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Numerical Solutions for Free Vibration Analysis of Thick Square Plates by the BEM

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Abstract: In this work, the BEM is applied to obtain the numerical solutions for free vibration analysis of thick square plates with two edges simply supported or clamped, and the other two edges free. A formulation based on Reissner's theory is used here, which includes the contribution of the additional translational inertia terms to the integral equation of displacements and internal forces. The boundary element method is used to discretize the space, where it is employed the static fundamental solution. In literature, the responses for the kind of problem addressed here are very important in the hydroelastic analysis of very large floating structures (VLFS) which are commonly modeled as plates with free edges. To verify the accuracy this formulation some analyses are presented at the end of the paper.

Keywords: BEM, free vibration, thick square plate, very large floating structure.

1 Introduction

A vast literature exists for free vibration analysis of square plates, but there are still some hypotheses either not introduced or not sufficiently tested that need to be studied. Plates, in general, are three dimensional structures used in many areas of applications such as in the civil engineering and in the aerospace, marine and nuclear industries, among others. In practical applications, two-dimensional theories may be considered to analyze plates.

In literature, there are three theories to study plates. The Kirchhoff's theory does not take into account the effect of shear deformation and rotatory inertia and is limited to thin plates, being it well described in Timoshenko and Woinowsky-Krieger

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(1959), Leissa (1973), Shames and Dym (1985). Another plate theory was proposed in the decade of 1940 [Reissner (1944); Reissner (1945); Reissner (1947)], and considers the effect shear deformation and requires three boundary conditions, instead of two, as established by the thin plates theory. The third is the Mindlin's theory that takes into account the effect of shear deformation based on a proposed displacement field through the plate thickness and incorporated the effect of rotatory inertia [Mindlin (1951); Mindlin, Schacknow, and Deresiewicz (1956)].

Due to difficulty in finding close solutions for free vibration analysis of plates, numerical methods are employed to compute the field variables such as: the collocation method [Takashi and Jin (1984)]; the boundary element method [Banerjee (1994)]; the finite element method [Bathe (1996)]; the meshless method [Gu and Liu (2001); Qian, Batra, and Chen (2003)]. Following this search line there are some methods that simulate plate as 3D bodies [Cheung and Chakrabarti (1972); Liew, Hung, and Lim (1993); Liew, Xiang, and Kitipornchai (1993); Malik and Bert (1998)].

Nowadays, we are facing a population growth and a corresponding expansion of urban centers as is the case of Japan. For solve this problem type, engineers have proposed the construction of very large floating structures (VLFS) to long coastlines. In literature, VLFS may be classified under two broad categories, namely the pontoon-type and the semi-submersible type. The pontoon-type VLFS may be modeled as plates with free edges [Wang, Wang, Watanabe, Utsunomiya, and Xiang (2001); Watanabe, Utsunomiya, Wang, and Xiang (2003); Watanabe, Utsunomiya, and Wang (2004); Chen, Wu, Cui, and Jensen (2006)], thus using boundary conditions adequate in the formulation, a set of equations is obtained, from which it is possible to compute modal shapes and stress-resultants.

Recently, two theories of shear deformable plate vibrations that account for the influence of the transverse normal stress components were presented in Batista (2011), being one of them based on the Mindlin's theory and the other on the Reissner's theory. Moreover, the transverse normal stress components are included in the boundary element method (BEM) for analysis of free and forced vibration of thick elastic plates [Pereira, Mansur, Karam, and Carrer (2011); Pereira, Karam, Carrer, and Mansur (2012)].

This work attempts to present accurate numerical results for free vibration analysis of thick square plates with two edges simply supported or clamped, and the other two edges free. For this, the BEM is applied to obtain the numerical solutions for free vibration analysis of thick square plates. A formulation based on Reissner's theory is used here, which includes the contribution of the additional translational inertia terms to the integral equation of displacements and internal forces. The boundary element method is used to discretize the space, where it is employed the static fundamental solution. In literature, the responses for the kind of problem addressed here are very important in the hydroelastic analysis of very large floating structures (VLFS) which are commonly modeled as plates with free edges.

