

## Arbitrary Stokes Flow About A Fixed or Freely-Suspended Slip Particle

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**Abstract:** The rigid-body migration of a slip and arbitrary-shaped solid particle freely suspended in a prescribed and arbitrary ambient Stokes flow is determined after calculating the hydrodynamic force and torque exerted on the particle when it either experiences a given rigid-body in a quiescent liquid or is held fixed in the ambient Stokes flow. It is also shown how one can subsequently obtain the velocity and surface traction on the particle boundary and thereafter, if necessary, the flow about the particle in the entire liquid domain. The advocated procedure extends a recent work (see Sellier (2012)) and consists in inverting at the most seven boundary problems involving coupled and regularized boundary-integral equations on the particle boundary. In addition to the numerical implementation, comparisons against analytical results for a spherical particle and numerical results for spheroids are given.

**Keywords:** Stokes flow, Ambient flow, Navier slip condition, Boundary-integral equation, Boundary Element Method.

### 1 Introduction

Suspensions, consisting in solid particles immersed in a Newtonian liquid with uniform viscosity  $\mu$  and density  $\rho$ , are encountered in several fields of interest. Accordingly, many works examine the macroscopic behaviour of either dilute or concentrated suspensions with attention paid, for instance, to the settling of a suspension in absence of ambient flow or the so-called effective viscosity of a flowing suspension subject to a prescribed ambient linear shear flow. Such investigations are tremendously involved if one solves the Navier-Stokes equations for a concentrated suspension.

Fortunately, for a dilute and unbounded suspension it is possible to confine the analysis to the case of a single solid particle embedded in a (quiescent or flowing) unbounded Newtonian liquid. Not surprisingly, the fluid flow past the solid particle surface  $S$  is strongly sensitive to the boundary conditions it satisfies there. Of

course, such key conditions are dictated by the surface properties and ability to let the liquid flow tangent to it. Although the usual no-slip condition (equal fluid and surface velocities at the boundary  $S$ ) is valid for most surfaces, it however breaks down for surfaces (such as hydrophobic ones) allowing a tangent slip. In most cases, one then resorts to the celebrated Navier (1823) slip condition in which (see (1)) the surface ability to let the fluid flow tangent to it is characterized by a so-called surface slip length  $\lambda \geq 0$ . For a Newtonian fluid with uniform viscosity  $\mu$ , velocity  $\mathbf{u}'$  and stress tensor  $\boldsymbol{\sigma}'$  flowing past the slip solid surface  $S$  moving with the rigid-body velocity  $\mathbf{v}$  and having unit normal  $\mathbf{n}$  directed into the liquid, this Navier slip condition reads

$$\mathbf{u}' = \mathbf{v} + \lambda \{ \boldsymbol{\sigma}' \cdot \mathbf{n} - (\mathbf{n} \cdot \boldsymbol{\sigma}' \cdot \mathbf{n}) \mathbf{n} \} / \mu \text{ on } S. \quad (1)$$

In other words (1) stipulates that  $(\mathbf{u}' - \mathbf{v}) \cdot \mathbf{n} = 0$  (impermeable surface) whilst the tangent velocity slip (tangent part of  $\mathbf{u}' - \mathbf{v}$ ) is a multiple of the tangent stress (with coefficient  $\lambda/\mu$ ). Note that, as evidenced in Churaev, Sobolev, and Somov (1994); Hutchins, Harper, and Felder (1995); Baudry, Charlaix, Tonck, and Mazuyer (2001), the slip condition (1) proposed by Navier in 1823 is confirmed by experiments.

For a particle with length scale  $a$  and a flow velocity with typical magnitude  $V$  the Reynolds number  $Re = \rho Va/\mu$  compares in the Navier-Stokes equation the inertial term with the viscous one. Whenever  $Re = \rho Va/\mu \ll 1$  one can therefore neglect all inertial effects and replace the Navier-Stokes equations with the *linear* (and thus much more tractable) steady creeping (Stokes) flow equations. Within this latter Low-Reynolds-Number flow convenient framework, a large body of literature has been devoted to the case of a slip solid particle experiencing a given rigid-body motion in a *quiescent* liquid. In that direction one can cite, among other works, three types of particle shapes :

(i) *Spherical or nearly-spherical slip particles*. The migrating slip sphere has been analytically treated by Basset (1961) while Palaniappan (1994); Ramkissoon (1997) and later Senchenko and Keh (2006); Chang and Keh (2009) asymptotically solved the case of a nearly-spherical slip particle.

(ii) *Axisymmetric slip particles*. Several different approaches have been employed to treat some axisymmetric shapes. For instance, using toroidal coordinates Williams (1987) treats the case of a slip torus translating or rotating parallel with its axis of revolution. Loyalka and Griffin (1994) and also Loyalka (1996) proposed and implemented a boundary approach to efficiently solve the case of a slip axisymmetric particle rotating parallel with its axis of revolution and provided numerical results for rotating slip torus and slip spheroid. Additional results for a slip spheroid experiencing a given *arbitrary* translational and/or angular velocity can also be obtained.

ned by combining several different works : Keh and Huang (2004); Keh and Chang (2008); Chang and Keh (2009, 2011).

(iii) *Arbitrary-shaped slip particles.*

Recently Sellier (2012) proposed a new boundary formulation to accurately handle the case of a slip and *arbitrary-shaped* solid particle experiencing a rigid-body migration in a quiescent liquid. The approach rests on the treatment of six boundary-integral equations on the particle surface which have been numerically inverted by implementing a boundary element technique. New results for different slip tori and non-spheroidal slip ellipsoids have then been obtained and discussed.

The previous works may be employed to predict the settling motion of a slip particle in a quiescent liquid. However, the case of a slip particle immersed in a prescribed ambient flow is also of the utmost importance for other applications such as, for instance, the evaluation of the effective viscosity of a dilute suspension in an unbounded liquid. For this latter example one indeeds might need to calculate other quantities such as the stresslet tensor exerted on a *freely-suspended* particle embedded in a given linear shear ambient flow. Despite such key applications, the interesting case of a slip particle held fixed or freely moving in a given and arbitrary ambient Stokes flow has not yet attracted so much attention. Moreover, the relevant literature actually solely deals with a *spherical* slip particle held fixed either in a Stokes flow with linear rate of strain (see Felderhof (1976)) or in an arbitrary Stokes flow (see Keh and Chen (1996)). Observe that Keh and Chen (1996) actually nicely extends to the case of a slip sphere ( $\lambda > 0$ ) the famous Faxen relations (Faxen (1922-1923)) derived for a no-slip ( $\lambda = 0$ ) sphere.

