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On a Method of Prediction of the Annular Pressure Buildup in Deepwater Wells for Oil & Gas

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Abstract: In deepwater wells for exploration and exploitation of oil & gas, the wellhead structure is of importance, and the annular pressure cannot be released after the casing hanger is set. The pressure changes in the casing annuli, caused by the temperature changes during the drilling and production processes, will increase the risk of failure of the casing. Therefore, a study of the safety of the casing in deepwater wells, by considering the complex engineering factors, will be of both academic as well as practical significance. In this paper, a model for the interactions among the casing-cement-formation system is established, by considering the thermal loading in a perfect well section. The deformation of the casing, as related to the temperature and the internal pressure of the casing, is obtained through a theoretical derivation. The theoretical model, established in the present paper, can be used to predict the annular pressure buildup caused by the thermal expansion of both the casing and the annular fluid. The present research shows that the annular pressure caused by a temperature change can lead to the failure of the casing, when the average temperature change is high enough. Therefore, the effects of temperature and pressure should be taken into account specially in the casing design for deepwater wells for oil & gas.

Keywords: Deepwater well; Casing Load; Annular Pressure Buildup; Casing Design; Casing Failure

1 Introduction

Pressure buildup in trapped annuli has long been the subject in the several studies by Adams [Adams (1991), Adams et al (1994)], Halal [Halal et al (1993)], Leach [Leach et al (1993)] and Wang Bo [Wang Bo (2006)]. There are two reasons which contribute to the pressure buildup in deepwater wells. First, the structure of

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the subsea wellhead is of a special construction in deepwater wells, and the annular pressure cannot be released after the casing hanger is set, so that the pressure buildup in the casing annuli, which is caused by the temperature change between drilling circulation and oil & gas production increases the risk of damage to the casing. Second, the change in the temperature of the annular fluid is more obvious. Based on the researches of Tang [Tang et al (2010)] and Gao [Gao Baokui et al(2006)], this condition is magnified in deepwater because the wellbore is cooled during drilling circulation, and is warmed up during the production or during the well testing for oil & gas.

In the previous researches by Adams [Adams (1991), Adams et al (1994)], the influences of the annular fluid temperature rise, and of the casing temperature rise were taken into account. However, the formation was thought to be rigid, with infinite stiffness. In practice, the formation and the cement around the casing, prevent the casing from expanding under the internal pressure loads, such that the smaller the displacement is, the higher the annular pressure build-up. The interaction among the casing-cement-formation system can be simulated by the mechanical model built by Li [Li et al (2009)] and Qian [Qian et al (2011)]. In this paper, a model for simulating the interactions among the casing-cement-formation system under a thermal loading is established in a perfect well section. Based on the study about the temperature distributions in a wellbore, the casing displacement which changes with the temperature and internal pressure of the casing, is obtained through a theoretical derivation. Thereafter, a theoretical model is established to predict the pressure buildup in different annuli of a deepwater well, by considering the effects of the casingwall thickness, and of the interactions of thermal expansions among the tubular strings, the fluid and the formation. The results for several cases show that the annular pressure may lead to the failure of the casing, when the average temperature rise is high enough.

2 Thermal Expansion of Annular Fluids

The thermal expansion in a wellbore consists of two parts, including the expansion of the annular fluid, and the expansion of the casing. The system consisting of the tubing, casing and annuli is considered to be elastic, and a computational method based on mechanics of elastic media can be used to compute the annular pressure buildup in a deepwater well. All the casings in the system, shown in Fig.1, are numbered as 0,1,2,3 from outside to inside, and the effect of casing couplings is neglected in this analysis. In practice, the fluid volume in annulus B is responsive to temperature and pressure, so that the total volume change ΔV_b^{pe} can be described

