phi^{'}(x_{ij})\sum_{j=1}^{N}\lambda_j^n\phi(x_{ij}) \\
A_2 &= \frac{\Delta t}{2}\phi^{'}(x_{ij}) \\
B_1 &= -\frac{\Delta t}{2}\phi^{'}(x_{ij}) \\
B_2 &= \phi(x_{ij}) \\
F_1 &= \lambda_j^n\phi^{''}(x_{ij})-\lambda_j^n\phi(x_{ij})+\frac{\Delta t}{2}\xi_j^n\phi^{'}(x_{ij}) \\
F_2 &= -\frac{\Delta t}{2}\lambda_j^n\phi^{'}(x_{ij})+\xi_j^n\phi(x_{ij})
\end{aligned} \tag{15}$$

This linear system (13) is solved by using Gaussian elimination with partial pivoting at each time step for the different test problems.

4 Numerical Examples and Comparisons

The SRLW equation has four invariants which are [Chen (1998); Seyler and Fenstermacher (1984)]

$$\begin{aligned}
I_1 &= \int_{-\infty}^{\infty} u(x,t)dx, \\
I_2 &= \int_{-\infty}^{\infty} \rho(x,t)dx, \\
I_3 &= \int_{-\infty}^{\infty} \left(u^2(x,t)+u_x^2(x,t)+\rho^2(x,t)\right)dx, \\
I_4 &= \int_{-\infty}^{\infty} \left(u(x,t)\rho(x,t)+\tfrac{1}{6}u^3(x,t)\right)dx.
\end{aligned} \tag{16}$$

In our algorithms, we used a finite solution domain so numerical values of the invariants were calculated in a finite interval $[a,b]$. Also the numerical values of these integrals were computed by using the rectangle rule.

Analytical solution of the single solitary wave motion of the SRLW equation is known, therefore the root mean square error L_2 and the maximum error L_∞ error norms can be calculated as follows

$$L_2 = \sqrt{h\sum_{j=0}^{N}\left|u_j^{exact}-u_j^{numerical}\right|^2}, \tag{17}$$

and

$$L_\infty = \max_j \left|u_j^{exact}-u_j^{numerical}\right|. \tag{18}$$

4.1 Test Problem 1: Single solitary wave motion

The SRLW equation system (4) has the single solitary wave solution as follows

$$\begin{aligned}
u(x,t) &= \frac{3(c^2-1)}{c}sech^2\left(\sqrt{\frac{c^2-1}{4c^2}}(x-ct)\right) \\
\rho(x,t) &= \frac{3(c^2-1)}{c^2}sech^2\left(\sqrt{\frac{c^2-1}{4c^2}}(x-ct)\right)
\end{aligned} \tag{19}$$

for boundary condition $u \to 0$ as $x \to \pm\infty$. To obtain the numerical results for the motion of single solitary wave in a finite domain, boundary conditions

$$u(a,t) = u(b,t) = 0, \quad \rho(a,t) = \rho(b,t) = 0 \tag{20}$$

and initial conditions

$$
\begin{aligned}
u(x,0) &= \frac{3(c^2-1)}{c} sech^2\left(\sqrt{\frac{c^2-1}{4c^2}}x\right)\\
\rho(x,0) &= \frac{3(c^2-1)}{c^2} sech^2\left(\sqrt{\frac{c^2-1}{4c^2}}x\right)
\end{aligned}
\tag{21}
$$

are used. In our computations, single solitary wave simulation is carried out over the solution domain $-20 \leq x \leq 80$ in the time period $0 \leq t \leq 40$ with time step $\Delta t = 0.005$, space step $h = 0.5$ for value $c = \sqrt{2}$.

