 CMES, vol.100, no.6, pp.463-475, 2014

Solution of Two-Dimensional Viscous Flow in a Rectangular Domain by the Modified Decomposition Method

Lei Lu[1,2,3], **Jun-Sheng Duan**[2] **and Long-Zhen Fan**[1]

Abstract: In this paper, the modified decomposition method (MDM) for solving the nonlinear two-dimensional viscous flow equations is presented. This study investigates the problem of laminar, isothermal, incompressible and viscous flow in a rectangular domain bounded by two moving porous walls, which enable the fluid to enter or exit during successive expansions or contractions. We first transform the original two-dimensional viscous flow problem into an equivalent fourth-order boundary value problem (BVP), then solve the problem by the MDM. The figures and tables clearly show high accuracy of the method to solve two-dimensional viscous flow.

Keywords: nonlinear differential equation, two-dimensional viscous flow, modified decomposition method, Adomian polynomials.

1 Introduction

Most scientific problems and phenomena are modeled by nonlinear ordinary or partial differential equations. Therefore, the study on the various methods used for solving the nonlinear differential equations is a very important topic for the analysis of engineering practical problems. There are a number of approaches for solving nonlinear equations, which range from completely analytical to completely numerical ones. Besides all the advantages of using numerical methods, closed form solutions appear more appealing because they reveal physical insights through the physics of the problem. Also, parametric studies become more convenient with applying analytical methods. Moreover, analytical solutions are generally required for the validation of numerical methods and computer softwares. Therefore, many different methods have been introduced to obtain analytical approximate solutions

[1] School of Management, Fudan University, Shanghai 200433, P.R. China.
[2] School of Sciences, Shanghai Institute of Technology, Shanghai 201418, P.R. China.
[3] Corresponding author. Email: lulei1698@sit.edu.cn; lulei1698@163.com.

for these nonlinear problems, such as the perturbation method [Holmes (2013); He (2000)], orthogonal polynomial and wavelet methods [Lakestani, Razzaghi, and Dehghan (2006)], methods of travelling wave solutions [Jafari, Borhanifar, and Karimi (2009)], the Adomian decomposition method (ADM) and the Variational iteration method.

One of the most applicable analytical techniques is the ADM [Lu and Duan (2014); Duan, Rach, and Wazwaz (2013); Fu, Wang, and Duan (2013); Lai, Chen, and Hsu (2008); Adomian (1983, 1986, 1989, 1994); Wazwaz (2009, 2011); Serrano (2011); Adomian and Rach (1983); Duan, Rach, Baleanu, and Wazwaz (2012); Rach (2012)]. It is a practical technique for solving nonlinear functional equations, including ordinary differential equations, partial differential equations, integral equations, integro-differential equations, etc. The ADM provides efficient algorithms for analytic approximate solutions and numeric simulations for real-world applications in the applied sciences and engineering without unphysical restrictive assumptions such as required by linearization and perturbation. The accuracy of the analytic approximate solutions obtained can be verified by direct substitution.

In the ADM, the solution $u(x)$ is represented by a decomposition series

$$u(x) = \sum_{n=0}^{\infty} u_n(x), \tag{1}$$

and the nonlinearity comprises the Adomian polynomials

$$Nu(x) = \sum_{n=0}^{\infty} A_n(x), \tag{2}$$

where the Adomian polynomials $A_n(x)$ is defined for the nonlinearity $Nu = f(u)$ as [Adomian and Rach (1983)]

$$A_n(x) = A_n(u_0, u_1, \ldots, u_n) = \frac{1}{n!} \frac{\partial^n}{\partial \lambda^n} f(\sum_{k=0}^{\infty} \lambda^k u_k(x)) \Bigg|_{\lambda=0}. \tag{3}$$

Different algorithms for the Adomian polynomials have been developed by Rach [Rach (2008, 1984)], Wazwaz [Wazwaz (2000)], Abdelwahid [Abdelwahid (2003)] and several others [Abbaoui, Cherruault, and Seng (1995); Zhu, Chang, and Wu (2005); Biazar, Ilie, and Khoshkenar (2006)]. Recently new algorithms and subroutines in MATHEMATICA for fast generation of the Adomian polynomials to high orders have been developed by Duan [Duan (2010b,a, 2011)].