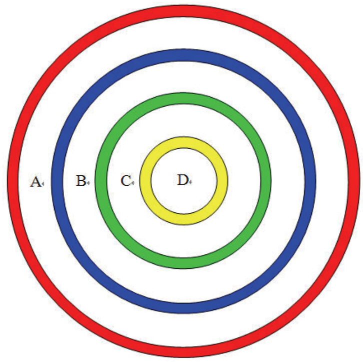

Figure 1: Schematic diagram of casing and annuli

as follows:

$$\Delta V_b^{pe} = \Delta V_b^e - \Delta V_b^p = \Delta V_b^e - \Delta P_b F_b \tag{1}$$

$$F_b = \frac{1}{E_{bf}} \sum_{s=1}^{n_b} \left[\frac{\pi L_s}{4} \left(D_{1i}^2 - D_{2o}^2 \right) \right]_s, E_{bf} = \frac{\Delta P_b V_b}{\Delta V_b^p}, \Delta V_b^e = \sum_{s=1}^{n_b} \left[\frac{\pi}{4} \left(D_{1i}^2 - D_{2o}^2 \right) L_s \beta_b \Delta T_b \right]_s$$

Where, $\Delta P_b F_b$ stands for the fluid volume change caused by ΔP_b; superscript 'p' stands for the pressure effect; superscript 'e' stands for the temperature effect; subscript 'L' refers to unit length, $n_b = D_b / L_s$; subscripts 'i' and 'o' refer to inner diameter and outer diameter respectively.

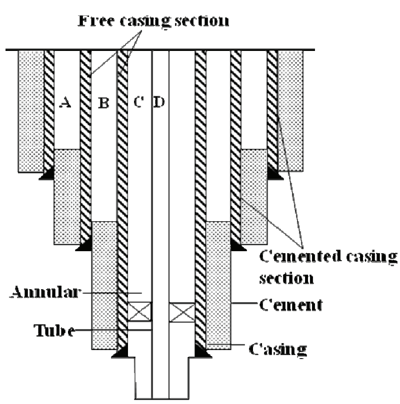

Figure 2: Schematic diagram of well structure

3 Thermal Expansion of the Casing

3.1 Free Casing Section

As is shown in Fig.2, for the whole free section between the annuli A and B, the volume change in the casing, in unit length, caused by diameter variation ΔD_1 can be described as follows:

$$\begin{cases} \Delta V_{1oL} = \frac{\pi}{4}\left[(D_{1o}+\Delta D_{1o})^2 - D_{1o}^2\right] L_s \simeq \frac{\pi L_s D_{1o} \Delta D_{1o}}{2} \\ \Delta V_{1iL} = \frac{\pi}{4}\left[(D_{1i}+\Delta D_{1i})^2 - D_{1i}^2\right] L_s \simeq \frac{\pi L_s D_{1i} \Delta D_{1i}}{2} \end{cases} \tag{2}$$

The total volume change of casing outer wall (taking No.1 casing as an example) can be described as follows:

$$\Delta V_{1o} = \sum_{s=1}^{n_b}\left(\frac{\pi L_s D_{1o} \Delta D_{1oa}}{2}\right)_s \tag{3}$$

Similarly, the total volume change of No.1 casing inner wall can be described as follows:

$$\begin{cases} \Delta V_{1ia} = \sum\limits_{s=1}^{n_a}\left(\frac{\pi L_s D_{1i} \Delta D_{1ia}}{2}\right)_s & 0 \le D \le D_a \\ \Delta V_{1iab} = \sum\limits_{s=1}^{n_{ab}}\left(\frac{\pi L_s D_{1i} \Delta D_{1iab}}{2}\right)_s & D_a < D \le D_b \end{cases} \tag{4}$$

Where $n_{ab} = (D_b - D_a)/L_s$.

The ΔD_1^e stands for the diameter variation caused by temperature variation ΔT_c in a free casing section. According to the thermoelastic theory, the casing wall displacements caused by annular pressure and temperature rise can be described as follows:

$$\begin{cases} \Delta D_{1oa}^{pe} = (1+v_c)\,\beta_c \Delta T_c D_{1o} + \frac{2\left(1-v_c^2\right) D_{1i}^2 D_{1o}}{E_c\left(D_{1o}^2 - D_{1i}^2\right)} \Delta P_b - \frac{\left(1+v_c^2\right)\left[(1-2v_c) D_{1o}^2 + D_{1i}^2\right] D_{1o}}{E_c\left(D_{1o}^2 - D_{1i}^2\right)} \Delta P_a \\ \Delta D_{1ia}^{pe} = (1+v_c)\,\beta_c \Delta T_c D_{1i} + \frac{(1+v_c)\left[(1-2v_c) D_{1i}^2 + D_{1o}^2\right] D_{1i}}{E_c\left(D_{1o}^2 - D_{1i}^2\right)} \Delta P_b - \frac{2\left(1-v_c^2\right) D_{1o}^2 D_{1i}}{E_c\left(D_{1o}^2 - D_{1i}^2\right)} \Delta P_a \end{cases} \tag{5}$$