In this work, an indicial notation is used, where Latin subscripts vary from 1 to 3, while the Greek subscripts range is from 1 to 2.

2 Basic equations

Consider a plate of thickness h made from homogeneous and isotropic elastic material with the modulus of elasticity E, Poisson's ratio ν and mass density ρ. The equilibrium equations governing its free vibrations, for an infinitesimal element of the thick elastic plate in a Cartesian coordinate system x_i, are given by

$$M_{\alpha\beta,\beta} - Q_\alpha = \frac{\rho h^3}{12}\ddot{\phi}_\alpha \tag{1a}$$

$$Q_{\alpha,\alpha} = \rho h \ddot{w} \tag{1b}$$

where overdots indicate to differentiation with respect to time t. $M_{\alpha\beta}$ and Q_α are the moments per unit length and the shear forces per unit length, respectively. The other variables involved in the problem are the generalized displacements which are the rotations ϕ_α and the vertical deflection w.

Due to hypothesis of Reissner (1944), it is admitted linear distribution of stresses through the thickness of the plate defined as

$$\sigma_{\alpha\beta} = \frac{12}{h^3} M_{\alpha\beta} x_3 \tag{2}$$

whose loading conditions at $x_3 = \pm h/2$, for upper and lower surfaces of the plate and the considerations made in this work, are given by

$$\sigma_{\alpha 3} = 0 \quad \text{and} \quad \sigma_{33} = 0 \tag{3}$$

Reissner (1945) in your work, consider which the generalized displacements can be represented by weighted averages over the thickness of the plate, related to real displacements u_α and u_3 defined as

$$\phi_\alpha = \frac{12}{h^3} \int_{-h/2}^{+h/2} u_\alpha x_3 \, dx_3 \tag{4a}$$

$$w = \frac{3}{2h} \int_{-h/2}^{+h/2} u_3 \left[1 - \left(\frac{2x_3}{h}\right)^2\right] dx_3 \tag{4b}$$

The equations of motion of the three-dimensional elastic theory, in terms of the real displacement, can be written as

$$\sigma_{\alpha\beta,\beta} + \sigma_{\alpha 3,3} = \rho\, \ddot{u}_{\alpha} \tag{5a}$$

$$\sigma_{3\beta,\beta} + \sigma_{33,3} = \rho\, \ddot{u}_{3} \tag{5b}$$

where σ_{ij} are the normal stresses for $i = j$ and the shear stresses for $i \neq j$.

To determine the stress-force relations, the five equations above are used. Thus, after all mathematics manipulation one has

$$\sigma_{\alpha 3} = \frac{3}{2h} Q_{\alpha} \left[1 - \left(\frac{2x_3}{h}\right)^2\right] \quad \text{and} \quad \sigma_{33} = -\frac{1}{2}\rho\, \ddot{w} \left[1 - \left(\frac{2x_3}{h}\right)^2\right] x_3 \tag{6}$$

Note that the expression of the normal stress σ_{33} presents one additional term of translational inertia, due to the hypothesis assumed, that is commonly disregarded in the thick elastic plate theory.

Based on Reissner's theory, the resultant moments and shear forces can be written as [Pereira, Mansur, Karam, and Carrer (2011)]

$$M_{\alpha\beta} = m_{\alpha\beta} - k\rho h \ddot{w}\delta_{\alpha\beta} \quad \text{with} \quad m_{\alpha\beta} = \frac{1}{2} D(1-\nu)\left[2\chi_{\alpha\beta} + \frac{2\nu}{(1-\nu)}\chi_{\gamma,\gamma}\delta_{\alpha\beta}\right] \tag{7a}$$

$$Q_{\alpha} = \frac{5}{6} G h \gamma_{\alpha} \tag{7b}$$

where $D = Eh^3/12(1-\nu^2)$ is the flexural rigidity of the plate, $G = E/2(1+\nu)$ is the shear modulus, $\delta_{\alpha\beta}$ is the Kronecker delta, and $k = \nu/6(1-\nu)\lambda^2$ with $\lambda^2 = 10/h^2$.

