Dam-breaking Flow Simulations by Particle-based Scheme Using Logarithmic Weighting Function

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Abstract: The application of a CPU/GPU-based particle method to dam-breaking incompressible viscous fluid flow problems is presented. The particle approach is based on the MPS (Moving Particle Semi-implicit) scheme using logarithmic weighting function to stabilize the spurious oscillatory solutions for solving the Poisson equation with respect to the pressure fields by using GPU-based SCG (Scaled Conjugate Gradient) method. The physics-based computer graphics for the results of three-dimensional simulation consist of the POV-Ray (Persistence of Vision Raytracer) rendering using marching cubes algorithm as polygonization. Numerical results demonstrate the workability and the validity of the present approach through the dam-breaking flow problem.

Keywords: GPU-based particle method, MPS, logarithmic weighting function, GPU-based SCG, dam-breaking flow.

1   Introduction

From a practical point of view, three-dimensional (3D) fluid flow simulations including free surfaces and moving interfaces are indispensable in the wide fields of engineering and science. The fluid flow problem of broken dam includes many interesting phenomena, such as large deformation of free-surfaces, very violent motions including splashing, and so forth. Some experimental data have been presented in the dam-breaking flow or the collapse of a liquid column [Martin and Moyce (1952); Koshizuka, Tamako and Oka (1995); Cruchaga, Celentano and Tezduyar (2007)]. The dam-breaking flow problem has been extensively used to verify the applicability and validity of the numerical methods.

There are various grid/mesh-based methods and gridless/meshless-based particle methods developed by many researchers for solving the complicated flow problems including free surfaces. As the approaches of the grid/mesh-based methods, the marker-and-cell (MAC) method [Harlow and Welch (1965)], the volume of

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fluid (VOF) method [Hirt and Nichols (1981)] and the level set (LS) method [Osher and Sethian (1988)] are well-known methodologies in fluid dynamics frameworks. The earliest numerical study for the dam-breaking flow was presented by Harlow and Welch [Harlow and Welch (1965)] using the MAC method. Afterwards, such flow problems have been numerically investigated by several researchers using the VOF methods [Hirt and Nichols (1981); Maronnier, Picasso and Rappaz (2003); Kim and Lee (2003); Greaves (2004); Greaves (2006)] and the LS methods [Yue, Lin and Patel (2003); Kohno and Tanahashi (2004); Lin, Lee, Lee and Weber (2005); Trontin, Vincent, Estivalezes and Caltagirone (2012)]. Both the VOF and LS methods provide worthwhile predictions by comparing their results with the data obtained from experiments. In the frameworks of finite element meshes with flexibility, the dam-breaking flows have also been carried out effectively by using Lagrangian finite element approaches [Ramaswamy and Kawahara (1987); Radovitzky and Ortiz (1998)] and the arbitrary Lagrangian-Eulerian (ALE) method [Hansbo (1992); Duarte, Gormaz and Natesan (2004); Nithiarasu (2005)]. Other interesting works have been performed efficiently using the pseudo-concentration method (PCM) [Thompson (1986); Kačeniauskas (2005)], the edge-tracked interface locator technique (ETILT) [Cruchaga, Celentano and Tezduyar (2005); Cruchaga, Celentano and Tezduyar (2007)], the constrained interpolation profile (CIP) method [Hu and Kashiwagi (2004)], and so forth [Murrone and Guillard (2005); Murrone and Guillard (2008); Nikitin, Olshanskii, Terekhov and Vassilevski (2011)], for capturing accurately the free-surfaces/interfaces of such flow problems.

On the other hand, there are also various kinds of gridless/meshless-based particle methods, such as SPH (Smoothed Particle Hydrodynamics) method [Lucy (1977); Gingold and Monaghan (1977)], MPS (Moving Particle Semi-implicit) method [Koshizuka and Oka (1996)], EFG (Element Free Galerkin) method [Belytschko, Lu and Gu (1994)], MLPG (Meshless Local Petrov-Galerkin) method [Atluri and Zhu (1998); Lin and Atluri (2000); Lin and Atluri (2001); Avila and Atluri (2009)], LMFE (Lagrangian Meshless Finite Element) method [Idelsohn, Storti and Oñate (2001); Idelsohn, Oñate and Pin (2003); Idelsohn, Oñate and Pin (2004)], and MP (Meshfree Particle) method [Li and Liu (2002)], to simulate effectively such complicated problems. The SPH methods for solving compressible fluid flows with gravity have been firstly developed in the field of astrophysics [Lucy (1977);Gingold and Monaghan (1977)], and applied successfully to a wide variety of complicated physical problems involving applications to the dam-breaking flow problem [Monaghan (1994); Colagrossi and Landrini (2003); Shao and Lo (2003); Sakai, Yang and Jung (2004); Lee, Moulinec, Xu, Violeau, Laurence and Stansby (2008); Molteni and Colagrossi (2009)]. The MPS method

