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On the Solution of Burgers-Huxley and Huxley Equation Using Wavelet Collocation Method

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Abstract: In this paper, Haar wavelet method is applied to compute the numerical solutions of non-linear partial differential equations like Huxley and Burgers-Huxley equation. The approximate solutions of the Huxley and Burgers-Huxley equations are compared with the exact solutions. The present scheme is very simple, effective and convenient with small computational overhead.

Keywords: Haar wavelets, Huxley equation, Burgers-Huxley equation.

1 Introduction

Numerical solutions of nonlinear differential equations are of great importance in physical problems since so far there exists no general technique for finding analytical solutions of nonlinear differential equations.

Generalized Burgers-Huxley equation[Ismail, Raslan, and Rabboh (2004); Javidi (2006); Wazwaz (2008)] is a nonlinear partial differential equation of the form

$$u_t + \alpha u^\delta u_x - u_{xx} = \beta u(1-u^\delta)(u^\delta - \gamma), \quad 0 \le x \le 1, t \ge 0 \tag{1}$$

where α, β, γ and δ are parameters, $\beta \ge 0$, γ, $\delta > 0$. When $\alpha = 0$, $\delta = 1$, Eq. (1) reduces to the Huxley equation. The Huxley equation [Wazwaz (2008); Hashemi, Daniali, and Ganji (2007); Zhou (2008)] is a nonlinear partial differential equation of second order of the form

$$u_t = u_{xx} + u(k-u)(u-1), k \neq 0 \tag{2}$$

This equation is an evolution equation that describes nerve pulse propagation in biology from which molecular CB properties can be calculated. Generalized Burgers-Huxley equation is of high importance for describing the interaction between reaction mechanisms, convection effects, and diffusion transport.

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Various powerful mathematical methods such as Adomian decomposition method [Ismail, Raslan, and Rabboh (2004); Hashim, Noorani, and Batiha (2006)], spectral collocation method [Javidi (2006)], the tanh-coth method [Wazwaz (2008)], homotopy perturbation method [Hashemi, Daniali, and Ganji (2007)], Exp-Function method [Zhou (2008)], variational iteration method [Batiha, Noorani, and Hashim (2007)] and Differential Quadrature method [Sari and Güraslan (2009)] have been used in attempting to solve the Burgers-Huxley and the Huxley equations. Most of the solitary wave solutions of the generalized Burgers-Huxley equation have been studied by the learned researchers [Wang, Zhu, and Lu (1990)] and [El-Danaf (2007)].

Yi and Chen (2012) proposed Haar wavelet operational matrix method to solve a class of fractional partial differential equations. They derive the Haar wavelet operational matrix of fractional order integration and also fractional order differentiation. Using operational matrix of fractional order differentiation, the fractional partial differential equations have been reduced to Sylvester equation.

Wei, Chen, Li, and Yi (2012) present a computational method for solving a class of space-time fractional convection-diffusion equations with variable coefficients which is based on the Haar wavelets operational matrix of fractional order differentiation. They also exhibit error analysis in order to show the efficiency of the method.

Zhi-Zhong, Yue-Sheng, and Zhang (2008) develop a numerical method based on wavelet theory for calculating band structures of 2D phononic crystals consisting of general anisotropic materials. They selected two types of wavelets, the Haar wavelet and biorthogonal wavelet. The method, combined with supercell technique, developed by the learned authors applied to compute the band structures of phononic crystals with point or line defects.

Zhou, Wang, Wang, and Liu (2011) present an efficient wavelet-based algorithm for solving a class of fractional vibration, diffusion and wave equations with strong nonlinearities. For that purpose they first suggest a wavelet approximation for a function defined on a bounded interval, in which expansion coefficients are just the function sampling at each nodal point. They use Laplace transform to convert fractional differential equations containing strong nonlinear terms and singular integral kernels into the second type Voltera integral equations with non-singular kernels. They use certain property of the integral kernel and the ability of explicit wavelet approximation to the nonlinear terms of the unknown function in the equation which enables to numerically decouple spatial and temporal dependencies during solution of those equations. They proposed an efficient numerical method without involving any matrix inversions for numerically solving the nonlinear fractional vibration, diffusion and wave differential equations.

