Uniform Loading of a Cracked Layered Composite Plate: Experiments and Computational Modelling

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Abstract: This paper examines the influence of a through crack on the overall flexural behaviour of a layered composite Carbon Fibre Reinforced Polymer (CFRP) plate that is fixed boundary along a circular boundary. Plates with different through crack configurations and subjected to uniform air pressure loading are examined both experimentally and computationally. In particular, the effect of crack length and its orientation on the overall pressure-deflection behaviour of the composite plate is investigated. The layered composite CFRP plate used in the experimental investigation consisted of 11 layers of a polyester matrix unidirectionally reinforced with carbon fibres. The bulk fibre volume fraction in the plate was approximately 61%. The stacking of cracked laminae is used to construct a model of the plate. The experimental results for the central deflection of the plate were used to establish the validity of a computational approach that also accounts for large deflections of the plate within the small strain range.

Keywords: Laminated composite plate, cracked plate, elasticity, finite element analysis, large deflection behavior, representative area element.

1 Introduction

Fibre-reinforced plates are used extensively in various engineering applications because of their high tensile strength, light weight, resistance to corrosion and high durability [Spencer (1972); Selvadurai and Moutafis (1975), Christensen (1979); Jones (1987); ten Busschen and Selvadurai (1995), Selvadurai and ten Busschen (1995), Hwu and Yu (2010); Nikopour and Nehdi (2011); Nehdi and Nikopour (2011)]. Although research on the mechanical behaviour of fibre-reinforced plates has made considerable progress over the past decades, there still remain a number

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of aspects of the mechanical behaviour of fibre-reinforced plates that need further investigation. In particular, the mechanics of fibre reinforced plates during progressive damage is less well understood in comparison to the significant progress that has been made in the modelling and experiments involving defect free composite plates [Reddy (2004)]. Damage in fibre-reinforced composites can take place at various scales ranging from matrix cracking, matrix yield, fibre fracture and interface debonding, depending upon the types of mechanical and environmental loads that are applied to the composite during its functional use. The importance of damage to the structural integrity of fibre-reinforced materials was discussed several decades ago by a number of researchers including Beaumont and Harris (1972), Bowling and Groves (1979), Aveston and Kelly (1980), and Backlund (1981). The investigations dealing with the modelling of flaw-bridging in composites were presented by Selvadurai (1983 a, b; 2010). This paper examines role of a through crack on the overall mechanical behaviour of a laminated fibre reinforced composite. In particular, the influence of crack length and orientation is examined experimentally using an apparatus that can apply a uniform pressure. The paper also presents the application of a computational procedure that accounts for large deflection effects, to model the plate flexure. The results of the computational modelling, which use the model for the elastic behaviour of a single unidirectionally reinforced laminae with an irregular representative fibre spatial arrangement and volume fraction, are compared with experimental results.

2 Plate lay-up and material properties

The fibre-reinforced plates used in this research were supplied by Aerospace Composite Products, California, USA. The tested plates measured 460.0 mm \times 360.0 mm \times 2.4 mm. Optical microscope investigations were made on samples measuring 25.0 mm \times 5.0 mm \times 2.4 mm to identify the fibre arrangement. Figure 1 shows the scanned results for the physical arrangement of fibres in the plate.

The image processing toolbox in the MATLABTM software was then used to estimate the fibre area fraction in a single layer. The fibre area fraction changes with the size, location and orientation of the chosen representative area; the results of image analysis indicate that the limiting fibre The plate consisted of 11 orthogonally oriented layers with a lay-up of $[(90^{\circ}/0^{\circ})_2, 90^{\circ}, 0^{\circ}, 90^{\circ}, (90^{\circ}/0^{\circ})_2]$ relative to the longitudinal direction of the plate. The diameter of a typical fibre was 8 μ m. Volume fraction was approximately 61% for a large square section that had an area greater than 40 times the area of a single fibre. As is evident from Fig. 2, the tensile failure of the fibre-reinforced material involves largely fibre breakage rather than fibre pull out. This points to a fibre-reinforced material with adequate fibre-matrix bond.



