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A Benchmark Problem for Comparison of Vibration-Based Crack Identification Methods

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Abstract: The vibration-based crack identification problem insists of finding a measured vibration parameter from a complete crack-detection-database constructed by numerical simulation. It is one of the classical optimization problems. Many intelligence methods, such as neural network (NN), genetic algorithm (GA), determinant transformation (DT), and frequency contour (FC) etc., have been extensively employed as optimization tools to achieve this task. The aim of this paper is to propose a benchmark problem to compare these extensive-used optimization methods in terms of crack identification precision and computational time. The merit and demerits for each method are discussed. The results suggest that FC is a visualized, stable and easily applied method for detecting crack in practice. The conclusions of current studies are useful to investigators in deciding which method should be chosen in their crack inspections.

Keywords: crack; identification; vibration-based method.

1 Introduction

Cracks present a serious threat to proper performance of structures. It is desirable to detect cracks when they are still very small. Nondestructive testing methods, such as ultrasonic testing, X-ray, acoustic emission, etc., are generally useful for this purpose. However, most of these methods are inconvenient in many situations due to the need for the investigator to have access to the component under analysis for crack detection [Naniwadekar, Naik and Maiti (2008)]. This inconvenience can be avoided through the use of vibration-based inspection because measurement and collection of vibration parameters like natural frequencies is easy. Additionally, vibration-based methods do not require the cleaning of local areas compared to others. Vibration-based methods have so far been intended for exploitation of struc-

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tural fault diagnosis [Wang (2011); Hajnayeb, Ghasemloonia, Khadem and Moradi (2011); Laurentys, Palhares and Caminhas (2011); Li, Chen and He (2005); Li and Meng (2008); Li, Meng and Ye (2008)]. The comparisons of vibration-based method with other nondestructive testing methods are showed in Table 1.

In the vibration-based approach, some signal features, such as change in natural frequencies, change in mode shapes, and change in amplitude of vibration have been taken into account. The natural frequency of structure is most easily measured from accessible point on the static component and convenient to use. Also, such measurement method is fast, easy and cheap. Hence frequency based method is frequently used algorithm in crack inspection.

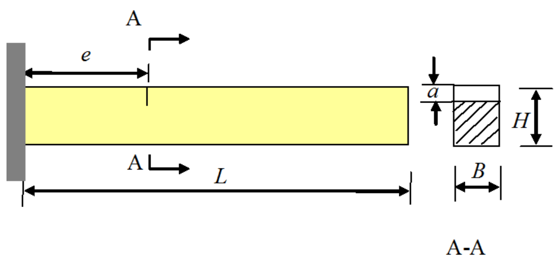


Figure 1: A cantilever beam with an open crack

For a uniform beam with an open crack located at $\beta = e/L$ (Fig.1), L, H, and B represent the length, highness and width of the beam respectively. e and a are the crack location and crack size respectively. β and α stand for the normalized crack position and normalized crack size respectively. The frequency based method includes two procedures [Li and He (2011)]. The first procedure is a forward problem, which comprises the construction of crack model exclusively for crack section and the construction of a numerically structural model to gain crack-detection-database for natural frequencies. That is the determination of function G_s relationship between the first three natural frequencies ω_s, crack normalized location β and crack normalized depth α, as follows

$$\omega_s = G_s(\beta, \alpha) \quad (s = 1,\ 2,\ 3) \tag{1}$$

The second procedure is an inverse problem, which consists of measuring structural frequency and finding a best similar solution to this frequency from a complete crack-detection-database built in the forward problem. That is the determination of crack normalized location β and depth α, as follows

$$(\beta, \alpha) = G_s^{-1}(\omega_s) \quad (s = 1,\ 2,\ 3) \tag{2}$$

Table 1: Comparisons of nondestructive testing methods

Items of comparison	Nondestructive testing methods					
	Vibration-based method	**Infiltrate**	**Magnetic particle**	**Eddy flow**	**Ultrasonic**	**Ray**
Test theory	Vibration response	Capillary phenomenon	Magnetism	Electromagnetic induction	Ultrasonic	Ray image
Test range	Metal, non-metal, etc.	Non-porosity material	Ferromagnetic material	Conduction material	Metal, non-metal, etc.	Metal, non-metal, etc.
Type of crack	Surface and inner crack	Surface crack	Surface and near surface crack	Surface and near surface crack	Surface and inner crack	Surface and inner crack
Active state of specimen	Static and dynamic test	Static test usually	Static test usually	Static test usually	Static and dynamic test	Static and dynamic test
Quantitative test	Ok	Hard	Hard	Hard	Yes	Yes
Test speed	Quick	Slow	Quick	Quick	Quick	Quick
Geometry of specimen	Complex structures	Bar, beam and plate, etc.	Bar, beam and plate, etc.	Bar, beam and plate, etc.	Bar, beam and plate, etc.	Complex structures
Sensitivity	High	High	High	Low	High	Middle
Pollution	Low	High	High	Low	Low	High

The scheme of crack identification problem can be depicted by Fig.2.