In this present paper, Wavelet collocation method for solving generalized Burger-Huxley and Huxley equations is analysed. This method consists of reducing the problem to a set of algebraic equation by expanding the term, which has maximum derivative, given in the equation as Haar functions with unknown coefficients. The operational matrix of integration and product operational matrix are utilized to evaluate the coefficients of Haar functions. This method gives us the implicit form of the approximate solutions of the problems.

This paper is systematized as follows: in Section 1, introduction to Burgers-Huxley and Huxley equation is described. In Section 2, the mathematical preliminaries of Haar wavelet is presented. Section 3 and 5 define the mathematical models of Huxley and Burgers-Huxley equation respectively. We applied the Haar wavelet method to Huxley and Burgers-Huxley equation in Section 4 and 6 respectively. The numerical results and discussions are discussed in Section 7 and Section 8 concludes the paper.

2 Haar wavelets and the operational matrices

The Haar wavelet family for $x \in [0,1)$ is defined as follows [Lepik (2007); Debnath (2002)]

$$h_i(x) = \begin{cases} 1 & x \in [\xi_1, \xi_2) \\ -1 & x \in [\xi_2, \xi_3) \\ 0 & \text{elsewhere} \end{cases} \tag{3}$$

where

$$\xi_1 = \frac{k}{m}, \xi_2 = \frac{k+0.5}{m}, \xi_3 = \frac{k+1}{m}.$$

In these formulae integer $m = 2^j$, $j = 0, 1, 2, \ldots, J$ indicates the level of the wavelet; $k = 0, 1, 2, \ldots, m-1$ is the translation parameter. Maximum level of resolution is J. The index i is calculated from the formula $i = m + k + 1$; in the case of minimal values $m = 1$, $k = 0$ we have $i = 2$. The maximal value of $i = 2M = 2^{J+1}$. It is assumed that the value $i = 1$ corresponds to the scaling function for which

$$h_i(x) = \begin{cases} 1 & \text{for } x \in [0,1) \\ 0 & \text{elsewhere} \end{cases} \tag{4}$$

In the following analysis, integrals of the wavelets are defined as

$$p_i(x) = \int_0^x h_i(x)\,dx, \quad q_i(x) = \int_0^x p_i(x)\,dx$$

This can be done with the aid of 3

$$p_i(x) = \begin{cases} x-\xi_1 & \text{for } x \in [\xi_1, \xi_2) \\ \xi_3 - x & \text{for } x \in [\xi_2, \xi_3) \\ 0 & \text{elsewhere} \end{cases} \tag{5}$$

$$q_i(x) = \begin{cases} 0 & \text{for } x \in [0, \xi_1) \\ \frac{1}{2}(x-\xi_1)^2 & \text{for } x \in [\xi_1, \xi_2) \\ \frac{1}{4m^2} - \frac{1}{2}(\xi_3 - x)^2 & \text{for } x \in [\xi_2, \xi_3) \\ \frac{1}{4m^2} & \text{for } x \in [\xi_3, 1] \end{cases} \tag{6}$$

The collocation points are defined as

$$x_l = \frac{l-0.5}{2M}, l = 1, 2, \ldots, 2M$$

It is expedient to introduce the $2M \times 2M$ matrices H, P, Q with the elements $H(i,l) = h_i(x_l)$, $P(i,l) = p_i(x_l)$, $Q(i,l) = q_i(x_l)$.

3 Huxley Equation

Huxley equation is a nonlinear partial differential equation of second order of the form

$$u_t = u_{xx} + u(k-u)(u-1) \tag{7}$$

with initial condition

$$u(x,0) = \frac{1}{2}\left[1 + \tanh\left(\frac{x}{2\sqrt{2}}\right)\right] \tag{8}$$

The exact solution of Eq. (7) is given by [Zhou (2008)]

$$u(x,t) = \frac{1}{2}\left[1 + \tanh\left\{\frac{1}{2\sqrt{2}}\left(x - \frac{2k-1}{\sqrt{2}}t\right)\right\}\right], k \neq 0 \tag{9}$$