as an incompressible fluid flow solver has been widely applied to the problem of breaking wave with large deformation [Koshizuka and Oka (1996); Ishii, Ishikawa and Tanabe (2006); Shakibaeinia and Jin (2010); Tanaka and Masunaga (2010); Kondo and Koshizuka (2011); Yamada, Sakai, Mizutani, Koshizuka, Oochi and Murozono (2011); Lee, Park, Kim and Hwang (2011)], the fluid-structure interaction problem, and so forth. However, the standard/original MPS approach leads to the unphysical numerical oscillation of pressure fields which are described by the discretized Poisson equation. To improve some shortcomings of the standard MPS method, Khayyer and Gotoh have proposed the modified MPS method for the prediction of wave impact pressure on a coastal structure to ensure more exact momentum conservation [Khayyer and Gotoh (2009)]. The improvement of stability in the standard MPS method has been recently achieved by adding some source terms into Poisson pressure equation [Kondo and Koshizuka (2011)]. Shao and Gotoh have also summarized the SPH and MPS turbulence models using large eddy simulation (LES) with a sub-particle scale [Shao and Gotoh (2005)]. They compared the performances of two particle models through experimental data of the dam-breaking flow and demonstrated the accuracy and robustness of two models. Atluri and Zhu [Atluri and Zhu (1998)] have developed the MLPG approach based on the local symmetric weak form and the moving least squares for solving accurately potential problems, and the approach was extended to deal with the problems for convection-diffusion equation [Lin and Atluri (2000)] and incompressible Navier-Stokes equations [Lin and Atluri (2001)] in fluid dynamics. Avila and Atluri [Avila and Atluri (2009)] have presented efficiently various numerical solutions of the non-steady, two-dimensional Navier-Stokes equations by using the MLPG method coupled with a fully implicit pressure-correction approach. They have also proposed a novel MLPG-mixed finite volume method for solving the steady-state Stokes flow involving complex phenomena between eccentric rotating cylinders [Avila, Han and Atluri (2011)]. Valuable overviews of the MLPG method involving applications to fluid flows have been presented in detail by Sladek et al. [Sladek, Stanak, Han, Sladek and Atluri (2013)]. A group of Idelsohn et al. [Idelsohn, Storti and Oñate (2001); Idelsohn, Oñate and Pin (2003); Idelsohn, Oñate and Pin (2004)] has developed expertly the LMFE method for solving incompressible fluid flows with free surfaces and applied to complex problems including the dam-breaking flow and fluid-structure interactions. Some reviews of meshfree/particle methods and their applications have been presented excellently by Li and Liu [Li and Liu (2002)].

Recently, the physics-based computer simulations on the GPU (Graphics Processing Units) have increasingly become an important strategy for solving efficiently various problems, such as fluid dynamics [Harris (2004); Crane, Llamas and Tariq

(2008); Harada, Masaie, Koshizuka and Kawaguchi (2008); Hori, Gotoh, Ikari and Khayyer (2011)], rigid body dynamics [Harada (2008)], and so forth. In our previous work, we have presented a GPU-based particle scheme using logarithmic weighting function for solving effectively two-dimensional problems of incompressible fluid flow [Kakuda, Nagashima, Hayashi, Obara, Toyotani, Katsurada, Higuchi and Matsuda (2012)]. The GPU-implementation consisted mainly of the five steps, namely, the search for neighboring particles in the influence area, the calculation of the particle number density, solving the Poisson equation with respect to the pressure fields, the calculation of the pressure gradient, and the modification of velocities and positions of the particles. We obtained that the performance on GPU with about $120,000$ particles led to approximately 12 times speed-up.