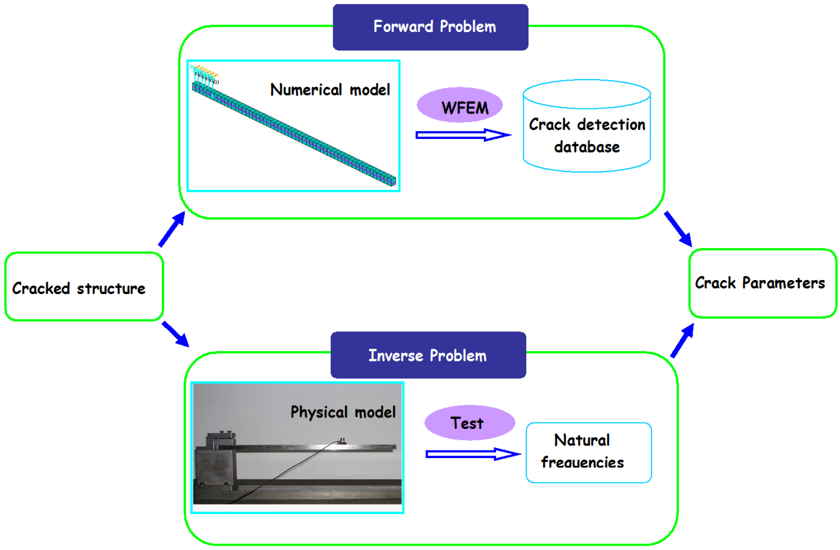


Figure 2: Scheme for crack identification

In the forward problem studies, Dong and Atluri [Dong and Atluri (2012a); Dong and Atluri (2013a); Dong and Atluri (2013b); Dong and Atluri (2013c)] developed symmetric Galerkin boundary element method (SGBEM) for modeling cracked 2D and 3D solid structures. Their methods significantly save computational costs and improve computational accuracy in fracture analyses of damaged structures. Nandwana [Naniwadekar, Naik and Maiti (2008)] modeled the crack as a rotational spring and gave an analytical solution for beams. Lele [Lele and Maiti (2005)] employed finite elements to make an efficient calculation for crack identification in a short beam with rectangle section. Meanwhile, wavelet finite element method (WFEM) was employed for the identification of a crack in structures due to the fact that wavelet multiresolution theory provides a powerful mathematical tool for function approximation of the displacement and stress field in crack tip [Li, Chen, and He (2005); Lele and Maiti (2005)]. 3D T-Trefftz Voronoi Cell Finite Elements were also constructed for micromechanical modeling and SHM of solids [Dong and Atluri (2012b); Dong and Atluri (2012c); Dong and Atluri (2012d); Dong and Atluri (2012e)]. According to linear fracture mechanics theory, the localized additional flexibility in crack vicinity can be represented by a lumped parameter element. The cracked beam was modeled by wavelet finite elements to gain accurate crack-detection-database.

The inverse problem insists of finding a measured vibration parameter from a complete crack-detection-database constructed by numerical simulation. It is one of the classical optimization problems. With accurately measured frequencies, several algorithms such as neural network (NN) [Dong, Chen and Li (2009); Lin, Zhao and Chen (2010); Xiang, Chen and Yang (2009); Wang and He (2007)], genetic algorithm (GA) [Li, Zhuo and He (2009); Xiang, Zhong, Chen and He (2008); Wang, Chen and He (2011); Vakil, Peimani and Sadeghi (2008)], determinant transformation (DT) [Li and He (2011)], and frequency contour (FC) [Li, Chen and He (2005); Rabinovich, Givoli and Vigdergauz (2007); Ye, He and Chen (2010); Yu and Chu (2009); Wang, Zhang and Ma (2008); Nahvi and Jabbari (2005)] were employed as optimization methods to minimize the errors between numerical simulation and experimental measurement. Numerous other methods are available, new ones have also been introduced at the 7^{th} vibration engineering meeting (VETOMAC2011). Hence it seems to be a reasonable to compare these methods in order to judge upon their performance in applications. The comparisons should help the investigator to decide whether he/she has to worry about the choice of the method at all and, if so, which method should be chosen.

In this paper, a benchmark problem of cantilever beam is given and the frequently-used vibration-based crack identification methods including of NN, GA, DT and FC are studied and compared in terms of crack identification precision and computational time (CPU time). The merit and demerits for each method are discussed. The conclusions of current studies are useful to the practitioners in deciding which method should be chosen in their crack inspections.

2 Crack identification methods

2.1 NN-based crack identification method

The solution of inverse problem for crack identification can be essentially an optimization problem. Neural network (NN) being recognized as a powerful optimization tool, it has gain considerable attentions in the structural damage identification studies [Dong, Chen and Li (2009); Lin, Zhao and Chen (2010); Xiang, Chen and Yang (2009); Wang and He (2007)]. However, both the modal frequencies and the structural response are needed for the training of NN to detect the structural damage in these researches [Dong, Chen and Li (2009); Lin, Zhao and Chen (2010); Xiang, Chen and Yang (2009)]. Li and Zhuo, et al., developed a simple methodology to detect crack location and size using WFEM and NN [Li, Zhuo and He (2009)]. Firstly, the first three natural frequencies of the beam with various crack locations and sizes are accurately found by means of WFEM. The only frequency information which is obtained by WFEM is used as training data for developing the

NN. Then for a particular crack location and size, the three frequencies of the beam are obtained under the situation that measured natural frequencies of crack beams are set as input of NN. The crack location and size can be identified through trained NN and WFEM prediction. The scheme of inverse problem using NN is depicted by Fig. 3 [Li, Zhuo and He (2009)].

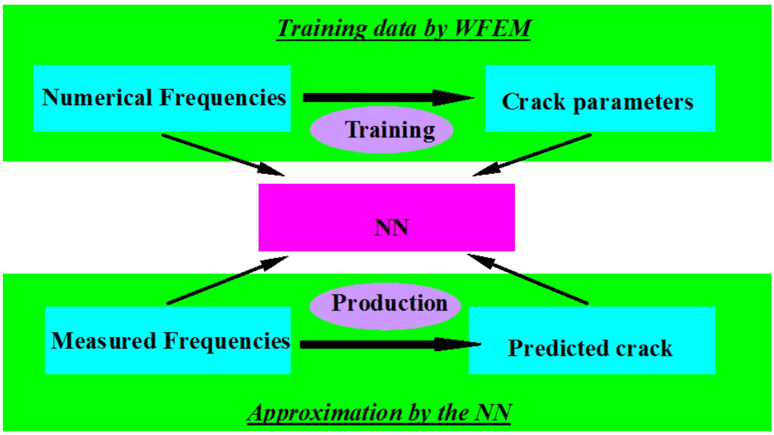


Figure 3: Scheme of NN-based method

2.2 GA-based crack identification method

The solution of inverse problem is essentially an optimization problem. Genetic algorithm (GA) can be employed as an optimization method to minimize the frequencies errors between numerical simulation and experimental measurement. In the genetic algorithm, this errors is used to evaluate the fitness of each individual in the population, the good, if not the best, individual achieved through evolution is just the solution to the inverse problem [Xiang, Zhong, Chen and He (2008)].

