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Prediction of High-frequency Vibro-acoustic Coupling in Anechoic Chamber Using Energy Finite Element Method and Energy Boundary Element Method

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Abstract: Energy finite element method(EFEM) is a promising method to solve high-frequency vibro-acoustic problem. Energy boundary element method (EBEM) is an effective way to compute high-frequency sound radiation in the unbounded medium. Vibro-acoustic coupling of cavity structure in anechoic chamber includes both the interior acoustic field and unbounded exterior acoustic field. In order to predict this kind of high-frequency vibro-acoustic coupling problem in anechoic chamber, an approach combined EFEM and EBEM is developed in this paper. As a numerical example, the approach is applied to solve the high-frequency vibro-acoustic coupling response of a cubic cavity structure excited by a point sound source in an anechoic chamber. The result shows the approach is of the feasibility.

Keywords: Energy finite element method; Energy boundary element method; anechoic chamber

1 Introduction

A series of tests for an aircraft must be performed on ground to ensure aircraft reliability before using. Testing of the acoustic environment is a very important part of this verification. Aircraft during flight can be considered as being in a free sound field, which can be simulated in an anechoic chamber on ground. For most anechoic chamber acoustic tests, test pieces are placed in the anechoic room, a sound source is placed at another location away from the test pieces, and the internal and external acoustic field characteristics and test piece vibration are measured. These experiments often have to be repeated multiple times to obtain reproducible results. Alternatively, testing time and cost can be reduced significantly with a proper nu-

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merical simulation test program. Therefore the study of anechoic chamber simulation methods is very meaningful. Finite element method (FEM) and boundary element method (BEM) [Brancati and Aliabadi (2012); Brancati et al. (2011)] are quite mature methods to deal with the vibro-acoustic problems in anechoic chamber. However, FEM and BEM are not always efficient and accurate, especially for high frequency ranges, since structural response is extremely sensitive to material and geometry details [Bitsie (1996)]. In order to capture the structural characteristic length, a much smaller mesh size is necessary for FEM or BEM models, which results in the procedure being computationally expensive or even prohibitive. Statistical energy analysis (SEA) is an alternative efficient method which is suitable for solving high-frequency problems and has been widely used as an analysis tool in practice. But local modeling details that concern designers are usually ignored in SEA due to the fact that SEA is based on the division of sub-structures.

The energy finite element method (EFEM) presented by Nefske *et al* [Nefske and Sung (1989)] is a new tool for high-frequency vibro-acoustic coupling analysis. In this method, governing differential equations are derived in terms of energy density variables which are solved by applying the finite element approach. Compared with SEA, EFEM can obtain the detail vibro-acoustic response information. It is a promising effective method to high-frequency vibro-acoustic prediction. This method can compute the high-frequency acoustic field generated from structural vibration. The radiation efficiency in EFEM determines the amount of acoustic power radiated in the fluid due to structural vibration and the total amount of the radiated acoustic power can be computed by integrating the acoustic intensity over all the structural elements which are in contact with the fluid. However, EFEM is not able to compute the acoustic field at unbound medium since the governing difference equation for acoustic space is based on plane wave approximation, and particularly valid for highly reverberant acoustical environment at high frequency.

Energy boundary element method (EBEM), which was developed along with EFEM, is an effective way to compute high-frequency sound radiation in the unbounded medium of an external structure. From EFEM analysis, the acoustic power radiated from each structural element is obtained. Since the vibration energy in the structural elements and the acoustic intensity radiating from structural elements in contact with the fluid are incoherent among different elements, the incoherent acoustic intensity on the interface between a structure and an acoustic cavity can be regarded as a boundary condition in high-frequency EFEM interior acoustic computations. Similarly, the incoherent acoustic intensities radiating from each element on the outer structure surface in the unbound medium comprise the boundary conditions for EBEM computations [Wang et al. (2004)]. Therefore, the acoustic field at specific field points around the structure can be computed by EBEM.