The purpose of this paper is in detail to present the application of the GPU-based particle method using logarithmic weighting function to 2D/3D dam-breaking incompressible fluid flow problem [Martin and Moyce (1952); Hirt and Nichols (1981); Ramaswamy and Kawahara (1987)]. As the physics-based computer graphics for 3D simulation, the polygonization of numerical data is also constructed by using the well-known marching cubes technique [Lorensen and Cline (1987)] as the most popular iso-surfacing extraction algorithm, and also the rendering is illustrated in using the generated polygons and POV-Ray [Kakuda, Obara, Toyotani, Meguro and Furuichi (2012)]. The workability and validity of the present approach are demonstrated through the dam-breaking flow problem, and compared with experimental data and other numerical ones.

Throughout this paper, the summation convention on repeated indices is employed. A comma following a variable is used to denote partial differentiation with respect to the spatial variable.

2   Statement of the particle-based flow simulation

Let Ω be a bounded domain in 2D/3D Euclidean space with a piecewise smooth boundary Γ. The unit outward normal vector to Γ is denoted by $\boldsymbol{n}$. Also, $\Im$ denotes a closed time interval.

The motion of an incompressible viscous fluid flow is governed by the following Navier-Stokes equations :

$$\frac{Du_i}{Dt} = -\frac{1}{\rho}p_{,i} + \nu u_{i,jj} + f_i \qquad in \; \Im \times \Omega \tag{1}$$

$$\frac{D\rho}{Dt} = 0 \qquad in \; \Im \times \Omega \tag{2}$$

where u_i is the velocity vector component, ρ is the density, p is the pressure, f_i is the external force, ν is the kinematic viscosity, and D/Dt denotes the Lagrangian

differentiation. In addition to Eq. 1 and Eq. 2, we prescribe the Dirichlet and Neumann boundary conditions, and the initial condition $u_i(\boldsymbol{x}, 0) = u_i^0$, where u_i^0 denotes the given initial velocity.

The particle interaction models of the MPS as illustrated in Fig. 1(a) are prepared with respect to differential operators, namely, gradient, divergence and Laplacian [Koshizuka and Oka (1996)]. The incompressible viscous fluid flow is calculated by a semi-implicit algorithm, such as SMAC (Simplified MAC) scheme [Amsden and Harlow (1970)]. For the standard MPS formulation, the selection of a weighting function is a key factor in the particle-based framework. If the distance r between the coordinates $\boldsymbol{r}_i$ and $\boldsymbol{r}_j$ is very close, then there is a possibility that the computation fails suddenly with unphysical numerical oscillations. Therefore, in order to stabilize such spurious oscillations generated by the standard MPS strategy, we adopt the following logarithmic-type weighting function as shown in Fig. 1(b), and also consider the reduction of *ad hoc* influence radius, r_e, for solving the pressure fields [Kakuda, Nagashima, Hayashi, Obara, Toyotani, Katsurada, Higuchi and Matsuda (2012)].

$$w(r) = \begin{cases} log(\frac{r_e}{r}) & (r < r_e) \\ 0 & (r \geq r_e) \end{cases} \tag{3}$$

The common logarithmic-type weighting function is also similar to the profile of the weighting function proposed by Kondo and Koshizuka to stabilize the pressure calculations [Kondo and Koshizuka (2011)](see Fig. 1(b)).

The particle number density n at particle i with the neighboring particles j is defined as

$$< n >_i = \sum_{j \neq i} w(|\boldsymbol{r}_j - \boldsymbol{r}_i|) \tag{4}$$

The model of the gradient vectors at particle i between particles i and j is weighted with the kernel function and averaged as follows :

$$< \boldsymbol{\nabla}\phi >_i = \frac{d}{n^0} \sum_{j \neq i} [\frac{\phi_j - \phi_i}{|\boldsymbol{r}_j - \boldsymbol{r}_i|^2} (\boldsymbol{r}_j - \boldsymbol{r}_i) w(|\boldsymbol{r}_j - \boldsymbol{r}_i|)] \tag{5}$$

where d is the number of spatial dimensions, ϕ_i and ϕ_j denote the scalar quantities at coordinates $\boldsymbol{r}_i$ and $\boldsymbol{r}_j$, respectively, and n^0 is the constant value of the particle number density.