To the author's very best knowledge, there is currently no work dealing with the challenging case of a slip and *arbitrary-shaped* solid particle held fixed or freely-suspended in a given and arbitrary ambient Stokes flow. The present paper fills the gap by introducing a suitable boundary approach to accurately solve this issue. As in Sellier (2012), it actually rests on the treatment of a few boundary-integral equations on the slip particle surface. More precisely, it is organized as follows. Challenging problems are introduced together with analytical solutions for a slip sphere in §2 while two different boundary approaches resulting in a few relevant boundary-integral equations on the particle surface are presented in §3 and §4. A possible numerical strategy together with numerical benchmarks and results are displayed in §5 whereas another interesting issue postponed to a future work is mentioned in §6.

## 2 Governing equations and analytical results for a slip sphere

This section introduces the adopted notations and the governing equations. It also shows how it is possible, by superposition, to reduce the task to the treatment of two problems when looking at the incurred rigid-body migration of a slip particle freely immersed in a prescribed ambient Stokes flow.

### 2.1 Key problems

As shown in Fig. 1, we consider a solid slip particle  $\mathcal{P}$  immersed in a Newtonian and unbounded liquid with uniform density  $\rho$  and viscosity  $\mu$ .

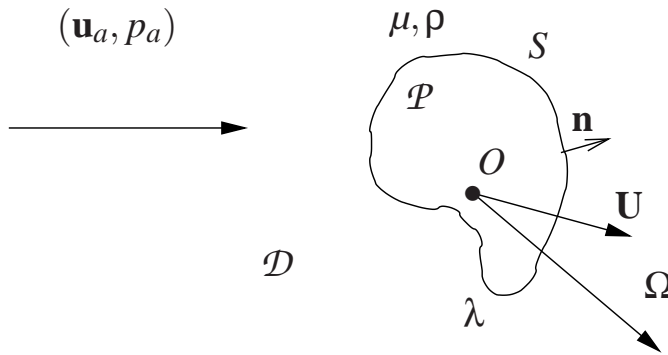


Figure 1 – A solid slip particle  $\mathcal{P}$  experiencing a rigid-body migration  $(\mathbf{U}, \mathbf{\Omega})$  and immersed in a prescribed arbitrary ambient Stokes flow  $(\mathbf{u}_a, p_a)$ .

This particle has attached point  $O$  and smooth slip surface  $S$  with unit normal  $\mathbf{n}$  directed into the liquid domain  $\mathcal{D}$ . Moreover, it is embedded in a steady and arbitrary Stokes flow with prescribed velocity field  $\mathbf{u}_a$ , pressure field  $p_a$  and stress tensor  $\sigma_a$ . For a Stokes flow these quantities obey

$$\mu \nabla^2 \mathbf{u}_a = \nabla p_a \quad \text{and} \quad \nabla \cdot \mathbf{u}_a = 0 \quad \text{in } \mathbb{R}^3. \tag{2}$$

The particle rigid-body motion has translational velocity  $\mathbf{U}$  (the velocity of  $O$ ) and angular velocity  $\mathbf{\Omega}$ . In addition, the disturbed flow about  $\mathcal{P}$  has velocity  $\mathbf{u}_a + \mathbf{u}$  and pressure  $p_a + p$  in the liquid domain  $\mathcal{D}$ . All inertial effects are neglected, i. e. the particle length scale  $a$  and the typical magnitude  $V > 0$  of the disturbed velocity

$\mathbf{u}_a + \mathbf{u}$  satisfy  $\text{Re} = \rho Va/\mu \ll 1$ . Accordingly, the flow  $(\mathbf{u}, p)$  with stress tensor  $\boldsymbol{\sigma}$  fulfills the following boundary-value problem

$$\mu \nabla^2 \mathbf{u} = \nabla p \quad \text{and} \quad \nabla \cdot \mathbf{u} = 0 \quad \text{in } \mathcal{D}, \quad (3)$$

$$(\mathbf{u}, p) \rightarrow (\mathbf{0}, 0) \quad \text{as } |\mathbf{x}| \rightarrow \infty, \quad (4)$$

$$\begin{aligned} \mathbf{u}(M) - \lambda \{ \boldsymbol{\sigma} \cdot \mathbf{n} - (\mathbf{n} \cdot \boldsymbol{\sigma} \cdot \mathbf{n}) \mathbf{n} \} / \mu = \\ - \mathbf{u}_a(M) + \lambda \{ \boldsymbol{\sigma}_a \cdot \mathbf{n} - (\mathbf{n} \cdot \boldsymbol{\sigma}_a \cdot \mathbf{n}) \mathbf{n} \} / \mu \\ + \mathbf{U} + \boldsymbol{\Omega} \wedge \mathbf{OM} \quad \text{on } S \end{aligned} \quad (5)$$

where  $\mathbf{x} = \mathbf{OM}$  and (5) is the Navier (1823) slip condition for the disturbed flow and  $\lambda \geq 0$  the particle surface slip length.

Because  $(\mathbf{u}_a, p_a)$  is a Stokes flow inside the particle (remind (2)) it exerts zero force and torque on it. Consequently, the hydrodynamic force  $\mathbf{F}$  and torque  $\mathbf{T}$  (about  $O$ ) experienced by the slip particle read

$$\mathbf{F} = \int_S [\boldsymbol{\sigma}_a + \boldsymbol{\sigma}] \cdot \mathbf{n} dS = \int_S \boldsymbol{\sigma} \cdot \mathbf{n} dS, \quad (6)$$

$$\mathbf{T} = \int_S \mathbf{x} \wedge [\boldsymbol{\sigma}_a + \boldsymbol{\sigma}] \cdot \mathbf{n} dS = \int_S \mathbf{x} \wedge \boldsymbol{\sigma} \cdot \mathbf{n} dS. \quad (7)$$