The expressions of the generalized strains for the linear theory, in terms of the generalized displacements that appear in Eq. (7) can be written as

$$\text{Flexural strains components:} \quad \kappa_{\alpha\beta} = 2\chi_{\alpha\beta} = \phi_{\alpha,\beta} + \phi_{\beta,\alpha} \tag{8a}$$

$$\text{Transverse shear strains components:} \quad \gamma_{\alpha} = \phi_{\alpha} + w_{,\alpha} \tag{8b}$$

For simplicity, from this point on, the generalized displacements ϕ_{α} and w will be written generically as u_i.

3 Boundary integral equations

Let us consider Ω as being the domain represented by the middle surface of the plate, and Γ be its contour. Thus, the following initial conditions are considered in the domain:

Initial displacements: $u_i(\mathbf{x},0) = u_i(\mathbf{x})$ (9a)

Initial velocity: $\dot{u}_i(\mathbf{x},0) = \dot{u}_i(\mathbf{x})$ (9b)

The prescribed boundary conditions on Γ for the three generalized directions of the plate are defined by

$$u_i = \bar{u}_i \quad \text{on} \quad \Gamma_u \tag{10a}$$

$$p_i = \bar{p}_i \quad \text{on} \quad \Gamma_p \tag{10b}$$

so that $\Gamma = \Gamma_u + \Gamma_p$, and p_i are the generalized surface forces, defined as

$$\bar{p}_\alpha = \bar{M}_{\alpha\beta} n_\beta \quad \text{and} \quad \bar{p}_3 = \bar{Q}_\beta n_\beta \tag{11}$$

where n_β is the outward normal vector to the boundary Γ.

From the above considerations, the integral equation for displacement can be written for three generic directions as [Pereira, Mansur, Karam, and Carrer (2011)]

$$\begin{aligned} c_{ij}(\xi)\, u_j(\xi,t) = & \int_\Gamma p_j(\mathbf{x},t)\, u^*_{ij}(\xi,\mathbf{x})\, d\Gamma(\mathbf{x}) - \int_\Gamma p^*_{ij}(\xi,\mathbf{x})\, u_j(\mathbf{x},t)\, d\Gamma(\mathbf{x}) \\ & - \frac{\rho h^3}{12}\int_\Omega \ddot{u}_\alpha(\mathbf{x},t) u^*_{i\alpha}(\xi,\mathbf{x})\, d\Omega(\mathbf{x}) - \rho h \int_\Omega \ddot{u}_3(\mathbf{x},t)\, u^*_{i3}(\xi,\mathbf{x})\, d\Omega(\mathbf{x}) \\ & + k\rho h \int_\Omega \ddot{u}_3(\mathbf{x},t)\, u^*_{i\alpha,\alpha}(\xi,\mathbf{x})\, d\Omega(\mathbf{x}) \end{aligned} \tag{12}$$

where ξ and $\mathbf{x}$ are source point and field point, respectively. The above equation holds for internal points with $c_{ij} = \delta_{ij}$ and for boundary points, with $c_{ij} = \delta_{ij}/2$ for smooth boundaries. In this equation, the asterisk indicates the corresponding terms refer to the static fundamental solution [Vander Weeën (1982)].

Eq. (12) presents three terms of inertia, where the last term corresponds to an additional part of the integral equations of BEM for free vibration analysis of thick elastic plate. In next section, will be shown terms like this within of the integral equations for moments and shear forces.