Genetic algorithm are stochastic search algorithm, which are based on the mechanics of nature selection and natural genetics, which is designed to efficiently search large, non-linear, discrete and poorly understood search space, where expert knowledge is scarce or difficult to model and where traditional optimization techniques fail. An individual corresponds to a solution for a problem, and consists of and array of gene values, its ‘chromosome’, and as in nature, an individual that is optimized for its environment is created by successive modification over a number of generation. Genetic algorithm have been frequently accepted as optimization methods in various fields, and have also proved their excellence in solving complicate, non-linear, discrete and poorly understood optimization problem [Xiang, Zhong, Chen, and He (2008)]. The generalized procedures of genetic algorithm are shown in Xiang et al. [Xiang, Zhong, Chen and He (2008)] and the scheme of GA is depicted by Fig. 4.

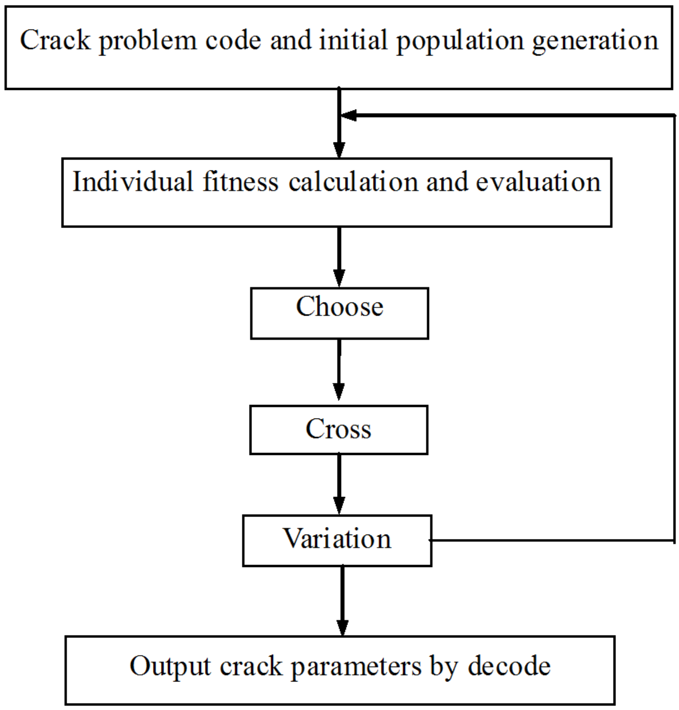


Figure 4: Scheme of GA-based method

In the crack detection in a shaft, the error function *Fit* is defined by the difference between the measured and the simulation frequencies. The explicit form is

$$Fit = -\left(\sum_{s=1}^{3} \left(G_s(\beta,\alpha) - \breve{\omega}_s\right)^2\right) \tag{3}$$

where $G_s(\beta,\alpha)$ is the frequency response function, and $\breve{\omega}_s$ denotes the measured frequency with the s order.

The candidate solution be searched is the normalized crack location β and depth α. Therefore, we can directly use bit strings to decode the candidate solution and the GAOT [Houck, Joines and Kay (1995)] toolbox of Matlab to solve inverse problem.

2.3 DT-based crack identification method

In the studies of forward problem for crack identification, the cracked beam is modeled by using of WFEM, where the crack is seen as a rotational spring with computable stiffness Kt. The values of Kt for various cross sections were given by Dimarogonas in [Dimarogonas (1996)]. Utilizing the determinant transformation (DT) method it transforms the vibration frequency equation into the quadratic equation with one unknown parameter: the rotational spring stiffness Kt. Finding the roots of quadratic equations at different crack locations, the three curves of spring stiffness versus crack location are plotted. The point of intersection of the curves identifies the location and size of the crack. The scheme of DT is depicted by Fig. 5.

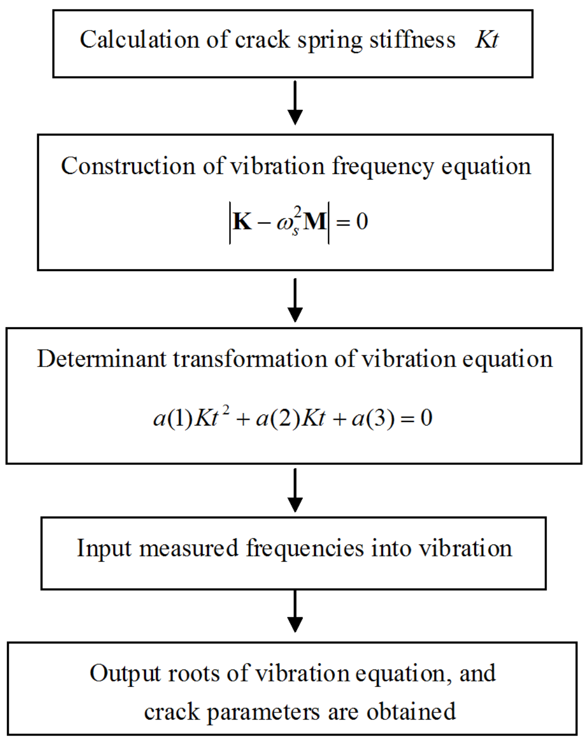


Figure 5: Scheme of DT-based method

Firstly, supposed that the crack is located between two wavelet-based finite elements, and the numbers of two nodes are i and $i+1$, respectively. The spring stiffness Kt was an unknown parameter of the following vibration frequency equation,