Taking $k = 1$, the boundary conditions are

$$\begin{aligned} u(0,t) &= \frac{1}{2}\left[1 - \tanh\left(\frac{t}{4}\right)\right] \\ u(1,t) &= \frac{1}{2}\left[1 + \tanh\left\{\frac{1}{2\sqrt{2}}\left(1 - \frac{t}{\sqrt{2}}\right)\right\}\right] \end{aligned} \tag{10}$$

4 Application of Haar wavelet method for solving Huxley equation

It is assumed that $\dot{u}^{''}(x,t)$ can be expanded in terms of Haar wavelets as

$$\dot{u}^{''}(x,t) = \sum_{i=1}^{2M} a_s(i)h_i(x) \quad \text{for } t \in [t_s, t_{s+1}] \tag{11}$$

where "$\cdot$" and "$'$" stands for differentiation with respect to t and x respectively.
Now, integrating Eq. (11) with respect to t from t_s to t and twice with respect to x from 0 to x the following equations are obtained

$$\begin{aligned}
u^{''}(x,t) &= (t-t_s)\sum_{i=1}^{2M} a_s(i)h_i(x) + u^{''}(x,t_s) \\
u^{'}(x,t) &= (t-t_s)\sum_{i=1}^{2M} a_s(i)p_i(x) + u^{'}(x,t_s) - u^{'}(0,t_s) + u^{'}(0,t) \\
u(x,t) &= (t-t_s)\sum_{i=1}^{2M} a_s(i)q_i(x) + u(x,t_s) - u(0,t_s) + x[u^{'}(0,t) - u^{'}(0,t_s)] + u(0,t) \\
\dot{u}(x,t) &= \sum_{i=1}^{2M} a_s(i)q_i(x) + x\dot{u}^{'}(0,t) + \dot{u}(0,t)
\end{aligned} \tag{12}$$

By using the boundary conditions, at $x = 1$, we have

$$\dot{u}^{'}(0,t) = -\sum_{i=1}^{2M} a_s(i)q_i(1) + \dot{u}(1,t) - \dot{u}(0,t) \tag{13}$$

From Eq. (6), it is obtained that,

$$q_i(1) = \begin{cases} 0.5 & \text{if } i = 1 \\ \frac{1}{4m^2} & \text{if } i > 1 \end{cases} \tag{14}$$

Discretising the results by assuming $x \to x_l$, $t \to t_{s+1}$, we obtain

$$\begin{aligned}
u^{''}(x_l,t_{s+1}) &= (t_{s+1}-t_s)\sum_{i=1}^{2M} a_s(i)h_i(x_l) + u^{''}(x_l,t_s) \\
u^{'}(x_l,t_{s+1}) &= (t_{s+1}-t_s)\sum_{i=1}^{2M} a_s(i)p_i(x_l) + u^{'}(x_l,t_s) - u^{'}(0,t_s) + u^{'}(0,t_{s+1}) \\
u(x_l,t_{s+1}) &= (t_{s+1}-t_s)\sum_{i=1}^{2M} a_s(i)q_i(x_l) + u(x_l,t_s) - u(0,t_s) \\
&\quad + x_l[u^{'}(0,t_{s+1} - u^{'}(0,t_s)] + u(0,t_{s+1}) \\
\dot{u}(x_l,t_{s+1}) &= \sum_{i=1}^{2M} a_s(i)q_i(x_l) + x_l\dot{u}^{'}(0,t_{s+1}) + \dot{u}(0,t_{s+1})
\end{aligned} \tag{15}$$

Substituting Eqs. (13), (14) and (15) in Eq. (7), we have

$$\sum_{i=1}^{2M} a_s(i)[q_i(x_l) - x_l q_i(1)] = u''(x_l,t_s) - u(x_l,t_s)[1-u(x_l,t_s)]^2 - \dot{u}(0,t_{s+1}) - x_l[\dot{u}(1,t_{s+1}) - \dot{u}(0,t_{s+1})] \tag{16}$$

From Eq. (16), the wavelet coefficients $a_s(i)$ can be successively calculated. This process started with

$$u(x_l,t_0) = \frac{1}{2}\left[1+\tanh\left(\frac{x_l}{2\sqrt{2}}\right)\right]$$
$$u'(x_l,t_0) = \frac{1}{4\sqrt{2}}\sec h^2\left(\frac{x_l}{2\sqrt{2}}\right)$$
$$u''(x_l,t_0) = -\frac{1}{8}\sec h^2\left(\frac{x_l}{2\sqrt{2}}\right)\tanh\left(\frac{x_l}{2\sqrt{2}}\right)$$