In 1996, an indirect boundary element method coupling structural vibration and acoustic field was presented by Bitsie [Bitsie (1996)], in which structural vibration was computed by EFEM and acoustic field was calculated with EBEM. In 2004 , Wang [Wang et al. (2004)] presented an EBEM formulation for calculating sound radiation at high frequency from a radiator with an arbitrary shape. In 2004, Choi and Dong [6-8] evaluated the exterior acoustic field of a vehicle excited by force by using of combing EFEM and EBEM. In 2006, Nicklos and Zhang [Vlahopoulos and Wang (2008)] studied the structural vibration and interior acoustic field excited by an exterior acoustic source. In 2007, Raveendra and Hong [Raveendra and Zhang (2007); Hong et al. (2007)] performed an analysis of interior acoustic and vibration characteristics of a simplified airplane cabin and van models using hybrid EFEM and EBEM.

For the cavity structure, the vibro-acoustic response includes both the exterior acoustic field and the interior acoustic field. If the exterior acoustic field is closed by absorbent material, means that it is a bounded space, then EFEA can be used to solve the interior and exterior acoustic field simultaneously by viewing the absorbent material as a boundary condition [Vlahopoulos and Wang (2008)]. However, in anechoic chamber, exterior acoustic field is in an unbounded medium, so the EBEA and EFEA must be combined to deal with this kind of problem.

In this paper, we study the interior and exterior acoustic fields of a cavity structure excited by an exterior acoustic source in an anechoic chamber using EFEM/EBEM. In this method, the anechoic chamber is taken as an unbounded acoustic medium, and the vibroacoustic system consist of interior medium, cavity structure and exterior medium. Coupling between the interior acoustic filed and the cavity structure vibration is computed by EFEM. Excitation imposed on the structure come from a point sound source placed in exterior medium. After this EFEA coupling analysis, the energy radiated from each structural element is determined, which is took as the source of EBEA to calculate the exterior acoustic field. As an example, a cubic shell excited by a point sound source in an anechoic chamber is studied to illustrate the feasibility. As an example, a cubic shell excited by a point sound source in an anechoic chamber is studied to illustrate the feasibility.

2 Basic theory of high-frequency structure-acoustic response

As stated in introduction, an approach combined EFEM and EBEM is developed to solve the anechoic chamber high-frequency vibro-acoustic coupling problem. The basic theory of EFEM and EBEM is as following.

2.1 Basic theory of EFEM

The governing equations of the structural-acoustic coupling a complex system are

$$
\begin{aligned}
& -\frac{C_{gB}^2}{(\eta_{sB}+\eta_{rad})\,\omega}\nabla^2\langle\bar{e}_{sB}\rangle+(\eta_{sB}+\eta_{rad})\,\omega\langle\bar{e}_{sB}\rangle=\langle\bar{\pi}_{sB}\rangle \\
& -\frac{C_{gL}^2}{\eta_{sL}\omega}\nabla^2\langle\bar{e}_{sL}\rangle+\eta_{sL}\omega\langle\bar{e}_{sL}\rangle=\langle\bar{\pi}_{sL}\rangle \\
& -\frac{C_{gT}^2}{\eta_{sT}\omega}\nabla^2\langle\bar{e}_{sT}\rangle+\eta_{sT}\omega\langle\bar{e}_{sT}\rangle=\langle\bar{\pi}_{sT}\rangle \\
& -\frac{c_a^2}{\eta_a\omega}\nabla^2\langle\bar{e}_a\rangle+\eta_a\omega\langle\bar{e}_a\rangle=\langle\bar{\pi}_a\rangle
\end{aligned}
\tag{1}
$$

where C is wave speed; subscripts B, L and T express the bending, longitudinal and transverse shear waves, respectively; subscript g means group speed; subscripts s and a denote the structure domain and the acoustic domain; η is the damping loss factor; η_{rad} is radiation damping; ω is the radian frequency of the harmonic excitation; $\langle\bar{e}\rangle$ is average of energy density over a time period and over a wavelength; and $\langle\bar{\pi}\rangle$ is average of input power density over a time period and over a wave length.