$$|\boldsymbol{\Theta}| = \left|\mathbf{K} - \omega_s^2\mathbf{M}\right| = 0 \tag{4}$$

or

$$\begin{vmatrix} k_{1,1}-\lambda_m m_{1,1}\cdots & \cdots & \cdots & \cdots k_{1,n}-\lambda_m m_{1,n} \\ \vdots \quad \vdots & \vdots & \vdots & \vdots \quad \vdots \\ \vdots \quad \vdots & k_{i,i}-\lambda_m m_{i,i}+Kt & k_{i,i+1}-\lambda_m m_{i,i+1}-Kt & \vdots \quad \vdots \\ \vdots \quad \vdots & k_{i+1,i}-\lambda_m m_{i+1,i}-Kt & k_{i+1,i+1}-\lambda_m m_{i+1,i+1}+Kt & \vdots \quad \vdots \\ \vdots \quad \vdots & \vdots & \vdots & \vdots \quad \vdots \\ k_{n,1}-\lambda_m m_{n,1}\cdots & \cdots & \cdots & \cdots k_{n,n}-\lambda_m m_{n,n} \end{vmatrix} = 0 \tag{5}$$

where, $|\bullet|$ denotes the determinant and $\lambda_s = \omega_s^2$, $(s = 1,\ 2,\ 3)$ are known natural frequency. **K** and **M** are the structural WFEM stiffness matrix and mass matrix respectively [[Li, Chen, and He (2005)].

According to the determinant calculation properties, the left determinant of Eq.(5) was expanded by ith column and $i+1$th column, and the quadratic equation with one unknown number could be obtained,

$$a(1)Kt^2 + a(2)Kt + a(3) = 0 \tag{6}$$

where

$$a(1) = \begin{vmatrix} & 0 & \\ \boldsymbol{\Theta}(1:n,1:i-1) & \mathrm{X} & \boldsymbol{\Theta}(1:n,i+2:n) \\ & 0 & \end{vmatrix} \tag{7}$$

$$a(2) = \begin{vmatrix} & 0 & \\ \boldsymbol{\Theta}(1:n,1:i-1) & \mathrm{H} & \boldsymbol{\Theta}(1:n,i+1:n) \\ & 0 & \end{vmatrix} + \begin{vmatrix} & 0 & \\ \boldsymbol{\Theta}(1:n,1:i) & \mathrm{N} & \boldsymbol{\Theta}(1:n,i+2:n) \\ & 0 & \end{vmatrix} \tag{8}$$

$$a(3) = |\boldsymbol{\Theta}| \tag{9}$$

$\boldsymbol{\Theta}(i:j,k:l)$ is the sub-matrix formed by the elements (from the ith row to jth row, and the kth column to lth column of $\boldsymbol{\Theta}$). $\mathrm{X} = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$, $\mathrm{H} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$, $\mathrm{N} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$.

The natural frequencies were taken into the Eq.(6), the corresponding spring stiffness Kt was obtained by finding the roots of Eq.(6). The same calculation was repeated in a different crack location β, so we can get three curves of $\beta - Kt$. The crossing points present the crack location β (horizontal ordinate) and spring stiffness Kt (longitudinal coordinate).

Because the crack stiffness Kt could be expressed as [Nandwana, and Maiti (1997)]

$$Kt = \frac{bh^2E}{72\pi(a/h)^2 f(a/h)} \tag{10}$$

where, E is Young's modulus, and

$$\begin{aligned} f(a/h) =& 0.6384 - 1.035\ (a/h) + 3.7201\ (a/h)^2 - 5.1773\ (a/h)^3 \\ & + 7.553\ (a/h)^4 - 7.332\ (a/h)^5 + 2.4909\ (a/h)^6 \end{aligned}, \tag{11}$$

taking Kt into the Eq.(10), we can get the crack depth a.

2.4 FC-based crack identification method

Due to the facts that frequency contour (FC) method [Li, Chen and He (2005); Rabinovich, Givoli and Vigdergauz (2007); Ye, He and Chen (2010); Yu and Chu (2009); Wang, Zhang and Ma (2008); Nahvi and Jabbari (2005)] is visualized and easy utilized in practice, it became mostly popular algorithm in crack identification problems. The crack identification procedure is briefly described as following, and the scheme of FC is depicted by Fig. 6.

Firstly, the WFEM are employed to model cracked structures. The crack is equivalent as a weightless rotational spring and the equivalent stiffness is evaluated by linear fracture mechanics approach. In accordance with the Saint-Venant principle, it is assumed that the crack only affects the region adjacent to it. So the element stiffness matrices, except for the cracked element, may be regarded as unchanged under a certain limitation of element size.

Secondly, by solving local crack stiffness matrix and adding the local crack stiffness matrix into the global stiffness matrix, the high performance wavelet-based model for crack identification is built up.

Thirdly, solve the first three natural frequencies under different normalized crack location and depth and then the influencing functions for diverse normalized crack parameters are obtained by means of surface-fitting techniques.

Finally, the first three identified natural frequencies are employed as the inputs of the inverse problem and the contour for the specified natural frequency can be plotted. Because the identifying system is confirmed, the parameters of a crack are certain. So there must be a common characteristic point in the three contours. While these curves are plotted in one coordinate, this common point must be the intersection, through which the crack parameters can be identified.