Figure 1: Results from the scanning electron microscope



Figure 2: The fracture topography

In a companion study [Selvadurai and Nikopour (2012)], the effective transversely isotropic properties of a unidirectionally fibre-reinforced carbon-fibre-polymer composite was investigated using a micro-mechanical computational simulation of the fibre arrangements over a representative over a representative area element of the composite. It was shown that the effective transversely isotropic elastic properties of the unidirectionally reinforced composite determined from experimental results coupled with computational simulations closely matched the results based on the theoretical relationships proposed by Hashin and Rosen (1964). The properties of both the fibre and matrix materials in their as-supplied condition were provided by the manufacturer and these are listed in Table 1.

Property	Specific	Tensile	Young's	Ultimate	Poisson's	
	Gravity	Strength	Modulus	Tensile Strain	Ratio	
Resin	1.20	78.6 MPa	3.1 GPa	3.4%	0.35	
Fibre	1.81	2450.4 MPa	224.4 GPa	1.6 %	0.20	

Table 1: Mechanical properties of resin matrix and fibre

Table 2 shows a comparison between the transversely isotropic elastic constants determined from the Hashin and Rosen model [Hashin and Rosen (1964)], which does not take into consideration any irregularity in the fibre arrangement with results obtained from the Selvadurai Nikopour RAE approach [Selvadurai and Nikopour (2012)] that considers irregular fibre arrangements.

The strain-stress relations can be expressed in terms of the five elastic constants,

Table 2: Transverse isotropic elasticity properties obtained from Hashin and Rosen(1964) and the RAE approach of Selvadurai and Nikopour (2012)

	Hashin and Rosen	RAE	Percentage difference
E_{11} [GPa]	149.17	146.26	2.0
<i>v</i> ₁₂	0.23	0.23	0.0
$E_{22}[\text{GPa}]$	12.72	12.11	5.0
V ₂₃	0.27	0.29	7.4
$G_{23}[\text{GPa}]$	5.01	4.85	3.3

 $E_{11}, E_{22}, v_{13}, v_{23}$ and G_{12} in the form

$ \left(\begin{array}{c} \boldsymbol{\varepsilon}_{11} \\ \boldsymbol{\varepsilon}_{22} \\ \boldsymbol{\varepsilon}_{33} \\ 2\boldsymbol{\varepsilon}_{31} \\ 2\boldsymbol{\varepsilon}_{12} \\ 2\boldsymbol{\varepsilon}_{23} \end{array}\right) $	> =	$\begin{bmatrix} \frac{1}{E_{11}} \\ -\frac{V_{12}}{E_{11}} \\ -\frac{V_{13}}{E_{11}} \\ 0 \\ 0 \\ 0 \end{bmatrix}$	$ \begin{array}{c} -\frac{v_{12}}{E_{11}} \\ \frac{1}{E_{11}} \\ -\frac{v_{13}}{E_{11}} \\ 0 \\ 0 \\ 0 \end{array} $	$ \begin{array}{c} -\frac{v_{13}}{E_{11}} \\ -\frac{v_{13}}{E_{11}} \\ \frac{1}{E_{33}} \\ 0 \\ 0 \\ 0 \end{array} $	$ \begin{array}{c} 0 \\ 0 \\ \frac{1}{G_{12}} \\ 0 \\ 0 \end{array} $	$ \begin{array}{c} 0 \\ 0 \\ 0 \\ \frac{1}{G_{12}} \\ 0 \end{array} $	$ \begin{array}{c} 0 \\ 0 \\ 0 \\ 0 \\ \underline{2(1+v_{23})} \end{array} $	$\left \begin{array}{c} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{31} \\ \sigma_{12} \\ \sigma_{23} \end{array}\right $	(1)
$(2\epsilon_{23})$			0	0	0	0	$\frac{Z(1+V_{23})}{E_{22}}$		

The relationship (1) can be inverted to obtain the stresses in terms of the strains. The thermodynamic requirements for positive definite strain energy is defined by the condition

$$U = \frac{1}{2}\sigma_{ij}\varepsilon_{ij} > 0 \tag{2}$$

We can substitute (1) into (2) and group terms such that the result has a quadratic form in ε_{ij} [see Selvadurai (2000) for the isotropic case]. The requirement for positive definite strain energy thus reduces to the multiplying factors of the quadratic terms be positive definite [Christensen (1979), Amadei et al. (1987)].

$$E_{11} > 0; \quad E_{33} > 0; \quad G_{23} > 0; \quad G_{12} > 0$$

$$-1 < v_{23} < 1/2; \quad v_{12} < \frac{E_{11}}{E_{33}}; \quad (1 + v_{23}) \left(1 - v_{23} - 2v_{12}^2 \frac{E_{22}}{E_{11}}\right) > 0$$
(3)

Elastic constants determined from both the Hashin and Rosen model and the Selvadurai Nikopour RAE approach satisfied the thermodynamic restrictions. The research presented in paper makes direct use of the properties determined from the RAE method to examine the mechanical behaviour of a laminated plate containing a through crack and exhibiting large deflections, within the small strain range, during flexure.