The solution components are determined by recursion scheme. The nth-stage approximation is given as $\phi_n(x) = \sum_{k=0}^{n-1} u_k(x)$.

We remark that the convergence of the Adomian series has already been proven by several investigators [Rach (2008); Abbaoui and Cherruault (1994, 1995); Abdelrazec and Pelinovsky (2011)]. For example, Abdelrazec and Pelinovsky [Abdelrazec and Pelinovsky (2011)] have published a rigorous proof of convergence for the ADM under the aegis of the Cauchy-Kovalevskaya theorem. In point of fact the Adomian decomposition series is found to be a computationally advantageous rearrangement of the Banach-space analog of the Taylor expansion series about the initial solution component function.

In this paper, we use the modified decomposition method(MDM) [Duan and Rach (2011)] to investigate the problem of laminar, isothermal, incompressible and viscous flow in a rectangular domain bounded by two moving porous walls. The paper is organized as follows. In Section 2, the mathematical formulation is presented. In Section 3, we extend the application of the MDM to construct the approximate solutions for the governing equation. Section 4 contains the results and discussion. The conclusions are summarized in Section 5.

2 Flow analysis and mathematical formulation

Studies of fluid transport in biological organisms often concern the flow of a particular fluid inside an expanding or contracting vessel with permeable walls. For a valved vessel exhibiting deformable boundaries, alternating wall contractions produce the effect of a physiological pump. The flow behavior inside the lymphatics exhibits a similar character. In such models, circulation is induced by successive contractions of two thin sheets that cause the downstream convection of the sandwiched fluid. Seepage across permeable walls is clearly important to the mass transfer between blood, air and tissue [Chang, Ha, Park, Kim, and Shin (1989)]. Therefore, a substantial amount of research work has been invested in the study of the flow in a rectangular domain bounded by two moving porous walls, which enable the fluid to enter or exit during successive expansions or contractions. Majdalani et al. [Majdalani, Zhou, and Dawson (2002)] studied the two-dimensional viscous flow between slowly expanding or contracting walls with weak permeability. Their study focused on the viscous flow driven by small wall contractions and expansions of two weakly permeable walls. Based on double perturbations in the permeation Reynolds number R_e and wall dilation rate α, they carried out their analytical procedure. Dauenhauer and Majdalani [Dauenhauer and Majdalani (1999)] studied the unsteady flow in semi-infinite expanding channels with wall injection. They are characterized by two nondimensional parameters, the expansion ratio of the wall α and the cross-flow Reynolds number R_e. Shooting method, coupled with a Runge-Kutta integration scheme, was utilized to numerically solve the resulting fourth-order differential equation. Majdalani and Zhou [Majdalani and Zhou

(2003)] studied moderate-to-large injection and suction driven channel flows with expanding or contracting walls. Using perturbations in cross-flow Reynolds number R_e, the resulting equation is solved both numerically and analytically. Boutros et al. [Boutros, AbdelMalek, Badran, and Hassan (2006)] studied the solution of the Navier-Stokes equations which described the unsteady incompressible laminar flow in a semi-infinite porous circular pipe with injection or suction through the pipe wall whose radius varies with time.

Consider the laminar, isothermal and incompressible flow in a rectangular domain bounded by two permeable surfaces that enable the fluid to enter or exit during successive expansions or contractions. The walls expand or contract uniformly at a time-dependent rate $\dot{a}$. At the wall, it is assumed that the fluid inflow velocity V_w is independent of position. The equations of continuity and motion for the unsteady flow are given as follows

$$\frac{\partial \bar{u}}{\partial \bar{x}}+\frac{\partial \bar{v}}{\partial \bar{y}}=0, \tag{4}$$

$$\frac{\partial \bar{u}}{\partial \bar{t}}+\bar{u}\frac{\partial \bar{u}}{\partial \bar{x}}+\bar{v}\frac{\partial \bar{u}}{\partial \bar{y}}=-\frac{1}{\rho}\frac{\partial \bar{p}}{\partial \bar{x}}+\nu[\frac{\partial^2 \bar{u}}{\partial \bar{x}^2}+\frac{\partial^2 \bar{u}}{\partial \bar{y}^2}], \tag{5}$$

$$\frac{\partial \bar{v}}{\partial \bar{t}}+\bar{u}\frac{\partial \bar{v}}{\partial \bar{x}}+\bar{v}\frac{\partial \bar{v}}{\partial \bar{y}}=-\frac{1}{\rho}\frac{\partial \bar{p}}{\partial \bar{y}}+\nu[\frac{\partial^2 \bar{v}}{\partial \bar{x}^2}+\frac{\partial^2 \bar{v}}{\partial \bar{y}^2}]. \tag{6}$$