5 Burgers-Huxley Equation

Consider the generalized Burgers-Huxley equation

$$u_t + \alpha u^\delta u_x - u_{xx} = \beta u(1-u^\delta)(u^\delta - \gamma), \quad 0 \le x \le 1, t \ge 0 \tag{17}$$

with initial condition

$$u(x,0) = \left(\frac{\gamma}{2} + \frac{\gamma}{2}\tanh[A_1,x]\right)^{1/\delta} \tag{18}$$

The exact solution of Eq. (17) is given by [Javidi (2006); Wang, Zhu, and Lu (1990)]

$$u(x,t) = \left(\frac{\gamma}{2} + \frac{\gamma}{2}\tanh[A_1(x-A_2t)]\right)^{1/\delta} \tag{19}$$

where

$$A_1 = \frac{-\alpha\delta + \delta\sqrt{\alpha^2+4\beta(1+\delta)}}{4(1+\delta)}\gamma,$$
$$A_2 = \frac{\gamma\alpha}{1+\delta} - \frac{(1+\delta-\gamma)(-\alpha+\sqrt{\alpha^2+4\beta(1+\delta)})}{2(1+\delta)} \tag{20}$$

where α, β, γ and δ are parameters with $\beta \ge 0$ and $\delta > 0$.

This exact solution satisfies the following boundary conditions

$$u(0,t) = \left[\frac{\gamma}{2} + \frac{\gamma}{2}\tanh(-A_1A_2t)\right]^{1/\delta}, t \ge 0$$
$$u(1,t) = \left[\frac{\gamma}{2} + \frac{\gamma}{2}\tanh(A_1(1-A_2t))\right]^{1/\delta}, t \ge 0 \tag{21}$$

6 Application of Haar wavelet method for solving Burgers-Huxley equation

Haar wavelet solution of $u(x,t)$ is sought by assuming that $\dot{u}^{''}$ can be expanded in terms of Haar wavelets as

$$\dot{u}^{''}(x,t) = \sum_{i=1}^{2M} a_s(i)h_i(x) \quad \text{for } t \in [t_s, t_{s+1}] \tag{22}$$

where "·" and "′" stands for differentiation with respect to t and x respectively. Integrating eq. (22) with respect to t from t_s to t and twice with respect to x from 0 to x the following equations are obtained

$$\begin{aligned} u^{''}(x,t) &= (t-t_s)\sum_{i=1}^{2M} a_s(i)h_i(x) + u^{''}(x,t_s) \\ u^{'}(x,t) &= (t-t_s)\sum_{i=1}^{2M} a_s(i)p_i(x) + u^{'}(x,t_s) - u^{'}(0,t_s) + u^{'}(0,t) \\ u(x,t) &= (t-t_s)\sum_{i=1}^{2M} a_s(i)q_i(x) + u(x,t_s) - u(0,t_s) + x[u^{'}(0,t) - u^{'}(0,t_s)] + u(0,t) \\ \dot{u}(x,t) &= \sum_{i=1}^{2M} a_s(i)q_i(x) + x\dot{u}^{'}(0,t) + \dot{u}(0,t) \end{aligned} \tag{23}$$

By using the boundary conditions, at $x = 1$, we have

$$\dot{u}^{'}(0,t) = -\sum_{i=1}^{2M} a_s(i)q_i(1) + \dot{u}(1,t) - \dot{u}(0,t) \tag{24}$$

It is obtained from eq. (6) that,

$$q_i(1) = \begin{cases} 0.5 & \text{if } i = 1 \\ \frac{1}{4m^2} & \text{if } i > 1 \end{cases} \tag{25}$$

