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An Application of Genetic Algorithms and the Method of Fundamental Solutions to Simulate Cathodic Protection Systems

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Abstract: The aim of this paper is to present numerical simulations of Cathodic Protection (CP) Systems using a Genetic Algorithm (GA) and the Method of Fundamental Solutions (MFS). MFS is used to obtain the solution of the associated homogeneous equation with the non-homogeneous equation subject to nonlinear boundary conditions defined as polarization curves. The adopted GA minimizes a nonlinear error function, whose design variables are the coefficients of the linear superposition of fundamental solutions and the positions of the source points, located outside the problem domain. In this work, the anodes added to the CP system are considered as point sources and therefore the integral that represents the particular solution can be obtained analytically. The results presented here include a comparison with a direct boundary element (BEM) solution procedure. Simulations are performed considering finite and infinite regions in R^2. For external problems a constant was added to the fundamental solution to impose the conservation of current between the anodes and cathodes of the problem.

Keywords: Optimization, MFS, BEM.

1 Introduction

Cathodic Protection (CP) is a technique used to control the corrosion of a metal surface by making it the cathode of an electrochemical cell. The technique is commonly used for protecting metallic structures placed in aggressive environments, e.g. ship hulls, offshore structures, storage tanks and underground pipelines.

In a CP system, the location and the impressed current of the anodes have to be determined with the goal of providing, as much as possible, a uniform potential distribution on the metal surface and below a critical potential (ϕ_c). The electrochemical potential problem is governed by the Poisson equation with boundary

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conditions given by a polarization curve, which is a non-linear relationship between the electrochemical potential (ϕ) and current density (i).

The BEM is one of the most appropriate techniques to solve problems involving CP systems. Several applications of BEM to study CP systems have been reported in the literature. Telles, Mansur, Wrobel, and Marinho (1990) described the PROCAT computer system based on the boundary element method, which can be applied to two-dimensional, axisymmetric or fully three-dimensional problems and makes simulations used to assist the design of cathodic protection systems. Santiago and Telles (1997) developed a BEM formulation for CP problems with dynamic polarization curves determined from potentiostatic data obtained from measurements. The mathematical model presented by Nisancioglu (1987) was used to represent the dynamic polarization curves. The commercial software Boundary Element Analysis System [BEASY (2000)] has also been used to predict the performance of cathodic protection systems for several metallic structures placed in an electrolyte. Also, Miltiadou and Wrobel (2004) presented a boundary element methodology coupled to Genetic Algorithms for: identification of polarization parameters, identification of coating holidays and the optimization of anode positioning and their impressed current.

The MFS is a technique which can similarly be applied to CP problems, even though not as many references can be found in the literature. The MFS belongs to the class of meshless methods. In the MFS, the approximate solution of the problem is represented in the form of a linear superposition of fundamental solutions with singular points located outside the domain of the problem. These singular points are called source points and form a "pseudo-boundary" having no common points with the actual boundary of the region. The essence of the MFS is the use of a fundamental solution which satisfies the associated homogeneous differential equation in every point except at the source point. The unknown source intensities, responsible for producing the approximate solution, are determined by imposing satisfaction of the boundary conditions at a set of boundary points (collocation points).

The basic ideas for the formulation of the MFS were first proposed by Kupradze and Aleksidze (1963). Its numerical implementation was carried out by Mathon and Johnson (1977). Just like BEM, MFS is applicable when a fundamental solution of the differential equation in question is known, with the advantage of not requiring any integration procedure or specific treatment for accommodating the singularities of the fundamental solution.

The problem for the application of the MFS is first the determination of the positions of the source points. Generally, in 2-D applications the arrangement of the source points is on a circular contour or on a contour geometrically similar to the

actual boundary of the region under consideration. However, the accuracy of the numerical solution is usually quite dependent on the radius of such a circle or on the distance from the source points over the geometrically similar boundary to the actual problem boundary. Nishimura, Nishihara, Nishimori, and Ishihara (2003), for instance, proposed a Genetic Algorithm for the optimal positioning of source points. The second difficulty commonly encountered is the ill-conditioning of the equations system. Here, the singular value decomposition (SVD) idea can also provide means to obtain acceptable solutions to the ill-conditioned equations system and has been successfully applied to MFS [Ramachandran (2002)].