In the above equations $\bar{u}$ and $\bar{v}$ indicate the velocity components in the x and y directions, $\bar{p}$ denotes the dimensional pressure, ρ, ν and t are the density, kinematic viscosity and time, respectively. The boundary conditions will be

$$\begin{aligned}
&\bar{u}=0,\ \bar{v}=-V_w=-\frac{\dot{a}}{c} \text{ at } \bar{y}=a(t),\\
&\frac{\partial \bar{u}}{\partial \bar{y}}=0,\ \bar{v}=0 \text{ at } \bar{y}=0,\\
&\bar{u}=0 \text{ at } \bar{x}=0,
\end{aligned} \tag{7}$$

where c $(c\equiv\frac{\dot{a}}{V_w})$ is the wall permeance or injection/suction coefficient, that is a measure of wall permeability. The stream function and mean flow vorticity can be introduced by putting

$$\bar{u}=\frac{\partial \bar{\psi}}{\partial \bar{y}},\ \bar{v}=-\frac{\partial \bar{\psi}}{\partial \bar{x}},\ \bar{\xi}=\frac{\partial \bar{v}}{\partial \bar{x}}-\frac{\partial \bar{u}}{\partial \bar{y}}, \tag{8}$$

$$\frac{\partial \bar{\xi}}{\partial t}+\bar{u}\frac{\partial \bar{\xi}}{\partial \bar{x}}+\bar{v}\frac{\partial \bar{\xi}}{\partial \bar{y}}=\nu[\frac{\partial^2 \bar{\xi}}{\partial \bar{x}^2}+\frac{\partial^2 \bar{\xi}}{\partial \bar{y}^2}]. \tag{9}$$

Due to mass conservation, a similar solution can be developed with respect to $\bar{x}$ [Majdalani, Zhou, and Dawson (2002)]. We define the transformation as

$$\bar{\psi} = \frac{\nu \bar{x} \bar{F}(y,t)}{a},\ \bar{u} = \frac{\nu \bar{x} \bar{F}_y}{a^2},\ \bar{v} = \frac{-\nu \bar{F}(y,t)}{a}, \tag{10}$$

$$y = \frac{\bar{y}}{a},\ \bar{F}_y \equiv \frac{\partial \bar{F}}{\partial y}. \tag{11}$$

Substituting Eqs.(10) and (11) into Eqs.(8) and (9) yields

$$\bar{u}_{\bar{y}t} + \bar{u}\bar{u}_{\bar{y}\bar{x}} + \bar{v}\bar{u}_{\bar{y}\bar{y}} = \nu \bar{u}_{\bar{y}\bar{y}\bar{y}}. \tag{12}$$

In order to solve Eq.(12), one uses the chain rule to obtain

$$\bar{F}_{yyyy} + \alpha(y\bar{F}_{yyy} + 3\bar{F}_{yy}) + \bar{F}\bar{F}_{yyy} - \bar{F}_y\bar{F}_{yy} - a^2\nu^{-1}\bar{F}_{yyt} = 0, \tag{13}$$

with the following boundary conditions

$$\bar{F} = 0,\ \bar{F}_{yy} = 0 \text{ at } y = 0, \tag{14}$$

$$\bar{F} = R_e,\ \bar{F}_y = 0 \text{ at } y = 1. \tag{15}$$

where $\alpha(t) \equiv \dot{a}a/\nu$ is the nondimensional wall dilation rate defined positive for expansion and negative for contraction. Furthermore, $R_e = aV_w/\nu$ is the permeation Reynolds number defined positive for injection and negative for suction through the walls. Eqs.(10), (11),(13),(14) and (15) can be normalized by putting

$$\psi = \frac{\bar{\psi}}{a\dot{a}},\ u = \frac{\bar{u}}{\dot{a}},\ v = \frac{\bar{v}}{\dot{a}},\ x = \frac{\bar{x}}{a},\ F = \frac{\bar{F}}{R_e}, \tag{16}$$