For the free casing section, the casing expansion can inevitably cause the annular pressure buildup. According to equations (4) and (5), the total casing volume change caused by the variations of temperature and pressure, can be described as follows:

$$\Delta V_{1oa}^{pe} = F_{1o}^{e} - F_{11}^{o} \Delta P_a + F_{12}^{o} \Delta P_b \tag{6}$$

$$\Delta V_{1ia}^{pe} = F_{1i}^{e} - F_{11}^{i}\Delta P_a + F_{12}^{i}\Delta P_b \tag{7}$$

Where

$$F_{11}^{o} = \sum_{s=1}^{n_a}\left[\frac{\pi L_s D_{1o}^2 (1+\nu_c)}{2E_c}\frac{\left[(1-2\nu_c) D_{1o}^2 + D_{1i}^2\right]}{\left(D_{1o}^2 - D_{1i}^2\right)}\right]_s,$$

$$F_{12}^{o} = \sum_{s=1}^{n_a}\left[\frac{\pi L_s D_{1o}^2}{E_c}\frac{\left(1-\nu_c^2\right) D_{1i}^2}{\left(D_{1o}^2 - D_{1i}^2\right)}\right]_s$$

$$F_{11}^{i} = \sum_{s=1}^{n_a}\left[\frac{\pi L_s D_{1o}^2}{E_c}\frac{\left(1-\nu_c^2\right) D_{1i}^2}{\left(D_{1o}^2 - D_{1i}^2\right)}\right]_s,$$

$$F_{12}^{i} = \sum_{s=1}^{n_a}\left[\frac{\pi L_s D_{1i}^2 (1+\nu_c)}{2E_c}\frac{\left[(1-2\nu_c) D_{1o}^2 + D_{1i}^2\right]}{\left(D_{1o}^2 - D_{1i}^2\right)}\right]_s$$

$$F_{1o}^{e} = \sum_{s=1}^{n_a}\left[\frac{\pi}{2} L_s \beta_c (1+\nu_c) D_{1o}^2 \Delta T_c\right]_s,$$

$$F_{1i}^{e} = \sum_{s=1}^{n_a}\left[\frac{\pi}{2} L_s \beta_c (1+\nu_c) D_{1i}^2 \Delta T_c\right]_s$$

3.2 Cemented casing section

For the cemented casing section, external pressure on the casing outer wall, caused by thermal expansion of formation and cement, can lead to an annular pressure buildup. According to appendix A, the volume change of casing inner wall in a cemented section can be described as follows:

$$\Delta V_{1iab}^{pe} = F_{1ab}^{o} q'_{ab} + F_{1ab}^{i}\Delta P_b + F_{1iab}^{e} \tag{8}$$

Where

$$F_{1ab}^{o} = \sum_{s=1}^{n_{ab}}\left[\frac{\left(1-\nu_c^2\right)}{E}\frac{\pi L_s D_{1o}^2 D_{1i}^2}{\left(D_{1o}^2 - D_{1i}^2\right)}\right]_s;$$

$$F_{1ab}^{i} = \sum_{s=1}^{n_{ab}}\left[\frac{\pi L_s (1+\nu_c)}{2E_c}\frac{\left\{(1-2\nu_c) D_{1i}^2 + \left[1-2(1-\nu_c) k_{i2}\right] D_{1o}^2\right\} D_{1i}^2}{D_{1o}^2 - D_{1i}^2}\right]_s;$$

$$F_{1iab}^{e} = \sum_{s=1}^{n_{ab}}\left[\frac{\pi}{2} L_s \beta_c (1+\nu_c) D_{1i}^2 \Delta T_c\right]_s.$$