The finite element method is used to numerically solve the governing equation (1). Using Galerkin weighted residual scheme and the Lagrangian shape function as the trial function, taking consider of the compatible condition at the discontinuous junctions, Equation (1) can be written in the form of matrix as

$$
\left(\begin{bmatrix} K_s & 0 \\ 0 & K_a \end{bmatrix}+J_{ss}+J_{sa}\right)\begin{pmatrix} E_s \\ E_a \end{pmatrix}=\begin{pmatrix} F_s \\ F_a \end{pmatrix}
\tag{2}
$$

K_s and K_a are the system matrices for the structural elements and acoustic elements, J_{ss} and J_{sa} are joint matrices between discontinuous junctions, $\{E\}$ is the vector of nodal energy density, and $\{F\}$ is the vector of input power.

2.2 Basic theory of EBEM

Energy density is defined as $e=\frac{1}{2}\left(\rho v^2+\frac{p^2}{\rho c^2}\right)$. Acoustic intensity is defined as $I=\frac{1}{2}Re\,(pv^*)$, where p is acoustic pressure, v is vibration velocity of the particle, ρ is acoustic medium's mass density, c is speed of sound in the medium, Re denotes real part of a complex number pv, and * denotes conjugation.

The ensemble averaging operator is applied to the expression of e and I, resulting in

$$\begin{aligned} \langle \bar{e} \rangle &= \int_S \sigma(p) \left(\frac{\rho}{64\pi^2 r^4} + \frac{k^2\rho}{32\pi^2 r^2} \right) dS \\ \langle \bar{I} \rangle &= \int_S \sigma(p) \frac{k^2\rho c}{32\pi^2 r^2} \vec{E}_r dS \end{aligned} \tag{3}$$

where $\sigma(p)$ denotes the source strength density at point p on the structure surface, r is stated as the distance between the field point and point p, and k is wavenumber. Structure surface S is divided into n quadrilateral or triangular elements and the source strength densityσ_j on each element is considered to be constant. Thus, Eq.(3) can be written in the discrete form

$$\begin{aligned} \langle \bar{e}_Y \rangle &= \sum_{j=1}^{n} \left[\sigma_j(p) \int_{S_j} \left(\frac{\rho}{64\pi^2 r(\xi,Y)^4} + \frac{k^2\rho}{32\pi^2 r(\xi,Y)^2} \right) dS \right] \\ \langle \bar{I}_Y \rangle &= \sum_{j=1}^{n} \left[\sigma_j(p) \int_{S_j} \frac{k^2\rho c}{32\pi^2 r(\xi,Y)^2} \vec{E}_r dS \right] \end{aligned} \tag{4}$$

where s_j is the surface of the jth element and ξ is a point of the$j$th element. Then the energy density expression in Eq.(4) can be written in matrix form

$$[K]\{\sigma\} = \{p\} \tag{5}$$

where $\{P\}_{n\times 1} = \left\{ \bar{p}_1 \quad \bar{p}_2 \quad \cdots \quad \bar{p}_n \right\}^T$ is acoustic power vector, $\{\sigma\}_{n\times 1} = \left\{ \sigma_1 \quad \sigma_2 \quad \cdots \quad \sigma_n \right\}^T$ is source strength density vector, which is an unknown variables vector, and K_{ij} of the matrix $[K]$ is solved by

$$K_{ij} = \begin{cases} \int_{S_i} \left(\int_{S_j} \frac{k^2\rho c}{32\pi^2 r(\xi,\eta)^2} \vec{E}_r \cdot \vec{n}_i dS \right) dS & i \neq j \\ A_i \frac{k^2\rho c}{16\pi} & i = j \end{cases} \tag{6}$$

The source strength density of each element is available by solving Eq.(5). Substituting the source strength density into EQ.(4), the acoustic energy density and intensity in exterior acoustic medium can be determined.