and so

$$\psi = \frac{xF}{c},\ u = \frac{xF'}{c},\ v = \frac{-F}{c},\ c = \frac{\alpha}{R_e}, \tag{17}$$

$$F^{(4)} + \alpha(yF''' + 3F'') + R_eFF''' - R_eF'F'' = 0. \tag{18}$$

The boundary conditions (14) and (15) will be

$$F = 0,\ F'' = 0 \text{ at } y = 0, \tag{19}$$

$$F = 1,\ F' = 0 \text{ at } y = 1. \tag{20}$$

The resulting Eq.(18) is the classic Berman's formula, with $\alpha = 0$ (channel with stationary walls).

3 The MDM solution

In Adomian's operator-theoretic notation, according to Eq.(18), we have

$$LF(y) = NF(y), \tag{21}$$

where

$$L(\cdot) = \frac{d^4}{dy^4}(\cdot),\ NF(y) = -\alpha(yF''' + 3F'') - R_e F F''' + R_e F' F''. \tag{22}$$

According to the Duan-Rach modified decomposition method for BVPs [Duan and Rach (2011)], we take the inverse linear operator as

$$L^{-1}(\cdot) = \int_0^y \int_0^y \int_0^y \int_0^y (\cdot) dydydydy. \tag{23}$$

Then, we have

$$L^{-1}LF(y) = \int_0^y \int_0^y \int_0^y \int_0^y F^{(4)}(y) dydydydy = F(y) - \Phi(y), \tag{24}$$

where

$$\Phi(y) = F(0) + yF'(0) + \frac{y^2}{2}F''(0) + \frac{y^3}{6}F'''(0). \tag{25}$$

Applying the operator $L^{-1}(\cdot)$ to both sides of Eq.(21) yields

$$F(y) = \Phi(y) + L^{-1}NF(y). \tag{26}$$

Using the boundary conditions (19),(20), we have from Eq.(25) as

$$\Phi(y) = yF'(0) + \frac{y^3}{6}F'''(0). \tag{27}$$

Upon substitution of the formula Eq.(27) into Eq.(26), we obtain

$$F(y) = yF'(0) + \frac{y^3}{6}F'''(0) + L^{-1}NF(y). \tag{28}$$

Before we design a modified recursion scheme, we determine the two undetermined coefficients $F'(0)$ and $F'''(0)$ in advance. Evaluating $F(y)$ at $y = 1$ and using the boundary condition $F(1) = 1$, we have

$$F'(0) + \frac{1}{6}F'''(0) + [L^{-1}NF(y)]_{y=1} = 1, \tag{29}$$

where this nonlinear Fredholm integral is

$$[L^{-1}NF(y)]_{y=1} = \int_0^1 \int_0^y \int_0^y \int_0^y NF(y)dydydydy. \tag{30}$$

Differentiating Eq.(28) then evaluating $F^{'}(y)$ at $y = 1$ and using the boundary condition $F^{'}(1) = 0$, we have

$$F^{'}(0) + \frac{1}{2}F^{'''}(0) + [\frac{dL^{-1}NF(y)}{dy}]_{y=1} = 0, \tag{31}$$

where this nonlinear Freddholm integrate is

$$[\frac{dL^{-1}NF(y)}{dy}]_{y=1} = \int_0^1 \int_0^y \int_0^y NF(y)dydydy. \tag{32}$$

From the system of Eqs.(29) and (31), which constitutes two linearly independent equations in two unknowns, we readily obtain

$$F^{'}(0) = -\frac{3}{2}[L^{-1}NF(y)]_{y=1} + \frac{1}{2}[\frac{dL^{-1}NF(y)}{dy}]_{y=1} + \frac{3}{2}, \tag{33}$$

$$F^{'''}(0) = 3[L^{-1}NF(y)]_{y=1} - 3[\frac{dL^{-1}NF(y)}{dy}]_{y=1} - 3. \tag{34}$$

Substituting Eqs.(33) and (34) into Eq.(28), we obtain the integral equation for the solution

$$\begin{aligned} F(y) &= \frac{3y}{2} - \frac{y^3}{2} - (\frac{3y}{2} - \frac{y^3}{2})[L^{-1}NF(y)]_{y=1} \\ &\quad +(\frac{y}{2} - \frac{y^3}{2})[\frac{dL^{-1}NF(y)}{dy}]_{y=1} + L^{-1}NF(y). \end{aligned} \tag{35}$$

Thus, we have converted the nonlinear BVP into an equivalent nonlinear integral equation without any undetermined coefficients.