For external problems governed by the Laplace equation, the Gauss condition needs especial attention since the integral of flux over the boundary is automatically balanced by a compensating flux from infinity [Brebbia, Telles, and Wrobel (1984)]. Telles, Mansur, and Wrobel (1984) added a constant was added to the fundamental solution, allowing to impose the conservation of current between the anodes and cathodes for CP problems in infinite regions solved by BEM. A generalization of this procedure, which ensures current conservation using BEM, even for finite domains, can be found in Telles and de Paula (1991).

The present work uses MFS to obtain the numerical solution of the associated homogeneous equation which, added to the particular solution, represents the electrochemical potential of metal surfaces immersed in electrolytes (domains). The aim is to simulate CP systems capable of providing an ideally homogeneous potential distribution on the metallic structure surface below the critical potential. The physical behaviour of metal surfaces is modelled by a nonlinear polarization curve which describes the nonlinear relation between potential and current densities. Thus, the unknown coefficients of the linear superposition of fundamental solutions and the positions of source points are determined by minimizing a nonlinear error function. This is here accomplished using a GA. Examples of application are presented considering finite and infinite regions in R^2 with different geometries. For external problems a constant was added to the fundamental solution to ensure the current conservation between the anodes and cathodes.

2 The MFS for CP

The mathematical model of the problem, within this conducting domain Ω (electrolyte), is based on a Poisson equation for the electrochemical potential:

$$k\nabla^2\phi(\mathbf{x}) = b(\mathbf{x}), \mathbf{x} \in \Omega, \tag{1}$$

where b is a known function representing the anodes as external sources and k is the conductivity of electrolyte.

In the present work, the metal surfaces are considered to be in direct contact with the electrolyte and therefore the boundary conditions related to Eq. 1 are given in the following form

$$i(\mathbf{x}) = F(\phi), \mathbf{x} \in \Gamma, \tag{2}$$

where Γ is the boundary of Ω, $i(\mathbf{x})$ is the current density in the outward normal direction n and $F(\phi)$ is a nonlinear function of ϕ.

The general solution (ϕ_g) of Eq. 1 is given by adding a particular solution (ϕ_p) to the solution of the associated homogeneous equation (ϕ_h), subjected to the corresponding homogeneous boundary conditions.

Any particular solutions of Eq. 1 can be written in integral form as follows

$$\phi_p(\mathbf{x}) = \int_\Omega G(\xi, \mathbf{x}) b(\xi) d\Omega. \tag{3}$$

The function $G(\xi, \mathbf{x})$ is a fundamental solution of Laplace's equation given by

$$G(\xi, \mathbf{x}) = \frac{1}{2\pi k} ln\left(\frac{1}{r}\right), \tag{4}$$

where r is the Euclidean distance between point ξ and the field point $\mathbf{x}$.

Treating the anodes as point sources, the term $b(\xi)$ is equal to

$$b(\xi) = \sum_{m=1}^{n_{ps}} P(\mathbf{x}_m^{ps}) \delta(\mathbf{x}_m^{ps}, \xi), \tag{5}$$

where $\mathbf{x}^{ps}$ are the coordinates of the point sources, $P(\mathbf{x}^{ps})$ is the intensity of the source given in amps (A), δ is the Dirac delta "function" and n_{ps} is the number of point sources inserted in the electrolyte. Therefore

$$\begin{aligned} \phi_p(\mathbf{x}) &= \sum_{m=1}^{n_{ps}} P(\mathbf{x}_m^{ps}) \int_\Omega G(\xi, \mathbf{x}) \delta(\mathbf{x}_m^{ps}, \xi) d\xi \\ &= \frac{1}{2\pi k} \sum_{m=1}^{n_{ps}} P(\mathbf{x}_m^{ps}) ln\left(\frac{1}{r}\right), \end{aligned} \tag{6}$$

where now r is the Euclidean distance between point $\mathbf{x}_m^{ps}$ and the point $\mathbf{x}$.

In addition, from Ohm's law, the particular solution for a current density is equal to

$$i_p(\mathbf{x}) = k \frac{\partial \phi_p}{\partial n} = -\frac{1}{2\pi} \sum_{m=1}^{n_{ps}} P(\mathbf{x}_m^{ps}) \frac{1}{r} \frac{\partial r}{\partial n}. \tag{7}$$

The numerical solution (ϕ_h) can be obtained by BEM or MFS. This paper presents the formulation of MFS.

The approximate solution of the problem by MFS is represented in the form of a linear superposition of fundamental solutions with singular points $(\mathbf{x}^{sp})$ located outside the domain of the problem. Thus, the electrochemical potential may be written by the summation

$$\phi_h(\mathbf{x}) = \sum_{j=1}^{n_{sp}} G(\mathbf{x}, \mathbf{x}_j^{sp}) c_j, \tag{8}$$

with n_{sp} being the number of source points and the coefficients that occur in the approximate solution are the unknown constants.