The acoustic power P come from vibration energy density of structure. The energy density is obtained by EFEM. The relation between acoustic power and the energy of structure can be expressed as follow

$$p_i = \int_{S_j} \eta_{rad} \omega e_{sB} h dS \tag{7}$$

where η_{rad} is radiation damping, ω is circle frequency and h is the thickness of the structure. The energy density e_{sB} on the nodes of the structure's surface and the acoustic power radiating from each element of a vibrating structure p_i is expressed by

$$P = WE_{sB} \tag{8}$$

the matrix W is written as

$$W_{ij} = \int_{S_i} \Delta_{ij} \eta_{rad}^{i} \omega h dS \tag{9}$$

where $\Delta_{ij} = 1$ if node j is located on the ith element, and $\Delta_{ij} = 0$ if node j is not on the ith element. Eq.(5) can now be written as

$$[K]\{\sigma\} = [W]\{e_{sB}\} \tag{10}$$

2.3 Calculation of input power

In the anechoic chamber, the cavity structure is excited usually by exterior acoustic source. The input power must be got if we predict the coupling response of the interior acoustic field and the vibration of cavity structure by using of EFEA. The input power at each structural element which is in contact with the exterior acoustic medium is provided by the information of energy density radiated by acoustic source and the transmission coefficient from acoustic field to structure.

The energy density of any acoustic field generated by m point acoustic sources is

$$e_{Y0} = \sum_{i=1}^{m} \left[\bar{\sigma}_i \left(\frac{\rho_0}{64\pi^2 r_i^4} + \frac{k^2 \rho_0}{32\pi^2 r_i^2} \right) \right] \tag{11}$$

σ_i is the source strength of the ith acoustic source. The incident power π_{inc} on each energy finite element is determined from the exterior acoustic energy density e_{Y0} using a plane wave approximation,

$$\pi_{inc} = e_{Y0} c_0 A \tag{12}$$

where A is the area of the structure element and c_0 is the wave speed in the acoustic medium. The power transmitted to a plate due to an impinging plane wave is evaluated as

$$\tau_{as} = \frac{\rho_0 c_0^3}{\rho_s c_B^2 h f} \sigma_{rad} \tag{13}$$

By using Eq. (12) and (13), the input power applied on an EFEM element due to the external acoustic source can be written as

$$\pi_{in} = \tau_{as} \pi_{inc} = \frac{\rho_0 c_0^4 A}{\rho_s c_B^2 h f} \sigma_{rad} e_{Y0} \tag{14}$$

3 Numerical example

3.1 Models

A cubic cavity structure model comprised of six plates, as shown in Fig.1, is constructed and studied. The dimensions of the cubic cavity structure are 1m x 1m x 2m and the thickness of the plates is 0.01m. The plates all have the same physical properties: the Young's modulus is 2.4e11Pa, Poisson's ratio is 0.30, Mass density is 7800Kg/m^3 and the hysteresis damping factor is 0.01. The cubic cavity structure is immersed in air and the acoustic space enclosed by the plates is filled with air with mass density of 1.225Kg/m^3, acoustic speed of 343m/s and hysteresis damping factor of 0.001. The distance between the center of the cubic cavity and point acoustic source is 5.5m. The location of the point acoustic source in relation to the cubic cavity structure is shown in Fig.1. Fig.2 shows the energy finite element model for the structure. The point acoustic source strength is unit strength. The response at several frequencies (2000Hz, 8000Hz 16000Hz and 32000Hz) is predicted.