The computed values of the error norms L_2, L_∞ and invariants are illustrated in Table 1 and presents a comparison with the earlier results in the literature. The invariants remained as unchanged at acceptable rate while time increases. Error norms have been observed as sensitive values when compared with the results of [Wang, Zhang, and Chen (2007)]. Using three different methods, which were two-level and nonlinear implicit scheme, three-level and linear-implicit scheme and an uncoupled linear-implicit conservative scheme numerical values of L_2 were computed as $0.0087, 0.0186, 0.0071$, respectively for time $t = 20$ and $h = \Delta t = 0.05$ in [Wang, Zhang, and Chen (2007)]. It is seen that Gaussian RBF collocation method gives very acceptable numerical results.

Error norms and invariants for the selected values of c at the domain $[0.5, 3]$ for time $t = 1$ are tabulated in Table 2. In Figure 3, it is seen that the sensitivity of the error depends on the choosing of the shape parameter c. The method satisfied acceptable results for conservations and provided higher accuracy for error norms for the values of c^2 at the domain $[0.5, 3]$.

The solitary wave profiles of solutions u and ρ are depicted at times $t = 0$, $t = 20$, $t = 40$ in Figure 1 and Figure 2.

Table 1: Invariants, error norms for a single wave motion

Method	t	I_1	I_2	I_3	I_4	L_2	L_∞
Gaussian	10	11.999140	27.150706	16.798448	8.485736	0.000609	0.000322
	20	11.998323	27.148637	16.796985	8.486198	0.000448	0.000257
	30	11.997829	27.146654	16.795583	8.486654	0.002173	0.001102
	40	11.997488	27.144649	16.794166	8.487162	0.001770	0.000964
[Dong (2011)]	10	12.0000038	27.1529477	16.8000335	8.4852811		
	20	12.0000081	27.1529497	16.8000348	8.4852811		
	30	12.0000124	27.1529515	16.8000361	8.4852811		
	40	12.0000166	27.1529533	16.8000374	8.4852811		

Table 2: Invariants and error norms for the values of shape parameter c^2 at time $t = 1$

c^2	I_1	I_2	I_3	I_4	L_2	L_∞
0.6	12.0000054894	27.1528496247	16.799964	8.485270	0.0003413214	0.0001728535
0.9	11.9999896227	27.1528643163	16.799974	8.485271	0.0003414409	0.0001735687
1.2	11.9999772543	27.1528608669	16.799972	8.485289	0.0003436166	0.0001759529
1.5	11.9999841968	27.1528934426	16.799995	8.485284	0.0003405083	0.0001732111
1.8	12.0000070013	27.1529442389	16.800031	8.485287	0.0003405500	0.0001709461
2.1	11.9999755778	27.1528675797	16.799979	8.485283	0.0003524318	0.0001796484
2.4	11.9999643575	27.1527872667	16.799938	8.485280	0.0004250652	0.0002149343
2.7	12.0000021701	27.1528072175	16.800022	8.485285	0.0007761969	0.0003762245
3.0	11.9999572108	27.1523426244	16.799923	8.485281	0.0019827573	0.0009474754

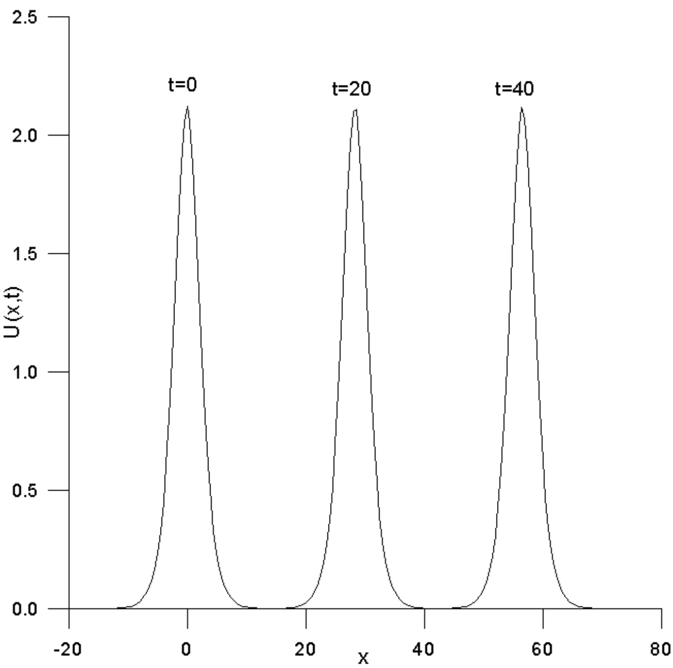


Figure 1: Motion of the single solitary wave at different times

4.2 Test Problem 2: Interaction of Two Solitary Waves

The interaction of two positive solitary waves is considered with initial condition