Next, we decompose the solution $F(y)$, and the nonlinearity $NF(y)$ as

$$F(y) = \sum_{m=0}^{\infty} F_m(y) \text{ and } NF(y) = \sum_{m=0}^{\infty} A_m(y). \tag{36}$$

By substitution (36) into Eq.(35) we have

$$\begin{aligned} \sum_{m=0}^{\infty} F_m(y) &= \frac{3y}{2} - \frac{y^3}{2} - (\frac{3y}{2} - \frac{y^3}{2})[L^{-1}(\sum_{m=0}^{\infty} A_m(y))]_{y=1} \\ &\quad +(\frac{y}{2} - \frac{y^3}{2})[\frac{dL^{-1}(\sum_{m=0}^{\infty} A_m(y))}{dy}]_{y=1} + L^{-1}(\sum_{m=0}^{\infty} A_m(y)). \end{aligned} \tag{37}$$

Using the modified recursion scheme, we have

$$F_0(y) = \frac{3y}{2} - \frac{y^3}{2}, \tag{38}$$

$$\begin{aligned} F_{m+1}(y) &= -(\frac{3y}{2} - \frac{y^3}{2})[L^{-1}A_m(y)]_{y=1} \\ &\quad +(\frac{y}{2} - \frac{y^3}{2})[\frac{dL^{-1}A_m(y)}{dy}]_{y=1} + L^{-1}A_m(y),\ m \geq 0. \end{aligned} \tag{39}$$

We can compute the solution components $F_m(y),\ m \geq 1$, where we can use any one of several efficient MATHEMATICA subroutine for generation of the Adomian polynomials.

The nth-stage approximate solution is

$$\phi_n(y) = \sum_{k=0}^{n-1} F_k(y). \tag{40}$$

Since the exact solution cannot be obtain in general for the case of most nonlinear operator equations, we instead consider the error remainder function in our context of the particular nonlinear differential equation $LF(y) - NF(y) = 0$,

$$\begin{aligned} ER_n(y) &= L\phi_n(y) - N\phi_n(y) \\ &= \phi_n^{(4)}(y) + \alpha(y\phi_n'''(y) + 3\phi_n''(y)) + R_e\phi_n(y)\phi_n'''(y) - R_e\phi_n'(y)\phi_n''(y), \end{aligned} \tag{41}$$

to verify the convergence of our solution and the maximal error remainder parameter

$$MER_n = \max_{0 \leq y \leq 1} \mid ER_n(y) \mid, \tag{42}$$

which can be conveniently computed by the MATHEMATICA native command 'NMaximize' for the nth-stage approximation $\phi_n(y)$.

4 Simulation results

The obtained analytical approximations include two parameters: the Reynolds number R_e and the expansion ratio of the wall α. Here we present simulation results of the proposed scheme for two-dimensional viscous flow in a rectangular domain.

We consider the error analytic function for two cases: (i) $\alpha = 1$ and $R_e = 3$, (ii) $\alpha = 2$ and $R_e = 1$. In Figs. 1(a) and 1(b), we plot the error remainder functions $ER_n(y)$ for $n = 5$ through 8 for two cases (i) and (ii). The maximal error remainder parameters MER_n for $n = 1$ through 10 for $\alpha = 0.5, R_e = 0.5$ are listed in Table 1. In Fig. 2, we display the logarithmic plots of the maximal error remainder parameters MER_n versus n for $\alpha = 0.5, R_e = 0.5$, where the points lie almost in a straight line, which indicates that the maximal error remainder parameters decrease approximately at an exponential rate.