The Laplacian model at particle i is also given by

$$< \boldsymbol{\nabla}^2\phi >_i = \frac{2d}{n^0 \lambda} \sum_{j \neq i} (\phi_j - \phi_i) w(|\boldsymbol{r}_j - \boldsymbol{r}_i|) \tag{6}$$

where λ is an *ad hoc* coefficient.

The Poisson equation for solving implicitly the pressure field at particle i is also given as follows [Kondo and Koshizuka (2011)]:

$$\frac{1}{\rho_0} < \boldsymbol{\nabla}^2 p >_i = - \frac{1-\beta}{\Delta t^2} \frac{<n^*>_i - 2 <n^k>_i + <n^{k-1}>_i}{n^0}$$
$$- \frac{\beta-\gamma}{\Delta t^2} \frac{<n^*>_i - <n^k>_i}{n^0} - \frac{\gamma}{\Delta t^2} \frac{<n^*>_i - n^0}{n^0} \qquad (7)$$

where ρ_0 is the density in the initial state, $<n^*>_i$ is an auxiliary particle number at particle i, and β and γ denote the adequate dimensionless parameters.

(a) Particle interaction models (3D)   (b) Profiles of weighting functions
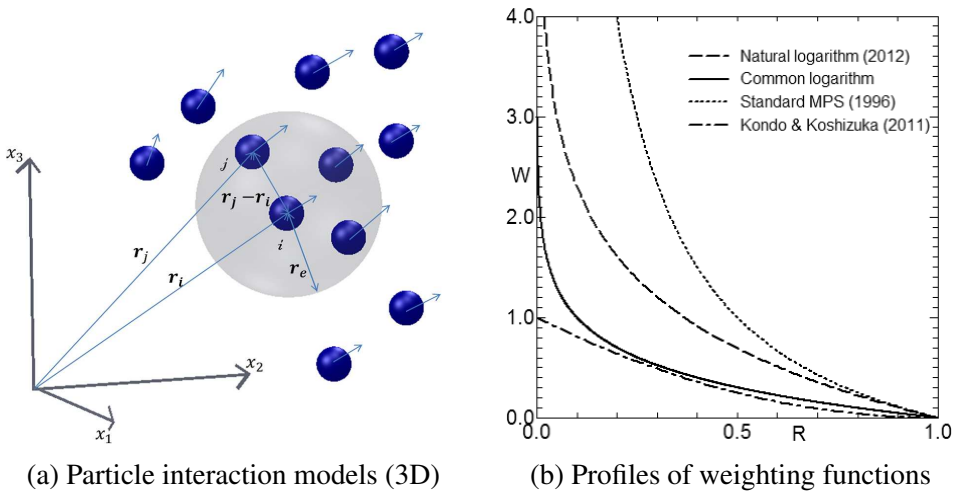
Figure 1: Particle interaction models and weighting functions

3   GPU-implementation using CUDA

The specification of CPU and GPU using CUDA is summarized in Tab. 1. A physical value at particle position is calculated as a weighted sum of the values of neighboring particles in the influence area. Therefore, we have to search for neighboring particles. The difficulty in implementing MPS on the GPU is that the neighborhood relationship among particles dynamically changes during the simulation. The GPU implementation consists mainly of the five steps as described to the reference [Kakuda, Nagashima, Hayashi, Obara, Toyotani, Katsurada, Higuchi and Matsuda (2012)]:

Table 1: A summary of the specification of CPU and GPU

CPU	Intel Core i7, 3.50GHz
Memory	DDR3 PC3-10600 16GB
OS	Cent OS 6.0 64bit
Bus	PCI Express 2.0x16
GPU	NVIDIA GeForce GTX580
Global Memory	1.5GB
Processor Clock	1544MHz
Streaming Multiprocessor (SM)	16
CUDA core	512
Memory Transfer Rate	192.4GB/s
Memory Interface	384bit
CUDA Driver	Version 4.10
Tool kit & SDK	Version 4.0