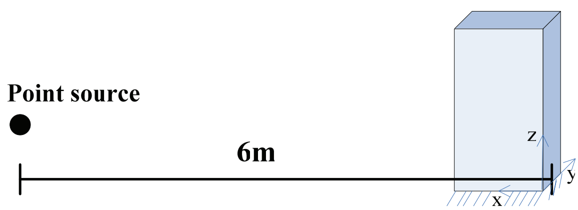

Figure 1: Schematic diagram of the point source and the structure

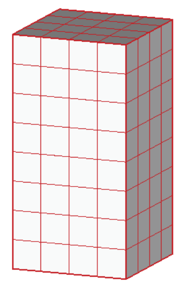

Figure 2: EFEA model for the structure

3.2 Results and discussion

Acoustic energy density is converted to the root mean square acoustic pressure with the approximation $p = c_0\sqrt{e\rho_0}$ and expressed in terms of decibel level using the acoustic pressure reference of $2 \times 10^{-5}Pa$. In the case of 2000Hz, the exterior acoustic pressures generated by structural vibration excited by the point source are shown in Fig.3. The distribution of interior acoustic pressure in the acoustic medium, the distribution of interior acoustic pressure at the y=0.5 symmetric plane, and the computed energy density distribution on the shell are shown in Fig.4, Fig.5 and Fig.6, respectively.

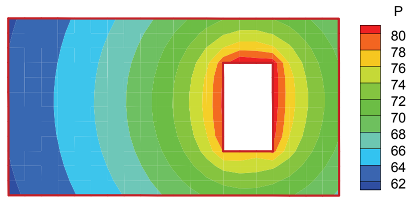

Figure 3: Exterior acoustic pressure generated by structural vibration

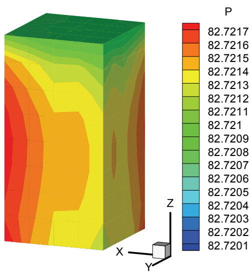

Figure 4: Distribution of interior acoustic pressure

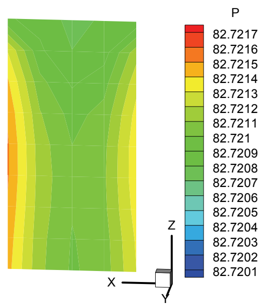

Figure 5: Distribution of interior acoustic pressure at the y=0.5 symmetric plane

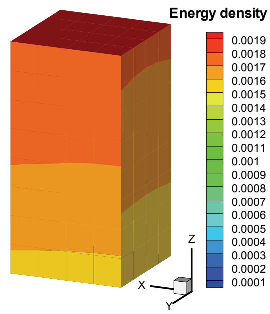

Figure 6: Distribution of energy density on the plate

In the case of 8000Hz, the corresponding results are shown in Fig.7-10.

In the case of 16000Hz, the corresponding results are shown in Fig.11-14.

In the case of 32000Hz, the corresponding results are shown in Fig.15-18.

The results show that:

1. With the increasing of frequency, the difference between maximum and minimum pressure inside the cubic cavity structure become small. The exterior pressure and the vibration energy get smaller and smaller as the frequency increase.

2. The variation of acoustic pressure in the acoustic medium enclosed by the plate is very small, but the distribution trend is acoustic pressure diminish from the boundary to the centre. Explanation for this is that energy is transmitted from the shell to the centre of the acoustic medium.

3. Energy density on the four lateral plates increases from the bottom to the top

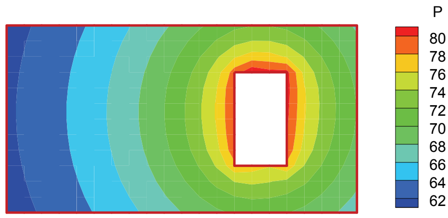

Figure 7: Exterior acoustic pressure generated by structural vibration

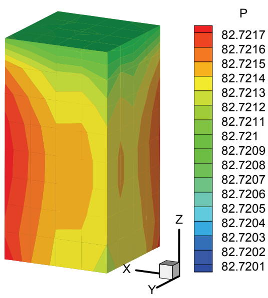

Figure 8: Distribution of interior acoustic pressure

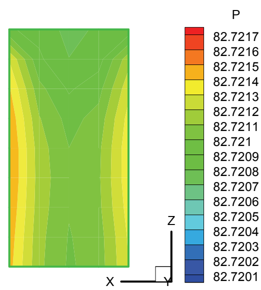

Figure 9: Distribution of interior acoustic pressure at the y=0.5 symmetric plane