Similarly, defining $H = k\frac{\partial G}{\partial n}$, the homogeneous solution for the current density (i_h) is given as

$$i_h(\mathbf{x}) = \sum_{j=1}^{n_{sp}} H(\mathbf{x}, \mathbf{x}_j^{sp}) c_j. \tag{9}$$

The aim of MFS is to determine the coefficients c_j satisfying the boundary condition at the collocation points.

The polarization curve of the structure describes data obtained by a series of experiments in a standard corrosion cell using the dc-potentiodynamic technique [Yan, Pakalapati, Nguyen, and White (1992)], and is given by the expression:

$$i = F(\phi) = e^{\frac{\phi+693.91}{\beta_1}} - \left[\frac{1}{i_1} + e^{\frac{\phi+521.6}{\beta_2}}\right]^{-1} - e^{-\frac{\phi+707.57}{\beta_3}}, \tag{10}$$

with ϕ and i having units mV and $\mu A/cm^2$, respectively. Here, β_1, β_2, β_3 and i_1 are given constant parameters: $\beta_1 = 24mV$, $\beta_2 = 23.47mV$, $\beta_3 = 55mV$ and $i_1 = 86.06\mu A/cm^2$. The conductivity of the electrolyte is equal to $k = 0.0479\Omega^{-1}cm^{-1}$ and the critical value of the electrochemical potential is $\phi_c = -850mV$ (*vs.* SCE).

The general solution of the problem must satisfy Eq. 10, i.e., $i_g = i_p + i_h = F(\phi_p + \phi_h) = F(\phi_g)$. This relationship results in a problem of nonlinear least squares with the design variables defined as the coefficients c_j's and the positions of the source points. The optimization is solved using a GA, which will minimize the following objective function

$$Z(\mathbf{c}, \mathbf{x}^{sp}) = \sqrt{\frac{1}{n_{cp}} \sum_{n=1}^{n_{cp}} \left[i_g^n - F(\phi_g^n)\right]^2}, \tag{11}$$

where Z represents the difference between the current density $(i_p + i_h)$ calculated values and the consistent values of Eq. 10, applied in the general solution of the electrochemical potential, at each collocation point. Also, n_{cp} is the number of collocation points and **c** is a vector containing the coefficients c_j.

The adopted GA used for the minimization of Eq. 11 has a binary representation and is inspired by the algorithm presented in Michalewicz (1996). However, some characteristics were included as the two-point crossover, the elitism and the probabilities of mutation and crossover will vary linearly over the generations.

3 BEM and MFS for external problems

In external problems the equation systems for BEM and MFS can be solved producing a solution that does not necessarily ensure conservation of current between the anodes and cathodes. This property can be included into the formulation through the satisfaction of

$$\int_{\Gamma^+} i(\mathbf{x}) d\Gamma(\mathbf{x}) = 0, \tag{12}$$

for the standard Laplace equation and

$$\int_{\Gamma^+} i(\mathbf{x}) d\Gamma(\mathbf{x}) = -\sum_{m=1}^{n_{ps}} P(\mathbf{x}_m^{ps}), \tag{13}$$

for the Poisson equation with point sources of intensity P, where Γ^+ represent the boundary Γ with positive orientation in relation to the infinite domain Ω.

A boundary integral equation equivalent to Eq. 1, Eq. 2 and Eq. 5 can be written as [Brebbia, Telles, and Wrobel (1984)]

$$\begin{aligned} d(\xi)\phi(\xi) &= \int_{\Gamma^+} G(\xi,\mathbf{x}) i(\mathbf{x}) d\Gamma(\mathbf{x}) - \int_{\Gamma^+} \phi(\mathbf{x}) H(\xi,\mathbf{x}) d\Gamma(\mathbf{x}) \\ &+ \sum_{m=1}^{n_{ps}} P(\mathbf{x}_m^{ps}) G(\mathbf{x}_m^{ps},\mathbf{x}),\ \xi,\ \mathbf{x} \in \Gamma, \end{aligned} \tag{14}$$

where $d(\xi)$ depends on the boundary geometry at the source point ξ.

In case of BEM, the conservation of current between the anodes and cathodes can be imposed in Eq. 14 by considering a constant K in the fundamental solution [Telles, Mansur, and Wrobel (1984)], using the following expression

$$\overline{G}(\xi,\mathbf{x}) = G(\xi,\mathbf{x}) + K, \tag{15}$$

where $G(\xi,\mathbf{x})$ is defined in Eq. 4.