In practice, when solving the problem (3)-(5) one may either know the particle rigid-body migration  $(\mathbf{U}, \boldsymbol{\Omega})$  (prescribed motion of the particle) or not (case of the freely-suspended particle)! As will be shown in §3.2, one can however find in this latter case the motion  $(\mathbf{U}, \boldsymbol{\Omega})$  by solely appealing to the solution proposed in Sellier (2012) for the *prescribed* rigid-body migration of a slip particle in a *quiescent* liquid. Now, once  $(\mathbf{U}, \boldsymbol{\Omega})$  is prescribed one may think about first determining the perturbation flow  $(\mathbf{u}, p)$  about the slip and arbitrary-shaped particle in the entire liquid domain  $\mathcal{D}$  and then subsequently calculate the resulting traction  $\boldsymbol{\sigma} \cdot \mathbf{n}$  on the particle surface. This permits one to obtain the previous vectors  $\mathbf{F}, \mathbf{T}$  together with other moments of the vector  $\mathbf{f}_t = (\boldsymbol{\sigma}_a + \boldsymbol{\sigma}) \cdot \mathbf{n}$ . For instance, in predicting the effective viscosity of a dilute suspension the following so-called stresslet tensor  $\mathbf{S}$  (here with respect to the point  $O$  attached to the particle) defined as (Brenner and Happel (1958); Pozrikidis (1992))

$$\mathbf{S} = \frac{1}{2} \int_S \left\{ \mathbf{x} \otimes \mathbf{f}_t + \mathbf{f}_t \otimes \mathbf{x} - \frac{2}{3} (\mathbf{x} \cdot \mathbf{f}_t) \mathbf{I} \right\} dS, \quad (8)$$

with  $\mathbf{x} = \mathbf{OM}$  and  $\mathbf{I}$  the usual identity tensor, is very likely to be needed.

By superposition, it is possible to reduce the task to the treatment of two different cases (i)-(ii) :

(i) Slip particle experiencing a prescribed rigid-body migration  $(\mathbf{U}, \Omega)$  in a *quiescent* liquid.

The resulting flow, obtained here for  $\mathbf{u}_a = \mathbf{0}$  and  $p_a = 0$ , exerts on the moving particle hydrodynamic force  $\mathbf{F}_h$  and torque  $\mathbf{T}_h$  given by (6)-(7). By linearity, one gets

$$\mathbf{F}_h = -\mu\{\mathbf{A}\cdot\mathbf{U} + \mathbf{B}\Omega\}, \quad \mathbf{T}_h = -\mu\{\mathbf{C}\cdot\mathbf{U} + \mathbf{D}\cdot\Omega\} \quad (9)$$

with second-rank resistance tensors  $\mathbf{A}, \mathbf{B}, \mathbf{C}$  and  $\mathbf{D}$  depending upon the particle geometry and slip length  $\lambda$  (in addition, tensors  $\mathbf{B}$  and  $\mathbf{C}$  are transposed while  $\mathbf{A}$  and  $\mathbf{D}$  are symmetric).

(ii) Particle *held fixed* in the arbitrary ambient Stokes flow  $(\mathbf{u}_a, p_a)$ .

Here,  $\mathbf{U} = \Omega = \mathbf{0}$  and one seeks the flow  $(\mathbf{u}, p)$  and the force and torque it exerts on the fixed particle given by (6)-(7) and further denoted by  $\mathbf{F}_a$  and  $\mathbf{T}_a$ , respectively.

Of course, once cases (i)-(ii) have been solved it is then straightforward to gain the rigid-body motion  $(\mathbf{U}, \Omega)$  of the slip particle when *freely-suspended* in the ambient Stokes flow  $(\mathbf{u}_a, p_a)$ . Indeed, requiring the particle with negligible inertia to be force-free and torque-free immediately yields for  $(\mathbf{U}, \Omega)$  the linear system

$$\mu\{\mathbf{A}\cdot\mathbf{U} + \mathbf{B}\cdot\Omega\} = \mathbf{F}_a, \quad \mu\{\mathbf{C}\cdot\mathbf{U} + \mathbf{D}\cdot\Omega\} = \mathbf{T}_a. \quad (10)$$

Therefore, it one contents oneself with obtaining the migration of the freely-suspended particle (i. e. solve problem (3)-(5) together with the additional conditions  $\mathbf{F} = \mathbf{T} = \mathbf{0}$ ) it is *sufficient* to solve cases (i) and (ii). In contrast, if the flow  $(\mathbf{u}, p)$  and/or the surface traction  $\boldsymbol{\sigma}\cdot\mathbf{n}$  are required it becomes necessary to solve this time (3)-(5) in which one prescribes on the right-hand side of (5) the value of  $(\mathbf{U}, \Omega)$  provided by solving the linear system (10). Actually, it will be shown in §4 how it is possible to gain the traction  $\boldsymbol{\sigma}\cdot\mathbf{n}$  without determining the flow  $(\mathbf{u}, p)$  in the entire liquid domain  $\mathcal{D}$ .

## 2.2 Analytical results for a spherical particle

For a spherical particle with radius  $a$  it is possible to gain analytical results. For case (i) the sphere experiences the rigid-body motion  $(\mathbf{U}, \Omega)$  in a *quiescent* liquid and Basset (1961) gets

$$\mathbf{F}_h = -6\pi\mu a \left[ \frac{1 + 2\lambda/a}{1 + 3\lambda/a} \right] \mathbf{U}, \quad (11)$$

$$\mathbf{T}_h = -8\pi\mu a^3 \left[ \frac{\Omega}{1 + 3\lambda/a} \right]. \quad (12)$$

Furthermore, one also analytically determines *whatever*  $(\mathbf{U}, \Omega)$  the flow  $(\mathbf{u}, p)$  about the moving sphere by putting at its center a Stokeslet, a potential dipole and a rotlet with strengths given in Sellier (2012). The case of a slip sphere immersed in a prescribed arbitrary ambient Stokes flow  $(\mathbf{u}_a, p_a)$  has been treated by Keh and Chen (1996) and the results read, for a sphere with center  $O$ ,

$$\mathbf{F}_a = 6\pi\mu a \left[ \frac{1+2\lambda/a}{1+3\lambda/a} \right] \mathbf{u}_a(O) + \pi\mu a^3 \left[ \frac{1}{1+3\lambda/a} \right] \nabla^2 \mathbf{u}_a(O), \quad (13)$$

$$\mathbf{T}_a = \left[ \frac{4\pi\mu a^3}{1+3\lambda/a} \right] (\nabla \wedge \mathbf{u}_a)(O). \quad (14)$$