3 Air pressure loading of a circular cracked plate

The test apparatus shown in Fig. 3, had provisions for examining the flexural behaviour of a composite plate that was fixed along a circular boundary with a diameter of 250 mm. Figure 4 shows a schematic view of the test setup and steel grips used to provide fixity. The plates were subjected to a monotonically increasing quasi-static air pressure using and air-flow controller (OmegaTM) with capacity of 10 litres /min, which was connected to 3 power supplies for ± 15 Volts DC control. A potentiometer was installed in the apparatus to measure the transverse deflection

at the center of the plate. Pressure in the system was monitored using a 1000 kPa pressure transducer (HoneywellTM). The applied pressure and the resulting displacements were monitored using a digital data acquisition system (Measurement COMPUTINGTM) connected to a computer that incorporated the TracerDAQTM software. The fixed boundary condition was achieved by clamping the CFRP plate between two steel plates of 10.0 mm thickness, using eight 4 mm screws. In order to prevent the air leakage and pressure loss in the system, four layers of thin adhesive film were placed between the steel plates and test frame and also between the steel plates and the CFRP plate. A thin natural rubber was incorporated between the lower steel plate and the cracked CFRP plate to prevent air leakage through the crack (Fig. 5).



Figure 3: Experimental setup

Through cracks were made using a 500 μ m thick slitting cutter with a diameter of 100 mm and 300 teeth which was connected to a milling machine (Fig. 6). Crack tips were later polished using a #1500-b sandpaper (Fig. 7) to eliminate the stress concentration effect. All tests were performed in a pressure control mode in a laboratory where the temperature was approximately 20°C with nominal variation of $\pm 1^{\circ}$ C and maximum air pressure p_{max} was limited to 120 kPa. The air pressurization was performed at a rate of $\dot{p} = 200$ Pa/sec to eliminate any dynamic or thermal effects. Each test was performed 3 times to establish repeatability of the nonlinear pressure-deflection results. Through crack patterns of various lengths were tested (Fig. 8d); the results were then compared with results of computational modelling, which was subsequently used to investigate flexure behaviour of plates with more complex crack patterns.



Figure 4: Schematic view of test setup



Figure 5: Schematic view of the detail at **B** in Fig. 4



(a) Crack forming setup (b) Slitting cutter Figure 6: Crack formation using a slitting cutter



(a) Before polishing(b) After polishingFigure 7: Optical images of a crack tip

4 Computational modelling of plates

The nonlinear theory of plates is well documented in the texts by Timoshenko and Woinowsky-Krieger (1959) and Reddy (2004). The theory of large deflections of laminated plates has also been investigated by Bathe and Bolourchi (1980), Punch and Atluri (1986) and Kam et al. (1996) using computational approaches. Wu and Erdogan (1993) analytically investigated the effect of through cracks on the stress intensity factor for orthotropic laminated plates under flexure. Baltacioglu and Civalek (2010) conducted a nonlinear analysis of anisotropic composite plates resting on nonlinear elastic foundations. The displacement field in a thin laminated plate undergoing large deflections is assumed to be of the form:

$$u_x(x, y, z) = u_0(x, y) + z \Psi_x(x, y)$$

$$u_y(x, y, z) = v_0(x, y) + z \Psi_y(x, y)$$

$$u_z(x, y, z) = w(x, y)$$

where u_x , u_y , u_z are the deflections in the *x*, *y*, *z* directions, respectively, u_0, v_0, w are the associated mid-plane deflections, and ψ_x and ψ_y the rotations due to shear. The strain-displacement relations in the von Karman plate theory can be expressed

in the form [Kam (1996)]:

