On the Continuum Modeling of the Tire/Road Dynamic Contact

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Abstract: The continuum modeling of tire/road vibro-contact dynamics is developed in this paper by assuming continuum relationship between the contact force and the deformation. An important aspect of this model is that the damping depends on the indentation. In the continuum approach, no difference is made between impact and contact, and the friction law can be other than the Coulomb's law. Since the road is rocky, a bristle model was chosen to take into account the effect of the road irregularities. The identification of the contact domain is performed by checking the minimum distance between bodies.

Keywords: Dynamic contact; Friction; Rocky road; Tire/road contact.

1   Introduction

Most of real systems exhibit vibro-impacts with the frictional slip. Railway brakes, chattering of machine tools, automotive seating, interaction between tire and the road, for example, have been extensively studied for nearly three decades showing a huge complexity of dynamical responses even for a simple impact oscillator without the frictional slip [Wiercigroch (2006)].

The interfaces between the bodies in contact can experience the vibro-impacts and frictional slips. The vibro-impacts are observed in the contact interfaces for high amplitude vibrations, and they are characterized by very brief duration, rapid dissipation of energy and large accelerations and decelerations. Including of friction into continuum modeling improves the interaction between the driver and the vehicle seat and makes the simulation more realistic. The frictional slips introduce nonlinearities in the stiffness and damping characteristics of the contact interfaces.

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The friction plays a dual role by transmitting energy from one surface to the other and by dissipating energy of relative motion.

The impact and the frictional slip mechanisms work together when they simultaneously develop in a contact interface. In this case, the distribution of forces can be manifested in many different forms, as shown by Jalali et al. (2011), in the problem of micro-vibro-impacts of a clamped Euler-Bernoulli beam. The vibro-impact dynamics changes the friction pattern by the appearance of slips consisting of elastic and plastic deformation. Comprehensive investigations of contact friction and vibro-impact mechanisms developing in contact interfaces have been performed in a series of relevant papers [Chen and Huber (2012); Ferri (1995); Berger (2002); Gilardi and Sharf (2002); Karnopp (1985); Menq et al. (1986)]. An important conclusion of these papers is that the continuous approach has several advantages over the discrete formulation. In the continuous approach no difference is made between impact and contact; therefore the methods of non-impact dynamics can be used to solve the problem. An advantage of the continuous approach over a discrete formulation is that it allows the application of the theory to multi-contact situations, as it is the case for the multi-body system. The Coulomb's law in the discrete approach leads to multiple solutions what is not happened in the continuum approach.

The contact force depends on the deformation and it is defined by the interference distance or penetration. Impacts between bodies are generally defined by the condition of impenetrability [Kim (1999]. The contact can be identified by checking the minimum distance between bodies [Gilardi and Sharf (2002)]

$$\min\left(\frac{1}{2}(r_1 - r_2)^T(r_1 - r_2)\right), \quad f_1(r_1) \leq 0, \ f_2(r_2) \leq 0, \tag{1}$$

where r_1 and r_2 are the position vectors of two points belonging to the tire and the road, respectively, and f_1 and f_2 are bounding surface constraints. The interference distance is defined as

$$\min(-d), \quad f_1(r_1) \leq -\frac{d}{2}e_1, \ f_2(r_2) \leq -\frac{d}{2}e_2, \tag{2}$$

where d is the interference distance and e_1 and e_2 are the unit vectors.

Following these ideas, a continuous approach of the vibro-contact dynamics is developed in this paper by using explicit relationships for contact and friction forces. The main aim of the paper is to model the dynamic contact between the tire and the road, when the slip and vibro-impact mechanisms are simultaneously developed in a contact interface between them.

The model takes as inputs the function of bristle displacement which characterizes the rocky road, and the vertical tire force, and produces as outputs, the identifica-

tion of the contact domain and the distribution of contact pressures in the contact domain.

The paper is organized as follows: Section 2 is devoted to the examination of the contact and friction forces, in context of the rocky road. The identification of the contact domain by checking the minimum distance between bodies is presented in Section 3. Finally, concluding remarks are provided in Section 4.