Accordingly, the migration of a sphere freely suspended in a Stokes flow  $(\mathbf{u}_a, p_a)$  is given by

$$\mathbf{U} = \mathbf{u}_a(O) + \left[ \frac{a^2}{6(1+2\lambda/a)} \right] \nabla^2 \mathbf{u}_a(O), \quad (15)$$

$$\Omega = \frac{1}{2} (\nabla \wedge \mathbf{u}_a)(O). \quad (16)$$

Note that Keh and Chen (1996) also gives the stresslet tensor  $\mathbf{S}$  when the Stokes flow  $(\mathbf{u}_a, p_a)$  is arbitrary (therefore extending to the case  $\lambda > 0$  the formula obtained in the  $\lambda = 0$  no-slip case by Batchelor and O'Neill (1972)) while Felderhof (1976) provides results analogous to (13)-(16) but restricted to a flow  $(\mathbf{u}_a, p_a)$  having a *linear* rate of strain  $[\nabla \mathbf{u}_a + (\nabla \mathbf{u}_a)^t]/2$ .

### 3 Auxiliary Stokes flows and resulting first boundary approach

This section introduces six auxiliary Stokes flows about the slip particle and also recalls the boundary-integral equation on the particle surface  $S$  obtained in Sellier (2012) which governs on  $S$  the traction for each of those flows. It also shows how the knowledge on  $S$  of the surface traction exerted by each auxiliary flow permits one to calculate the hydrodynamic force  $\mathbf{F}_a$  and torque  $\mathbf{T}_a$  applied by the Stokes flow  $(\mathbf{u}_a, p_a)$  on a *fixed* particle *without* solving the associated problem (2)-(5) with  $\mathbf{U} = \Omega = \mathbf{0}$ .

#### 3.1 Auxiliary Stokes flows and associated boundary-integral equations

Henceforth, we use Cartesian coordinates  $(O, x_1, x_2, x_3)$  with the notations  $\mathbf{x} = \mathbf{OM}$ ,  $x_i = \mathbf{x} \cdot \mathbf{e}_i$  for  $i = 1, 2, 3$  and  $r = |\mathbf{x}| = \{x_1^2 + x_2^2 + x_3^2\}^{1/2}$ . For convenience we shall also use the usual tensor summation convention with, for instance,  $\mathbf{x} = x_i \mathbf{e}_i$  and  $\mathbf{n} = n_j \mathbf{e}_j$ .

At that stage it is useful to introduce (for  $i = 1, 2, 3$ ) six auxiliary Stokes flows  $(\mathbf{u}_t^{(i)}, p_t^{(i)})$ , with stress tensor  $\boldsymbol{\sigma}_t^{(i)}$ , and  $(\mathbf{u}_r^{(i)}, p_r^{(i)})$ , with stress tensor  $\boldsymbol{\sigma}_r^{(i)}$ , solution to (3)-(5) for  $(\mathbf{u}_a, p_a) = (\mathbf{0}, 0)$  and  $(\mathbf{U}, \boldsymbol{\Omega}) = (\mathbf{e}_i, \mathbf{0})$  or  $(\mathbf{U}, \boldsymbol{\Omega}) = (\mathbf{0}, \mathbf{e}_i)$ , respectively. In other words, those flows are produced by a migrating slip particle when it either translates or rotates at the velocities  $\mathbf{e}_1, \mathbf{e}_2$  or  $\mathbf{e}_3$  in a quiescent liquid. As shown in Sellier (2012), the resulting tractions  $\mathbf{f}_t^{(i)} = \boldsymbol{\sigma}_t^{(i)} \cdot \mathbf{n}$  and  $\mathbf{f}_r^{(i)} = \boldsymbol{\sigma}_r^{(i)} \cdot \mathbf{n}$  on the particle surface  $S$  are governed by a boundary problem involving coupled and regularized boundary-integral equations. More precisely, consider the Stokes flow  $(\mathbf{u}, p)$ , with stress tensor  $\boldsymbol{\sigma}$ , about the particle when it experiences a given rigid-body motion  $(\mathbf{U}, \boldsymbol{\Omega})$  in a quiescent fluid. For such a flow fulfilling (3)-(5) one introduces the surface quantities

$$d = \mathbf{n} \cdot \boldsymbol{\sigma} \cdot \mathbf{n} / \mu, \mathbf{d} = [\boldsymbol{\sigma} \cdot \mathbf{n} - (\mathbf{n} \cdot \boldsymbol{\sigma} \cdot \mathbf{n})\mathbf{n}] / \mu = d_i \mathbf{e}_i \tag{17}$$

here obtained by solving the boundary problem (see details in Sellier (2012))

$$L_i[d, \mathbf{d}] = [\mathbf{U} + \boldsymbol{\Omega} \wedge \mathbf{OM}] \cdot \mathbf{e}_i \text{ for } \mathbf{x} \text{ on } S (i = 1, 2, 3), \tag{18}$$

$$\mathbf{d} \cdot \mathbf{n} = 0 \text{ for } \mathbf{x} \text{ on } S \tag{19}$$

with linear operators  $L_i$  defined (for  $i = 1, 2, 3$ ) as

$$\begin{aligned} 8\pi L_i[d, \mathbf{d}] = & -8\pi\lambda d_i(\mathbf{x}) - \int_S G_{ki}(\mathbf{y}, \mathbf{x}) d_k(\mathbf{y}) dS(\mathbf{y}) - \int_S G_{ki}(\mathbf{y}, \mathbf{x}) n_k(\mathbf{y}) d(\mathbf{y}) dS(\mathbf{y}) \\ & + \lambda \int_S [d_k(\mathbf{y}) - d_k(\mathbf{x})] T_{kil}(\mathbf{y}, \mathbf{x}) n_l(\mathbf{y}) dS(\mathbf{y}) \end{aligned} \tag{20}$$

where, denoting by  $\delta$  the Kronecker delta symbol,

$$G_{ij}(\mathbf{x}, \mathbf{y}) = \frac{\delta_{ij}}{|\mathbf{x} - \mathbf{y}|} + \frac{[(\mathbf{y} - \mathbf{x}) \cdot \mathbf{e}_i][(\mathbf{y} - \mathbf{x}) \cdot \mathbf{e}_j]}{|\mathbf{x} - \mathbf{y}|^3}, \tag{21}$$

$$T_{ijk}(\mathbf{y}, \mathbf{x}) = -\frac{6[(\mathbf{y} - \mathbf{x}) \cdot \mathbf{e}_i][(\mathbf{y} - \mathbf{x}) \cdot \mathbf{e}_j][(\mathbf{y} - \mathbf{x}) \cdot \mathbf{e}_k]}{|\mathbf{x} - \mathbf{y}|^5}. \tag{22}$$