Considering this new fundamental solution, the following extra term is added in Eq. 14

$$K\left(\int_{\Gamma^+} i(\mathbf{x})d\Gamma(\mathbf{x}) + \sum_{m=1}^{n_{ps}} P(\mathbf{x}_m^{ps})\right), \tag{16}$$

this term is associated with the equilibrium presented in Eq. 13. Hence, Eq. 14 with the new term added produces a system of equations yielding a solution with the conservation of current between anodes and cathodes. The added constant K multiplied by the imbalance is considered as an unknown value in the equations system of BEM.

In the case of MFS, the general solution is the contribution of the source points plus the contribution of the point sources, i.e.,

$$\phi_g(\mathbf{x}) = \sum_{j=1}^{n_{sp}} G(\mathbf{x},\mathbf{x}_j^{sp})c_j + \sum_{m=1}^{n_{ps}} G(\mathbf{x},\mathbf{x}_m^{ps})P(\mathbf{x}_m^{ps}). \tag{17}$$

The adoption of the new fundamental solution adds to Eq. 17 the following term

$$K\left(\sum_{j=1}^{n_{sp}} c_j + \sum_{m=1}^{n_{ps}} P(\mathbf{x}_m^{ps})\right), \tag{18}$$

which now is associated with the equilibrium:

$$\sum_{j=1}^{n_{sp}} c_j = -\sum_{m=1}^{n_{ps}} P(\mathbf{x}_m^{ps}). \tag{19}$$

In CP systems, Eq. 19 will ensure that the structure to be protected becomes the cathode of an electrolytic cell while the source points and point sources have an anodic behaviour. The right-had-side of Eq. 19 is zero for the standard Laplace equation case.

Notice that conservation of current between the anodes and cathodes is satisfied in finite regions, whereas for infinite regions with cavities the following integral can be written

$$\int_{\Gamma^-} i(\mathbf{x})d\Gamma(\mathbf{x}) = -\sum_{j=1}^{n_{sp}} c_j, \tag{20}$$

where Γ^- represents the part Γ with negative orientation (cavities) in relation to the infinite domain.

Considering that the identity below is satisfied by the MFS solution:

$$\int_{\Gamma^+} i(\mathbf{x}) d\Gamma(\mathbf{x}) = -\int_{\Gamma^-} i(\mathbf{x}) d\Gamma(\mathbf{x}), \tag{21}$$

and using Eq. 19 and Eq. 20, it is possible to write the following expression

$$\int_{\Gamma^+} i(\mathbf{x}) d\Gamma(\mathbf{x}) = -\int_{\Gamma^-} i(\mathbf{x}) d\Gamma(\mathbf{x}) = \sum_{j=1}^{n_{sp}} c_j = -\sum_{m=1}^{n_{ps}} P(\mathbf{x}_m^{ps}). \tag{22}$$

Hence, Eq. 22 proves that the MFS solution satisfies the current conservation between anodes and cathodes for infinite regions when the Eq. 18 is included in the formulation.

The constant K added to the fundamental solution is a design variable of the optimization problem, $Z = Z(\mathbf{c}, \mathbf{x}^{sp}, K)$. Furthermore, all the candidate solutions of GA must satisfy the equation:

$$\sum_{j=1}^{n_{sp}} c_j = -\sum_{m=1}^{n_{ps}} P(\mathbf{x}_m^{ps}) + \varepsilon, \tag{23}$$

where ε is a given tolerance. Thus, every individuals of the GA will satisfy approximately Eq. 19. In the simulations, the authors used $\varepsilon = \pm 2000 \mu A$. In addition, the following objective function is considered to external problems

$$Z(\mathbf{c}, \mathbf{x}^{sp}, K) = \sqrt{\frac{1}{n_{cp}} \sum_{n=1}^{n_{cp}} \left[i_g^n - F(\phi_g^n) \right]^2} + \left| \sum_{j=1}^{n_{sp}} c_j - \left(-\sum_{m=1}^{n_{ps}} P(\mathbf{x}_m^{ps}) \right) \right|. \tag{24}$$

Eq. 24 provides a solution that satisfies the boundary conditions and the conservation of current for infinite regions.