2   Contact and friction forces

Let us consider the contact between two contacting bodies, namely the tire and the road. These bodies have at the moment time t, a contact interface $\Omega(t)$. We suppose that $\Omega(t)$ simultaneously undergoes the frictional slip and vibro-impact mechanisms when tire is operating o the road. The tire load and velocity generate forces at $\Omega(t)$. These forces act in three directions. There is a vertical component, the contact force $F_c = F_{cz}$ acting in z direction, the longitudinal component of the friction force F_{tx} acting in the x direction, and the lateral component of the friction force F_{ty} acting in the y direction, respectively. A typical contact between the tire and a rocky road is displayed in Fig. 1.

The lateral friction force F_{ty} tend to close the grooves of the tire, remaining fairly constant [Yap (1989)]. The longitudinal friction force F_{tx} is directed towards the center of the footprint and exhibit some changes in its direction close to the midpoints of $\Omega(t)$. In contrast to longitudinal and lateral friction pressures, the vertical contact force is generally non-uniform over $\Omega(t)$ especially because the $\Omega(t)$ is composed by a sum of subdomains $D_{cj}(t) \subset \Omega(t)$, $j = 1, 2, 3..., p$.

Three problems are important to be discussed here, i.e. the modeling of the unknown contact domains, and the definitions of contact and friction forces in Ω.

To shape of the unknown contact domain D_c is taken as the ellipse shape defined by the Lamé curve

$$\left(\frac{x}{a(t)}\right)^n + \left(\frac{y}{b(t)}\right)^n = 1, \quad n > 0, \tag{3}$$

where x and y define the envelope of the contact area, a is half of the contact length, and b is half of the contact width (radii of the oval shape are depending of time), and n the power of the ellipsoid. The case of $n = 2/3$ corresponds to a squashed astroid, $n = 1$ to a squashed diamond, $n = 2$ to ellipse and $n \to \infty$ to rectangles [Gardner (1977)]. The advantage of the Lamé curve consists in the effect of n to rounding the sharp corners. It provides a smooth transition between the oval and the rectangle shape.
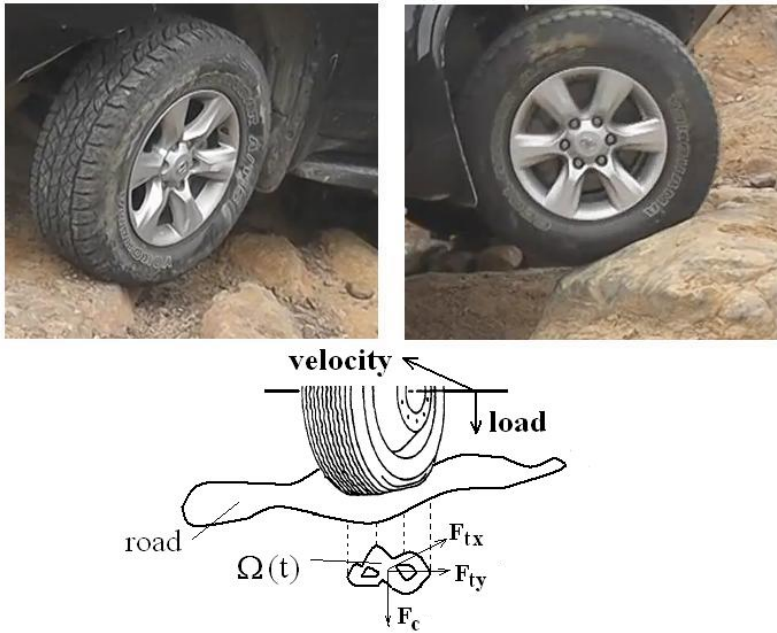
Figure 1: The contact between the tire and the rocky road.

The parametric representation of (3) is

$$x(t) = a(t)\cos^{2/n}\theta, \quad y(t) = b(t)\sin^{2/n}\theta. \tag{4}$$

For area of the ellipsoid (3), we find

$$A(t) = 4b\int_0^a \left(1 - \left(\frac{x}{a}\right)^n\right)^{1/n} dx = \frac{4^{1-1/n}a(t)b(t)\sqrt{\pi}\Gamma\left(1 + \frac{1}{n}\right)}{\Gamma\left(\frac{1}{2} + \frac{1}{n}\right)}, \tag{5}$$

where Γ is the Gamma function

$$\Gamma(z) = \lim_{n\to\infty} \frac{n!n^z}{z(z+1)...(z+n)}, \quad (z \neq 0, -1, -2, ...).$$

In what concerns the contact forces, we consider that they act in a continuous manner during the dynamic contact, and thus, the analysis is performed by adding the contact forces to the equations of motion [Gilardi and Sharf (2002); Sharf and Zhang (2006)]. The case of contact interfaces with corners written in terms of the stress intensity factors was studied by Hwu et al. (2009).