4 Calculation of the Annular-Pressure Buildup

Based on the annulus B, the above equations are derived through the mechanics of elastic media, and the equations for other annuli can be analogously derived by this method. The following equations can be obtained by considering thermal expansions of the fluid and the casing comprehensively:

$$\begin{cases} \Delta V_a^{pe} = \Delta V_{0ia}^{pe} - \Delta V_{1o}^{pe} \\ \Delta V_b^{pe} = \Delta V_{1ia}^{pe} + \Delta V_{1iab}^{pe} - \Delta V_{2ob}^{pe} \\ \Delta V_c^{pe} = \Delta V_{2ib}^{pe} + \Delta V_{2ibc}^{pe} - \Delta V_{3oc}^{pe} \\ \Delta V_d^{pe} = \Delta V_{3ic}^{pe} \end{cases} \tag{9}$$

In the above equations, the left side represents the volume change in the annular-fluid, and the right side represents the volume change in the casing. The Eq.(14) can be expressed as follows:

$$\begin{bmatrix} \Delta V_a^e + F_{1o}^e - F_{0a}^o q_a' - F_{0ia}^e \\ -F_{1i}^e + F_{2o}^e + \Delta V_b^e - F_{1ab}^o q_{ab}' - F_{1iab}^e \\ -F_{2i}^e + F_{3o}^e + \Delta V_c^e - F_{2bc}^o q_{bc}' - F_{2ibc}^e \\ -F_{3i}^e + \Delta V_d^e \end{bmatrix} = \begin{bmatrix} F_{11}^o + F_a + F_{0a}^i & -F_{12}^o & 0 & 0 \\ -F_{11}^i & F_{12}^i + F_{1ab}^i + F_{21}^o + F_b & -F_{22}^o & 0 \\ 0 & -F_{21}^i & F_{22}^i + F_{2bc}^i + F_{31}^o + F_c & -F_{32}^o \\ 0 & 0 & -F_{31}^i & F_{32}^i + F_d \end{bmatrix} \begin{bmatrix} \Delta P_a \\ \Delta P_b \\ \Delta P_c \\ \Delta P_d \end{bmatrix} \tag{10}$$

It can be also rewritten as:

$$[\Delta P^e] = [F]^{-1} [\Delta V^e] \tag{11}$$

5 Examples

5.1 Parameters

Parameters of the casing-cement-formation system are listed in the following Tables 1 & 2.

5.2 An Analysis of the Strength of the Casing

Taking into account the average temperature rise in the analysis of casing strength may be a valid approach, and is convenient for discussion. The computed results based on the above models are shown in Fig. 3. The average temperature rise

Table 1: Physical parameters

Materials	Elastic Modulus	Coefficient of Thermal Expansion	Poissen's Ratio
Casing	2×10^5	1.2×10^{-5}	0.3
Formation	1.2×10^4	5.8×10^{-6}	0.25
Cement	1×10^4	6×10^{-6}	0.15
Annular fluid	2.2×10^3	6×10^{-4}	

Table 2: Data of well structure

Numbers of pipe	Outer Diameter in	Depth m	TOC m	Density of Drilling Fluids kg/m^3	Fracture Pressure MPa
	20	2070	1479	1036	23.58
1	13-3/8	2365	1970	1108	28.88
2	9-5/8	3590	2265	1192	54.38
3	5-1/2	4094	3490	1306	67.31

leads to an increase of the casing annuli pressure. The rational approach used for conventional casing design is to consider the worst one under all working conditions. But, the neglected pressure buildup in annuli may lead to leak-off at the prior casing shoe. Assuming that the fracture pressure at No.0 casing shoe is p_{af}, the distribution of external pressure on No.1 casing can be expressed as follows:

$$p_o = \Delta P_a + \rho_m g D \text{ or } p_o = p_{af} - \rho_m g D \tag{12}$$

Assuming that the fracture pressure at No.1 casing shoe is p_{bf}, the distribution of internal pressure on No.1 casing can be expressed as follows:

$$p_i = \Delta P_b + \rho_m g D \text{ or } p_i = p_{bf} - \rho_m g D \tag{13}$$

Down-hole accidents may be caused by a collapsed or burst casing, when the maximum value of the differential pressure, $|p_o - p_i|$, exceeds the strength rating of the casing. In a practical design, the strength rating of a casing is the ratio of the casing strength to the design coefficient. The differential pressure is shown as Fig.4 for the average temperature rise of 30° and shown as Fig.5 for the average temperature rise of 40°. It can be seen from Fig.5 that the annular pressure buildup in the annulus can lead to a casing failure and the effects of temperature and pressure should not be neglected in the casing design in deepwater wells.