$$\begin{aligned} \varepsilon_{x} &= \frac{\partial u_{0}}{\partial x} + \frac{1}{2} \left(\frac{\partial w}{\partial x} \right)^{2} + z \frac{\partial \psi_{x}}{\partial x} = \varepsilon_{x}^{0} + z \kappa_{x} \\ \varepsilon_{y} &= \frac{\partial v_{0}}{\partial y} + \frac{1}{2} \left(\frac{\partial w}{\partial y} \right)^{2} + z \frac{\partial \psi_{y}}{\partial y} = \varepsilon_{y}^{0} + z \kappa_{y} \\ \varepsilon_{s} &= \frac{\partial u_{0}}{\partial y} + \frac{\partial v_{0}}{\partial x} + \frac{\partial w}{\partial x} \frac{\partial w}{\partial y} + z \left(\frac{\partial \psi_{x}}{\partial y} + \frac{\partial \psi_{y}}{\partial x} \right) = \varepsilon_{s}^{0} + z \kappa_{s} \end{aligned}$$

$$\begin{aligned} \varepsilon_{4} &= \psi_{y} + \frac{\partial w}{\partial y} \\ \varepsilon_{5} &= \psi_{x} + \frac{\partial w}{\partial x} \end{aligned}$$

$$(5)$$

where ε_i^0 (*i* = *x*, *y*, *s*) are in-plane strains; ε_j (*j* = 4, 5) are transverse shear strains, and κ_i (*i* = *x*, *y*, *s*) are bending curvatures. The associated second Piola-Kirchoff stress vector σ is

$$\boldsymbol{\sigma} = \left[\boldsymbol{\sigma}_{x}, \boldsymbol{\sigma}_{y}, \boldsymbol{\sigma}_{s}, \boldsymbol{\sigma}_{4}, \boldsymbol{\sigma}_{5}\right]^{T}$$
(6)

The constitutive equations for the plate can be written as

$$\begin{cases} N_i \\ M_i \end{cases} = \begin{bmatrix} A_{ij} & B_{ij} \\ B_{ij} & D_{ij} \end{bmatrix} \begin{cases} \varepsilon_j^0 \\ \kappa_i \end{cases} \quad (i, j = x, y, s)$$
 (7a)

$$\begin{cases} Q_2 \\ Q_1 \end{cases} = \begin{bmatrix} \bar{A}_{44} & \bar{A}_{45} \\ \bar{A}_{45} & \bar{A}_{55} \end{bmatrix} \begin{cases} \varepsilon_j^0 \\ \kappa_i \end{cases} \quad (i, j = x, y, s)$$
 (7b)

In (4), $a_{ij}, B_{ij}, D_{ij}(i, j = x, y, s)$ and $\bar{A}_{ij}(i, j = 4, 5)$ are the in-plane, bending coupling, bending or twisting, and thickness-shear stiffness coefficients. N_i, M_i, Q_1 and Q_2 are the stress resultants defined by

$$(N_i: M_i) = \int_{-h/2}^{h/2} (1:z) \sigma_i dz$$
(8a)

$$(Q_1:Q_2) = \int_{-h/2}^{h/2} (\sigma_5:\sigma_4) dz$$
(8b)

and

$$(A_{ij}: B_{ij}: D_{ij}) = \sum_{m=1}^{NL} \int_{z_m}^{z_{m+1}} Q_{ij}^{(m)}(1:z:z^2) dz; \ (i,j=x,y,s)$$
(9a)

$$\bar{A}_{ij} = \sum_{m=1}^{NL} \int_{z_m}^{z_{m+1}} k_{\alpha} k_{\beta} Q_{ij}^{(m)} dz; \ (i, j = 4, 5; \alpha = 6; \beta = 6 - j)$$
(9b)

where z_m is the distance from the mid-plane to the lower surface of the m^{th} layer, NL is the total number of layers, Q_{ij} are material constants, and k_{α} are the shear correction coefficients which are set as $k_1 = k_2 = \sqrt{5/6}$, [Kam et al. (1996)]. The basis of the formulation of the governing equations of the plate is the principle of minimum total potential energy in which the total potential energy π is expressed as the sum of strain energy, U, and the potential energy, P:

$$\pi = U + P \tag{10}$$

Performing the through thickness integration, the strain energy can be rewritten as [Kam et al. (1996)]:

$$U = \frac{1}{2} \iint \left[\varepsilon^{0T} A \varepsilon^0 + 2 \varepsilon^{0T} B \kappa + \kappa^T D \kappa + \gamma^T \bar{A} \gamma \right] dx dy$$
(11)