Discretising the results by assuming $x \to x_l$, $t \to t_{s+1}$ we obtain

$$\begin{aligned}
u''(x_l,t_{s+1}) =& (t_{s+1}-t_s)\sum_{i=1}^{2M} a_s(i)h_i(x_l) + u''(x_l,t_s) \qquad (26)\\
u'(x_l,t_{s+1}) =& (t_{s+1}-t_s)\sum_{i=1}^{2M} a_s(i)p_i(x_l) + u'(x_l,t_s) - u'(0,t_s) + u'(0,t_{s+1})\\
u(x_l,t_{s+1}) =& (t_{s+1}-t_s)\sum_{i=1}^{2M} a_s(i)q_i(x_l) + u(x_l,t_s) - u(0,t_s) +\\
& x_l[u'(0,t_{s+1}) - u'(0,t_s)] + u(0,t_{s+1})\\
\dot{u}(x_l,t_{s+1}) =& \sum_{i=1}^{2M} a_s(i)q_i(x_l) + x_l\dot{u}'(0,t_{s+1}) + \dot{u}(0,t_{s+1})
\end{aligned}$$

Substituting Eqs. (24), (25) and (26) in Eq. (17), we have

$$\begin{aligned}
\sum_{i=1}^{2M} a_s(i)[q_i(x_l) - x_l q_i(1)] =& u''(x_l,t_s) + u(x_l,t_s)[1-u(x_l,t_s)][u(x_l,t_s) - 0.001] -\\
& u(x_l,t_s)u'(x_l,t_s) - \dot{u}(0,t_{s+1}) - x_l[\dot{u}(1,t_{s+1}) - \dot{u}(0,t_{s+1})]
\end{aligned} \qquad (27)$$

From the above equation the wavelet coefficients $a_s(i)$ can be successively calculated. This process started with

$$\begin{aligned}
u(x_l,t_0) &= 1 + \tanh\left(\frac{x_l}{2}\right)\\
u'(x_l,t_0) &= \frac{1}{2}\sec h^2\left(\frac{x_l}{2}\right)\\
u''(x_l,t_0) &= -\frac{1}{2}\sec h^2\left(\frac{x_l}{2}\right)\tanh\left(\frac{x_l}{2}\right)
\end{aligned}$$

7 Numerical Results and discussions

The following tables show the comparisons of the exact solutions with the approximate solutions of Burgers-Huxley equation taking $\alpha = 1$, $\beta = 1$, $\gamma = 0.001$, $\delta = 1$ and different values of t. In tables 1-4, J is taken as 3 i.e. $M = 8$ and Δt is taken as 0.0001.

Table 1: The absolute errors for Burgers-Huxley equation for various collocation points of x with $t = 0.4$ and $\gamma = 0.001$.

x	**Approximate value** (u_{approx})	**Exact value** (u_{exact})	**Absolute Error**
0.03125	0.00050006	0.000500054	6.5661e-9
0.09375	0.000500121	0.000500062	5.90949e-8
0.15625	0.000500234	0.000500069	1.64153e-7
0.21875	0.000500399	0.000500077	3.21739e-7
0.28125	0.000500617	0.000500085	5.31854e-7
0.34375	0.000500887	0.000500093	7.94498e-7
0.40625	0.00050121	0.000500101	1.10967e-6
0.46875	0.000501586	0.000500109	1.47737e-6
0.53125	0.000502014	0.000500116	1.8976e-6
0.59375	0.000502495	0.000500124	2.37036e-6
0.65625	0.000503028	0.000500132	2.89565e-6
0.71875	0.000503613	0.00050014	3.47347e-6
0.78125	0.000504251	0.000500148	4.10381e-6
0.84375	0.000504942	0.000500155	4.78669e-6
0.90625	0.000505685	0.000500163	5.5221e-6
0.96875	0.000506481	0.000500171	6.31009e-6

Table 2: The absolute errors for Burgers-Huxley equation for various collocation points of x with $t = 0.6$ and $\gamma = 0.001$.