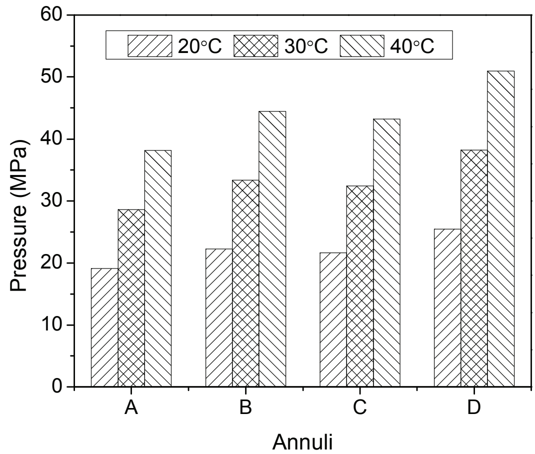

Figure 3: The pressures of confined annuli

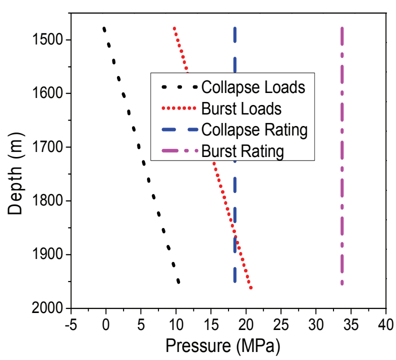

Figure 4: Casing strength check (30°)

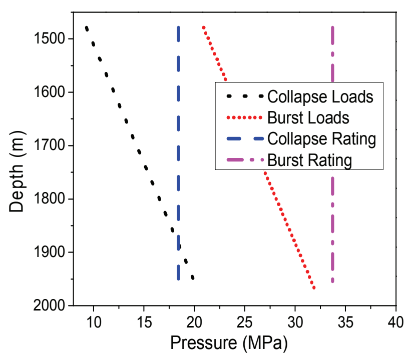

Figure 5: Casing strength check (40°)

6 Conclusions

1. The model for interactions in the casing-cement-formation system has been established, under thermal loading in a perfect well section. The casing displacement due to the temperature and internal pressure of the casing has been obtained through theoretical analysis. The present theoretical model can be used to predict the annular pressure buildup caused by thermal expansion of both the casing and the annular fluid.

2. The annular pressure in a cemented casing section is generally overvalued, because of the effects of thermal expansion and internal pressure. It is shown in this paper that the accuracy of the annular pressure prediction will be improved by considering the interactions among the casing-cement-formation system.

3. The annular pressure buildup is taken into account in the checking for the safety of casings in a deepwater well. The annular pressure buildup in annulus can lead to casing failure, so that the effects of temperature and pressure should not be neglected in the design of casings in deepwater wells.

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Nomenclature

r	radius, m (ft);
r_o	inner radius, m (ft);
a_o	inside radius of casing, m (ft);
a	outside radius of casing, m (ft);
b	outside radius of the formation, m (ft);
ΔT_c	temperature change of casing, ˚C (˚F);
σ_r	radial stress, MPa (psi);
σ_θ	tangential stress, MPa (psi);
ε_r	radial strain, MPa (psi);
ε_θ	tangential strain, MPa (psi);
u	radial displacement, m (ft);
u_c, u_{cem}, u_s	radial displacements of casing, cement, formation, m (ft);
E_c, E_{cem}, E_s	elastic modulus of casing, cement, formation, MPa (psi);