Table 1: The maximal error remainder parameters MER_n for $\alpha = 0.5, R_e = 0.5$

n	1	2	3	
MER_n	7.5	3.40714	0.565377	
n	4	5	6	
MER_n	0.0394957	0.00143823	0.000153683	
n	7	8	9	10
MER_n	$8.87219 * 10^{-6}$	$9.06599 * 10^{-7}$	$6.25602 * 10^{-8}$	$5.96985 * 10^{-9}$

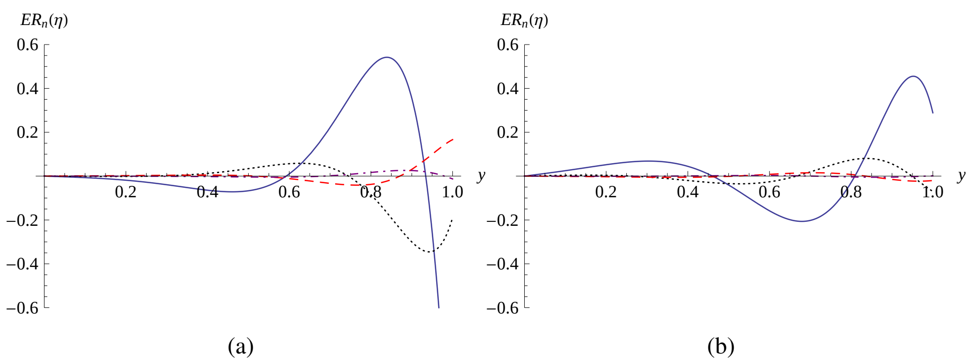

Figure 1: Curves of $ER_n(y)$ versus y for $n = 5$ (solid line), $n = 6$ (dot line), $n = 7$ (dash line), $n = 8$ (dot-dash line), and for (a) $\alpha = 1, R_e = 3$, (b) $\alpha = 2, R_e = 1$.

In Fig. 3, we plot the curves of ϕ_5 versus y for $R_e = 20$ and different values of α. For the fixed R_e, increase in values of α is cause of increasing in velocity. In Figs. 4(a) and 4(b), we plot the curves of ϕ_5 versus y for different values of R_e and for $\alpha = 1$ and 9, respectively. For this case, we find that for the fixed $\alpha = 1$, increase in values of R_e is cause of decreasing in velocity. When $\alpha = 9$, increase in values of R_e is cause of increasing in velocity. The effects of the model parameters R_e and α on the dimensionless velocity are investigated.

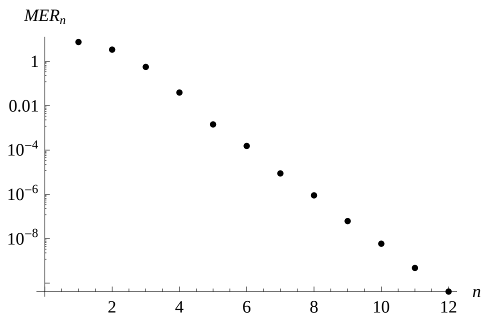

Figure 2: Logarithmic plots of the maximal errors remainder parameters MER_n versus n for $n = 1$ through 12.

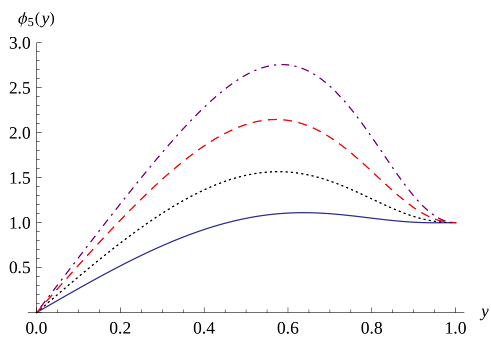

Figure 3: The curves of ϕ_5 versus y for $R_e = 20$ and $\alpha = 3$ (solid line), $\alpha = 5$ (dot line), $\alpha = 7$ (dash line),$\alpha = 9$ (dot-dash line).