Different models have been postulated in time to represent the contact force of two contacting bodies [Hunt and Crossley (1975); Stronge (1995); Barkan (1974); Brach (1991)]. Hertz (1986)] uses the elastostatic theory to calculate local indentation without the use of damping. Dubowsky and Freudenstein (1971) assume a linear viscous damping law and a Hertzian spring for modeling the contact interfaces. Hunt and Crossley (1975) proposed a model based on Hertz's theory of contact with a non-linear damping force defined in terms of local penetration and the corresponding rate. Other damping models have been proposed to describe totally or partially plastic impacts [Barkan (1974); Van Vliet, Sharf and Ma (2000); Sharf and Nahon (1995); Lim and Stronge (1999); Johnson (1985); Goldsmith (1960)].

In the literature the indentation δ is the principal factor in defining the contact force [Demiral et al, 2010]

$$F_c = f(\delta, \dot{\delta}). \tag{6}$$

A particular form of (6) is

$$F_c = k\delta + b\dot{\delta}, \tag{7}$$

with k and b constants depending on the material and geometry [Sharf and Zhang (2006); Van Vliet, Sharf and Ma (2000)]. This model has some limitations [Gilardi and Sharf (2002)]. Firstly, the contact force at the start of the impact is discontinuous, due to the damping term. When the contacting bodies are separating when the indentation is tending to zero, their relative velocity tends to be negative. As a result, a negative force holding the objects together is present. The equivalent coefficient of restitution defined for this model does not depend on impact velocity. But both elastic and damping forces should be initially at zero and increase over time, and the experimentally demonstrated that the coefficient of restitution e depends on velocity [Goldsmith (1960)].

Another particular form of (7) is the Hertz model

$$F_c = k\delta^n, \tag{8}$$

with k and n constants depending on the material and geometry. In this model, $e = 1$, because the dissipation energy is not present. However, this model can be used only for low impact speeds and hard materials.

Hunt and Crossley (1975) reported another version for (6)

$$F_c = k\delta^n + b\delta^p \dot{\delta}^q, \tag{9}$$

where n, p, q are constants, coefficient k depends on the material and the geometric properties of the bodies in contact, and b is defined with respect to the coefficient

of restitution $0 \leq e \leq 1$. These coefficients are calculated based on the viscoelastic theory. For example, $n = 3/2$ in the case of two spheres in central impact and k is defined in terms of Poisson's ratios, Young's moduli and the radii of the two spheres. The standard values are $p = n$ and $q = 1$. In the case of central impact between two bodies, the coefficient of restitution is $e = 1 - 2b\dot{\delta}_0/3k$ [Gilardi and Sharf (2002)]. An important aspect of this model is that the contact area increases with deformation and a plastic region can appear for larger indentation, i.e. the damping depends on the indentation. Another advantage of (9) is that the contact force has no discontinuities at initial contact and separation, and it begins and finishes with the value of zero.

The energy W_r released during restitution at impact can be calculated as the negative work done by F_c during the collision

$$W_r = - \int_{F_c} \dot{\delta} dt. \tag{10}$$

The coefficient of restitution can be calculated as

$$e = \sqrt{\frac{W_r}{-W_c}}, \tag{11}$$

where W_r is the energy released during restitution and W_c is the energy absorbed during compression.

In this paper we consider the contact pressure distribution obtained by extending the Maugis-Dugdale model [Johnson (1985, 1997); Maugis (1992); Maugis and Barquins (1978); Dugdale. (1960]. The effect of adhesive forces in the contact between the tire and the road is to increase the contact radius above that prescribed by the Hertz theory.

Dugdale approximation is that the adhesive force intensity σ_0 is constant until a separation h_0 is reached, whereupon it falls to zero. Intimate contact is maintained over the domain D_c. Adhesive forces of intensity σ_0 extend the contact domain to the adhesive domain $D_c \subset D_a$, and value $\max(a, b)$ to a value $c > \max(a, b)$, c being half of the contact length (or width).