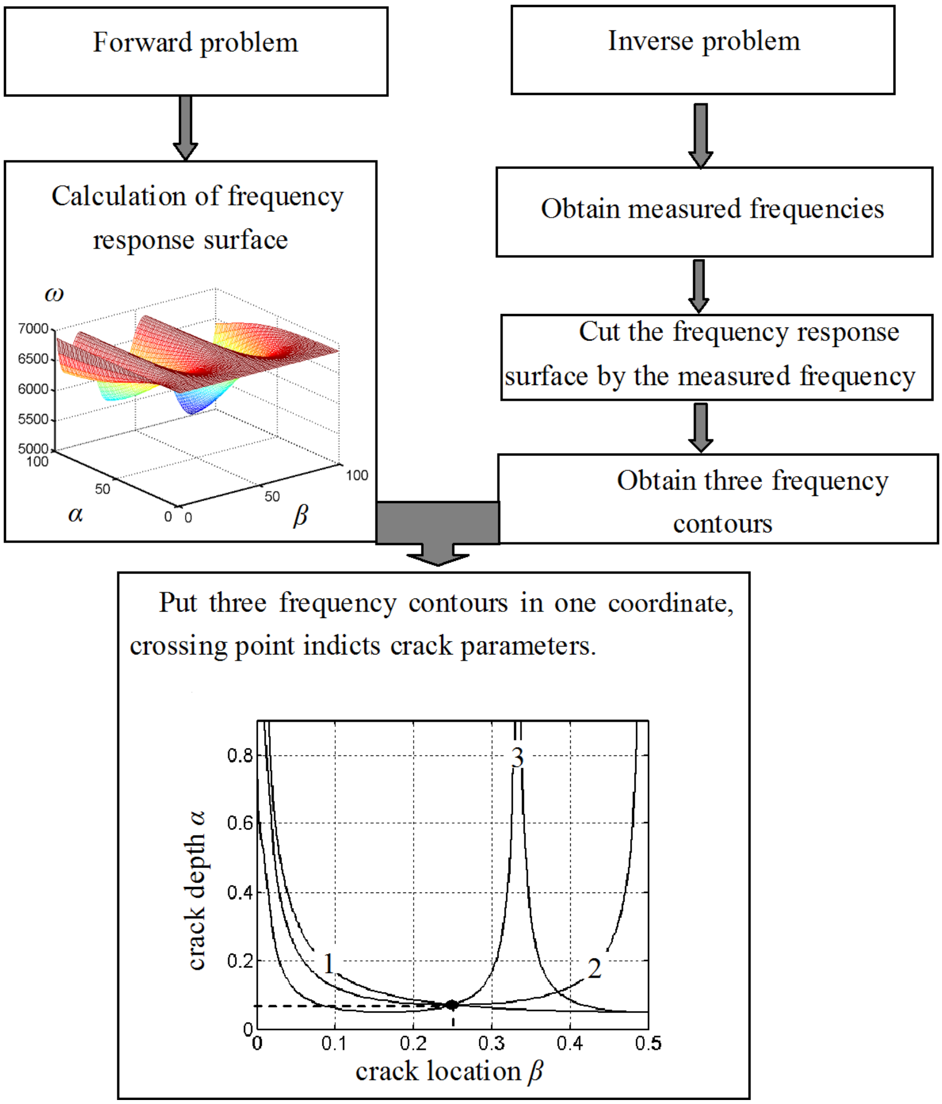


Figure 6: Scheme of FC-based method

3 Benchmark problem

3.1 Description of a benchmark problem and evaluation of forward problem

In order to compare four frequency-based methods of crack identification including of NN, GA, DT and FC, a benchmark problem of cantilever beam with rectangle cross-section is used.

The beam in Fig. 1 is steel beam with 0.02 m $\times$ 0.012 m rectangular cross-section and 0.5 m long. The corresponding material properties were: Young's modulus $E =$ 210 GPa, Poisson's rate $\upsilon = 0.3$, and material density $\rho = 7860$ kg/m^3. Here, the crack in the beam is simulated by a cut normal to the beams' longitudinal axis, with a depth (as listed in Table 2.). The first three natural frequencies were calculated by WFEM, and are listed in Table 3. The analytical solutions are listed in Table 4 [Nandwada and Maiti (1997)].

Table 2: Four crack cases.

case No.	crack location β	crack depth α
I	0.1	0.1
II	0.1	0.2
III	0.4	0.1
IV	0.4	0.2

3.2 Calculations of inverse problem

The method for crack identification is verified for several combinations of crack positions and crack sizes listed in Table 2. The first three natural frequencies calculated using analytic method of vibration mechanics are used as input in this case, and the values are listed in Table 4. Four methods including of NN, GA, DT, and FC are employed to detect crack respectively, and the results of calculation precision and cpu time are compared each other.

(1) NN-based crack identification method

In NN-based crack identification, the location $\beta$and the depth α of crack are called the identification parameters P and the first natural frequencies $\omega_s \quad (s = 1,\ 2,\ 3)$ are called the vibration parameters T. When the P are considered in first stage, the resulting T can be obtained by WFEM and this process is forward analysis. A pair of these P and T is called a training data. In the second stage, these training data are used to develop the NN. When the P is given to the input layer, the NN is trained

Table 3a: First order natural frequency ω_1 (rad/s).