4 Integral equations of the stress-resultants

The integral equations for moments and shear forces at internal point ξ are obtained from Eq. (7), where the differentiation of displacements is substituted by the derivatives of the integral equation (12). Thus, the resulting equations can be written as

$$\begin{aligned} M_{\alpha\beta}(\xi,t) = &\int_\Gamma p_k(\mathbf{x},t)u^*_{\alpha\beta k}(\xi,\mathbf{x})\,d\Gamma(\mathbf{x}) - \int_\Gamma u_k(\mathbf{x},t)p^*_{\alpha\beta k}(\xi,\mathbf{x})\,d\Gamma(\mathbf{x}) \\ &- \frac{\rho h^3}{12}\int_\Omega \ddot{u}_\eta(\mathbf{x},t)\,u^*_{\alpha\beta\eta}(\xi,\mathbf{x})\,d\Omega(\mathbf{x}) - \rho h\int_\Omega \ddot{u}_3(\mathbf{x},t)\,u^*_{\alpha\beta 3}(\xi,\mathbf{x})\,d\Omega(\mathbf{x}) \\ &+ k\rho h\int_\Omega \ddot{u}_3(\mathbf{x},t)\,z^*_{\alpha\beta}(\xi,\mathbf{x})\,d\Omega(\mathbf{x}) \end{aligned} \tag{13a}$$

and

$$\begin{aligned} Q_{\beta}(\xi,t) = &\int_\Gamma p_k(\mathbf{x},t)u^*_{3\beta k}(\xi,\mathbf{x})\,d\Gamma(\mathbf{x}) - \int_\Gamma u_k(\mathbf{x},t)p^*_{3\beta k}(\xi,\mathbf{x})\,d\Gamma(\mathbf{x}) \\ &- \frac{\rho h^3}{12}\int_\Omega \ddot{u}_\eta(\mathbf{x},t)\,u^*_{3\beta\eta}(\xi,\mathbf{x})\,d\Omega(\mathbf{x}) - \rho h\int_\Omega \ddot{u}_3(\mathbf{x},t)\,u^*_{3\beta 3}(\xi,\mathbf{x})\,d\Omega(\mathbf{x}) \\ &+ k\rho h\int_\Omega \ddot{u}_3(\mathbf{x},t)\,z^*_{3\beta}(\xi,\mathbf{x})\,d\Omega(\mathbf{x}) \end{aligned} \tag{13b}$$

where the tensors of the third order that appear with the asterisk in Eq. (13) represent the static fundamental solution [Karam and Telles (1988); Pereira (2009)].

5 Numerical procedure

As shown in Fig. 1, the boundary Γ is discretized by quadratic elements, each one denoted by Γ_j, and the domain Ω is discretized by constant triangular cells, with Ω_l being the domain of each cell, and thus, the integral equation present in the previous section can be solved.

BEM guidelines consider boundary element and internal cell approximations as follows. The displacements $\mathbf{u}^j$ and surface forces $\mathbf{p}^j$ for an element j are computed from its nodal values, $\mathbf{u}^n$ and $\mathbf{p}^n$, according to the following approximations

$$\mathbf{u}^j = \mathbf{N}\mathbf{u}^n \quad \text{and} \quad \mathbf{p}^j = \mathbf{N}\mathbf{p}^n, \tag{14a}$$

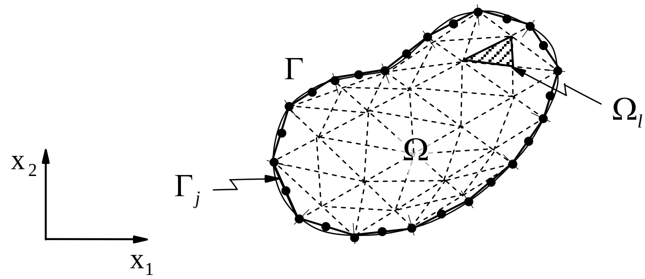

Figure 1: Discretization with boundary elements and internal cells.