We consider that the contact pressure in a point comprises two terms, the contact pressure p_1 and the adhesion pressure p_a

$$p = p_z + p_a. \tag{12}$$

The contact pressure p_z is determined from the integral

$$F_c = - \int_A \frac{p_z dA}{}, \tag{13}$$

computed on the area A given by (5) of the contact domain D_c for a time value, with F_c is given by (9). The adhesive pressure p_a is given by

$$p_a = -\frac{\sigma_0}{\pi} \arccos f, \quad \text{for} \quad r \leq \max(a,b) \tag{14}$$

with $r = \sqrt{x^2 + y^2}$ in a rectangular system of coordinates. Adhesive pressure extends the contact domain to $D_c \subset D_a$, with $c > \max(a,b)$, half of the contact length. In the Hertz theory, the function f is given as

$$f = \frac{2\max(a,b)^2 - c^2 - r^2}{c^2 - r^2}. \tag{15}$$

Friction modeling is another key aspect of the vibro-impacts with the frictional slip situation. The friction can stop and/or reverse the motion as well as, it contributes to energy dissipation. The traditionally law used to determine the force of dry friction is the Coulomb's law, which states that the frictional force F_t, is related to the normal force (contact force) F_c, through a friction coefficient, and its direction is always opposite to the relative tangential motion.

Friction F_t occurring at the contact point during sticking can be defined as [Johnson (1997)]

$$F_t = k_t \delta_t, \tag{16}$$

where δ_t is the tangential component of displacement at the contact point and k_t is the tangential stiffness.

For the friction force F_t we propose a bristle model which is able to represent the effect of road irregularities on the tire by using bristles [Haessig and Friedland (1991); Ma (1995)]. Such a model is compatible to continuous contact dynamics modeling because it effectively calculate F_t as a function of time (through dependence on the road irregularities and F_c). The friction force is defined explicitly and uniquely during sticking at the contact domain. If the road is rocky as in this paper, the road bumpiness has to be taken into account. It is not unusual for automotive designers to test virtual models of cars on virtual models of bumpy roads

The bristle model for the friction force is given by

$$F_t = \begin{cases} k_f s(t_0) + \displaystyle\int_{t_0}^{t} v_t dt, & |s| < s_{\max}, \\[2ex] \mu \dfrac{|F_c|}{k_f} \dfrac{v_t}{|v_t|}, & \text{otherwise}, \end{cases} \tag{17}$$

where k_f is the bristle stiffness, $s(t)$ the function of the bristle displacement, t_0 is the start time of the last sticking at that contact point, v_t the relative tangential velocity and parameter s_{max} is the maximum allowable deflection of the bristle.

Others attempts to develop on-road and off-road capable tire models for vehicle dynamics simulations are reported in [Chan (2008); Lacombe (2000); Zeggelaar (1998)].

3   Identification of the contact domain

In the case of a rocky road, the tire is rather deformable than rigid when operating on the road. The identification of the contact patches in the interval T_0 is performed by checking the minimum distance between bodies according to (1) and (2). The shape of the patches changes in time from an oval shape ($n = 2$) at very low values of the vertical loads to almost rectangular shape ($n = 4$) at higher values of vertical load (Fig. 2).

$$n = 2.33 \qquad n = 2.71 \qquad n = 3.01 \qquad n = 3.93 \qquad n = 4.11$$
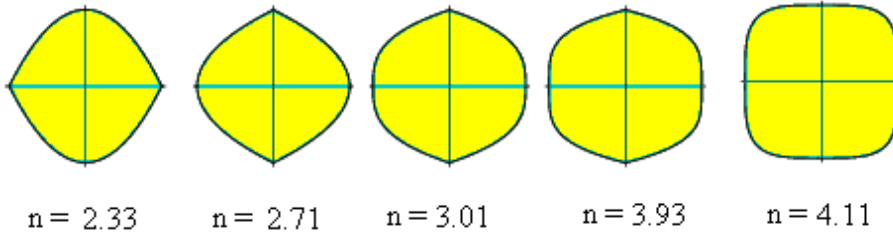
Figure 2: Different shapes of the contact patches.