x	**Approximate value** (u_{approx})	**Exact value** (u_{exact})	**Absolute Error**
0.03125	0.000500089	0.000500079	9.84903e-9
0.09375	0.000500175	0.000500087	8.86412e-8
0.15625	0.000500341	0.000500094	2.46226e-7
0.21875	0.000500585	0.000500102	4.82602e-7
0.28125	0.000500908	0.00050011	7.97771e-7
0.34375	0.00050131	0.000500118	1.19173e-6
0.40625	0.00050179	0.000500126	1.664e-6
0.46875	0.00050235	0.000500134	2.216e-6
0.53125	0.000502988	0.000500141	2.847e-6
0.59375	0.000503705	0.000500149	3.556e-6
0.65625	0.0005045	0.000500157	4.343e-6
0.71875	0.000505375	0.000500165	5.21e-6
0.78125	0.000506328	0.000500173	6.155e-6
0.84375	0.00050736	0.00050018	7.18e-6
0.90625	0.000508471	0.000500188	8.283e-6
0.96875	0.000509661	0.000500196	9.465e-6

Table 3: The absolute errors for Burgers-Huxley equation for various collocation points of x with $t = 1$ and $\gamma = 0.001$.

x	**Approximate value** (u_{approx})	**Exact value** (u_{exact})	**Absolute Error**
0.03125	0.000500145	0.000500129	1.6414e-8
0.09375	0.000500284	0.000500137	1.47726e-7
0.15625	0.000500555	0.000500144	4.10351e-7
0.21875	0.000500957	0.000500152	8.04288e-7
0.28125	0.00050149	0.00050016	1.32954e-6
0.34375	0.000502154	0.000500168	1.9861e-6
0.40625	0.00050295	0.000500176	2.774e-6
0.46875	0.000503877	0.000500183	3.694e-6
0.53125	0.000504935	0.000500191	4.744e-6
0.59375	0.000506125	0.000500199	5.926e-6
0.65625	0.000507445	0.000500207	7.238e-6
0.71875	0.000508898	0.000500215	8.683e-6
0.78125	0.000510481	0.000500223	1.0258e-5
0.84375	0.000512196	0.00050023	1.1966e-5
0.90625	0.000514042	0.000500238	1.3804e-5
0.96875	0.00051602	0.000500246	1.5774e-5

Table 4: The absolute errors for Burgers-Huxley equation for various collocation points of x with $t = 0.4$ and $\gamma = 2$.

x	**Approximate value** (u_{approx})	**Exact value** (u_{exact})	**Absolute Error**
0.03125	1.01558	0.817686	0.197938
0.09375	1.04679	0.848061	0.198731
0.15625	1.07792	0.878725	0.199192
0.21875	1.10889	0.909622	0.199267
0.28125	1.13965	0.940695	0.198957
0.34375	1.17015	0.971882	0.198263
0.40625	1.20032	1.00312	0.197191
0.46875	1.23011	1.03436	0.19575
0.53125	1.25948	1.06553	0.193949
0.59375	1.28838	1.09657	0.191803
0.65625	1.31676	1.12743	0.189327
0.71875	1.34458	1.15804	0.186538
0.78125	1.37181	1.18835	0.183456
0.84375	1.39841	1.2183	0.180104
0.90625	1.42436	1.24785	0.176504
0.96875	1.44964	1.27695	0.172693

The error function is given by

$$\text{Error function} = ||u_{\text{approx}}(x_l,t) - u_{\text{exact}}(x_l,t)|| = \sqrt{\sum_{l=1}^{2M} (u_{\text{approx}}(x_l,t) - u_{\text{exact}}(x_l,t))^2}$$

$$\text{Global error estimate} = \text{R.M.S.error} = \frac{||u_{\text{approx}}(x_l,t) - u_{\text{exact}}(x_l,t)||}{\sqrt{2M}}$$
$$= \frac{1}{\sqrt{2M}}\sqrt{\sum_{l=1}^{2M}(u_{\text{approx}}(x_l,t) - u_{\text{exact}}(x_l,t))^2} \tag{28}$$

In case of $\gamma = 0.001$, the *R.M.S. error* between the numerical solutions and the exact solutions of Burgers-Huxley equations for $t = 0.4, 0.6$ and 1 are 0.00000300204, 0.00000450295 and 0.00000750449 respectively and for $\gamma = 2$ and $t = 0.4$, the *R.M.S. error* is 0.19142.

In the following tables 5-7 also J has been taken as 3 i.e. $M = 8$ and Δt is taken as 0.0001. Similarly, tables 5-7 exhibit the absolute errors for Huxley equation by taking $k = 1$ and different values of t. Again, the *R.M.S. error* can be calculated from eq. 28 for different values of t. For $t = 0.4, 0.6$ and 1, the *R.M.S. error* between the numerical solutions and the exact solutions of Huxley equation are 0.0209303, 0.0354936 and 0.060677 respectively.