4 Numerical example

In this section we present numerical results obtained from applications of the above-mentioned numerical method to incompressible viscous fluid flow problems, namely dam-breaking flow problem involving free surface and gravity. The initial velocities are assumed to be zero everywhere in the interior domain. In 2D/3D simulations, we set the CFL condition $u_{max}\Delta t/l_{min} \leq C$, where C is the Courant number. The kernel sizes for the particle number density and the gradient/Laplacian models are $r_e = 4.0l_0$ and $\bar{r}_e = 2.0l_0$ for velocity and pressure calculations, respectively, in which l_0 is the distance between two neighboring particles in the initial state. In this case, we set $l_0 = 0.012m$ and also $(\beta, \gamma) = (0.5, 0.05)$.
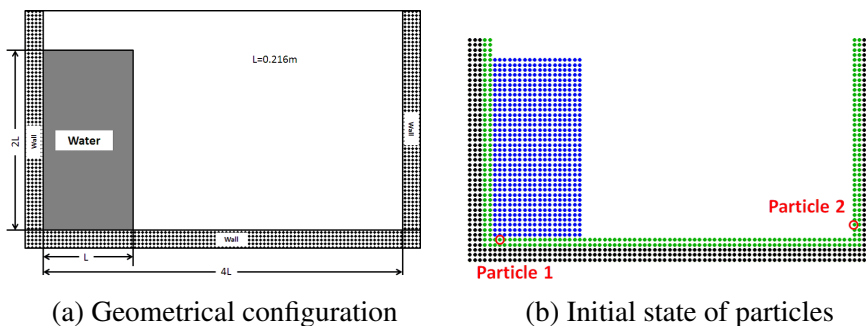

(a) Geometrical configuration   (b) Initial state of particles

Figure 2: Dam-breaking flow configuration for 2D

4.1   2D dam-breaking flow simulation

Fig. 2 shows the geometry and the initial state of particles for 2D flow in the dam-breaking problem. In this two-dimensional CPU-based simulation, we set 1,458 particles in the initial configuration. The standard MPS method leads to irregular pressure distributions at early times (see, Fig. 3(a)), while the present distributions are slightly improved as well as the results of Fig. 3(b). The pressure distributions are also smoother in the results obtained by means of the improved scheme with Eq. 7 (see, Fig. 3(c)). Fig. 4 shows the time histories of the pressure at particles 1 and 2 as shown in Fig. 2(b). We can see from Fig. 4 that the pressure behaviors at particles 1 and 2 in Fig. 4(c) are smoother than the standard MPS calculations of Fig. 4(a).