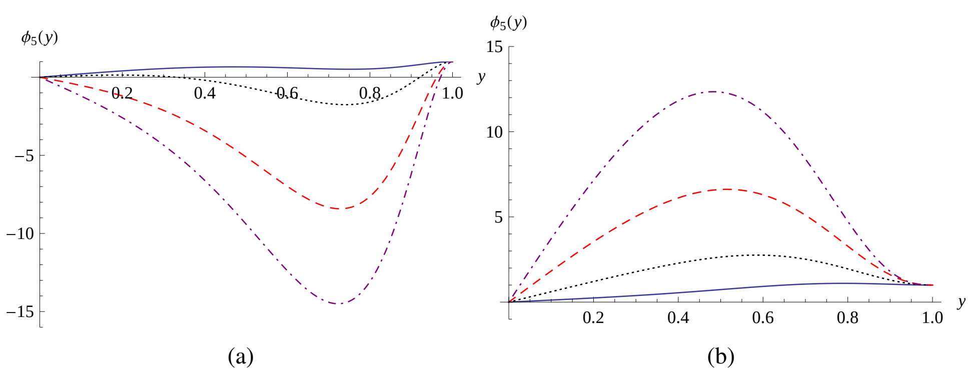

Figure 4: The curves of ϕ_5 versus y for (a) $R_e = 40$ (solid line), $R_e = 60$ (dot line), $R_e = 80$ (dash line),$R_e = 90$ (dot-dash line) and for $\alpha = 1$, (b) $R_e = 10$ (solid line), $R_e = 20$ (dot line), $R_e = 30$ (dash line),$R_e = 40$ (dot-dash line) and for $\alpha = 9$.

5 Conclusions

In this research, the modified decomposition method was applied successfully to find the analytical solution of two-dimensional viscous flow in a rectangular domain bounded by two moving porous walls. The figures and tables clearly show high accuracy of the method to solve two-dimensional viscous flow. Consequently, the present success of the modified decomposition method for the highly non linear problem verifies that the method is a useful tool for non linear problems in science and engineering.

Acknowledgement: This work was supported by the Natural Science Foundation of Shanghai (No. 14ZR1440800) and the Innovation Program of the Shanghai Municipal Education Commission (No. 14ZZ161).

References

Abbaoui, K.; Cherruault, Y. (1994): Convergence of Adomian's method applied to differential equations. *Comput Math Appl.*, vol. 28, pp. 103–109.

Abbaoui, K.; Cherruault, Y. (1995): New ideas for proving convergence of decomposition methods. *Comput Math Appl.*, vol. 29, pp. 103–108.

Abbaoui, K.; Cherruault, Y.; Seng, V. (1995): Practical formulae for the calculus of multivariable Adomian polynomials. *Math Comput Model.*, vol. 22, pp. 89–93.

Abdelrazec, A.; Pelinovsky, D. (2011): Convergence of the Adomian decomposition method for initial-value problems. *Numer Methods Partial Differential Equations*, vol. 27, pp. 749–766.

Abdelwahid, F. (2003): A mathematical model of Adomian polynomials. *Appl Math Comput.*, vol. 141, pp. 447–453.

Adomian, G. (1983): *Stochastic Systems*. Academic, New York.

Adomian, G. (1986): *Nonlinear Stochastic Operator Equations*. Academic, Orlando.

Adomian, G. (1989): *Nonlinear Stochastic Systems Theory and Applications to Physics*. Kluwer Academic, Dordrecht.

Adomian, G. (1994): *Solving Frontier Problems of Physics: The Decomposition Method*. Kluwer Academic, Dordrecht.

Adomian, G.; Rach, R. (1983): Inversion of nonlinear stochastic operators. *J Math Anal Appl.*, vol. 91, pp. 39–46.

Biazar, J.; Ilie, M.; Khoshkenar, A. (2006): An improvement to an alternate algorithm for computing Adomian polynomials in special cases. *Appl Math Comput.*, vol. 173, pp. 582–592.

Boutros, Y.; AbdelMalek, M.; Badran, N.; Hassan, H. (2006): Lie-group method for unsteady flows in a semi-infinite expanding or contracting pipe with injection or suction through a porous wall. *J. Comput. Appl. Math.*, vol. 197, no. 2, pp. 465–494.

Chang, H.; Ha, J.; Park, J.; Kim, I.; Shin, H. (1989): Velocity field of pulsatile flow in a porous tube. *J. Biomech*, vol. 22, pp. 1257–1262.

Dauenhauer, E. C.; Majdalani, J. (1999): Unsteady flows in semi-infinite expanding channels with wall injection. *in: AIAA paper*, pp. 3499–3523.

Duan, J. S. (2010): An efficient algorithm for the multivariable Adomian polynomials. *Appl Math Comput.*, vol. 217, pp. 2456–2467.