The irregularities of the road (bumps, corners, peaks, shallows, etc.) are schematically shown in Fig. 3. At each moment several contact patches are identified in the given interval of time. For each contact point $j = 1, 2, ...,$ the contact force F_{cj}, is given by (9)

$$F_{cj} = k\delta^{n_j} + b\delta^{p_j}\dot{\delta}^{q_j}, \quad j = 1, 2, 3, ..., \tag{18}$$

where $n_j, p_j, q_j, j = 1, 2, 3, ...,$ are constants, the coefficient k depends on the tire's material and the geometric properties of the tire, and b is defined with respect to the coefficient of restitution $0 \le e \le 1$. The friction force F_{tj} in the contact points
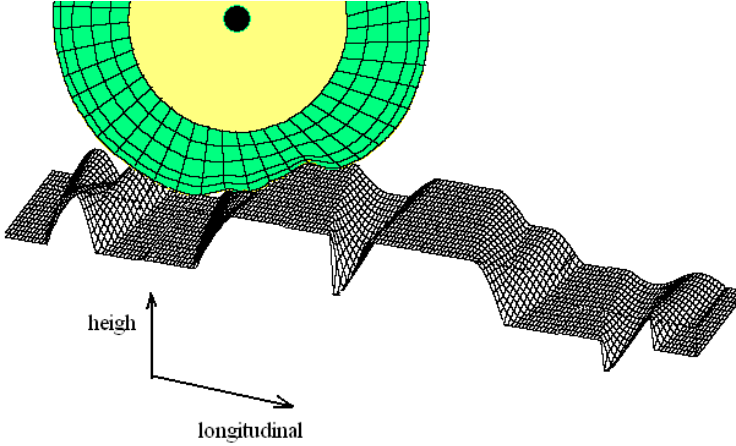
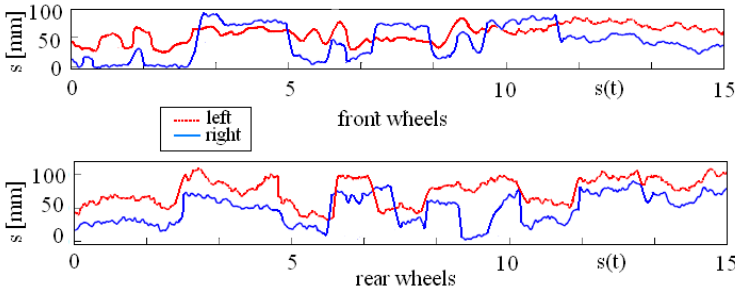Figure 3: Typically irregularities of the rocky road.

Figure 4: The function of bristle-displacement $s(t)$.

$j = 1, 2, ...$ are given by (17)

$$F_{tj} = \begin{cases} k_f s(t_0) + \displaystyle\int_{t_0}^{t} v_{tj} \mathrm{d}t, & |s| < s_{\max}, \\ \mu \dfrac{|F_{cj}|}{k_f} \dfrac{v_{tj}}{|v_{tj}|}, & \text{otherwise,} \end{cases} \tag{19}$$

where k_f is the bristle stiffness, $s(t)$ the function of bristle displacement, t_0 is the start time of the last sticking at that contact point, v_{tj} the relative tangential velocity in tires and parameter $s_{\max}$ is the maximum allowable deflection of the bristle. For $t = 0$ and $s = 0$ it results $f_j(0) = 0$.

The function $s(t)$ of the bristle-displacement is displayed in Fig. 4.

Numerical simulation of detection of the contact points between the tire and the road in the interval T_0, is achieved by checking the minimum distance between points belonging to the tire and the road, respectively. As results, for a speed of the vehicle of 10 m/s, a number of more than 500 contact points were detected. By defining a contact patch consisting from a minimum nearest 5 contact points, a number of more than 100 contact patches were identified.

Table 1 shows, for example, the characteristics of the first 4 detected contact patches in $t \in [0; 5\,\text{sec}]$. The maximum value of the contact pressures for the first contact patch, with respect to a/a_0 and b/b_0, are plotted in Figs. 5 and 6, respectively. The a_0 and b_0 are the reference radii of the oval shape of D_c.

The enhanced size of the contact patches in $D_c \subset D_a$, $c > \max(a, b)$, because of the adhesive pressures are presented in Table 2. We observe that the adhesive pressures given by (14), represent about 65-72% from the corresponding contact pressures.

Table 1: Dimensions of first 4 contact patches in the interval T_0.