Considering the laminated composite plate discretized into *NE* elements, the strain energy and potential energy of the plate can be expressed in the form:

$$U = \sum_{i=1}^{NE} U_e = \frac{1}{2} \sum_{i=1}^{NE} \left\{ \int_{\Omega_e} \left[\varepsilon^{0T} A \varepsilon^0 + 2\varepsilon^{0T} B \kappa + \kappa^T D \kappa + \gamma^T \bar{A} \gamma \right] d\Omega \right\}_i$$
(12)

and

$$P = -\sum_{i=1}^{NE} \left\{ \int_{\Omega_{\varepsilon}} q(x, y) w(x, y) d\Omega \right\}_{i}$$
(13)

where Ω_e , U_e are, respectively, the element area and the strain energy per element. The mid-plane displacements and rotations $(u_0, v_0, w, \psi_x, \psi_y)$ within an element are given as a function of $5 \times n$ discrete nodal deflections and in matrix form they are:

$$u = \sum_{i=1}^{n} [\Phi_i I] \nabla_{ei} = \Phi \tilde{\nabla}_e \tag{14}$$

where *n* is the number of nodes of the element; Φ_i are the shape functions; *I* is a 5 × 5 unit matrix; Φ is the shape function matrix; $\tilde{\nabla}_e = \{\nabla_{e1}, \nabla_{e2}, ..., \nabla_{eq}\}^{\mathrm{T}}$; and the nodal displacements ∇_{ei} at a node are:

$$\nabla_{ei} = \{u_{0i}, v_{0i}, w_i, \psi_{xi}, \psi_{yi}\}^{\mathrm{T}}, \ i = 1, ..., n$$
(15)

The first variation of equation (10) in terms of the nodal displacements can be expressed as:

$$\delta \pi = \delta(U+P) = \sum_{i=1}^{NE} \left[\delta \tilde{\nabla}_e^{\mathrm{T}} F_e \tilde{\nabla}_e \right]_i - \sum_{i=1}^{NE} \left[\delta \tilde{\nabla}_e^{\mathrm{T}} P_e \right]_i = 0$$
(16)

where $F_e \tilde{\nabla}_e$, P_e are the element internal and nodal force vectors.

In this research, computational modelling of the large deflection behaviour of the plates, which incorporated the above developments, was performed using the generalpurpose finite element code ABAQUS TM . The objective here is to model cracked CFRP plates with different through crack patterns that are subjected to uniform pressure, compare the computational predictions with experimental observations for the central deflection of the plate and to observe the influence of the through crack on the deflection contours. Considering the stacking arrangement for the composite, the circular CFRP plate was modelled as a three-dimensional domain. Transversely isotropic stiffness coefficients were determined [see e.g. Lekhnitskii (1987), Hearmon (1961), Green and Zerna (1968); Maceri (2010)] and each layer was modelled as a homogenous transversely isotropic material with a principal axis along its fibre direction. Table 3 presents the transversely isotropic stiffness coefficients calculated on the basis of the effective estimates derived computationally using the RAE approach of Selvadurai and Nikopour (2012) and the analytical estimates obtained from Hashin and Rosen (1964) for the effective elasticity properties of the transversely isotropic elastic layer. Perfect interface bonding was assumed to exist between the layers forming the composite plate. As the thickness of sealing rubber was small, (1 mm in the undeformed state and 0.3 mm in the deformed state after the steel screws and the steel bolts where completely tightened). fixed-edge boundary conditions were used for the boundaries of the composite layers. The computational modelling of the test plate was performed using standard 15-noded quadratic triangular prism elements available in the element library of ABAQUSTM. Each node has three displacement and three rotational degrees of freedom. The second-order form of the quadrilateral elements was selected because it provides greater accuracy for problems that do not involve complex contact conditions. Second-order elements also have extra mid-side nodes in each element making computations of large deflection behaviour more effective. The large deflections in the modelling are assumed to be mainly due to bending action and the procedures can be extended to include shear deformations of the plate [Rajapakse and Selvadurai (1986)]. Although in principle the crack tip contains a stress singularity, in the current modelling that focuses on the flexure problem, no singular elements were incorporated. A study with progressive increases in the mesh refinement was conducted to determine convergent results and whether or not there

should be a mesh refinement, to accommodate high stress gradient at the crack tip, or use a coarser mesh to reduce the computing time. Mesh configuration for composite plates with different types of crack patterns (Fig. 8) with a/R=0.8 are presented in Fig. 9. Denser meshes were implemented close to the crack tips. Scanning electron microscopy was used to check the possibility of crack development/extension close to the crack tips. Crack extension was not observed at the maximum pressure level chosen for testing the plates, consequently crack extension was not considered in the computational modelling.