Duan, J. S. (2010): Recurrence triangle for Adomian polynomials. *Appl Math Comput.*, vol. 216, pp. 1235–1241.

Duan, J. S. (2011): Convenient analytic recurrence algorithms for the Adomian polynomials. *Appl Math Comput.*, vol. 217, pp. 6337–6348.

Duan, J. S.; Rach, R. (2011): A new modification of the Adomian decomposition method for solving boundary value problems for higher oder nonlinear differential equations. *Appl Math Comput.*, vol. 218, pp. 4090–4118.

Duan, J. S.; Rach, R.; Baleanu, D.; Wazwaz, A. M. (2012): A review of the Adomian decomposition method and its applications to fractional differential equations. *Commun Fract Calc.*, vol. 3, pp. 73–99.

Duan, J. S.; Rach, R.; Wazwaz, A. M. (2013): A new modified Adomian decomposition method for higher-order nonlinear dynamical systems. *CMES-Comput. Model. Eng. Sci.*, vol. 94, no. 1, pp. 77–118.

Fu, S. Z.; Wang, Z.; Duan, J. S. (2013): Solution of quadratic integral equations by the Adomian decomposition method. *CMES-Comput. Model. Eng. Sci.*, vol. 92, no. 4, pp. 369–385.

He, J. H. (2000): A review on some new recently developed nonlinear analytical techniques. *Int J Non-linear Sci Numer Simul.*, vol. 1, pp. 51–70.

Holmes, M. H. (2013): *Introduction to Perturbation Methods*. 2nd edn. Springer, New York.

Jafari, H.; Borhanifar, A.; Karimi, S. A. (2009): New solitary wave solutions for the bad Boussinesq and good Boussinesq equations. *Numer Methods Partial Differential Equations*, vol. 25, pp. 1231–1237.

Lai, H. Y.; Chen, C. K.; Hsu, J. C. (2008): Free vibration of non-uniform Euler-Bernoulli beams by the Adomian modified decomposition method. *CMES-Comput. Model. Eng. Sci.*, vol. 34, pp. 87–116.

Lakestani, M.; Razzaghi, M.; Dehghan, M. (2006): M. Numerical solution of the controlled Duffing oscillator by semi-orthogonal spline wavelets. *Physica Scripta*, vol. 74, pp. 362–366.

Lu, L.; Duan, J. S. (2014): How to select the value of the convergence parameter in the Adomian decomposition method. *CMES-Comput. Model. Eng. Sci.*, vol. 97, no. 1, pp. 35–52.

Majdalani, J.; Zhou, C. (2003): Moderate-to-large injection and suction driven channel flows with expanding or contracting walls. *ZAMM– Z. Angew. Math. Mech.*, vol. 83, pp. 181–196.

Majdalani, J.; Zhou, C.; Dawson, C. A. (2002): Two-dimensional viscous flow between slowly expanding or contracting walls with weak permeability. *J. Biomech*, vol. 35, pp. 1399–1403.

Rach, R. (1984): A convenient computational form for the Adomian polynomials. *J Math Anal Appl.*, vol. 102, pp. 415–419.

Rach, R. (2008): A new definition of the Adomian polynomials. *Kybernetes*, vol. 37, pp. 910–955.

Rach, R. (2012): A bibliography of the theory and applications of the Adomian decomposition method. *Kybernetes*, vol. 41, pp. 1087–1148.

Serrano, S. E. (2011): *Engineering Uncertainty and Risk Analysis: A Balanced Approach to Probability, Statistics, Stochastic Modeling, and Stochastic Differential Equations*. 2nd revised edn. HydroScience, Ambler.

Wazwaz, A. M. (2000): A new algorithm for calculating Adomian polynomials for nonlinear operators. *Appl Math Comput.*, vol. 111, pp. 53–69.

Wazwaz, A. M. (2009): *Partial Differential Equations and Solitary Waves Theory*. Higher Education Press, Beijing.

Wazwaz, A. M. (2011): *Linear and Nonlinear Integral Equations: Methods and Applications*. Higher Education Press, Beijing.

Zhu, Y.; Chang, Q.; Wu, S. (2005): A new algorithm for calculating Adomian polynomials. *Appl Math Comput.*, vol. 169, pp. 402–416.