Once  $d$  and  $\mathbf{d}$  are gained by inverting (18)-(19) it is straightforward to get the velocity on the particle surface  $S$  from the Navier slip boundary condition (5), i. e. from the equality

$$\mathbf{u} = \mathbf{U} + \boldsymbol{\Omega} \wedge \mathbf{OM} + \lambda \{ \boldsymbol{\sigma} \cdot \mathbf{n} - (\mathbf{n} \cdot \boldsymbol{\sigma} \cdot \mathbf{n})\mathbf{n} \} / \mu \text{ on } S. \tag{23}$$



### 3.2 Use of the reciprocal identity

As outlined in §2.1, one key step consists in calculating the net force  $\mathbf{F}_a$  and torque  $\mathbf{T}_a$  exerted by a Stokes flow  $(\mathbf{u}_a, p_a)$  on a *fixed* slip particle. The perturbation flow  $(\mathbf{u}, p)$  with stress tensor  $\boldsymbol{\sigma}$  obeys (3)-(5) with  $\mathbf{U} = \boldsymbol{\Omega} = \mathbf{0}$  in (5). Exploiting (6) easily gives

$$\mathbf{F}_a \cdot \mathbf{e}_i = \int_S (\mathbf{e}_i \cdot \mathbf{n})(\mathbf{n} \cdot \boldsymbol{\sigma} \cdot \mathbf{n}) dS + \int_S [\boldsymbol{\sigma} \cdot \mathbf{n} - (\mathbf{n} \cdot \boldsymbol{\sigma} \cdot \mathbf{n})\mathbf{n}] \cdot \mathbf{e}_i dS. \quad (24)$$

By definition, remind that one has the following boundary conditions

$$\mathbf{u}_t^{(i)} = \mathbf{e}_i + \lambda \{ \boldsymbol{\sigma}_t^{(i)} \cdot \mathbf{n} - (\mathbf{n} \cdot \boldsymbol{\sigma}_t^{(i)} \cdot \mathbf{n})\mathbf{n} \} / \mu \text{ on } S, \quad (25)$$

$$\mathbf{u}(M) - \lambda \{ \boldsymbol{\sigma} \cdot \mathbf{n} - (\mathbf{n} \cdot \boldsymbol{\sigma} \cdot \mathbf{n})\mathbf{n} \} / \mu = -\mathbf{u}_a(M) + \lambda \{ \boldsymbol{\sigma}_a \cdot \mathbf{n} - (\mathbf{n} \cdot \boldsymbol{\sigma}_a \cdot \mathbf{n})\mathbf{n} \} / \mu \text{ on } S. \quad (26)$$

Furthermore, the usual reciprocal identity (Happel and Brenner (1991); Kim and Karrila (1991)) gives the additional relation

$$\int_S \mathbf{u}_t^{(i)} \cdot \boldsymbol{\sigma} \cdot \mathbf{n} dS = \int_S \mathbf{u} \cdot \boldsymbol{\sigma}_t^{(i)} \cdot \mathbf{n} dS \quad (27)$$

which, exploiting the normal component of (25)-(26), becomes

$$\begin{aligned} & \int_S (\mathbf{e}_i \cdot \mathbf{n})(\mathbf{n} \cdot \boldsymbol{\sigma} \cdot \mathbf{n}) dS + \int_S \mathbf{u}_t^{(i)} \cdot [\boldsymbol{\sigma} \cdot \mathbf{n} - (\mathbf{n} \cdot \boldsymbol{\sigma} \cdot \mathbf{n})\mathbf{n}] dS \\ &= - \int_S (\mathbf{u}_a \cdot \mathbf{n})(\mathbf{n} \cdot \boldsymbol{\sigma}_t^{(i)} \cdot \mathbf{n}) dS + \int_S \mathbf{u} \cdot [\boldsymbol{\sigma}_t^{(i)} \cdot \mathbf{n} - (\mathbf{n} \cdot \boldsymbol{\sigma}_t^{(i)} \cdot \mathbf{n})\mathbf{n}] dS. \end{aligned} \quad (28)$$

Combining (24) with (28) then yields

$$\begin{aligned} \mathbf{F}_a \cdot \mathbf{e}_i &= - \int_S (\mathbf{u}_a \cdot \mathbf{n})(\mathbf{n} \cdot \boldsymbol{\sigma}_t^{(i)} \cdot \mathbf{n}) dS - \int_S [\mathbf{u}_t^{(i)} - \mathbf{e}_i] \cdot [\boldsymbol{\sigma} \cdot \mathbf{n} - (\mathbf{n} \cdot \boldsymbol{\sigma} \cdot \mathbf{n})\mathbf{n}] dS \\ & \quad + \int_S \mathbf{u} \cdot [\boldsymbol{\sigma}_t^{(i)} \cdot \mathbf{n} - (\mathbf{n} \cdot \boldsymbol{\sigma}_t^{(i)} \cdot \mathbf{n})\mathbf{n}] dS \end{aligned} \quad (29)$$

Appealing again to (25)-(26) several terms on the right-hand side of (29) cancel and one finally easily ends up with the key relation

$$\mathbf{F}_a \cdot \mathbf{e}_i = - \int_S \mathbf{u}_a \cdot \boldsymbol{\sigma}_t^{(i)} \cdot \mathbf{n} dS + \frac{\lambda}{\mu} \int_S (\boldsymbol{\sigma}_a \cdot \mathbf{n}) \cdot [\boldsymbol{\sigma}_t^{(i)} \cdot \mathbf{n} - (\mathbf{n} \cdot \boldsymbol{\sigma}_t^{(i)} \cdot \mathbf{n})\mathbf{n}] dS. \quad (30)$$