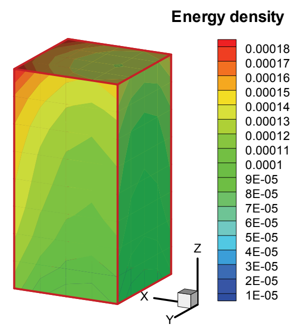

Figure 10: Distribution of energy density on the plate

of the cubic cavity structure because the bottom plate is fixed. However, the contour is not parallel to the bottom plate; the energy density of the plate which is closer to the load location is large.

4. If a sound absorbing material is utilized, EBEA don't suit any more. In this case, EFEA is adapted, and the absorbing material is taken as boundary element in EFEA which is similar with the method in the paper of Brancati, A.

3.3 Comparison of EFEA with SEA and FEA

In order to prove that the hybrid method of EFEA and EBEA is the more suitable method to deal with the high frequency vibro-acoustic in anechoic chamber, comparison of EFEA with SEA and FEA has been done in terms of accuracy and calculating time.

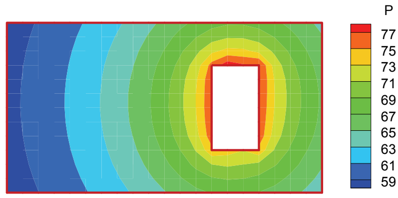

Figure 11: Exterior acoustic pressure generated by structural vibration

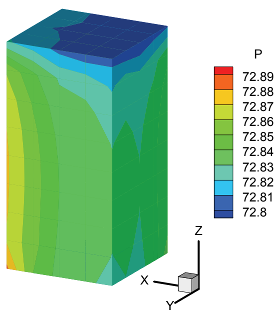

Figure 12: Distribution of interior acoustic pressure

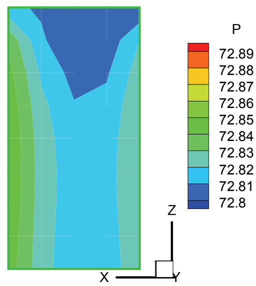

Figure 13: Distribution of interior acoustic pressure at the y=0.5 symmetric plane

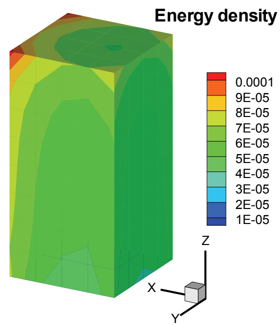

Figure 14: Distribution of energy density on the plate

3.3.1 Comparison of EFEA with FEA

In FEA, the biggest factor affecting the computation time is the number of elements, so the number of elements can roughly determine the computation time. For the cuboid, according to the accuracy requirement, the required number of elements is shown as table 1. It can be seen that there need more and more elements as the frequency increasing.

Table 1: Required number of elements in FEA

Frequency	element of FEA
2000Hz	3240
4000Hz	6656
8000Hz	12816
16000Hz	25806

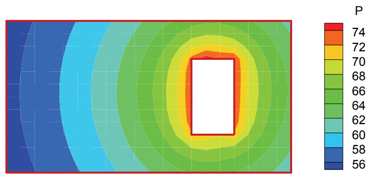

Figure 15: Exterior acoustic pressure generated by structural vibration

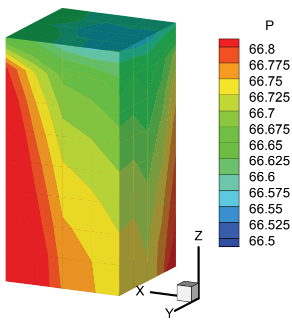

Figure 16: Distribution of interior acoustic pressure

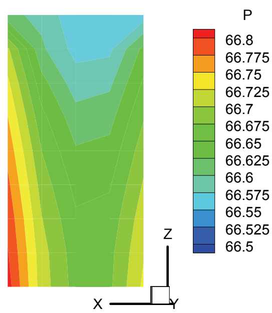

Figure 17: Distribution of interior acoustic pressure at the y=0.5 symmetric plane