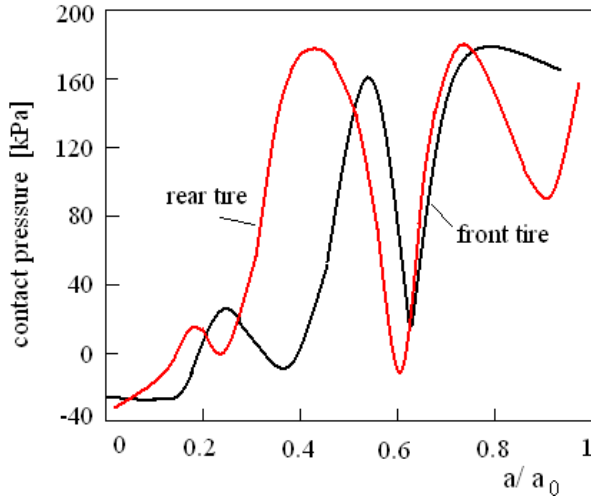| Contact patch	Vertical load F_z[N]	a [mm]	b[mm]	n
Front tire	2000	35	46	2.02
		38.3	51.2	2.18
Rear tire		39.4	52.0	2.22
		41.2	52.4	2.33
Front tire	3000	45.2	53.9	2.41
		50.2	54.8	2.55
Rear tire		52.7	57.0	2.67
		63.4	61.2	2.71
Front tire	5000	68.0	63.2	2.88
		73.6	65.5	2.91
Rear tire		77.8	67.2	2.98
		79.4	67.9	3.01
Front tire	8000	89.5	72.3	3.15
		92.5	74.9	3.93
Rear tire		98.3	75.0	4.04
		100.1	76.3	4.11

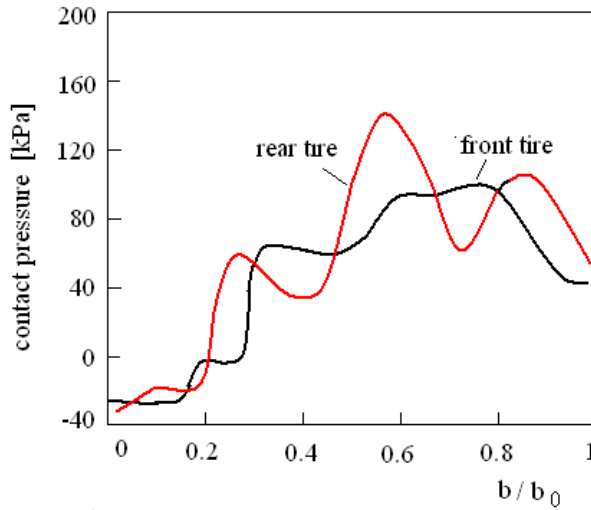Figure 5: The maximum contact pressure in the first contact patch.

Figure 6: The maximum contact pressure in the first contact patch.

Table 2: Enhanced dimensions of first 4 contact patches in the interval T_0.

Contact patch	Vertical load F_z[N]	c [mm]	Contact pressure p_z[kPa]	Adhesive pressure p_a[kPa]
Front tire	2000	47	141	100
		51.7	145	103
Rear tire		53.4	153	101
		53.2	154	103
Front tire	3000	55.4	155	105
		55.9	156	107
Rear tire		57.2	161	108
		61.9	166	115
Front tire	5000	68.4	173	114
		73.9	175	114
Rear tire		78.5	176	115
		80.3	181	120
Front tire	8000	90.0	185	125
		92.8	186	125
Rear tire		98.8	186	125
		101.7	189	127

4   Conclusions

The continuous approach of the modeling the dynamic contact between the tire and the road has several advantages over the discrete formulation. In the continuous approach no difference is made between impact and contact; therefore the methods of non-impact dynamics can be used to solve the problem. An advantage of the continuous approach over a discrete formulation is that it allows the application of the theory to multi-contact situations, as it is the case for the multi-body system. The Coulomb's law in the discrete approach leads to multiple solutions what is not happened in the continuum approach.

This paper is assuming continuum laws for the contact and friction forces. The continuum modeling of tire/road vibro-contact dynamics is developed in this paper in order to identify the contact domain by checking the minimum distance between bodies. The model takes as inputs the function of bristle displacement and the vertical tire force amd produces, as outputs, the distribution of contact pressure in the interfaces between the tire and the road in a short interval of time $T_0 = [0, \, 15 \, \text{sec}]$.

The contact pressure comprises two terms, the contact pressure and the adhesion pressure. The adhesive forces extend the contact domain to the enhanced adhesive domain $D_c \subset D_a$, by a value $c > \max(a,b)$, where c is half of the contact length. We observe that the that the adhesive pressures represent about 65-72% from the corresponding contact pressures.

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