(a) Standard MPS method

(b) MPS using logarithmic-type weighting function
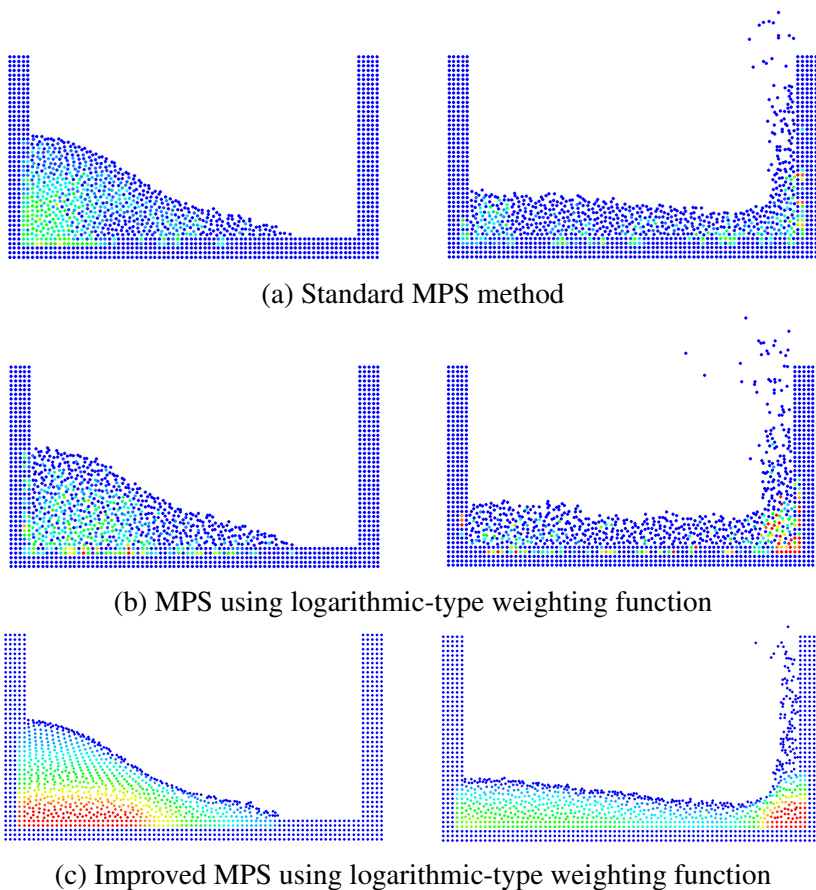
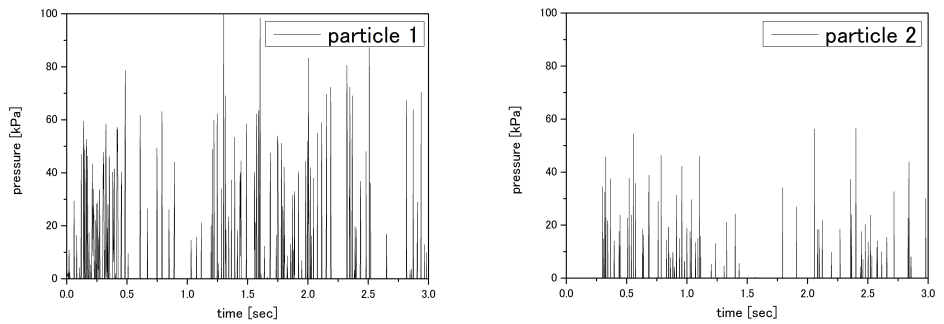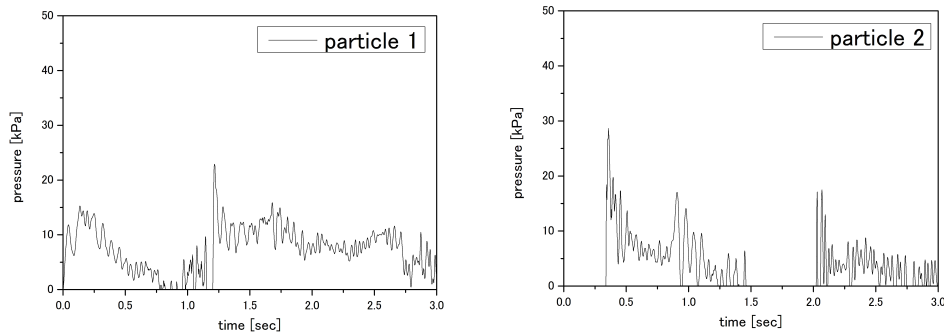(c) Improved MPS using logarithmic-type weighting function
Figure 3: Particle and pressure behaviors at time 0.25 (left) and 0.5 (right)
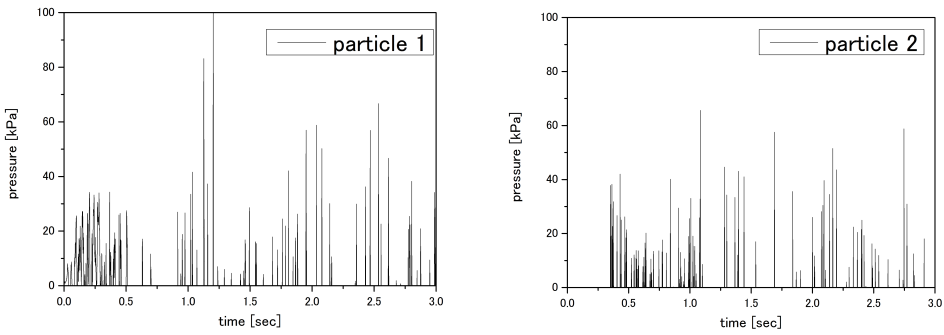
(a) Standard MPS method


(b) MPS using logarithmic-type weighting function

(c) Improved MPS using logarithmic-type weighting function

Figure 4: Time histories of the pressure at particles 1 (left) and 2 (right)