In a similar fashion, using this time the flow  $(\mathbf{u}_r^{(i)}, p_r^{(i)})$  also gives for the applied hydrodynamic torque (about point  $O$ )

$$\mathbf{T}_a \cdot \mathbf{e}_i = - \int_S \mathbf{u}_a \cdot \boldsymbol{\sigma}_r^{(i)} \cdot \mathbf{n} dS + \frac{\lambda}{\mu} \int_S (\boldsymbol{\sigma}_a \cdot \mathbf{n}) \cdot [\boldsymbol{\sigma}_r^{(i)} \cdot \mathbf{n} - (\mathbf{n} \cdot \boldsymbol{\sigma}_r^{(i)} \cdot \mathbf{n})\mathbf{n}] dS. \quad (31)$$

According to the formulae (30)-(31) and (9)-(10) it is sufficient to solve for the auxiliary Stokes flows the boundary problems (18)-(20) in order to obtain the force  $\mathbf{F}_a$  and torque  $\mathbf{T}_a$  exerted on a slip particle *held fixed* in the ambient flow and the resulting incurred particle rigid-body migration  $(\mathbf{U}, \Omega)$  when it is freely-suspended in the ambient flow. This task, termed the first boundary approach in the present work, actually solely appeals to the material detailed in Sellier (2012) plus the new relations (30)-(31). These relations may be cast into the more convenient forms (13)-(14) for a *spherical* slip particle and hold for a solid and *arbitrary-shaped* slip particle.

#### 4 Second boundary approach

As outlined before giving the definition (8), it is sometimes also necessary to compute the traction  $\mathbf{f}_t = (\sigma_a + \sigma) \cdot \mathbf{n}$  arising on the slip particle when the flow  $(\mathbf{u}, p)$  obeys (3)-(5). This section shows how such a task can also be done by solving another boundary problem.

##### 4.1 Velocity integral representations

By superposition, we can confine the analysis to the obtention of  $\mathbf{f}_t$  when the particle is *held fixed* in the quiescent liquid, i. e. when the perturbation flow  $(\mathbf{u}, p)$  with stress tensor  $\sigma$  satisfies (26). Reminding the usual tensor summation convention and setting  $\mathbf{u} = u_i \mathbf{e}_i$ , we then have the following integral representation (see Sellier (2012) for further details) in the entire liquid domain

$$8\pi\mu u_j(\mathbf{x}) = \mu \int_S [\mathbf{u}(\mathbf{y}) - \mathbf{u}(\mathbf{x})] \cdot \mathbf{e}_i T_{ijk}(\mathbf{y}, \mathbf{x}) n_k(\mathbf{y}) dS(\mathbf{y}) - \int_S [\mathbf{e}_i \cdot \sigma \cdot \mathbf{n}](\mathbf{y}) G_{ij}(\mathbf{y}, \mathbf{x}) dS(\mathbf{y}) \text{ for } \mathbf{x} \text{ in } \mathcal{D}. \tag{32}$$

Because the ambient flow  $(\mathbf{u}_a, p_a)$  is a Stokes flow *inside* the particle  $\mathcal{P}$  one can also prove (use the material in Pozrikidis (1992)) the additional integral representation

$$0 = \mu \int_S [\mathbf{u}_a(\mathbf{y}) - \mathbf{u}_a(\mathbf{x})] \cdot \mathbf{e}_i T_{ijk}(\mathbf{y}, \mathbf{x}) n_k(\mathbf{y}) dS(\mathbf{y}) - \int_S [\mathbf{e}_i \cdot \sigma_a \cdot \mathbf{n}](\mathbf{y}) G_{ij}(\mathbf{y}, \mathbf{x}) dS(\mathbf{y}) \text{ for } \mathbf{x} \text{ in } \mathcal{P}. \tag{33}$$

The second boundary approach developed in this section rests on the integral representations (32) and (33).

#### 4.2 Resulting boundary-integral equations for a slip particle held fixed in a prescribed ambient Stokes flow

This time the unknown quantities on the particle surface  $S$  are the quantity  $d'$  and vector  $\mathbf{d}'$  defined as

$$d' = \mathbf{n} \cdot [\boldsymbol{\sigma}_a + \boldsymbol{\sigma}] \cdot \mathbf{n} / \mu, \quad (34)$$

$$\mathbf{d}' = \{ [\boldsymbol{\sigma}_a + \boldsymbol{\sigma}] \cdot \mathbf{n} - (\mathbf{n} \cdot [\boldsymbol{\sigma}_a + \boldsymbol{\sigma}] \cdot \mathbf{n}) \mathbf{n} \} / \mu. \quad (35)$$

These quantities are found to satisfy a boundary problem on the surface  $S$ . This is established by noting that both (32) and (33) still hold when  $\mathbf{x}$  is located on the particle surface  $S$  (see definitions (21)-(22)), combining (32) and (33) there and exploiting the boundary condition (26). Recalling the definition (20), one then immediately arrives at the following boundary problem

$$L_i[d', \mathbf{d}'] = -\mathbf{u}_a \cdot \mathbf{e}_i \text{ for } \mathbf{x} \text{ on } S (i = 1, 2, 3), \quad (36)$$

$$\mathbf{d}' \cdot \mathbf{n} = 0 \text{ for } \mathbf{x} \text{ on } S. \quad (37)$$

Accordingly, the traction  $(\boldsymbol{\sigma}_a + \boldsymbol{\sigma}) \cdot \mathbf{n}$  is obtained by inverting a boundary problem similar to the one previously encountered for the auxiliary Stokes flows. Our second boundary approach consists in directly solving the problem (36)-(37). Once this is done, the force  $\mathbf{F}_a$  and the torque  $\mathbf{T}_a$  are evaluated by the relations (recall (6)-(7))

$$\mathbf{F}_a = \int_S [\boldsymbol{\sigma}_a + \boldsymbol{\sigma}] \cdot \mathbf{n} dS, \quad \mathbf{T}_a = \int_S \mathbf{x} \wedge [\boldsymbol{\sigma}_a + \boldsymbol{\sigma}] \cdot \mathbf{n} dS. \quad (38)$$

Before closing this subsection, one should also note that once the traction  $\boldsymbol{\sigma} \cdot \mathbf{n}$  and also (use (26)) the velocity  $\mathbf{u}$  are known on the particle surface  $S$  it is subsequently straightforward to determine the flow velocity  $\mathbf{u}$  also in the entire liquid domain  $\mathcal{D}$  by appealing to the integral representation (32).