v_c, v_{cem}, v_s	poisson's ratio of casing, cement, formation;
$\beta_c, \beta_{cem}, \beta_s$	thermal expansion coefficients of casing, cement, formation, 1/°C (1/°F);
k_c, k_s	stiffness of casing, formation, MPa/m (psi/ft);
ΔV_b	volume in annulus B, m^3 (ft^3);
ΔV_b^e	tvolume change caused by thermal expansion, m^3 (ft^3);
ΔV_b^p	volume change caused by pressure, m^3 (ft^3);
$\Delta V_a^{pe}, \Delta V_b^{pe}, \Delta V_c^{pe}, \Delta V_d^{pe}$	total fluid volume changes in annular A, B, C, D, m^3 (ft^3);
ΔV_{1oa}^{pe}	total outer volume change of casing in free section, m^3 (ft^3);
ΔV_{lia}^{pe}	total inner volume change of casing in free section, m^3 (ft^3);
ΔV_{1iab}^{pe}	total inner volume change of casing in cemented section, m^3 (ft^3);
$\Delta P_a, \Delta P_b, \Delta P_c, \Delta P_d$	values of pressure buildup in annular A, B, C, D, MPa (psi);
E_{bf}	bulk modulus of fluid, MPa (psi);
D_a	length of annulus A, m (ft);
D_b	length of annulus B, m (ft);
D_{ab}	length of cemented casing, m (ft);
D	length of any annulus, m (ft);
L_s	unit length in free section, m (ft);
β_b	thermal expansion coefficient of annulus B, $^oC^{-1}$ ($^oF^{-1}$);
D_{1i}	inner diameter of No.1 casing, m (ft);
D_{1o}	outer diameter of No.1 casing, m (ft);
ΔD_{1o}	outer diameter change of No.1 casing, m (ft);
ΔD_{1ia}	inner diameter change of No.1 casing, m (ft);
ΔD_{1iab}	inner diameter change of No.1 casing in D_{ab} section, m (ft);
ΔT_b	temperature change of annulus B, ^oC (^oF);
ΔV_{1oL}	outer wall volume change of No.1 casing in unit length, m^3 (ft^3);
ΔV_{1iL}	inner wall volume change of No.1 casing in unit length, m^3 (ft^3);
ΔV^e	volume change after thermal expansion, m^3 (ft^3);
ΔP^e	pressure change, MPa (psi);
p_o	external pressure, MPa (psi);
p_i	internal pressure, MPa (psi);
p_{af}	fracture pressure at No.0 casing shoe, MPa (psi);
p_{bf}	fracture pressure at No.1 casing shoe, MPa (psi);
ρ_m	mud weight, g/cm^3 (lbm/ft^3).

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Appendix A: Interaction model for the casing-cement-formation system

The casing deformation is under restrictions after cementing so that the spatial problem can be considered as a plane one. A reasonably accurate description of the behavior of these stresses in the elastic range can be provided by assuming that casing, cement sheath and formation form a rotational symmetric cylinder model.

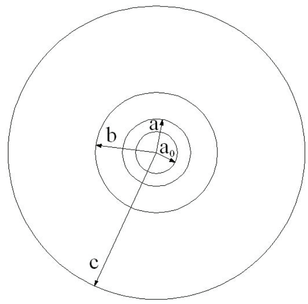

Figure 6: Casing-Cement-Formation System

As shown in Fig.6, it is supposed that inside radius of casing is 'a_0' and outside radius 'a', and outside radius of cement sheath is 'b', and the formation radius is a constant-temperature boundary 'c'. In cylindrical coordinate, the physical equations can be described as follows:

$$\begin{cases} \varepsilon_r = \frac{1-\nu^2}{E}\left[\sigma_\mathrm{r} - \frac{\nu}{1-\nu}\sigma_\theta\right] + (1+\nu)\,\beta\,(r)\,T\,(r) \\ \varepsilon_\theta = \frac{1-\nu^2}{E}\left[\sigma_\theta - \frac{\nu}{1-\nu}\sigma_\mathrm{r}\right] + (1+\nu)\,\beta\,(r)\,T\,(r) \end{cases} \tag{A1}$$

Geometrical equations can be described as follows:

$$\varepsilon_r = \frac{\partial u_r}{\partial r},\ \varepsilon_\theta = \frac{u_r}{r} \tag{A2}$$

Differential equations of equilibrium can be described as follows:

$$\frac{d\sigma_\mathrm{r}}{dr} + \frac{\sigma_\mathrm{r} - \sigma_\theta}{r} = 0 \tag{A3}$$

The following expression can be obtained by substituting geometrical equations and physical equations into differential equations of equilibrium:

$$u = \frac{1+\nu}{1-\nu}\frac{1}{r}\int_{r_0}^{r} r\beta(r)T(r)dr + C_1 r + \frac{C_2}{r} \tag{A4}$$