4.2 3D dam-breaking flow simulation

Let us consider the GPU-based simulation using the improved approach for 3D flow in the dam-breaking problem. Fig. 5 shows the geometry and the initial state of particles $89,168$ for the dam-breaking flow problem. The particle and pressure
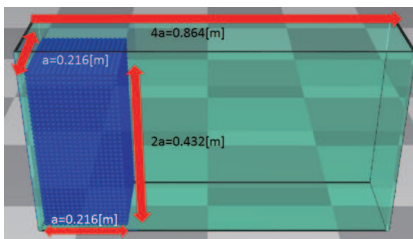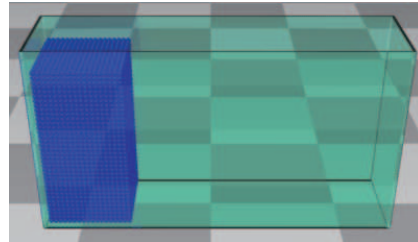
(a) Geometrical configuration   (b) Initial state of particles
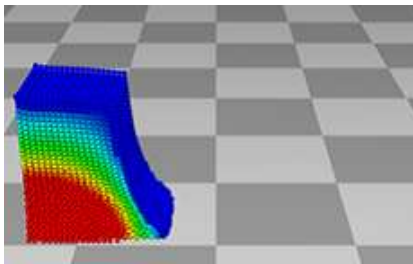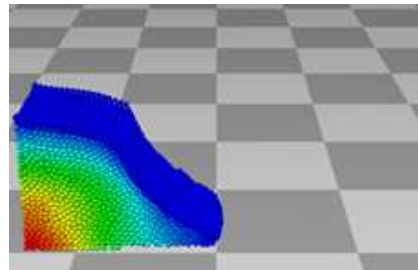
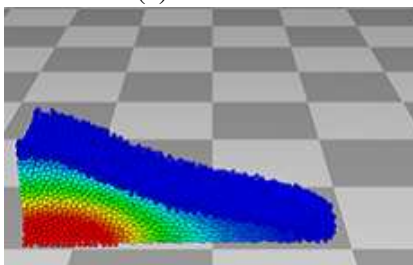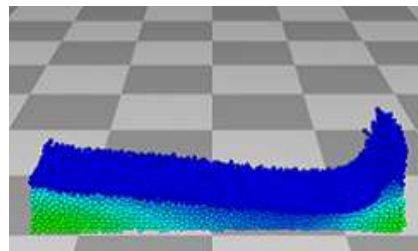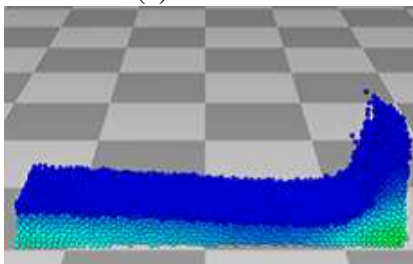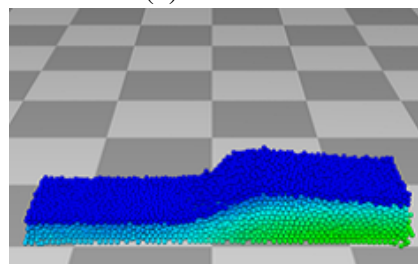Figure 5: Dam-breaking flow configuration for 3D