### 5 Numerical implementation and results

This section briefly describes the implemented numerical treatment and also gives numerical results for spheroidal slip particles immersed in pure linear or quadratic ambient shear flows.

#### Numerical strategy

Since it is detailed in Sellier (2012), we briefly present the boundary element technique employed to numerically invert the encountered regularized boundary problems (18)-(19) and (36)-(37) and refer the reader to Brebbia, Telles, and Wrobel (1984); Beskos (1998); Bonnet (1999) and also Sellier (2011) for more general issues related to the Boundary Element Technique. As in Sellier and Pasol (2006) or

Sellier (2007, 2008), we use on the particle surface  $S$  a  $N$ -node mesh consisting of 6-node curvilinear and triangular boundary elements. At each nodal point, where the unit normal  $\mathbf{n}$  and two unit vectors  $\mathbf{t}_1$  and  $\mathbf{t}_2$  tangent to the particule surface  $S$  such that  $\mathbf{t}_1 \cdot \mathbf{t}_2 = 0$  are calculated, one then (for instance for the problem (18)-(19)) ends up with three unknown quantities :  $d$  and also  $d_1^t$  such  $d_2^t$  that  $\mathbf{d} = d_1^t \mathbf{t}_1 + d_2^t \mathbf{t}_2$ . This choice ensures the property (19) and the unknown triplet  $(d, d_1^t, d_2^t)$  at each nodal point is found by inverting the discretized counterpart of the coupled and regularized boundary-integral equations (18). Those equations result in a linear system with  $3N \times 3N$  non-symmetric and dense matrix  $A$ . Such a linear system is then solved by Gaussian elimination.

### 5.1 Numerical benchmarks and results

We consider spheroidal slip particles, with surface having equation  $(x_1/a)^2 + (x_2/a)^2 + (x_3/b)^2 = 1$ , immersed in linear or quadratic shear flows  $(\mathbf{u}_a, p_a) = (k_s x_3 \mathbf{e}_1, 0)$  or  $(\mathbf{u}_a, p_a) = (k_q x_3^2 \mathbf{e}_1, 2\mu k_q x_1)$  shear flows. For such slip particles and flows symmetry easily show that  $\mathbf{U} = U_1 \mathbf{e}_1, \mathbf{\Omega} = \Omega_2 \mathbf{e}_2$  and also that

$$\mathbf{F}_h = -6\pi\mu a f_1 U_1 \mathbf{e}_1, \quad \mathbf{T}_h = -8\pi\mu a^3 c_2 \Omega_2 \mathbf{e}_2, \quad (39)$$

$$\mathbf{T}_a = 4\pi\mu a^3 k_s c_s \mathbf{e}_2, \quad \mathbf{F}_a = \mathbf{U} = \mathbf{0} \text{ (linear shear),} \quad (40)$$

$$w_s = \Omega_2 / k_s = c_s / (2c_2) \text{ for linear shear,} \quad (41)$$

$$\mathbf{F}_a = 2\pi\mu a^3 k_q f_q \mathbf{e}_1, \quad \mathbf{T}_a = \mathbf{\Omega} = \mathbf{0} \text{ (quadratic shear),} \quad (42)$$

$$u_q = U_1 / (k_q a^2) = f_q / (3f_1) \text{ for quadratic shear} \quad (43)$$

with dimensionless force factors  $f_1, f_q$ , torque factors  $c_2, c_s$  and translational and angular velocities  $u_q$  and  $\omega_s$ .

For a sphere ( $b = a$ ) the analytical results (13)-(16) give  $f_1 = (1 + 2\lambda/a)/(1 + 3\lambda/a), f_q = c_s = c_2 = 3u_q = (1 + 3\lambda/a)^{-1}$  and  $\omega = 1/2$ . As shown in Table 1, our computations converge to those results as the number  $N$  of nodes put on the sphere boundary increases. Note that results obtained using either the relations (30)-(31) or solving the boundary problem (36)-(37) and appealing to (38) are reported for comparison purposes. Clearly, numerical and analytical results perfectly match and the results provided by the two boundary approaches nicely agree.

Note that for a sphere  $w_s$  does not depend upon  $\lambda/a$ . This is not the case any more for spheroidal particles as illustrated in Table 2 for two oblate ( $b/a = 0.8$ ) and prolate ( $b/a = 1.2$ ) spheroids. Note that  $\omega_s$  increases or decreases as  $\lambda/a$  increases for the prolate or oblate spheroid, respectively.

We plot in Fig. 2-5 versus the normalized slip length  $s = \lambda/a$  the friction coefficients  $c_s, f_q$  and the dimensionless velocities  $w_s$  and  $u_s$  for a slip sphere with radius

Table 1 – Computed coefficients  $c_s, f_q, \omega_s$  and  $u_q$  for a sphere with radius  $a$  versus the number  $N$  of nodal points for  $\lambda/a = 0.5, 2$ . The values computed after solving (36)-(37) for  $\mathbf{d}'$  are indicated with overlined symbols while the values given by (30)-(31) are indicated by symbols.

$N$	$\lambda/a$	$c_s$	$f_q$	$\omega_s$	$u_q$	$\bar{c}_s$	$\bar{f}_q$	$\bar{\omega}_s$	$\bar{u}_q$
74	0.5	0.39758	0.39973	0.49926	0.16596	0.39685	0.38526	0.498343	0.15995
242	0.5	0.39992	0.39972	0.50006	0.16651	0.39977	0.39584	0.499864	0.16490
1058	0.5	0.40001	0.39997	0.50002	0.16665	0.39999	0.39967	0.499996	0.16652
<i>exact</i>	0.5	0.4	0.4	0.5	0.16667	0.4	0.4	0.5	0.16667
74	2	0.14355	0.15021	0.50588	0.06950	0.14146	0.13915	0.49852	0.06438
242	2	0.14280	0.14303	0.50005	0.06670	0.14276	0.13927	0.49992	0.06494
1058	2	0.14285	0.14285	0.49999	0.06666	0.14285	0.14252	0.5	0.06651
<i>exact</i>	2	0.14286	0.14286	0.5	0.06667	0.14286	0.14286	0.5	0.06667

Table 2 – Computed coefficients  $c_s, f_q, \omega_s$  and  $u_q$  for one oblate (ob) spheroid with  $b = 0.8a$  and one prolate (pro) spheroid with  $b = 1.2a$ . Those results have been obtained by using the first boundary approach, i. e formulae (30)-(31).