β α	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.0	419.70	419.70	419.70	419.70	419.70	419.70	419.70	419.70	419.70	419.70
0.1	419.70	417.06	417.83	418.46	418.94	419.29	419.52	419.64	419.69	419.71
0.2	419.70	409.73	412.58	414.94	416.78	418.12	418.97	419.45	419.65	419.70
0.3	419.70	397.49	403.63	408.84	412.98	416.00	418.00	419.11	419.57	419.70
0.4	419.70	379.70	390.24	399.44	406.97	412.67	416.41	418.55	419.46	419.70
0.5	419.70	356.56	372.10	386.25	398.29	407.65	414.04	417.69	419.27	419.69
0.6	419.70	329.88	350.18	369.51	386.79	400.81	410.71	416.48	418.99	419.66
0.7	419.70	302.71	326.72	350.67	373.15	392.33	406.45	414.92	418.64	419.65
0.8	419.70	278.14	304.51	331.91	358.88	383.01	401.61	413.08	418.25	419.60
0.9	419.70	258.44	286.04	315.63	345.88	374.13	396.80	411.25	417.83	419.58

Table 3b: Second order natural frequency ω_2 (rad/s)

β α	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.0	2630.3	2630.3	2630.3	2630.3	2630.3	2630.3	2630.3	2630.3	2630.3	2630.3
0.1	2630.3	2624.2	2630.2	2628.1	2622.6	2618.9	2619.9	2624.1	2628.3	2630.1
0.2	2630.3	2607.9	2629.9	2621.8	2600.9	2587.3	2590.6	2606.4	2622.4	2629.6
0.3	2630.3	2581.8	2629.4	2611.0	2564.3	2534.1	2540.5	2575.3	2611.9	2628.6
0.4	2630.3	2546.8	2628.6	2594.8	2509.9	2455.8	2465.0	2526.2	2594.8	2626.9
0.5	2630.3	2505.6	2627.7	2572.8	2437.7	2352.5	2362.2	2455.1	2568.5	2624.4
0.6	2630.3	2463.7	2626.5	2546.2	2352.8	2232.1	2237.9	2361.6	2531.3	2620.8
0.7	2630.3	2426.4	2625.4	2518.0	2265.3	2108.8	2105.4	2252.8	2483.3	2616.1
0.8	2630.3	2396.8	2624.4	2491.8	2186.2	1997.5	1981.3	2141.8	2428.1	2610.5
0.9	2630.3	2375.7	2623.6	2470.5	2123.3	1909.0	1879.5	2043.9	2373.3	2604.8

Table 3c: Third order natural frequency ω_3 (rad/s)

β α	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.0	7364.9	7364.9	7364.9	7364.9	7364.9	7364.9	7364.9	7364.9	7364.9	7364.9
0.1	7364.9	7362.3	7355.7	7338.6	7351.5	7365.5	7348.3	7329.8	7342.5	7362.2
0.2	7364.9	7353.6	7328.6	7264.7	7313.1	7365.4	7301.0	7231.9	7277.1	7352.7
0.3	7364.9	7340.0	7282.7	7143.9	7250.0	7365.3	7223.7	7071.1	7162.2	7335.6
0.4	7364.9	7322.0	7215.1	6973.7	7160.9	7365.1	7114.8	6843.9	6982.2	7307.1
0.5	7364.9	7301.3	7126.2	6763.4	7050.0	7364.9	6980.4	6562.5	6726.4	7262.4
0.6	7364.9	7280.7	7023.4	6537.3	6930.1	7364.7	6836.2	6260.9	6405.2	7196.4
0.7	7364.9	7262.8	6919.9	6325.7	6817.2	7364.4	6702.0	5981.1	6056.8	7106.6
0.8	7364.9	7248.8	6828.6	6151.0	6723.5	7364.2	6591.9	5753.2	5732.3	6996.9
0.9	7364.9	7239.0	6757.8	6022.3	6654.2	7364.0	6511.3	5588.1	5472.2	6881.0

Table 4: Natural frequencies of cracked beam (rad/s)

case No.	analytical solutions ω_s^* (rad/s)		
	ω_1	ω_2	ω_3
I	417.06	2624.2	7364.8
II	409.72	2607.9	7353.8
III	418.95	2622.6	7351.6
IV	416.78	2600.9	7312.5

by using an error back propagation algorithm until the T are obtained at the output layer. When other P is given to the input layer of this trained network, the network can give the unknown T in the third stage. This means that the developed NN can be used as a tool for the crack identification.

The crack is represented by a rotational spring. Thus, the natural frequencies ω_s for various given crack parameters $(\beta, \alpha = 0,\ 0.1,\ 0.2, \ldots,\ 0.9)$ are obtained through WFEM in Table 3. That is, the 100 training data are obtained. The NN for crack identification is composed of the input layer, the hidden layers and the output layer, and each layer has several units as shown in Fig. 7. In the Fig. 7, the P are the first three natural frequencies $(\omega_1, \omega_2, \omega_3)$. The hidden layer has two layers with 33 units. The output layer has the 2 units (β, α).

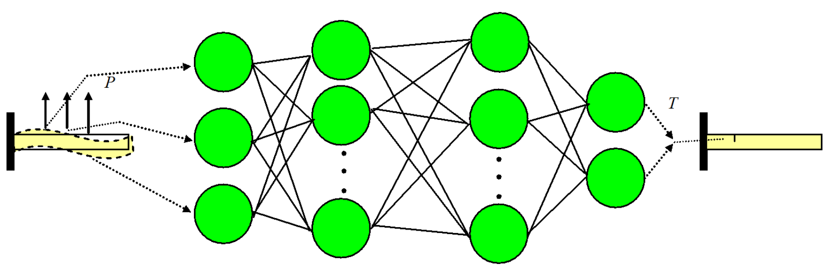


Figure 7: Data processing in neural network

The analytic natural frequencies ω_s^* are used as the input parameters in order to produce the predicted crack variables (β, α). The estimations of crack variables are given by Table 5.