The following relationships can be obtained by substituting Eq.(A3) into geometrical equations and physical equations:

$$\begin{aligned} &\sigma_r(r) = -\frac{E}{1-\nu}\frac{1}{r^2}\int_{r_0}^{r} r\beta(r)T(r)dr + \frac{C_1E}{(1+\nu)(1-2\nu)} - \frac{C_2E}{(1+\nu)r^2} \\ &\sigma_\theta(r) = \frac{E}{1-\nu}\frac{1}{r^2}\int_{r_0}^{r} r\beta(r)T(r)dr - \frac{E}{1-\nu}\beta(r)T(r) + \frac{C_1E}{(1+\nu)(1-2\nu)} + \frac{C_2E}{(1+\nu)r^2} \\ &\sigma_z(r) = \frac{2C_1\nu E}{(1+\nu)(1-2\nu)} - \frac{E}{1-\nu}\beta(r)T(r) \end{aligned} \tag{A5}$$

A.1 Casing

Casing wall is so thin that can be heated up quickly and simplified into the uniform temperature change. Supposed that q_0 and q_1 are respectively the external pressure and internal pressure, the following relationships can be obtained by substituting these boundary conditions into Eq.(A5):

$$\begin{cases} C_1 = \frac{(1+\nu_c)(1-2\nu_c)}{E_c}\frac{a_0^2 q_0 + a^2 q_1 + a^2 M_c}{a^2 - a_0^2} \\ C_2 = \frac{(1+\nu_c)}{E_c}\frac{a_0^2 a^2}{a^2 - a_0^2}\left(q_0 + q_1 + M_c\right) \end{cases} \tag{A6}$$

Where $M_c = \frac{E_c}{1-\nu_c}\frac{a^2-a_0^2}{2a^2}\beta_c \Delta T_c$. The radial displacement of casing can be described as follows:

$$u_c(r) = \frac{1+\nu_c}{1-\nu_c}\frac{1}{r}\beta_c \Delta T_c \int_{a_0}^{r} r dr + C_1 r + \frac{C_2}{r} \tag{A7}$$

Thus, the radial displacement at outside boundary can be expressed as follows:

$$u_c\left(a\right) = k_{i1} q_0 + k_c q_1 + u_{cT} \tag{A8}$$

Where $k_{i1} = \frac{2\left(1-\nu_c^2\right)}{E_c}\frac{a_0^2 a}{a^2-a_0^2}$; $k_c = \frac{1+\nu_c}{E_c}\frac{\left[(1-2\nu_c)a^2+a_0^2\right]a}{a^2-a_0^2}$; $u_{cT} = \left(1+\nu_c\right) a\beta_c \Delta T_c$.

A.2 Cement

Assuming that both boundaries of the cement sheath are restricted, cement inside boundary 'a' subjects to external pressure -q_1, and outside boundary 'b' subjects to external pressure q_2. The following expressions can be obtained by substituting these boundary conditions into Eq.(A5):

$$\begin{cases} C_1 = \frac{(1+\nu_{cem})(1-2\nu_{cem})}{E_{cem}}\frac{a^2 q_1 + b^2 q_2 + b^2 M_{cem}}{b^2 - a^2} \\ C_2 = \frac{(1+\nu_{cem})}{E_{cem}}\frac{a^2 b^2}{b^2 - a^2}\left(q_1 + q_2 + M_{cem}\right) \end{cases} \tag{A9}$$

Where $M_{cem} = \frac{E_{cem}}{1-\nu_{cem}}\frac{1}{b^2}\int_a^b r\beta_{cem}(r) T_{cem}(r) dr$. Thus, the radial displacement can be expressed as follows:

$$u_{cem}(r) = \frac{1+\nu_{cem}}{1-\nu_{cem}}\frac{1}{r}\int_a^r r\beta_{cem}(r) T_{cem}(r) dr + C_1 r + \frac{C_2}{r} \tag{A10}$$

The radial displacement at the outside and inside boundary can be described as follows:

$$\begin{cases} u_{cem}\left(a\right) = A_{cema} q_1 + B_{cema} q_2 + U_{cemT}\left(a\right) \\ u_{cem}\left(b\right) = A_{cemb} q_1 + B_{cemb} q_2 + U_{cemT}\left(b\right) \end{cases} \tag{A11}$$