while the translational inertia term $\ddot{\mathbf{u}}^l$ at internal points are approximated by

$$\ddot{\mathbf{u}}^l = \bar{\mathbf{N}}\ddot{\mathbf{u}}^m \tag{14b}$$

By substituting Eq. (14) into (12) and from these resulting equations, we write the equations for all boundary nodes and for all cell nodes, with nodes being collocation points ξ. Then, the following system of equations is formed:

$$\begin{bmatrix} \mathbf{H}^{bb} & \mathbf{0} \\ \mathbf{H}^{db} & \mathbf{I} \end{bmatrix} \begin{Bmatrix} \mathbf{u}^b \\ \mathbf{u}^d \end{Bmatrix} = \begin{bmatrix} \mathbf{G}^{bb} & \mathbf{0} \\ \mathbf{G}^{db} & \mathbf{0} \end{bmatrix} \begin{Bmatrix} \mathbf{p}^b \\ \mathbf{0} \end{Bmatrix} - \begin{bmatrix} \mathbf{0} & \mathbf{M}^{bd} \\ \mathbf{0} & \mathbf{M}^{dd} \end{bmatrix} \begin{Bmatrix} \mathbf{0} \\ \ddot{\mathbf{u}}^d \end{Bmatrix} \tag{15}$$

The superscripts b and d in the above matrix equation correspond to the boundary and domain, respectively. Moreover, the first superscript corresponds to the source point, while the second concerns to the field point.

The integrals over the boundary elements and internal cells are computed numerically, using Gaussian quadrature. In the case of singular integrals, special procedures can be used for the integrals on the boundary [Telles (1987); Karam (1992)], while that the finite part numerical quadrature is utilized for the integrals in the domain [Kutt (1975)].

On the hypothesis of harmonic response, the displacement field is expressed as

$$\mathbf{u} = \boldsymbol{\phi}(\mathbf{x})\, sin\,(\omega t) \tag{16}$$

where $\boldsymbol{\phi}(\mathbf{x})$ is the amplitude of the nodal displacement and ω is the natural frequency of plate.

By substituting Eq. (16) into (15) yields to the following eigenvalue problem:

$$(\mathbf{A} - \lambda\,\mathbf{I})\boldsymbol{\phi}^d = 0 \quad \text{with} \quad \lambda = 1/\omega^2 \tag{17a}$$

and

$$\mathbf{A} = \mathbf{M}^{dd} - \mathbf{K}^{*}\mathbf{K}^{-1}\mathbf{M}^{bd} \tag{17b}$$

In the BEM method, **A** is a real matrix that is commonly sparse, non-symmetric and non-positive definite. This form, to evaluate the natural frequency of plate is necessary to use an iterative algorithm to solve the eigenvalue problem [Smith, Boyle, Dongarra, Garbow, Ikebe, Klema, and Moler (1976)].

6 Numerical examples

Consider a square plate with side length $b = 1.0$, mass density $\rho = 1.0$, modulus of elasticity $E = 1.0$ and Poisson's ratio $\nu = 0.3$. Fig. 2 shows the two problems analyzed, as depicted in Fig. 2(a) and Fig. 2(b). In both cases the plate is discretized by a mesh with 40 boundary elements and 400 internal cells, as shown in Fig. 2(c).

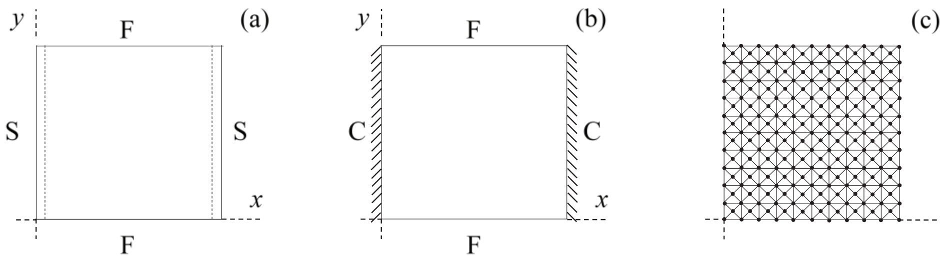

Figure 2: Problems for the analysis: (a) Square plate with SFSF boundary conditions, (b) Square plate with CFCF boundary conditions, (c) Mesh with 40 boundary elements and 400 internal cells.