$$\begin{aligned} u(x,0) &= u_1(x-x_0,0)+u_2(x+x_0,0) \\ u_1(x-x_0,0) &= \frac{3(c_1^2-1)}{c_1} sech^2\left(\sqrt{\frac{c_1^2-1}{4c_1^2}}(x-x_0)\right) \\ u_2(x+x_0,0) &= \frac{3(c_2^2-1)}{c_2} sech^2\left(\sqrt{\frac{c_2^2-1}{4c_2^2}}(x+x_0)\right) \end{aligned} \tag{22}$$

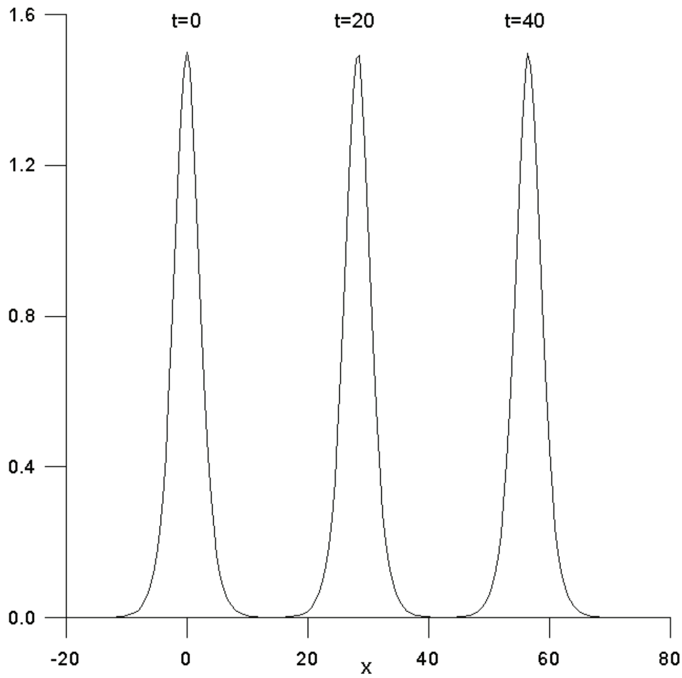


Figure 2: Simulation of the $\rho(x,t)$ at different times

and boundary conditions $u(-30,t) = u(120,t) = 0$. Calculations are carried out in the time period $0 \leq t \leq 16$ with parameters $\Delta t = 0.005$, $h = 0.5$, $c_1 = 2$, $c_2 = 6$ and $x_0 = 12$.

Simulations of two solitary waves profiles are depicted at different times in Figure 4 and also interactions of two solitary waves are depicted as detailed at times $t = 4.6$, $t = 5.5$, $t = 6.8$ in Figure 5. Initially the peak points of two waves are located at the positions $x = -12$ and $x = 12$. After, two solitary waves are propagating to the right with velocities depend upon their magnitudes. While time increasing the larger wave reaches to smaller one and has passed through and distances between waves will be become longer as time increases because of their magnitudes. This interaction is presented as detailed and interaction ended about time $t = 8$. At the end of the running time $t = 16$, it is observed that waves regained their original amplitudes. The values of invariants are computed as $I_1 = 89.749192744$, $I_2 = 1043.878432522$, $I_3 = 1974.310412439$ and $I_4 = 22.224463997$ at the initial time and at the end of running time are computed as $I_1 = 89.753782742$, $I_2 = 1043.774402991$, $I_3 = 1974.033723008$ and $I_4 = 22.222907315$. From this calculated results it is seen that invariants are satisfactorily preserved.