In several simulations involving external problems, overflows have been detected as a result of the exponential terms present in Eq. 10. This fact occurred due to the addition of the constant K to the fundamental solution and the non exact satisfaction of Eq. 18, in particular at the beginning of the optimization process. For this reason, only potential values within the interval $-2000.0mV < \phi < \phi_{corrosion}$ have been allowed to occur ($\phi_{corrosion} \approx -611.0mV$ *vs.* SCE - corresponds to zero current). Fig. 1 and Fig. 2 show the behaviour of the polarization curve in the intervals $-1000.0mV < \phi < \phi_{corrosion}$ and $-2000.0mV < \phi < \phi_{corrosion}$, respectively. It can be noticed that $i \to -\infty$ when $\phi \to -\infty$.

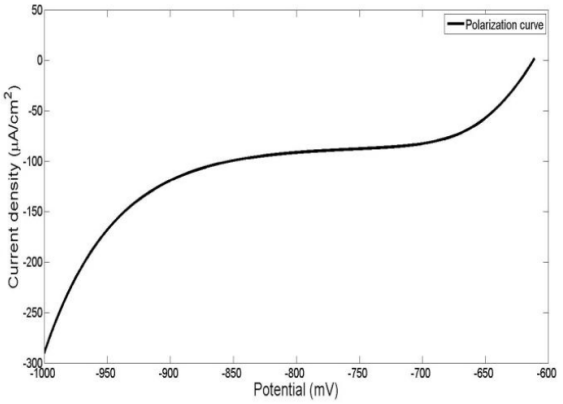

Figure 1: Polarization curve with $-1000.0mV < \phi < -611.0mV$

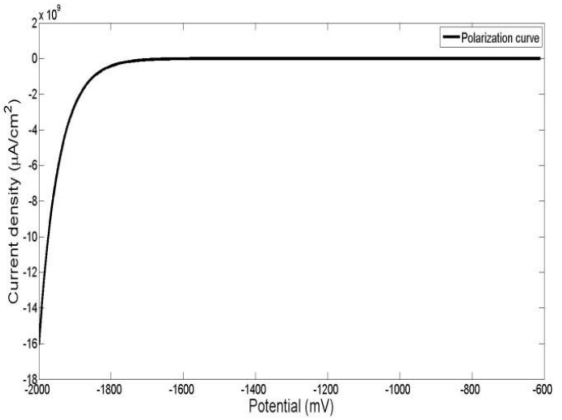

Figure 2: Polarization curve with $-2000.0mV < \phi < -611.0mV$

4 Applications

Simulations have been performed considering metal surfaces in contact with an electrolyte and modelled by polarization curves defined by Eq. 10. Anodes are placed in the electrolyte with the goal of providing a potential distribution on the metal surface under the critical potential $\phi_c = -850mV$ (*vs.* SCE).

In the following applications, $n_{sp} = \frac{n_{cp}}{2}$. The regular number of design variables is equal to $3n_{sp}$, it includes the coefficients c_j and the positions of source points, $\mathbf{x}^{sp} = (x^{sp}, y^{sp})$. The number of design variables is increased to $3n_{sp}+1$ for external problem simulations.

All simulations have been carried out considering finite and infinite regions in R^2. The range of c_j values is assumed to be $[-5000.0, 5000.0]$ for finite regions and $[0.0, 1000.0]$ for the infinite region example. In addition, the following values are used for the GA: population size=30, initial crossover probability=60, final crossover probability=50, initial mutation probability=0.1 and final mutation probability=0.5.

4.1 Finite regions

4.1.1 Example 1

In the first simulation a metallic structure in the form of a rectangle with a localized reentrance in the upper right corner is studied. The dimensions of the structure are $100cm \times 50cm$. Anodes are placed in the electrolyte with a current intensity of $-8200.00\mu A$, sufficient to maintain the potential on the metal surface below the critical potential. The positions of anodes were obtained using an optimization performed in Santos, Santiago, and Telles (2011).

The calculations were performed for $n_{cp} = 150$ collocation points, $n_{sp} = 75$ source points and 225 design variables. The distance between any source points and the geometry was kept not to be greater than $30cm$.

The plot of the source points arrangement determined by GA after 5000 generations is presented in Fig. 3. Fig. 4 presents the potential distribution on the metal determined by MFS and by BEM with 150 constant boundary elements. The average difference - defined in Eq. 11 - between the potential values on the boundary determined by BEM and MFS is $0.27748mV$. The computational time for convergence was approximately $2.3min$.