Where

$$\begin{cases} A_{cema} = \frac{1+\nu_{cem}}{E_{cem}} \frac{(1-2\nu_{cem})a^2+b^2}{b^2-a^2} a, B_{cema} = \frac{2\left(1-\nu_{cem}^2\right)}{E_{cem}} \frac{b^2}{b^2-a^2} a \\ A_{cemb} = \frac{2\left(1-\nu_{cem}^2\right)}{E_{cem}} \frac{a^2}{b^2-a^2} b, B_{cemb} = \frac{1+\nu_{cem}}{E_{cem}} \frac{(1-2\nu_{cem})b^2+a^2}{b^2-a^2} b \end{cases}$$

and

$$U_{cemT}(a) = \frac{2(1+\nu_{cem})a}{b^2-a^2} \int_a^b r\beta_{cem}(r) T_{cem}(r) dr$$
$$U_{cemT}(b) = \frac{2(1+\nu_{cem})b}{b^2-a^2} \int_a^b r\beta_{cem}(r) T_{cem}(r) dr$$

A.3 Formation

Supposed that boundary "b" of the formation inside is subjected to load "$-q_1$" and its outside boundary is infinitely far away from the wellbore. Then, the following relationships can be obtained by substituting these boundary conditions into Eq.(A5):

$$\begin{cases} C_1 = 0 \\ C_2 = \frac{(1+\nu_s)}{E_s} b^2 q_2 \end{cases} \tag{A12}$$

The displacement distribution can be described as the following relationship by substituting Eq.(A12) into Eq.(A4),:

$$u(r) = \frac{1+\nu_s}{1-\nu_s} \frac{1}{r} \int_b^r r\beta_s(r) T_s(r) dr + \frac{(1+\nu_s)}{E_s} \frac{b^2}{r} q_2 \tag{A13}$$

Thus, the radial displacement at the outside boundary can be expressed as follows:

$$u_s(b) = k_s q_2 \tag{A14}$$

Where $k_s = \frac{(1+\nu_s)}{E_s} b$.

A.4 Continuity Conditions

In order to investigate the impact of high temperature on annular pressure buildup, the following continuity conditions can be established for casing-cement-formation system under the perfect cementing condition:

$$\begin{cases} u_s(b) = u_{cem}(b) \\ u_{cem}(a) = u_c(a) \end{cases} \tag{A15}$$

The following relationships are obtained by substituting Eq.(A19), (A21) and (A14) into Eq.(A15):

$$\begin{cases} A_{cema} q_1 + B_{cema} q_2 + U_{cemT}(a) = k_i q_0 + k_c q_1 + u_{cT} \\ A_{cemb} q_1 + B_{cemb} q_2 + U_{cemT}(b) = k_s q_2 \end{cases} \tag{A16}$$

A.5 Solutions

By solving the above equations, the pressures at boundaries can be expressed as follows:

$$\begin{cases} q_1 = \frac{[U_{cemT}(a)-u_{cT}-k_{i1}q_0](k_s-B_{cemb})+B_{cema}U_{cemT}(b)}{(A_{cema}-k_c)(B_{cemb}-k_s)-A_{cemb}B_{cema}} \\ q_2 = \frac{(k_c-A_{cema})U_{cemT}(b)+A_{cemb}[U_{cemT}(a)-u_{cT}-k_{i1}q_0]}{(A_{cema}-k_c)(B_{cemb}-k_s)-A_{cemb}B_{cema}} \end{cases} \tag{A17}$$

Substituting geometrical equations, physical equations and boundary conditions into equilibrium equations, the casing displacement can be described as follows:

$$\begin{aligned} u_c(a_0) = {} & \frac{(1+\nu_c)\,a_0\left\{(1-2\nu_c)\,a_0^2+[1-2\,(1-\nu_c)\,k_{i2}]\,a^2\right\}}{E_c\left(a^2-a_0^2\right)}q_0 \\ & + \frac{2\left(1-\nu_c^2\right)a_0a^2}{E_c\left(a^2-a_0^2\right)}q_1' + (1+\nu_c)\,a_0\beta_c\Delta T_c \end{aligned} \tag{A18}$$

Where $q' = \frac{[U_{cemT}(a)-u_{cT}](k_s-B_{cemb})+B_{cema}U_{cemT}(b)}{(A_{cema}-k_c)(B_{cemb}-k_s)-A_{cemb}B_{cema}}$.
The above equations shows that the value of $u_c(a_0)$ can be determined by the physical dimensions of casing, internal and external pressures, wellbore temperature and cement material property, etc.