Table 3: Transversely isotropic stiffness coefficients for a composite layer obtained from Hashin and Rosen [Hashin and Rosen (1964)] and RAE [Selvadurai and Nikopour (2012)] methods.



Elastic Coefficient	D ₁₁₁₁	D_{2222}	D ₁₁₂₂	D_{2233}	D ₁₂₁₂	D ₂₃₂₃
RAE Method (GPa)	143.77	13.35	4.33	3.97	55.20	4.85
Hashin and Rosen (GPa)	142.08	13.86	4.42	3.84	55.20	5.01



Figure 8: Intact plate and various crack patterns (R= 125 m; Crack length = 2a; Plate lay-up in x- direction: [(90°/0°)₂, 90°, 0°, 90°, (90°/0°)₂]).

5 Results and discussion

Figures 10-15 show the experimental results for pressure versus central deflection for the intact plate and plates with C-90° crack pattern and crack length ratios (a/R) of 0.2, 0.4, 0.6, 0.8 and 1. Plates with higher stiffness and lower crack length



Figure 9: Mesh configurations for the intact CFRP plate and cracked CFRP plates with a/R=0.8

showed a more observable nonlinear pressure-deflection trend compared to less stiff plates. This characteristic response is considered to be an effect that results from large deflections. Plates with lower crack lengths had larger crack opening at the maximum pressure loading; however, as mentioned earlier, no evidence of the extension of the through crack was observed.

Having the experimental results for intact and C-90⁰ cracked plates, computational modelling was verified by comparing numerical results with experimental results. The reasonable predictions of the experimental responses using computational modelling then allowed further simulation of the other possible crack patterns. Figure 16 presents the computational results for deflection contours of plates with various types of crack patterns with a constant crack length ratio, a/R, of 0.8 under a constant pressure of 120 kPa. The cracked plate C-0° had the smallest increase and C-0°-45° cracked plate had the largest increase in their maximum deflection, with respect to the intact plate.

It should be mentioned that it is expected that by increasing the number of layers, the results for C- 0^0 , C- 45^0 and C- 90^0 will converge to a unique number similar to



Figure 10: Experimental and computational results for pressure versus transverse deflection for intact CFRP plate.



Figure 11: Experimental and computational results for pressure versus deflection for C-90° plate with crack length ratio, a/R=0.2



Figure 12: Experimental and computational results for pressure versus deflection for C-90° plate with crack length ratio, a/R=0.4



Figure 13: Experimental and computational results for pressure versus deflection for C-90° plate with crack length ratio, a/R=0.6



Figure 14: Experimental and computational results for pressure versus deflection for C-90° plate with crack length ratio, a/R=0.8



Figure 15: Experimental and computational results for pressure versus deflection for C-90° plate with crack length ratio, a/R=1.0



Figure 16: Computational results for deflection of other cracked CFRP plates with a/R= 0.8 under a pressure of 120 kPa

that of an isotropic plate.

6 Concluding remarks

The mechanical behaviour of a CFRP composite plate, with various crack patterns and crack lengths, subjected to uniform loading was examined using both experiments and computational simulations. It was found that the RAE method developed for estimating the elastic properties of uni-directional fibre reinforced plates provides reasonable estimates for the finite element modelling of a composite plate at the macro-scale. The limitation of the RAE method is the need to perform optical experiments to determine the fibre arrangement prior to developing the RAE element. The RAE approach, however, captures the geometric features of the fibre configuration, which is absent in effective property estimates proposed in the literature. The fact that both approaches give close results suggests that the theoretical estimates can be used with confidence in instances where scanning electron microscope images and data are unavailable. The other important observation was that plates with higher stiffness and smaller crack lengths were more likely to exhibit a nonlinear trend in the applied pressure-transverse deflection. At the level of applied pressure, the regions in the vicinity of the crack tip remained intact and there were no observations of crack extension. The computational modelling that incorporates effect of large deflections is essential for examining the deflection behaviour of the plate.

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