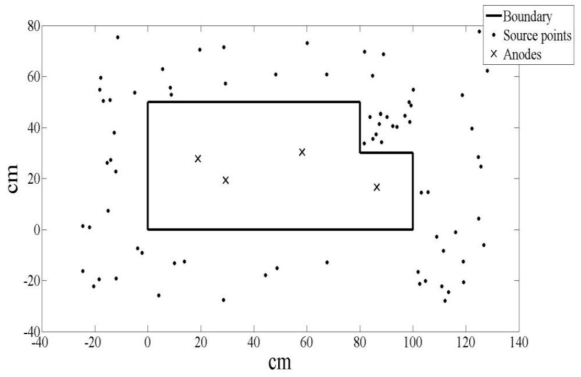

Figure 3: Anodes ($\times$) and source points arrangement by GA

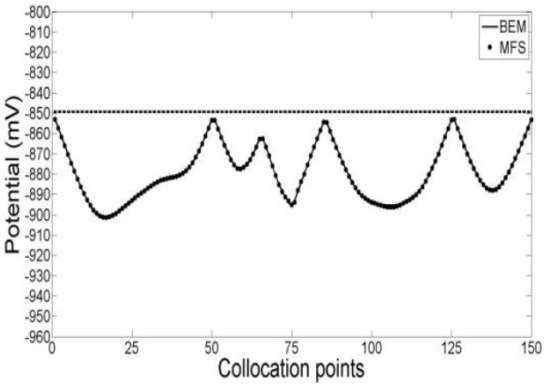

Figure 4: Potential distribution on the metal

In Fig. 5 the potential distribution in the electrolyte using BEM is presented and in Fig. 6 the MFS counterpart result is depicted. The average difference between the potential values at the internal points determined by BEM and MFS is $0.11275mV$.

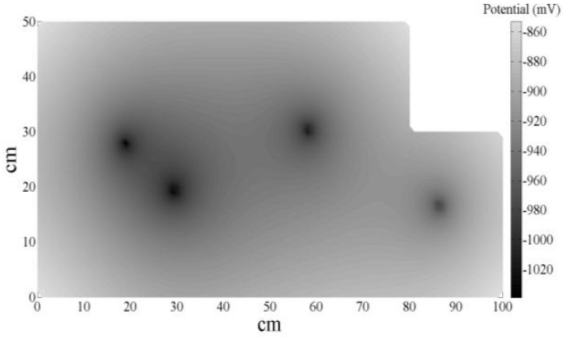

Figure 5: Potential in the electrolyte using BEM

The results show a good agreement between the potential values obtained by BEM and GA MFS. However, due to the large number of design variables (225), the computational time for convergence was also large (2.3 min).

4.1.2 Example 2

The next simulation this a metallic structure in the form of a square cross. The dimensions of the structure are $150cm \times 150cm$. Similarly to the first example, anodes are placed in the electrolyte with current intensity given by $-16600.00\mu A$.

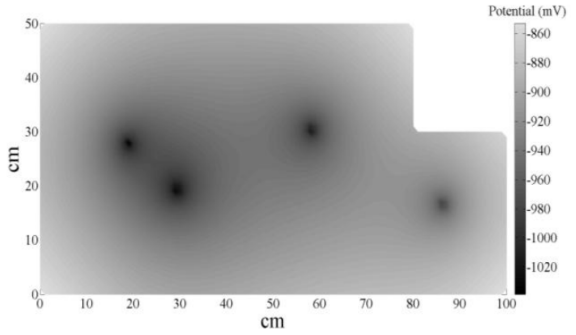

Figure 6: Potential in the electrolyte using MFS

The calculations were performed for $n_{cp} = 168$ collocation points, $n_{sp} = 84$ source points and 252 design variables. The distance between source points and the boundary geometry has been restricted to be less than $25cm$.

The plot of source points arrangement determined by GA after 10000 generations is presented in Fig. 7. In Fig. 8 the potential distribution on the metal determined by MFS and by BEM with 168 constant boundary elements is presented. The difference between the potential values on the boundary determined by BEM and the MFS is $0.429267mV$. The computational time for convergence was approximately $5.6min$.