$\lambda/a$	$c_s(ob)$	$f_q(ob)$	$\omega_s(ob)$	$u_q(ob)$	$c_s(pro)$	$f_q(pro)$	$\omega_s(pro)$	$u_q(pro)$
0	0.61483	0.58828	0.39016	0.21338	1.49872	1.55486	0.59034	0.48039
0.2	0.32776	0.31031	0.33912	0.13029	1.02335	1.06734	0.62285	0.37307
0.5	0.14987	0.13647	0.23998	0.06326	0.74875	0.78707	0.68911	0.29857
0.8	0.06602	0.05357	0.14092	0.02611	0.62052	0.65709	0.75785	0.25967

$a$  and those oblate ( $b/a = 0.8$ ) and prolate ( $b/a = 1.2$ ) spheroids. It is seen that  $f_q, c_s$  and  $u_q$  decrease as the slip length increases. By contrast, the angular velocity  $w_s$  for a slip spheroid immersed in the linear shear flow increases (oblate spheroid) or decreases (prolate spheroid) as the slip increases.

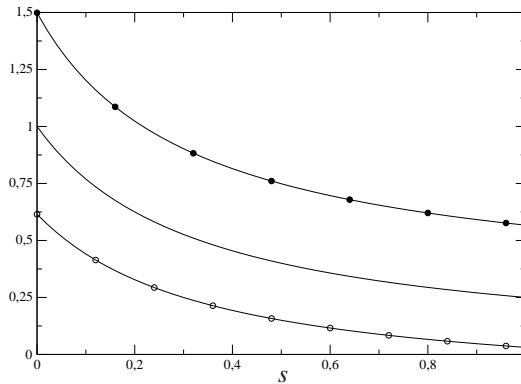


Figure 2 – Torque friction coefficient  $c_s$  versus  $s = \lambda/a$  for the linear shear flow. Solid line for the sphere, (o) for the prolate ( $b/a = 1.2$ ) spheroid, (●) for the oblate ( $b/a = 0.8$ ) spheroid.

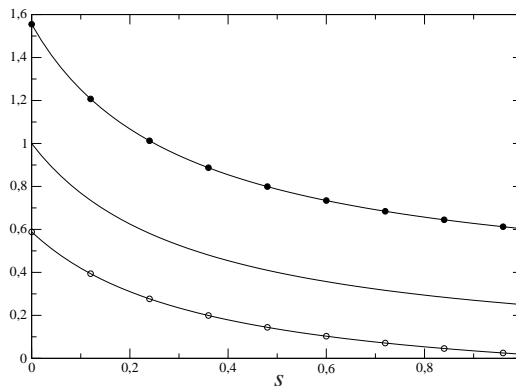


Figure 3 – Force friction coefficient  $f_q$  versus  $s = \lambda/a$  for the quadratic shear flow. Solid line for the sphere, (o) for the prolate ( $b/a = 1.2$ ) spheroid, (●) for the oblate ( $b/a = 0.8$ ) spheroid.

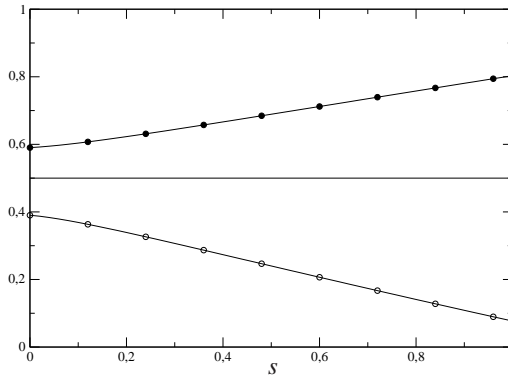


Figure 4 – Normalized angular velocity  $\omega_s$  versus  $s = \lambda/a$  for the linear shear flow. Solid line for the sphere, ( $\circ$ ) for the prolate ( $b/a = 1.2$ ) spheroid, ( $\bullet$ ) for the oblate ( $b/a = 0.8$ ) spheroid.

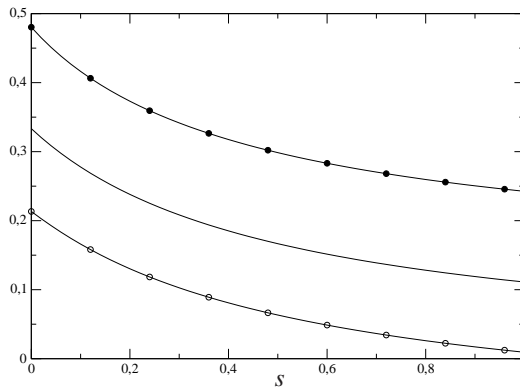


Figure 5 – Normalized translational velocity  $u_q$  versus  $s = \lambda/a$  for the quadratic shear flow. Solid line for the sphere, ( $\circ$ ) for the prolate ( $b/a = 1.2$ ) spheroid, ( $\bullet$ ) for the oblate ( $b/a = 0.8$ ) spheroid.

## 6 Conclusions

A relevant boundary approach has been proposed to accurately compute the force and torque applied on a slip solid particle held fixed in an arbitrary Stokes flow and, if necessary, not only the resulting rigid-body migration of a slip particle but

also the surface traction arising on the slip particle surface when the particle is freely-suspended in a given ambient Stokes flow. It has been possible to confine the task to the treatment of at the most seven boundary problems involving regularized boundary-integral equations on the particle boundary. Accordingly, it is sufficient to mesh the particle boundary when solving the problem. The advocated boundary element and collocation point technique implemented to accurately invert those boundary problems has been nicely tested against the analytical results established elsewhere for a spherical particle and new numerical results have been given for slip spheroidal particles immersed in a linear or quadratic ambient shear flow. Amazingly, the angular velocity of a prolate or oblated spheroid freely suspended in the linear shear flow is seen to increase or decrease with the slip, respectively.

Finally, the proposed second boundary method which provides the surface traction exerted by the disturbed flow about a slip particle *held fixed* in a prescribed and arbitrary Stokes flow opens the way to the evaluation of the effective viscosity of a dilute suspension of slip and either spherical or non-spherical particles. Such a key issue however requires further investigations. It will be the subject of another work.

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