(2) GA-based method

For the discrete samples in Table 2, analytic natural frequencies in Table 4 are used to be input parameters of fitness function for GA. 80 initial populations are chosen

Table 5: Predicted crack variables for the four crack cases using NN

case No.	β (error %)	α (error %)	cpu time (s)
I	0.143 (4.3)	0.185 (8.5)	20.7
II	0.037 (6.3)	0.175 (2.5)	18.4
III	0.317 (8.3)	0.179 (7.9)	20.1
IV	0.434 (3.4)	0.195 (0.5)	20.5

Table 6: Predicted crack variables for the four crack cases using GA

case No.	β (error %)	α (error %)	cpu time (s)
I	0.05 (5.0)	0.05 (5.0)	174.4
II	0.06 (6.0)	0.13 (7.0)	169.8
III	0.48 (8.0)	0.11 (9.0)	176.7
IV	0.43 (3.0)	0.15 (5.0)	174.5

Table 7: Predicted crack variables for the four crack cases using DT

case No.	β (error %)	α (error %)	cpu time (s)
I	0.101 (0.1)	0.199 (0.1)	8.1
II	0.099 (0.1)	0.101 (0.1)	8.2
III	0.401 (0.1)	0.099 (0.1)	8.3
IV	0.397 (0.3)	0.204 (0.4)	8.7

stochastically, the crack location and depth are predicted through the calculations of choose, cross, and variation procedures. The program of GA is GAOT toolbox [Houck, Joines and Kay (1995)] in Matlab software, and the results of identification are predicted in Table 6.

(3) DT-based method

The analytical natural frequencies ω_s^*, $(s = 1, 2, 3)$ are used as the input parameters in order to produce the predicted crack variables (β, α). The intersection of the three curves in Figs. 8 indicates the possible crack position and crack size. When the three curves do not meet exactly, the midpoint of the three pairs of intersections is taken as the crack position and crack size. The estimations of crack variables are given by Table 7.

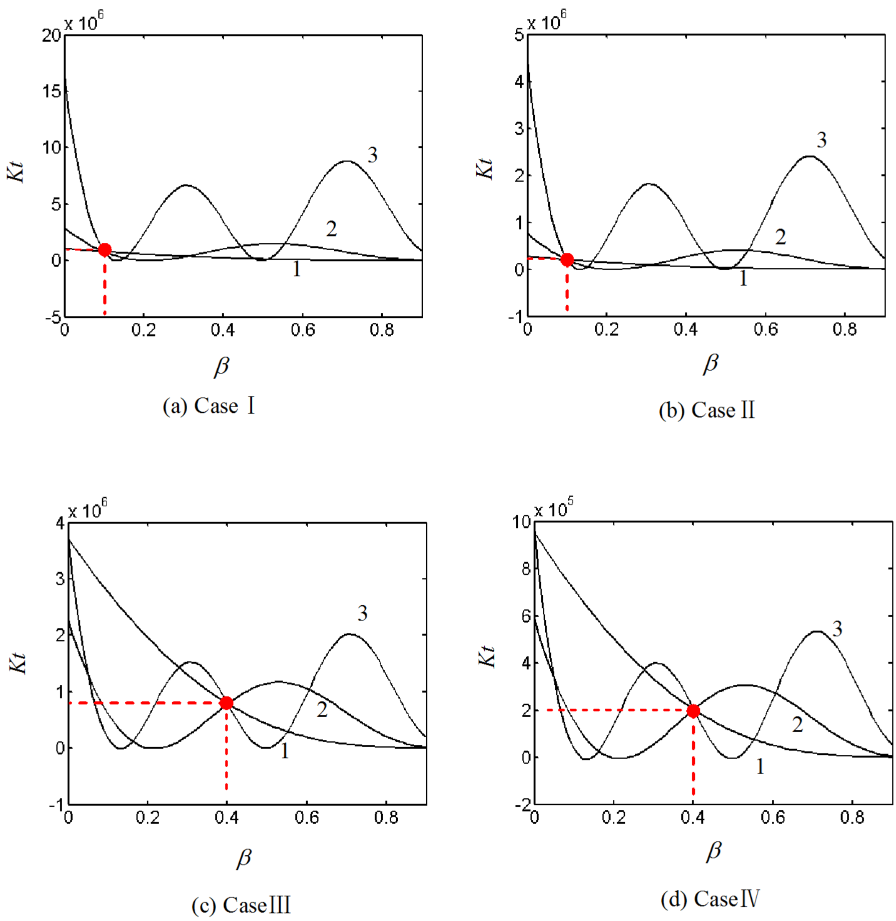


Figure 8: Predicted crack variables using DT for the four crack cases (1: The first order; 2: The second order; 3: The third order)

(4) FC-based method

The first three natural frequencies using analytical method are used as input in this case. Using FC method, the variation of cracks size α and crack position β are plotted for the three modes in Fig. 9. The intersection of the three curves indicates the possible crack position and crack size. When the three curves do not meet exactly, the centroid of the three pairs of intersections is taken as the crack position and crack size. The predicted crack positions and crack sizes are presented in Table 8.