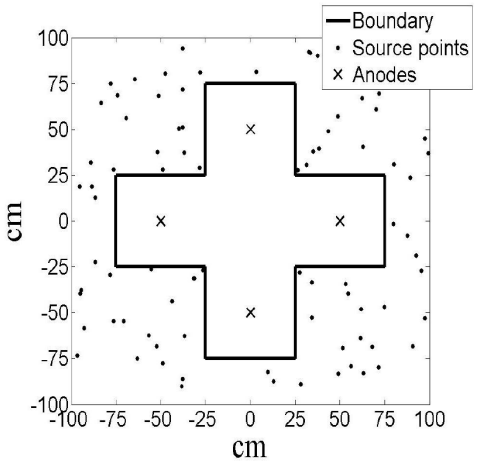

Figure 7: Anodes ($\times$) and source points arrangement by GA

Fig. 9 presents the potential distribution on the electrolyte using BEM and Fig. 10

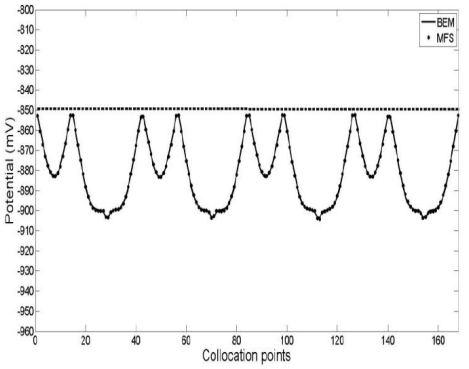

Figure 8: Potential distribution on the metal

depicts the MFS solution. The difference between the potential values at the internal points determined by BEM and MFS is $0.191975mV$.

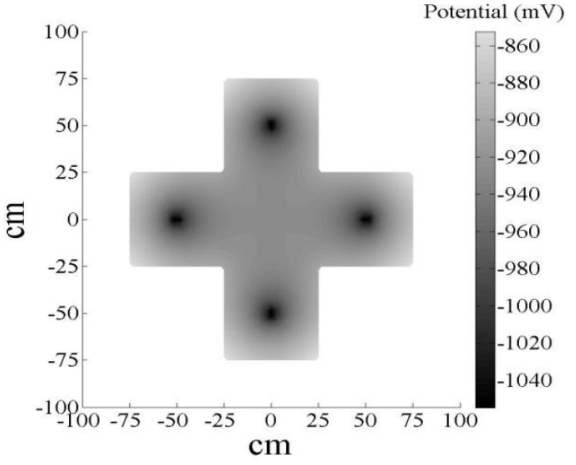

Figure 9: Potential in the electrolyte using BEM

In the second example, it was necessary to make more generations with the GA. Only with 10000 generations was possible to reach a error of the order of $0.1mV$. This fact, can be explicated due to complex geometry and the increasing number of design variables (252).

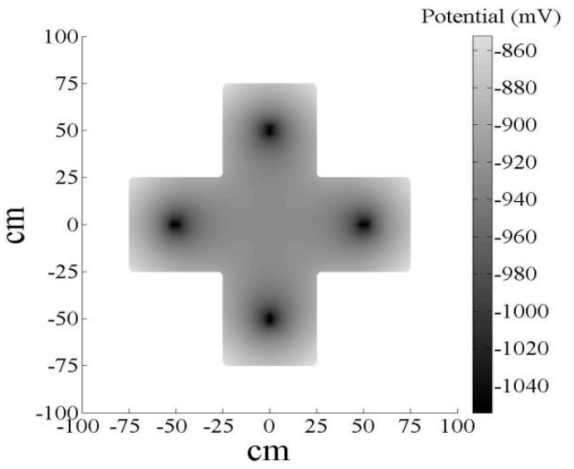

Figure 10: Potential in the electrolyte using MFS

4.2 Infinite regions

4.2.1 Example 1

The last simulation this a submerged metallic structure with square shape. The dimensions of the structure are $100cm \times 100cm$. Anodes are placed in the electrolyte with current intensity given by $-11000.00\mu A$.

The calculations were performed for $n_{cp} = 152$ collocation points, $n_{sp} = 76$ source points and 229 design variables. The source points are localized outside the problem domain, i.e., these are localized inside the square. Therefore, the range for the position of the source points can be assumed as $[-49.9cm, 49.9cm]$. The new design variable K had a range of $[10.0, 5000.0]$.

The plot of source points arrangement determined by GA after 5000 generations is presented in Fig. 11. In Fig. 12 the potential distribution on the metal surface determined by MFS and by BEM with 152 constant boundary elements is presented. The discrepancy between the potential values on the boundary determined by BEM and the MFS is $0.186397mV$. The computational time for convergence was approximately $4.0min$.

Fig. 13 presents the potential distribution in the electrolyte using BEM and Fig. 14 shows the MFS solution. The difference between the potential values at internal points determined by BEM and MFS is $0.788147mV$.