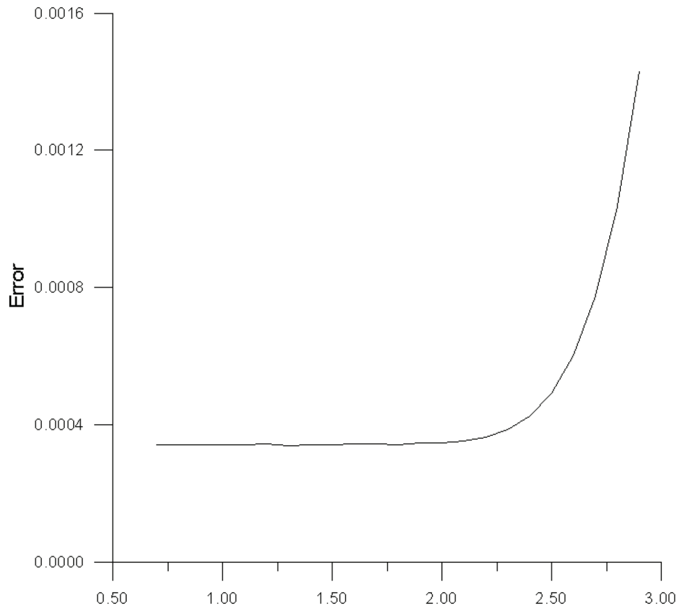


Figure 3: Error values at time $t = 1$ for the different values of shape parameter c^2

4.3 Test Problem 3: The clash of two solitary waves

In this test problem, the clash of two solitary waves which are of exactly the same form but different signs move to each other is studied over the solution domain $-90 \leq x \leq 90$ in the time period $0 \leq t \leq 12$ with time step $\Delta t = 0.005$, space step $h = 0.5$. Initially peak point of the wave with positive amplitude is located at $x = -20$ and other one is located to $x = 20$. The clash of waves occurs time increases and new wave pairs are occurred at the opposite directions. During running time, three wave pairs which were the same form but different signs were occurred. The generation of wave pairs will be continue time increases as stated in the [Dong (2011)].

Profiles of the clash of waves are plotted in Figure 6 at different times, so wave generations can be seen as detailed.

Invariants values were calculated as $I_1 = -0.000008715$, $I_2 = 12911.837906482$, $I_3 = -0.007362215$, $I_4 = 23.946607641$ at the initial time, but at the end of running time $t = 12$ due to clash the values of invariants changed as $I_1 = 0.014620833$, $I_2 = 8403.119945036$, $I_3 = -8.786076510$ and $I_4 = 23.720686649$.

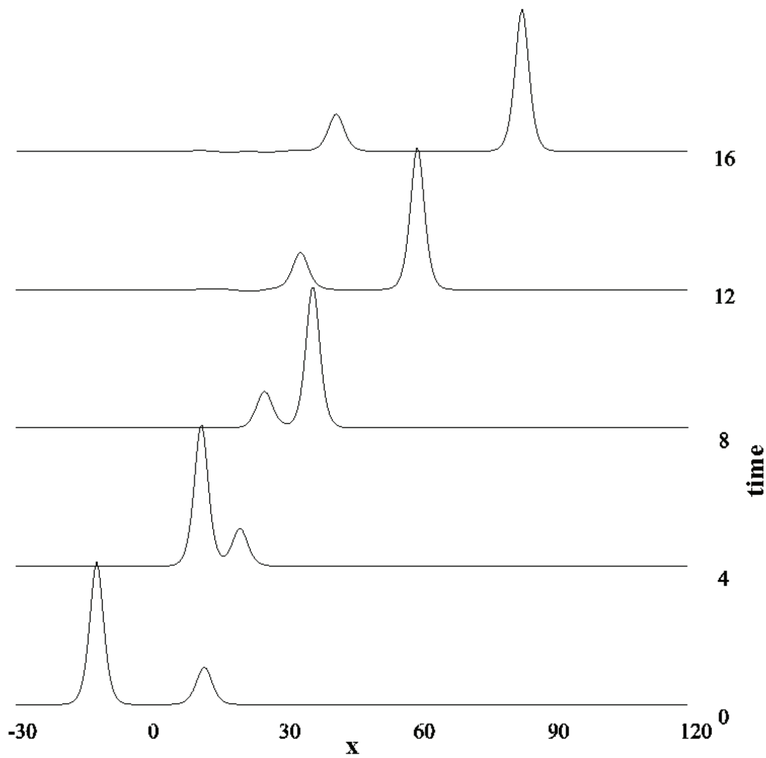
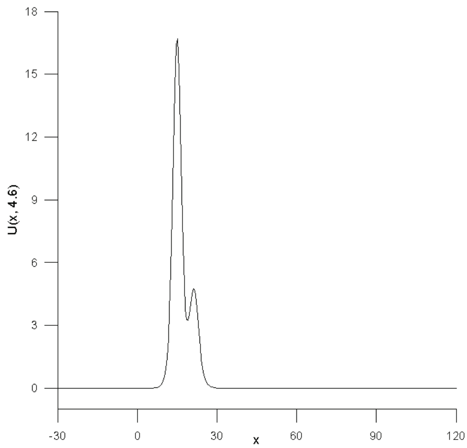