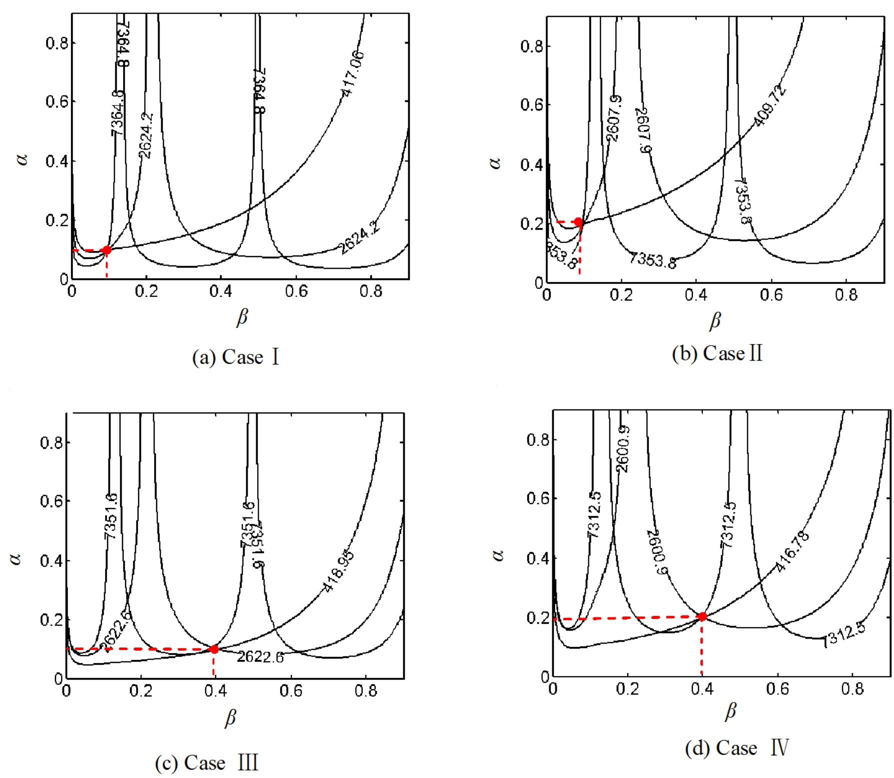
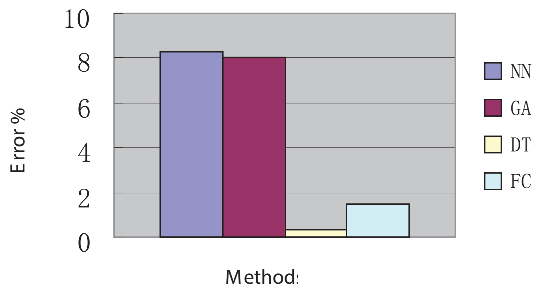


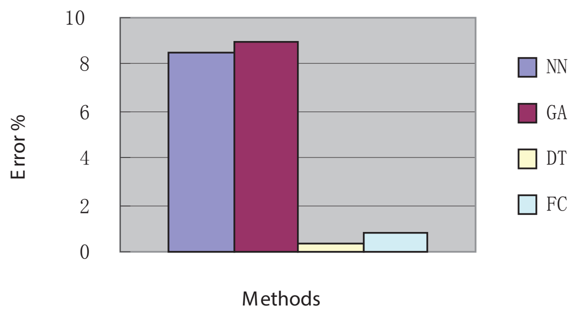
Figure 9: Predicted crack variables using FC for the four crack cases

Table 8: Predicted crack variables for the four crack cases using FC

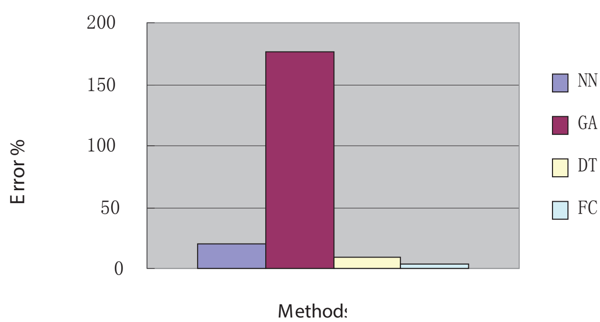
case No.	β (error %)	α (error %)	cpu time (s)
I	0.095 (0.5)	0.097 (0.3)	0.53
II	0.096 (0.4)	0.195 (0.5)	0.54
III	0.385 (1.5)	0.092 (0.8)	0.53
IV	0.396 (0.4)	0.201 (0.1)	0.53



(a) Comparison of crack location prediction error for different methods



(b) Comparison of crack depth prediction error for different methods



(c) Comparison of cpu time for different methods

Figure 10: Comparisons of NN, GA, DT and FC

3.3 Assessment of crack identification methods

The four vibration-based crack identification methods including of NN, GA, DT and FC are compared in terms of crack identification precision and computational time for a given benchmark problem. The comparisons given by means of graphically displaying results are shown in Fig. 10.

(1) NN and GA can output crack prediction results directly without a procedure of looking for the crossing point in DT and FC, however, the CPU time and calculation precision of crack location and depth are worse than DT and FC based methods.

(2) DT is a direct method compared with others. It doesn't need build big crack fault samples or train neural network firstly. So DT has high calculation efficiency. The ability in the computing precision and cpu time both are better than NN and GA. However, the demerit of DT is sensitive to disturb of input parameters in root finding procedure. In addition, the determinant transform and root finding can hardly be achieved for complicate structures crack identification, such as bridge, steel frame building and so on.

(3) The three-dimensional surfaces of the natural frequencies are based on macro-calculation in different crack locations and sizes, which influence the efficiency of FC-based crack identification method. However, due to the facts that frequency contour method is visualized, stable and easy utilized in practice, it became mostly popular algorithm in crack identification problems. In addition, the crack fault samples can only be calculated one time, and need not duplicate compute. So FC is relative optimum method in the group of NN, GA, DT and FC.

4 Conclusions

The problem of crack identification can be essentially an optimization problem. In this paper, a benchmark problem of cantilever beam was given and the four frequently-used crack identification methods including of NN, GA, DT and FC are studied and compared in terms of identification precision and computational time. NN and GA can output crack prediction results directly without a procedure of looking for the crossing point in DT and FC, however, the CPU time and calculation precision of crack location and depth are worse than DT and FC based methods. DT's abilities in the computing precision and CPU time both are better than NN and GA. However, DT is sensitive to disturb of input parameters in root finding procedure. Due to the facts that frequency contour method is visualized, stable and easy utilized in practice, FC became mostly popular algorithm in crack identification problems. In addition, the crack-detection-database can only be calculated one time, and need not duplicate compute. So FC is relative optimum method in the group of NN, GA, DT and FC. The conclusions of current studies are useful to investigators in deciding which method should be chosen in their crack inspections.

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