In this work, two symbolism are used for identify a square plate with the edges $x = 0$, $y = 0$, $x = b$, $y = b$ as having boundary conditions. For example, the symbolism SFSF having simply supported, free, simply supported, and free boundary conditions, respectively. Thus, boundary conditions must be satisfied for a simply supported edge, $M_x = \phi_y = w = 0$, and for a free edge, $M_{xy} = M_y = Q_y = 0$. The another symbolism CFCF having clamped, free, clamped, and free boundary conditions, respectively, whose boundary conditions must be satisfied for clamped edge, $\phi_x = \phi_y = w = 0$.

The boundary element method described in this work has been applied to compute the non-dimensional frequency parameter, $\Delta = \left(\omega b^2/\pi^2\right)\sqrt{\rho h/D}$. Thus, the two examples presented here tries to establish the influence of the additional translational inertia term in the formulation. Numerical results are made to various thick-

Table 1: Frequency parameters Δ for square plate with SFSF boundary conditions

h/b	Method	Frequency parameters							
		Δ_1	Δ_2	Δ_3	Δ_4	Δ_5	Δ_6	Δ_7	Δ_8
0.01	3D DQM	**0.9755**	**1.6316**	**3.7118**	**3.9422**	**4.7268**	**7.1459**	**7.6074**	**8.8985**
	BEM	0.9758	1.6349	3.7339	3.9523	4.7446	7.1978	7.7008	8.9493
	Present	0.9758	1.6349	3.7340	3.9523	4.7446	7.1979	7.7010	8.9496
0.1	3D DQM	**0.9571**	**1.5603**	**3.4361**	**3.6919**	**4.3454**	**4.9400**	**6.3161**	**6.5234**
	2D Ritz	0.9565	1.5593	3.4307	3.6838	4.3358	-	6.2971	-
	BEM	0.9573	1.5623	3.4514	3.6949	4.3544	-	6.3451	-
	Present	0.9576	1.5627	3.4539	3.6989	4.3590	-	6.3544	-
0.2	3D DQM	**0.9120**	**1.4309**	**2.4697**	**2.9650**	**3.1888**	**3.2617**	**3.6651**	**4.6127**
	2D Ritz	0.9102	1.4280	-	2.9521	3.1684	-	3.6435	-
	BEM	0.9109	1.4311	-	2.9684	3.1780	-	3.6592	-
	Present	0.9118	1.4324	-	2.9747	3.1883	-	3.6699	-
0.3	3D DQM	**0.8523**	**1.2855**	**1.6462**	**2.1745**	**2.5168**	**2.7010**	**3.0500**	**3.0752**
	BEM	0.8497	1.2837	-	-	2.5118	2.6812	3.0360	-
	Present	0.8513	1.2857	-	-	2.5206	2.6953	3.0491	-
0.4	3D DQM	**0.7883**	**1.1466**	**1.2344**	**1.6309**	**2.1469**	**2.3019**	**2.3064**	**2.5665**
	BEM	0.7843	1.1437	-	-	2.1377	2.2779	-	2.5499
	Present	0.7865	1.1460	-	-	2.1475	2.2936	-	2.5633
0.5	3D DQM	**0.7261**	**0.9874**	**1.0223**	**1.3047**	**1.8451**	**1.8502**	**1.9872**	**2.1938**
	BEM	0.7211	-	1.0190	-	1.8395	-	1.9622	2.1783
	Present	0.7238	-	1.0212	-	1.8494	-	1.9781	2.1902