Figure 4: Simulations of interaction of two waves at different times

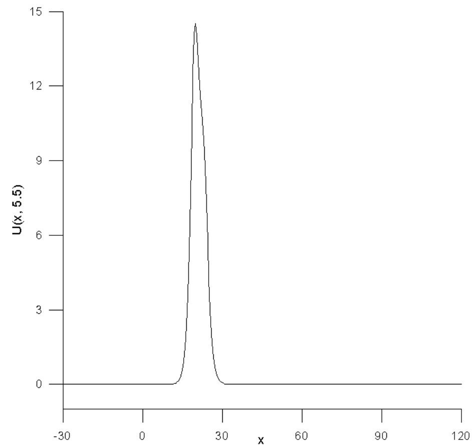
5 Conclusion

In this study, the symmetric regularized long wave (SRLW) equation was solved for various test problems numerically by using radial basis functions collocation method. Single solitary wave motion, the interaction of two positive solitary waves and the clash of waves were studied. For the single solitary wave motion error norms were calculated and compared with other numerical results. Also for the all test problems invariants were calculated. Invariants remained unchanged for first and second test problems. However, invariants changed considerably at the clash of waves problem because of generation of new waves pairs.

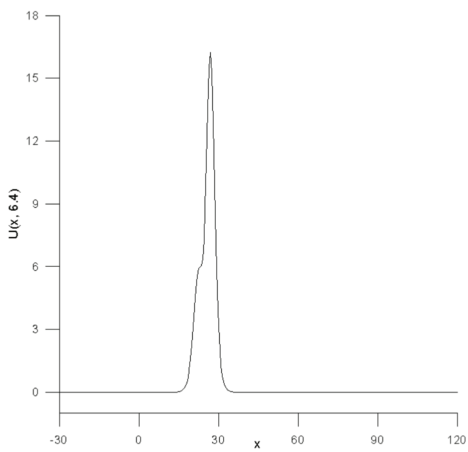
Determination of the optimal value of shape parameter is very important problem for the meshless radial basis functions collocation method. Effects of the shape parameter on the error were investigated in this study. In a certain domain, error values were obtained as stable and acceptable but at the outside of domain very high error values were calculated as depend on the values of shape parameter c.



(a)



(b)



(c)

Figure 5: Observing of collision of two solitary waves at times 4.6, 5.5, 6.4.

Therefore it was shown that the sensitivity of the error depended on the choosing of the shape parameter c.

According to obtained datum, the method satisfied highly acceptable results for the numerical solution of the symmetric regularized long wave equation. So it is shown that the radial basis functions collocation method is very efficient method and successfully can be applied to this type nonlinear partial differential equations systems in order to find numerical results.