(a) $t \approx 0.15s$ (b) $t \approx 0.20s$

(c) $t \approx 0.30s$ (d) $t \approx 0.50s$

(e) $t \approx 0.60s$ (f) $t \approx 1.0s$

Figure 6: Particle and pressure behaviors at different time

(a) $t \approx 0.15s$

(b) $t \approx 0.20s$

(c) $t \approx 0.30s$

(d) $t \approx 0.50s$

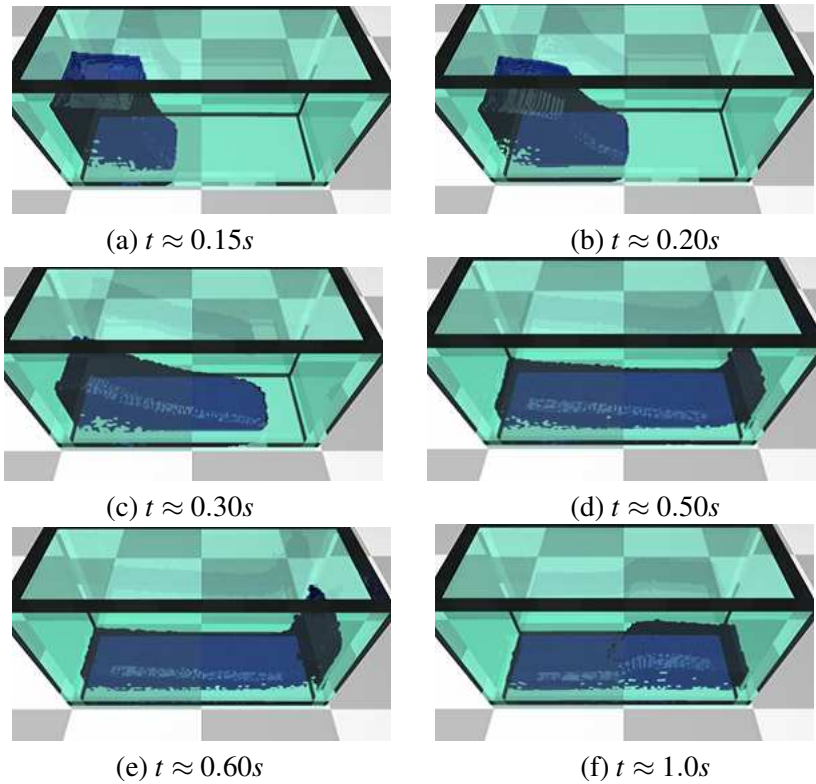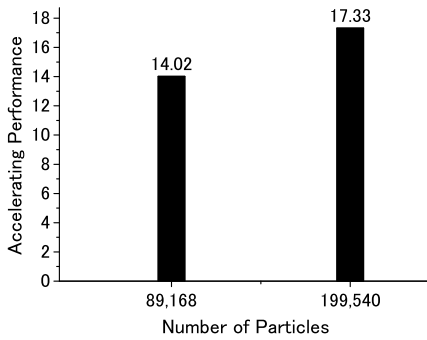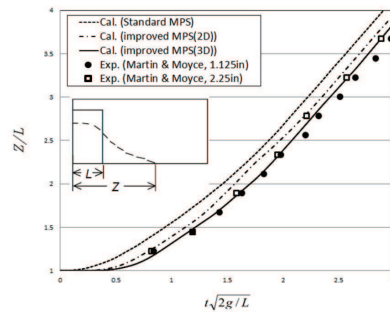(e) $t \approx 0.60s$

(f) $t \approx 1.0s$

Figure 7: POV-Ray rendering representations at different time

(a) GPU-accelerating performance

(b) Comparisons with other data

Figure 8: GPU-accelerating performance and comparisons with experimental data

behaviors at different time are shown in Fig. 6, and more smoother results are obtained certainly by means of the improved scheme. The convincing representations are obtained when the iso-surface of the color field is visualized using the POV-Ray rendering with the marching cubes algorithm [Lorensen and Cline (1987)] as illustrated in Fig. 7 [Kakuda, Obara, Toyotani, Meguro and Furuichi (2012)]. Using the generated polygons and the POV-Ray, our computational results show satisfactory rendering effects. Fig. 8 shows the accelerating performance of GPU for 3D simulation, and also the time evolutions of the leading-edge of the water using the present approach and the standard MPS method through comparison with experimental data [Martin and Moyce (1952)]. We can see from Fig. 8 that the performance with 89,168 particles leads to approximately 14.02 times speed-up. The agreement between the present results and the experimental data appears also satisfactory.

5   Conclusions

We have presented the CPU/GPU-based MPS approach using logarithmic weighting function for solving numerically 2D/3D incompressible viscous fluid flow of the broken dam problem. The standard MPS scheme has been widely utilized as a particle strategy for free surface flow, the problem of moving boundary, and multi-physics/multi-scale ones. To overcome spurious oscillations in the standard MPS method, we have proposed to utilize the logarithmic weighting function and also consider the influence radius reduction for solving an auxiliary Poisson equation for the pressure field. The GPU implementation consists of the five steps, namely, the search for neighboring particles, the calculation of the particle number density, solving the Poisson equation with respect to the pressure fields by using GPU-based SCG method, the calculation of the pressure gradient, and the modification of velocities and positions of the particles.

As the numerical example, the well-known 2D/3D dam-breaking flow simulations were carried out and compared with experimental data and standard MPS data. The qualitative agreement between our 3D simulation and experimental data appears satisfactory. The GPU-performance with about 90,000 and 200,000 particles led to approximately 14 and 17 times speed-up, respectively. The polygonization of numerical 3D data has been also constructed by using the marching cubes algorithm, and then the rendering has been significantly illustrated in using the generated polygons and POV-Ray as the physics-based computer graphics.

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