Table 2: Frequency parameters Δ for square plate with CFCF boundary conditions

h/b	Method	Frequency parameters							
		Δ_1	Δ_2	Δ_3	Δ_4	Δ_5	Δ_6	Δ_7	Δ_8
0.01	3D DQM	**2.2482**	**2.6743**	**4.4083**	**6.1972**	**6.7985**	**8.0653**	**8.8531**	**12.1530**
	BEM	2.2451	2.6757	4.4324	6.2052	6.8172	8.1639	8.9123	12.2112
	Present	2.2451	2.6757	4.4324	6.2052	6.8173	8.1641	8.9125	12.2116
0.1	3D DQM	**2.1050**	**2.4489**	**3.9234**	**5.3859**	**5.8272**	**5.9500**	**6.9678**	**7.3581**
	2D Ritz	2.0904	2.4342	3.9055	5.3392	5.7811	-	6.9368	7.3046
	BEM	2.0910	2.4377	3.9285	5.3549	5.8052	-	7.0149	7.3615
	Present	2.0917	2.4385	3.9313	5.3607	5.8113	-	7.0261	7.3720
0.2	3D DQM	**1.7996**	**2.0363**	**2.9770**	**3.1909**	**4.1062**	**4.4084**	**5.3313**	**5.4325**
	2D Ritz	1.7772	2.0151	-	3.1652	4.0413	4.3472	-	5.3939
	BEM	1.7778	2.0186	-	3.1825	4.0532	4.3659	-	5.4323
	Present	1.7793	2.0201	-	3.1887	4.0637	4.3761	-	5.4510
0.3	3D DQM	**1.4999**	**1.6639**	**1.9858**	**2.6089**	**3.1647**	**3.4031**	**3.5611**	**3.6551**
	BEM	1.4747	1.6443	-	2.5943	3.1062	3.3562	-	-
	Present	1.4765	1.6458	-	2.6023	3.1182	3.3669	-	-
0.4	3D DQM	**1.2582**	**1.3764**	**1.4901**	**2.1845**	**2.5321**	**2.6747**	**2.7373**	**2.7411**
	BEM	1.2320	1.3572	-	2.1677	2.4739	-	2.6915	-
	Present	1.2338	1.3585	-	2.1765	2.4865	-	2.7016	-
0.5	3D DQM	**1.0723**	**1.1601**	**1.1926**	**1.8710**	**2.0957**	**2.1419**	**2.1928**	**2.2801**
	BEM	1.0463	1.1422	-	1.8542	2.0398	-	-	2.2371
	Present	1.0479	1.1431	-	1.8633	2.0528	-	-	2.2464

3D DQM: Liew, Hung, and Lim (1993)
2D Ritz: Liew, Xiang, and Kitipornchai (1993)
BEM: without the inertia terms
Present: with the inertia terms

nesses of plates, where are considered thickness-side ratios h/b covering the range 0.01 to 0.5.

To validate the accuracy of the present method, comparison studies have been carried out for cases where the solutions for exact three-dimensional analysis (3D DQM) and the Mindlin theory analysis (2D Ritz) are available.

Table 1 presents the lowest eight frequency parameters for square plate with SFSF boundary conditions with different thickness-side ratios h/b. In this table, are presented two responses using the boundary element method, i.e., the results with the additional translational inertia terms (Present), and the results without these terms (BEM). It should be observed that for small values of the thickness-side ratio, the additional terms have no influence. While that for the others thickness-side ratios of plate, the responses already show some differences. For thickness-side ratios $h/b = 0.1$ and 0.2, one may observe which the responses of the present formulation with the inertia terms are better than 2D Ritz method. In Table 2, the same observations can be made for square plate with CFCF boundary conditions.

7 Conclusion

In this work, the BEM is applied to obtain the numerical solutions for free vibration analysis of thick square plates with two edges simply supported or clamped, and the other two edges free. A formulation based on Reissner's theory is used here, which includes the contribution of the additional translational inertia terms to the integral equation of displacements and internal forces. The boundary element method is used to discretize the space, where it is employed the static fundamental solution. The numerical simulations carried out with the additional term considered by the present formulation modified results in comparison with results obtained without this term and its contribution in the analyses carried out here was more relevant for thickness-side ratio $h/b = 0.20$. The responses for the kind of problem addressed here are very important in the hydroelastic analysis of very large floating structures (VLFS) which are commonly modeled as plates with free edges.

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