The new design variable converged to $K = 2442.5727$. The summation of the coefficients that occur in the MFS solution was $\sum_{j=1}^{n_{sp}} c_j = 43999.5727278276$. The

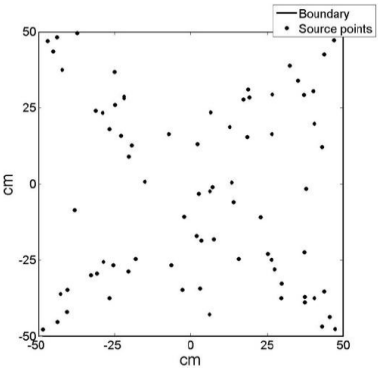

Figure 11: Source points arrangement by GA

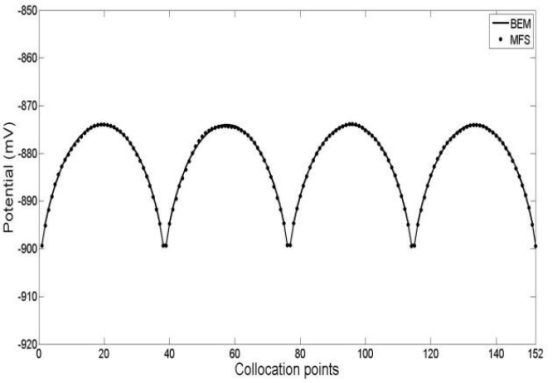

Figure 12: Potential distribution on the metal

summation of the four point sources is $\sum_{m=1}^{n_{ps}} P(x_m^{ps}) = -44000.0\mu A$.

The flux integral was calculated using simple Gauss quadrature rules with a large number of integration points. The flux integral obtained was $\int_{\Gamma^+} i(\mathbf{x}) d\Gamma(\mathbf{x}) = 43999.5727278268$.

In this simulation, the computational time has increased mainly due to the extra condition imposed to all the candidate solutions of GA, see Eq. 23, and the recursive calculation of the new objective function, see Eq. 24.

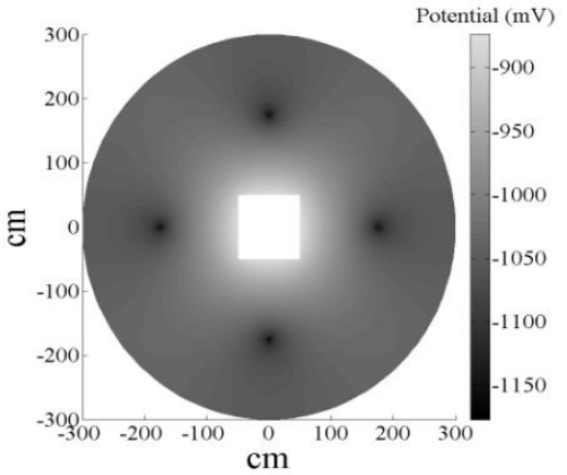

Figure 13: Potential in the electrolyte using BEM

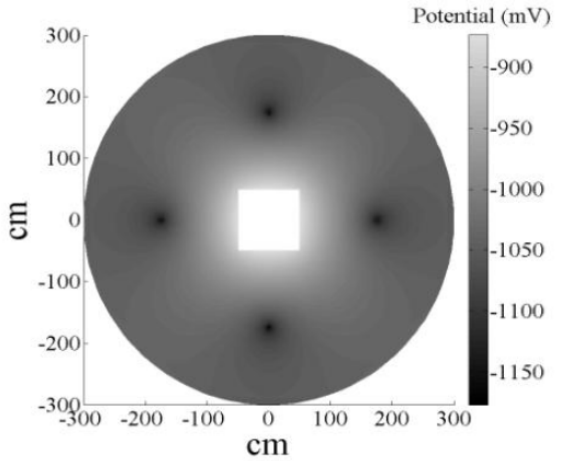

Figure 14: Potential in the electrolyte using MFS

5 Conclusions

The analyzes performed using constant elements BEM and the proposed GA MFS indicate a good agreement between the electrochemical potential distribution on the metal surface and within the electrolyte. The results found confirm GA as a robust optimization procedure to work on such problems. In the first example were

determined 225 design variables, in the second example there were 252 design variables and in the last example there were 229 design variables.

The GA MFS model converged only after many generations due to the large number of design variables. It is important to consider new search algorithms for this non-linear optimization. For example, a deterministic nonlinear least square procedure can be coupled with GA to improve runtime efficiency of the algorithm. This will be the subject of a future publication.

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