Table 5: The absolute errors for Huxley equation for various collocation points of x with $k = 1$ and $t = 0.4$.

x	**Approximate value** (u_{approx})	**Exact value** (u_{exact})	**Absolute Error**
0.03125	0.455737	0.455641	0.0000955529
0.09375	0.467153	0.466623	0.000530592
0.15625	0.478927	0.477636	0.0012907
0.21875	0.491048	0.488672	0.00237582
0.28125	0.503505	0.499718	0.00378609
0.34375	0.516287	0.510765	0.00552185
0.40625	0.529385	0.521802	0.0075836
0.46875	0.542789	0.532817	0.00997205
0.53125	0.556488	0.5438	0.012688
0.59375	0.570474	0.554741	0.0157326
0.65625	0.584736	0.565629	0.019107
0.71875	0.599266	0.576454	0.0228126
0.78125	0.614057	0.587206	0.026851
0.84375	0.629099	0.597876	0.031223
0.90625	0.645576	0.608453	0.037123
0.96875	0.668244	0.61893	0.049314

Table 6: The absolute errors for Huxley equation for various collocation points of x with $k = 1$ and $t = 0.6$.

x	**Approximate value** (u_{approx})	**Exact value** (u_{exact})	**Absolute Error**
0.03125	0.431155	0.430968	0.000187446
0.09375	0.442789	0.441837	0.000951983
0.15625	0.454998	0.452763	0.00223552
0.21875	0.467771	0.463734	0.00403766
0.28125	0.481098	0.47474	0.0063583
0.34375	0.494968	0.485771	0.0091976
0.40625	0.509372	0.496816	0.012556
0.46875	0.524298	0.507863	0.0164343
0.53125	0.539737	0.518904	0.0208334
0.59375	0.55568	0.529925	0.0257546
0.65625	0.572117	0.540918	0.0311994
0.71875	0.589041	0.551871	0.0371696
0.78125	0.606441	0.562774	0.043667
0.84375	0.62431	0.573616	0.050694
0.90625	0.645842	0.584389	0.061453
0.96875	0.683829	0.595081	0.088748

Table 7: The absolute errors for Huxley equation for various collocation points of x with $k = 1$ and $t = 1$.

x	**Approximate value** (u_{approx})	**Exact value** (u_{exact})	**Absolute Error**
0.03125	0.383171	0.382747	0.00042376
0.09375	0.395066	0.393241	0.00182475
0.15625	0.407796	0.403834	0.00396201
0.21875	0.421352	0.414518	0.00683392
0.28125	0.435721	0.425282	0.0104393
0.34375	0.450895	0.436118	0.0147773
0.40625	0.466863	0.447015	0.0198477
0.46875	0.483614	0.457964	0.0256507
0.53125	0.501139	0.468953	0.0321868
0.59375	0.519429	0.479972	0.0394572
0.65625	0.538474	0.491011	0.0474633
0.71875	0.558265	0.502058	0.056207
0.78125	0.578795	0.513104	0.065691
0.84375	0.600054	0.524137	0.075917
0.90625	0.632639	0.535146	0.097493
0.96875	0.718961	0.546121	0.17284

In case of Burgers-Huxley equation, the figures 1-3 cite the comparison graphically between the numerical and exact soutions for different values of t and γ. Similarly, in case of Huxley equation, the figures 4-6 present the comparison graphically between the numerical and exact solutions for different values of t and $k = 1$.

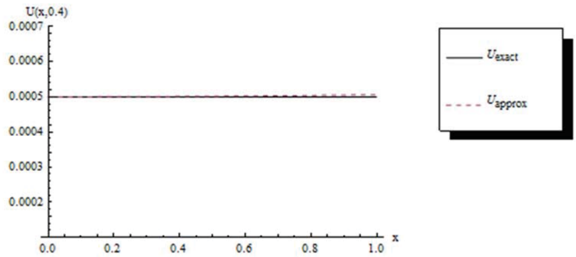

Figure 1: Comparison of Numerical solution and exact solution of Burgers-Huxley equation when $t = 0.4$ and $\gamma = 0.001$.