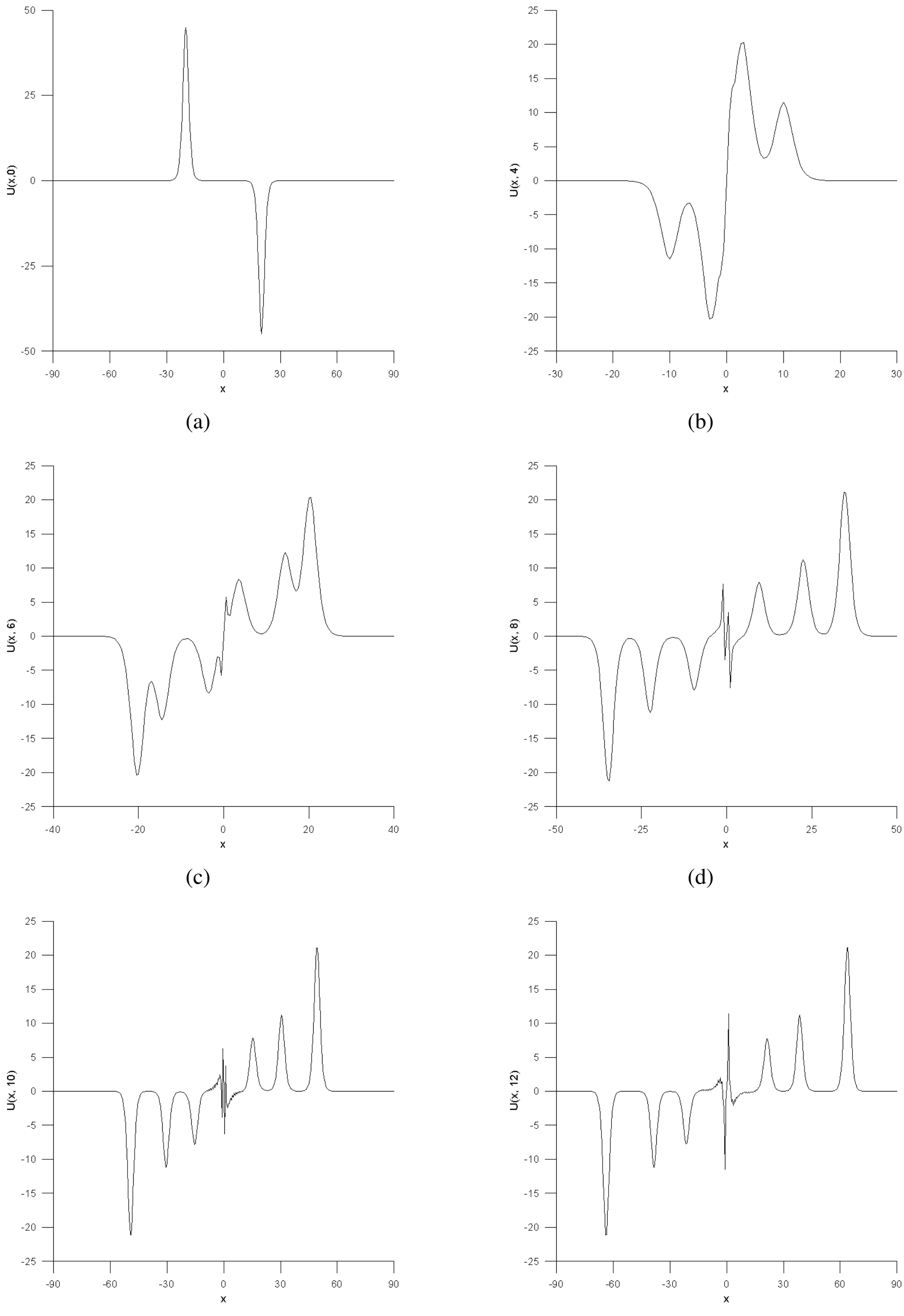


Figure 6: Profiles of the clash of waves at different times.

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