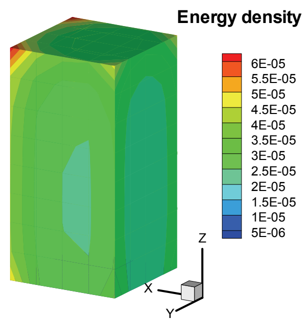

Figure 18: Distribution of energy density on the plate

However, in EFEA, the elastic wave propagation problem in EFEM is similar as the heat transfer problem in FEM. So, there is no need to change the mesh size with frequency. Frequency need not to be considered in EFEM mesh models. So more high frequency, more convenient energy finite element analysis is.

3.3.2 Comparison of EFEA with SEA

In order to make clear the advantage of EFEA over SEA, the cubic cavity structure excited by exterior acoustic source is calculated by using of SEA. At 2000Hz, the results are shown in Fig.19.

From the Fig.19, the vibration energy of the top plate and the front plate is $1.712 \times 10^{-5}J$ and $3.131 \times 10^{-5}J$, respectively.

From the Fig.6, the energy density calculated by EFEA is obtained. In order to compare the result of EFEA to SEA, the energy density is converted to energy by the way of multiplying energy density by corresponding volume. Thus, the

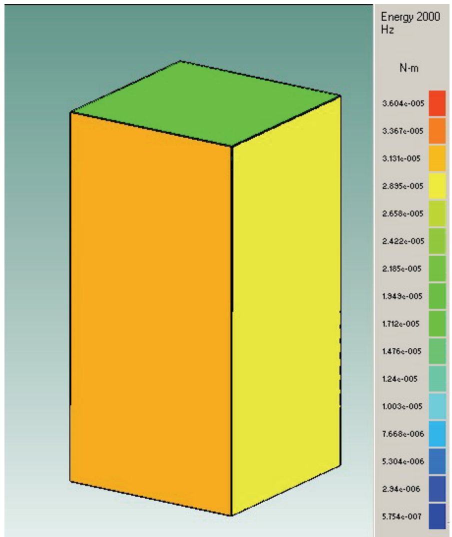

Figure 19: The distribution of energy calculated by SEA

vibration energy of the top plate and the front plate calculated by EFEA is $1.700 \times 10^{-5} J$ and $3.000 \times 10^{-5} J$, respectively. It can be seen that EFEA and SEA has approximate results, however, the distribution of energy density on structure can be obtained by EFEA.

4 Conclusion

Considering a cavity structure excited by a point sound source in an anechoic chamber, the high frequency vibro-acoustic response is hard to reach with conventional finite element analysis because of the computational cost. EFEA based on energy conservation is effective. An approach solving the anechoic chamber high-frequency vibro-acoustic coupling problem using EFEA and EBEA is presented in this paper. The anechoic chamber is taken as an unbounded acoustic medium. The exterior/interior acoustic fields of the structure and the vibration energy density distribution within an anechoic chamber are obtained using an EFEM/EBEM developed in this paper. As an example, a cubic shell excited by a point sound source in an anechoic chamber is studied. The result actually is reasonable and indicates that it is feasible and convenient to solving vibro-acoustic coupling of the whole structural-acoustic system in anechoic chamber using EFEM/EBEM proposed in this paper. It is to believe that the method described here can accurately simulate anechoic chamber high-frequency vibration-acoustic test results and provide an alternative to chamber testing of structural components.

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