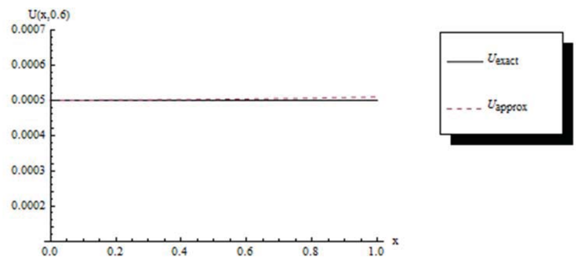

Figure 2: Comparison of Numerical solution and exact solution of Burgers-Huxley equation when $t = 0.6$ and $\gamma = 0.001$

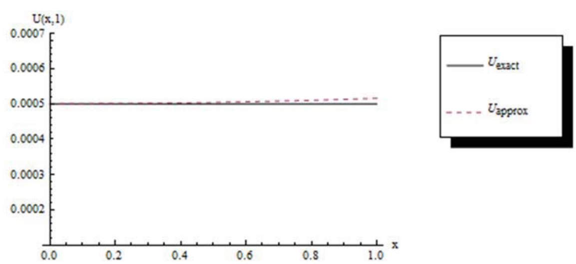

Figure 3: Comparison of Numerical solution and exact solution of Burgers-Huxley equation when $t = 1$ and $\gamma = 0.001$

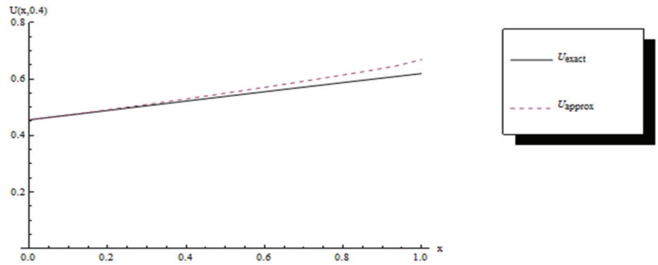

Figure 4: Comparison of Numerical solution and exact solution of Huxley equation for $t = 0.4$ and $k = 1$

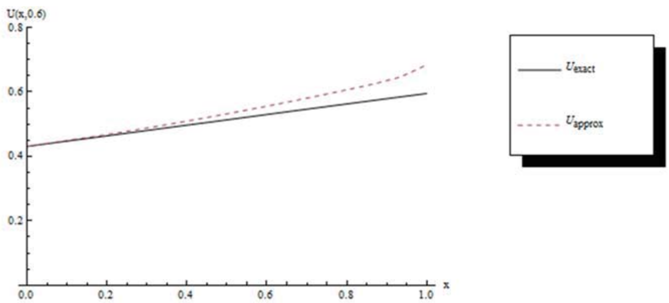

Figure 5: Comparison of Numerical solution and exact solution of Huxley equation for $t = 0.6$ and $k = 1$

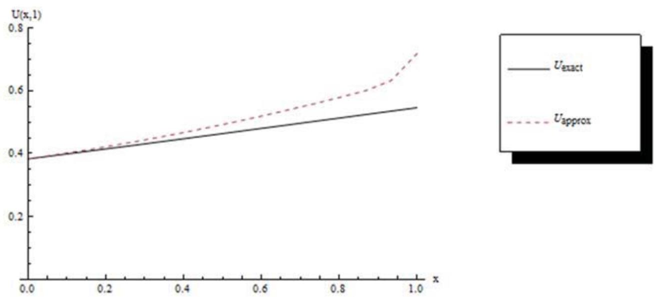

Figure 6: Comparison of Numerical solution and exact solution of Huxley equation for $t = 1$ and $k = 1$

8 Conclusions

In this paper, the generalized Burgers-Huxley equation and Huxley equation have been solved by Haar wavelet method. The obtained results are then compared with exact solutions. These have been cited in Tables and also graphically. These results demonstrated in Tables justify the accuracy and efficiency of Haar wavelet method. This method is reliable and convenient for solving Burgers-Huxley and Huxley equations. The main advantage of this method is its simplicity and small computational overhead.

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